
Optimal Maintenance Probabilities and Preventive Replacement Maintenance Policy for Photocopy Machines

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Abstract

The need for suitable replacement policies are essential to minimize down time, maintenance cost and maximize the availability and reliability of equipment. On this premise, this work models the failure rate of Photocopy machines and obtain its optimal preventive maintenance policy that would prevent damage and its attendant losses to both users and end-product consumers. The failure distribution of the machine was shown to follow the Log-Logistic distribution with shape parameter, $\hat{\alpha} = 1.723339368$ and scale parameter, $\hat{\beta} = 763.9219635$. Optimal probabilities of the distribution were obtained and utilized in both the cumulative failure function and cumulative hazard function-based replacement models to formulate a replacement maintenance policy for the machine. The failure cumulative function-based replacement model was found to be a better model which yields optimal replacement maintenance time of 166 *hours* at a minimum cost of 113 *Naira* for maintaining the machine per cycle time with 96% availability, 94% reliability and 0.07% chance of failure occurrence in the machine.

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1 Introduction

All equipment fail, degrade or age during operation. Equipment failure is economically harmful to both users and end-product consumers, hence, the need to determine a suitable replacement maintenance policy in this work that would minimize down time, maintenance cost and maximize the availability and reliability of equipment. Failure models have been applied successfully in reliability theory of both repairable and non-repairable systems to model the failure data of equipment. Various distributions are exhibited by different systems based on their failure mode. Examples of these models are the Normal, Log-Normal, Log-Logistic, Weibull, Birnbaum-Saunders (fatigue life), Gamma, Gumbel models, etc. Consequently, the choice of a life distribution model can either depend on the failure mode or the goodness-of-fit test of sample data.

The Log-Logistic distribution is the probability distribution of a random variable whose logarithm has a logistic distribution. It is a very popular distribution pioneered to model population growth; Verhulst (1838). It has attracted a wide applicability in survival and reliability analysis over the last few decades, particularly for events whose rate increases initially and decrease later; Mahmoud and Mohammed (2013). For instance, it has been used in modeling mortality from cancer following diagnosis or treatment; Gupta et al. (1999). Anderson, McClure, Baird-Parker and Cole (1996) used this model to describe the thermal inactivation of clostridium botulinum 213B at different temperature levels. In income inequality literature, the Log-Logistic model is well-known as the Fisk distribution due to Fisk (1961). It has also been widely used in many areas such as survival analysis, actuarial science, economics, engineering and hydrology. In hydrology, it has been used to model stream flow and precipitation; see, Shoukri *et al* (1988) and Ashkar and Mahdi (2006), and for modeling flood frequency; Ahmad et al. (1988). Furthermore, it is also used to model censored data usually common in reliability and life-testing experiments; Tahir et al. (2014). The log-logistic distribution is a derivation from the logistic distribution as stated in Clark and El-Taha (2015) and can be alternatively used in place of lognormal distribution; Akhter and Khan (2014).

Tahara and Nishida (1975) introduced the maintenance policy, “replace the unit at the first failure after t_0 hours of operation or when the total operating time reaches T_0 ($0 \leq t_0 \leq T_0$) whichever occurs first. If $t_0 = 0$, it becomes the basic age replacement policy. Nakagawa (1984) extended the age replacement policy (T-policy) to replacing equipment at time, T or at number of failures, N whichever occurs first, and undergoes minimal repair at failure between replacements (T-N policy). While Segawa et al. (1992) investigated the optimal age replacement problem with minimal repairs under the average cost criterion, and showed that among all allowable policies, an optimal policy is a T -policy. Wang and Pham (1996) made another extension of age replacement policy, called “mixed age preventive maintenance policy.” Bahrami et al. (2000) proposed a new perspective of block and age replacement models based on the failure cumulative function, which were both applied on a hydraulic jack in the crankshaft line in a car engine manufacturing company. Also, Cassady and Pohl (2003) proposed an age replacement model based on cumulative hazard function, which was applied on a drilling machine to optimize the time to failure and the cost of replacement of any component of the machine.

Huynh et al. (2012) dealt with age replacement policies with minimal repairs for single-unit repairable systems which are subject to competing and dependent failures due to degradation and traumatic shocks, while Mahdavi and Mahdavi (2009) proposed a new optimal age policy to maximize system reliability. Chouhan et al. (2013) summarizes, classifies, and compares varied existing maintenance policies for each single and multiple-unit systems. Other works on the application of replacement models in the literature include; Jibril and Ekundayo (2015), Lamberson (2013) and Staye (2014).

1.1 Assumptions of the Study

- i. Observed failures at each time, t are continuously and independently distributed.
- ii. The failure of a life distribution of the machine, $f(t)$ is assumed to occur at the end of time, t .
- iii. The failures that occur at each time, t are independently and continuously distributed.
- iv. Failures occur at random in the machine.
- v. Failure of a component implies failure of the Machine.

2 Methodology

2.1 The Failure Distribution Function

The inter-failure times of Photocopy Machine was shown to follow the Log-Logistic distribution using chi-square goodness-of-fit test with the aid of Easyfit (5.6) version software having the rank of 1. This implies that it is the best fit model for the sample data in the family of life distribution functions.

2.2 The Log-Logistic Probability Density Function

Let a random variable Y follows a logistic distribution with parameter μ and σ^2 ; $Y \sim L(\mu, \sigma^2)$ where $-\infty < \mu < \infty$ and $\sigma^2 > 0$ with pdf;

$$g(y) = \frac{\pi}{\sigma\sqrt{3}} \frac{e^{-\frac{\pi}{\sigma\sqrt{3}}(y-\mu)}}{\left(1 + e^{-\frac{\pi}{\sigma\sqrt{3}}(y-\mu)}\right)^2}; \quad -\infty < \mu < \infty \quad (1)$$

The pdf of the Logistic distribution can also be written as;

$$g(y) = \frac{\pi}{\sigma\sqrt{3}} \frac{e^{\left(-\frac{\pi}{\sigma\sqrt{3}}y + \frac{\pi}{\sigma\sqrt{3}}\mu\right)}}{\left(1 + e^{-\frac{\pi}{\sigma\sqrt{3}}y + \frac{\pi}{\sigma\sqrt{3}}\mu}\right)^2} = \frac{\pi}{\sigma\sqrt{3}} \frac{e^{-\frac{\pi}{\sigma\sqrt{3}}y} e^{\frac{\pi}{\sigma\sqrt{3}}\mu}}{\left(1 + e^{-\frac{\pi}{\sigma\sqrt{3}}y} e^{\frac{\pi}{\sigma\sqrt{3}}\mu}\right)^2}$$

To obtain the probability density function (pdf) of a Log-Logistic distribution, it is useful to make the following substitutions; see, Clark and El-Taha (2015);

Let $\beta = e^\mu$; ($\beta > 0$)
and $\alpha = \frac{\pi}{\sigma\sqrt{3}}$; ($\alpha > 0$)

$$g(y) = \alpha \frac{e^{-\alpha y} e^{\alpha\mu}}{\left(1 + e^{-\alpha y} e^{\alpha\mu}\right)^2} = \alpha \frac{e^{-\alpha y} \beta^\alpha}{\left(e^{-\alpha y} \beta^\alpha\right)^2 \left[\frac{1}{e^{-\alpha y} \beta^\alpha} + 1\right]^2}$$

$$\therefore g(y) = \frac{\alpha e^{\alpha y} \beta^{-\alpha}}{\left(1 + e^{\alpha y} \beta^{-\alpha}\right)^2} \quad (2)$$

Also, we let Y be the logarithm of another random variable T {i.e. $T = e^Y$; $\Rightarrow y = \ln(t)$ }. Then the pdf of T can be obtained using the following transformation technique;

$$f(t) = \left| \frac{dy}{dt} \right| g(y), \quad \text{where } y = \ln(t)$$

$$\begin{aligned}
 f(t) &= \left| \frac{d \ln(t)}{dt} \right| \alpha \frac{e^{\alpha(\ln(t))} \left(\frac{1}{\beta}\right)^\alpha}{\left(1 + e^{\alpha(\ln(t))} \left(\frac{1}{\beta}\right)^\alpha\right)^2} \\
 &= \frac{1}{t} \frac{\alpha \left(\frac{1}{\beta}\right)^\alpha t^\alpha}{\left(1 + \left(\frac{1}{\beta}\right)^\alpha t^\alpha\right)^2} = \frac{t^{-1} \alpha \left(\frac{t}{\beta}\right)^\alpha}{\left(1 + \left(\frac{t}{\beta}\right)^\alpha\right)^2}
 \end{aligned}$$

Since $\frac{d \ln(t)}{dt} = \frac{1}{t}$ and $e^{\alpha \ln(t)} = e^{\ln(t^\alpha)} = t^\alpha$

But $\frac{\beta}{\beta} = \left(\frac{1}{\beta}\right) \left(\frac{1}{\beta}\right)^{-1} = 1$

$$\begin{aligned}
 \Rightarrow f(t) &= \frac{t^{-1} \alpha \left(\frac{t}{\beta}\right)^\alpha \left(\frac{1}{\beta}\right) \left(\frac{1}{\beta}\right)^{-1}}{\left(1 + \left(\frac{t}{\beta}\right)^\alpha\right)^2} \\
 \therefore f(t) &= \frac{\left(\frac{t}{\beta}\right)^{\alpha-1} \left(\frac{\alpha}{\beta}\right)}{\left(1 + \left(\frac{t}{\beta}\right)^\alpha\right)^2}; 0 \leq t \leq \infty; \alpha > 1; \beta > 0. \quad (3)
 \end{aligned}$$

Where $f(t)$ is the Log-Logistic pdf with Continuous scale parameter β and continuous shape parameter α .

2.3 Log-Logistic Cumulative Density Function

From the pdf of T , we can also obtain its cumulative density function (cdf) as;

$$F(t) = \int_0^t f(u) du = \int_0^t \frac{\left(\frac{u}{\beta}\right)^{\alpha-1} \left(\frac{\alpha}{\beta}\right)}{\left(1 + \left(\frac{u}{\beta}\right)^\alpha\right)^2} du = \frac{\left(\frac{t}{\beta}\right)^\alpha}{1 + \left(\frac{t}{\beta}\right)^\alpha}$$

Multiply and divide by $\left(\frac{t}{\beta}\right)^{-\alpha}$

$$\begin{aligned}
 F(t) &= \frac{1}{1 + \left(\frac{t}{\beta}\right)^{-\alpha}} \\
 \therefore F(t) &= \frac{1}{1 + \left(\frac{\beta}{t}\right)^\alpha} \quad (4)
 \end{aligned}$$

2.4 Estimation of the Log-Logistic Distribution Parameters

The method of moments was used to estimate the mean, μ and variance, σ^2 of the log-logistic distribution. The use of data for this purpose, without personal identifiers, was proposed to the Institutional Review Board, which ruled it exempt from further review; Clark and El-Taha (2015). As an example, theoretical log-logistic distribution was fitted in this way to data from 333 cases of Coronary Artery Bypass Grafting (CABG) performed at the Maine Medical Center in 2013. Therefore, the following substitutions were made;

$$\hat{\beta} = e^{\hat{\mu}} \tag{5}$$

$$\hat{\alpha} = \frac{\pi}{\hat{\sigma}\sqrt{3}} \tag{6}$$

where

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n \ln(t) \tag{7}$$

and

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \left[\sum_{i=1}^n [\ln(t)]^2 - n\hat{\mu}^2 \right]} \tag{8}$$

2.5 Mean and Variance of a Log-logistic Distribution with Parameters α, β ($LL(\alpha, \beta)$)

The r th moment of T exist for $r < \beta$, and is expressed as;

$$E(T^r) = \int_0^\infty t^r f(t) dt = \int_0^\infty t^r \left\{ \frac{\frac{\alpha t^{\alpha-1}}{\beta^\alpha}}{\left(1 + \left(\frac{t}{\beta}\right)^\alpha\right)^2} \right\} dt$$

$$\text{Let } U = \left(\frac{t}{\beta}\right)^\alpha; \Rightarrow dU = \frac{\alpha t^{\alpha-1}}{\beta^\alpha} dt$$

$$E(T^r) = \int_0^\infty t^r \frac{dU}{(1+U)^2}$$

$$\text{But } U = \left(\frac{t}{\beta}\right)^\alpha \Rightarrow \beta U^{\frac{1}{\alpha}} = t$$

$$\text{Hence, } t^r = \beta^r U^{\frac{r}{\alpha}}$$

$$E(T^r) = \int_0^\infty \frac{\beta^r U^{\frac{r}{\alpha}}}{(1+U)^2} dU = \beta^r \frac{\left(\frac{r\pi}{\alpha}\right)}{\sin\left(\frac{r\pi}{\alpha}\right)};$$

$$\text{since } \int_0^\infty \frac{x^k}{(1+x)^2} dx = \frac{(k\pi)}{\sin(k\pi)}$$

$$\therefore E(T^r) = \frac{\beta^r \left(\frac{r\pi}{\alpha}\right)}{\sin\left(\frac{r\pi}{\alpha}\right)} \tag{9}$$

If $\alpha > 1$, (9) becomes;

$$E(T) = \frac{\beta \left(\frac{\pi}{\alpha}\right)}{\sin\left(\frac{\pi}{\alpha}\right)} \tag{10}$$

$$E(T^2) = \frac{\beta^2 \left(\frac{2\pi}{\alpha}\right)}{\sin\left(\frac{2\pi}{\alpha}\right)};$$

$$Var(T) = \frac{\beta^2 \left(\frac{2\pi}{\alpha}\right)}{\sin\left(\frac{2\pi}{\alpha}\right)} - \left[\frac{\beta \left(\frac{\pi}{\alpha}\right)}{\sin\left(\frac{\pi}{\alpha}\right)} \right]^2 \tag{11}$$

2.6 Log-Logistic Survival (or Reliability) Function

This is the probability that a variate takes on a value greater than t which denotes the probability that a unit survives beyond time, t :

$$S(t) = P(T > t) = 1 - F(t) = 1 - \frac{1}{1 + \left(\frac{\beta}{t}\right)^\alpha} = \frac{\left(\frac{\beta}{t}\right)^\alpha}{1 + \left(\frac{\beta}{t}\right)^\alpha}$$

Multiply and divide by $\left(\frac{\beta}{t}\right)^{-\alpha}$

$$S(t) = \frac{1}{1 + \left(\frac{t}{\beta}\right)^\alpha} \tag{12}$$

2.7 The Hazard Function, $h(t)$ of the Log-Logistic Distribution

The hazard function is used in reliability applications to describe the instantaneous failure rate at any point in time. It varies with time for this distribution

and is given as;

$$h(t) = \frac{f(t)}{S(t)} = \frac{\left\{ \frac{\left(\frac{t}{\beta}\right)^{\alpha-1} \left(\frac{\alpha}{\beta}\right)}{\left(1 + \left(\frac{t}{\beta}\right)^\alpha\right)^2} \right\}}{\left\{ \frac{\left(\frac{\beta}{t}\right)^\alpha}{1 + \left(\frac{\beta}{t}\right)^\alpha} \right\}} = \frac{\left(\frac{t}{\beta}\right)^{\alpha-1} \left(\frac{\alpha}{\beta}\right)}{1 + \left(\frac{t}{\beta}\right)^\alpha} \quad (13)$$

2.8 The Cumulative Hazard Function of the Log-Logistic Distribution

$$\text{Recall: } f(t) = \frac{d}{dt}F(t) = \frac{d}{dt}(1 - S(t)) = -S'(t)$$

$$\text{Then, } h(t) = -\frac{S'(t)}{S(t)} = -\frac{d}{dt}\ln(S(t))$$

$$\int h(t)dt = \int -\frac{d}{dt}\ln(S(t))dt = -\ln(S(t)) = H(t)$$

$$\therefore H(t) = -\ln(S(t)) = -\ln\left(\frac{1}{1 + \left(\frac{t}{\beta}\right)^\alpha}\right) \quad (14)$$

2.9 Replacement Models for DC-6240L Triumph-Adler Photocopy Machine

In developing a replacement model, the decision criterion is defined by $E[C(t)]$, which is the expected cost per cycle of replacing a part of the system in cycle period $(0, t]$. The expected number of failure occurring in the cycle period $(0, t]$ is equal to the probability of occurrence of a failure before time, t . Bahrami (2000) denoted the number of failures occurring during the period $(0, t]$ by $N(t)$, which is a discrete random variable with probability distribution function defined as;

$$P[N(t) = n] = G(n); n = 1, 2, 3, \dots$$

The mean number of failure during the cycle period $(0, t]$ is;

$$E[N(t)] = \sum_{N(t)=0}^{N(t)=n} N(t) * G(N(t))$$

$G(N(t))$ is the probability distribution function of $N(t)$ failures occurring in the period $(0, t]$ with the assumption that each interval is made as short as possible so that the probability of having more than one failure is negligible. That is;

$$P[N(t) = 2] < P[N(t) = 1]; P[N(t) = 3] < P[N(t) = 2];$$

$$P[N(t) = 4] < P[N(t) = 3], \text{ etc.}$$

Therefore, $P[N(t) = 1] > P[N(t) = 2] > P[N(t) = 3] > P[N(t) = 4] > \dots$

In the case of preventive replacement at time, τ the expected number of failures in the period $(0, \tau)$ can be estimated from the following;

$$G(1) = P[N(\tau) = 1] \cong F(\tau)$$

$$G(0) = P[N(\tau) = 0] \cong 1 - F(\tau)$$

$$E(N(\tau)) = \left\{ \sum_{n=0}^{\infty} n * G(n) \right\} = \{0 * [1 - F(\tau)]\} + [1 * F(\tau)] = F(\tau)$$

$$\therefore E(N(\tau)) = F(\tau)$$

Therefore, the mean number of failures occurring during the cycle period $(0, \tau]$ is equal to the probability of occurrence of a failure before time, τ .

Let C_p be the total cost of planned (or preventive) replacement; C_u be the total cost of unplanned (or failure) replacement; and τ be the replacement time with optimal value, τ^* . The total expected cost per unit time for preventive replacement at replacement time, τ is defined as;

$$E[C(\tau)] = \frac{\text{total expected cost}}{\text{replacement time}} = \frac{C_p + C_u E(N(\tau))}{\tau} \quad (15)$$

The expected number of failure occurring during the cycle $(0, \tau]$ is equal to the probability of occurrence of a failure before time, τ . Hence, the expected cost per unit time for preventive replacement is given by;

$$E[C(\tau)] = \frac{C_p + C_u F(\tau)}{\tau} \quad (16)$$

Also, Cassady and Pohl (2003), stated that the mean number of failures occurring during the cycle $(0, \tau]$ is equal to the cumulative hazard rate at time, τ using the concept of non-homogeneous Poisson process. Hence the expected cost per unit time for preventive replacement was given by;

$$E[C(\tau)] = \frac{C_p + C_u H(\tau)}{\tau} \quad (17)$$

2.9.1 Minimization of the expected cost function

Minimizing (15);

$$\begin{aligned} \frac{d}{d\tau}E[C(\tau)] &= \frac{\tau C_u \frac{d}{d\tau}E(N(\tau)) - \{C_p + C_u E(N(\tau))\}\{1\}}{\tau^2} = 0 \\ \therefore \tau^* &= \frac{\frac{C_p}{C_u} + E(N(\tau))}{\frac{d}{d\tau}E(N(\tau))} \end{aligned} \quad (18)$$

2.9.2 Optimal Replacement Time, τ^* for the Cumulative Failure Function-based Replacement Model

Recall; $E[N(\tau)] = F(\tau) \Rightarrow \frac{d}{d\tau}F(\tau) = f(\tau)$

$$\therefore \tau^* = \frac{\frac{C_p}{C_u} + F(\tau)}{f(\tau)} \quad (19)$$

2.9.3 Optimal Replacement Time, τ^* for the Cumulative Hazard Function-based Replacement Model

$$E[N(\tau)] = H(\tau); \Rightarrow \frac{d}{d\tau}H(\tau) = h(\tau)$$

$$\therefore \tau^* = \frac{\frac{C_p}{C_u} + H(\tau)}{h(\tau)} \quad (20)$$

2.10 Availability of the System

Availability is the probability that a system will work as required during a particular period of time. Let $A(\tau^*)$ denotes the availability of a system at optimal time, (τ^*) ; $E(\uparrow)$ denotes the expected uptime at optimal time, (τ^*) ; $E(\downarrow)$ denotes the expected downtime at optimal time, (τ^*) ; D_p denotes the average downtime for planned (preventive) replacement; D_u denotes the average uptime for unplanned (failure) replacement; $R(\tau^*)$ denotes the reliability at optimal time (τ^*) and $F(\tau^*)$ the cumulative failure at optimal time, (τ^*) .

$$\text{Then, } A(\tau) = \frac{E(\uparrow)}{E(\uparrow) + E(\downarrow)}$$

According to Cassady and pohl (2003);

$$E[\uparrow] = \int_0^{\tau^*} x f(x) dx + \tau^* S(\tau^*)$$

$$= \tau^* F(\tau^*) + \tau^* [1 - F(\tau^*)] = \tau^* F(\tau^*) + \tau^* - \tau^* F(\tau^*) = \tau^*$$

$$\therefore E[\uparrow] = \tau^*$$

And,

$$E[\downarrow] = D_p F(\tau^*) + D_u S(\tau^*) = F(\tau^*) [D_p - D_u] + D_u$$

$$\therefore E[\downarrow] = F(\tau^*) [D_p - D_u] + D_u$$

Where,

$$D_p = \frac{1}{n_1} \sum_{j=1}^{n_1} D_j \tag{21}$$

And,

$$D_u = \frac{1}{n_2} \sum_{i=1}^{n_2} D_i \tag{22}$$

Therefore,

$$A(\tau^*) = \frac{E(\uparrow)}{E(\uparrow) + E(\downarrow)} = \frac{\tau^*}{\tau^* + [F(\tau^*) [D_p - D_u] + D_u]}$$

$$= \frac{\tau^*}{\tau^* + \left[\frac{D_p - D_u}{1 + \left(\frac{\beta}{\tau^*}\right)^\alpha} \right] + D_u} \tag{23}$$

3 Analysis and Results

3.1 Estimation of the Log-Logistic Parameters of the DC-6240L Triumph-Adler Photocopy Machine

We use the inter-failure times, t (in hours) to estimate the parameters of the Log-Logistic distribution as follows;

Recall (7), (8), (9) and (11) for $n = 29$;

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^n \ln(t) = 6.638465642$$

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \left\{ \left[\sum_{i=1}^n (\ln(t))^2 \right] - n\hat{\mu}^2 \right\}} = 1.052914737$$

Hence, $\hat{\beta} = 763.9219635$ and $\hat{\alpha} = 1.723339368$.

The Easyfit (5.6) version software was used to validate the parameters estimates.

3.2 Evaluation of Probability Functions of the Photocopy Machine Using Log-Logistic Distribution

The estimated shape parameter, $\hat{\alpha} = 1.723339368$ and scale parameter, $\hat{\beta} = 763.9219635$ were used to obtain the optimal probabilities for the failure function, $f(t)$; failure cumulative function, $F(t)$; survival (reliability) function, $S(t)$; hazard function, $h(t)$; and the cumulative hazard function, $H(t)$ shown in Table 1.

3.3 Evaluation of Optimal Replacement Time and Minimum Expected Preventive Replacement Maintenance Cost of Photocopy Machine

3.3.1 Replacement model based on cumulative failure function

The cost values were obtained as $C_u = \sum_{i=1}^{23} C_i = 157550$ and $C_p = \sum_{j=1}^6 C_j = 8150$ and the data on Table 2 were applied to (16) and (19) at $t_p = 100$ and (17) and (20) at $t_p = 110$ respectively to obtain values for minimum expected cost and optimal replacement time of the machine for the cumulative failure and cumulative hazard-based replacement models. The results are also shown in Table 1.

3.4 Discussion of Results

i. Estimated parameters of the Log-Logistic distribution: The estimates $\hat{\alpha} = 1.723339368$ and $\hat{\beta} = 763.9219635$ are the shape and scale parameters of the Log-Logistic distribution respectively. The scale parameter is also known as the median of the distribution.

Table 1 Optimal probabilities and expected costs for the replacement maintenance models

Probability Function	Cumulative Failure-Based Replacement Model with $\tau^* = 166$	Cumulative Hazard-Based Replacement Model with $\tau^* = 161$
$F(\tau^*)$	0.067192365	Not applicable
$H(\tau^*)$	Not applicable	0.066100607
$h(\tau^*)$	0.000697562	0.000684661
$R(\tau^*)$	0.935807635	0.936036688
$A(\tau^*)$	0.959651769	0.958355733
$E[C(\tau^*)]$	113	115

ii. The failure density function: Figure 1 shows the failure function of the DC-6240L Triumph-Adler Photocopy Machine. It is skewed to the right and is unimodal. Also the failure function increases from the point: 0.00039507 to 0.00080629 within the time interval of 72 hours to 336 hours, and then there is a gradual decrease from 0.00080629 to 0.0000090519 within the time interval of 336 hours to 5664 hours. The shape of the curve typifies a density function of Log-Logistic distribution.

iii. The failure cumulative function: Figure 2 shows the failure cumulative function of the machine as an increasing function. There is an increase from 0.01678764 to 0.57574533 within the time interval of 72 hours to 912 hours, and then a continual rapid increase from the point: 0.57574533 to 0.96930777 within the time interval of 912 hours to 5664 hours.

iv. The survival (reliability) function: Figure 3 shows the reliability function of the machine reducing gradually from the point: 0.98321236 to 0.40282036 within the time interval of 72 hours to 960 hours, and then continues to reduce rapidly from the point: 0.40282036 to 0.03069223 within the time interval of 960 hours to 5664 hours.

v. The hazard function or failure rate: Figure 4 shows the hazard rate of the machine which increases slowly from 0.00040182 to 0.00114147 within the time interval: 72 hours to 600 hours. Then, there is a gentle decrease from 0.00114147 to 0.00029492 within the time interval: 600 hours to 5664 hours.

vi. The cumulative hazard function: This is a strictly increasing function of the machine as shown in Figure 5. It shows a rapid increase from point: 0.01693015 to 3.48374565 within the time interval: 72 hours to 5664 hours.

vii. Replacement model based on failure cumulative function: Figure 6 is the plot of the replacement time, τ against equal interval of failure time, t_p for the replacement model based on the failure cumulative function. It has a minimum preventive maintenance replacement time of 166 hours. Table 1 shows that preventive replacement maintenance at this optimum time is at a minimum cost of 113 Naira per cycle time. Also, the availability of the Machine at the optimum replacement maintenance time is 96% and 94% reliability with 0.07% chance of failure occurrence.

viii. Replacement model based on cumulative hazard function: Figure 7 is the plot of replacement time, τ against equal interval of failure time, t_p for the cumulative hazard-based replacement model with a minimum preventive maintenance replacement time of 161 hours. Table 1 shows that the associated

cost of preventive maintenance per cycle time at the optimum time is 115 Naira for this model. Also, the availability of the Machine at the optimum replacement maintenance time is 96% and is 94% reliable with 0.07% chance of failure occurrence.

ix. Model comparison: The two preventive replacement maintenance models have same values of maximum availability, reliability and percentage chance of failure occurrence in the machine as summarized in Table 1. However, the replacement model based on failure cumulative function is a better model because it allows for a longer optimal time of 166 hours before replacement maintenance at a lower expected cost per cycle time of 113 naira, while the model based on the cumulative hazard function yields a shorter optimal replacement maintenance time of 161hours at a higher minimum cost of 115 naira. It is clear that the model based on the failure cumulative function gives a longer time before replacement maintenance and at a lower minimum cost than the one obtained from the cumulative hazard-based model.

3.5 Maintenance policy for the DC-6240L Triumph-Adler Photocopy Machine

The DC-6240L Triumph-Adler Photocopy Machine with 96% availability level, 94% reliability and 0.07% chance of failure occurrence should be optimally operated for $T \leq 166$ hours with a minimum maintenance cost of 113 Naira, and failure replacement should be performed at any time, $t < T$.

4 Conclusion

A proposed preventive replacement maintenance policy that maximizes availability and reliability of photocopy machines and similar equipment at minimum cost has been formulated in this work. The Log-logistic distribution following from a chi-squared goodness-of-fit test with rank one was used to modeled the inter-failure distribution of the machine to obtain optimal failure and reliability probabilities for the machine. Also, the cumulative failure function-based replacement model by Bahrami (2000) was found to be better than the hazard-based replacement model by Cassady and Phol (2003) because it allows for a longer optimal operational time of 166 hours before replacement maintenance at a lower expected cost per cycle time of 113 naira. Hence, it was used to formulate the maintenance policy for Photocopy Machines and similar equipment to guarantee seamless operation,

reduce downtime, save cost of maintenance and increase profit turnover in a competitive business environment.

Appendix

Table 2 Table of probability functions evaluation

i	t	$f(t)$	$F(t)$	$S(t)$	$h(t)$	$H(t)$
1	72	0.00039507	0.01678764	0.98321236	0.00040182	0.01693015
2	96	0.00047615	0.02726764	0.97273236	0.00048949	0.0276463
3	120	0.00054551	0.03954924	0.96045076	0.00056797	0.04035256
4	312	0.0008013	0.17607264	0.82392736	0.00097254	0.19367291
5	336	0.00080629	0.19537233	0.80462767	0.00100206	0.21737563
6	384	0.00080464	0.23409115	0.76590885	0.00105057	0.26669211
7	408	0.00079898	0.25334031	0.74665969	0.00107008	0.29214577
8	480	0.00076776	0.3098543	0.6901457	0.00111247	0.37085255
9	480	0.00076776	0.3098543	0.6901457	0.00111247	0.37085255
10	480	0.00076776	0.3098543	0.6901457	0.00111247	0.37085255
11	504	0.00075381	0.32811599	0.67188401	0.00112193	0.39766956
12	504	0.00075381	0.32811599	0.67188401	0.00112193	0.39766956
13	600	0.00068783	0.3974148	0.6025852	0.00114147	0.50652622
14	672	0.00063336	0.44498744	0.55501256	0.00114117	0.58876453
15	768	0.00056097	0.50229379	0.49770621	0.00112711	0.69774531
16	912	0.00046157	0.57574533	0.42425467	0.00108794	0.85742137
17	960	0.00043183	0.59717964	0.40282036	0.00107202	0.90926458
18	984	0.00041765	0.60737252	0.39262748	0.00106373	0.934894
19	1152	0.00033078	0.66994343	0.33005657	0.0010022	1.10849121
20	1248	0.00029015	0.69970005	0.30029995	0.0009662	1.20297348
21	1392	0.00023956	0.73770175	0.26229825	0.0009133	1.33827306
22	1416	0.00023218	0.74336206	0.25663794	0.00090471	1.36008896
23	1872	0.00013343	0.8241359	0.1758641	0.00075869	1.73804376
24	2088	0.00010536	0.84977167	0.15022833	0.00070136	1.89559896
25	2112	0.00010274	0.85226872	0.14773128	0.00069543	1.9123603
26	2136	0.00010019	0.85470371	0.14529629	0.00068958	1.92898026
27	3216	3.8299E-05	0.92252622	0.07747378	0.00049435	2.55781572
28	4368	1.7747E-05	0.95279076	0.04720924	0.00037591	3.05316565
29	5664	9.0519E-06	0.96930777	0.03069223	0.00029492	3.48374565

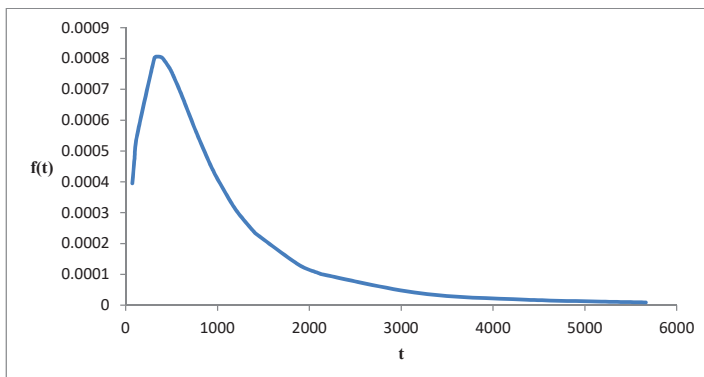


Figure 1 A graph of the Log-Logistic density function $f(t)$ against time t for DC-6240L Triumph-Adler Photocopy Machine.

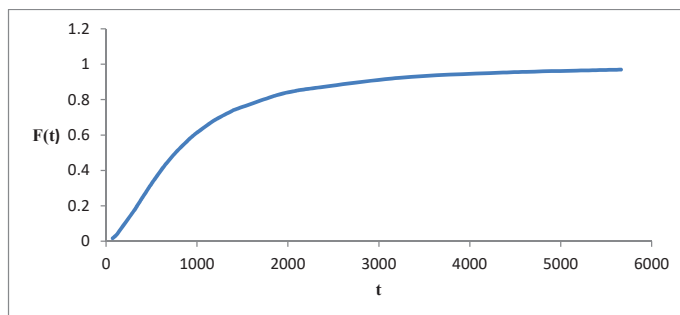


Figure 2 A graph of the Log-Logistic cumulative density function $F(t)$ against time t for DC-6240L Triumph Adler Photocopy Machine.

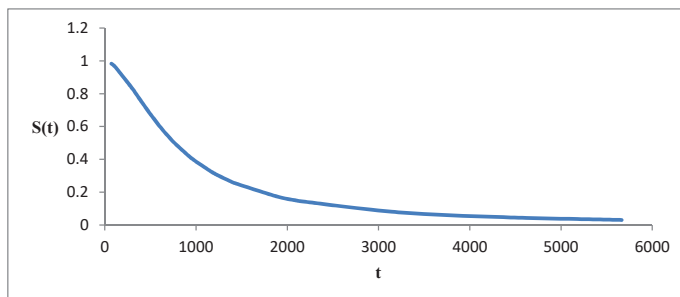


Figure 3 A graph of the Log-Logistic Survival (Reliability) function $S(t)$ against time t for DC-6240L Triumph-Adler Photocopy Machine.

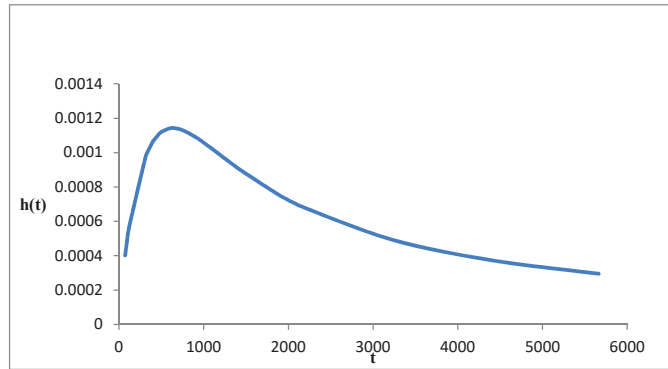


Figure 4 A graph of the Log-Logistic hazard function $h(t)$ against time t for DC-6240L Triumph-Adler photocopy Machine.

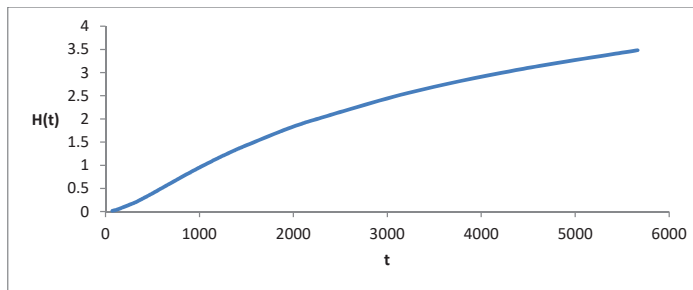


Figure 5 A graph of the Log-Logistic cumulative hazard function $H(t)$ against time t for DC-6240L Triumph-Adler Photocopy Machine.

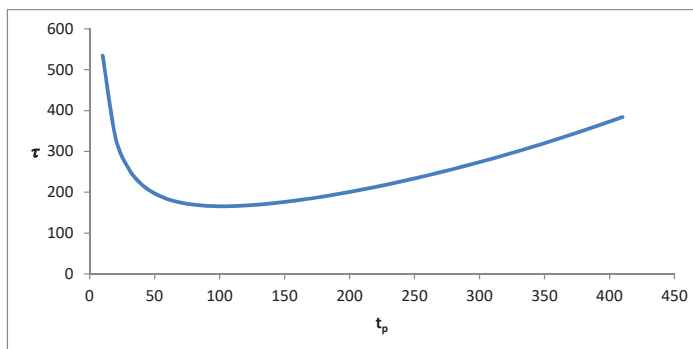


Figure 6 A graph of the replacement time τ against equal interval of failure time t_p for replacement model based on the cumulative failure function.

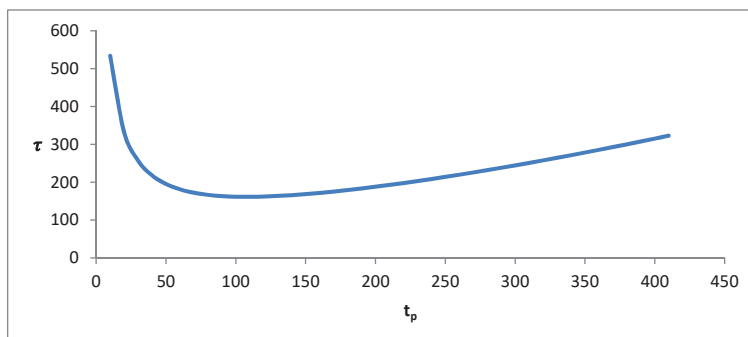


Figure 7 A graph of the replacement time τ against equal interval of failure time t_p for replacement model based on the cumulative hazard function.

References

- [1] Ahmad M. I., Sinclair C. D. and Werritty A. (1988). Log-logistic food frequency analysis. *Journal of Hydrology*, 98: 205–224.
- [2] Akhtar, M. T., and Khan, A. A. (2014). Log-logistic distribution as a reliability model: A Bayesian analysis. *American Journal of Mathematics and Statistics*, 4(3): 162–170.
- [3] Anderson, W. A., McClure P. J., Baird-Parker A. C. and Cole M. B. (1996). The application of a log-logistic model to describe the thermal inactivation of *Clostridium botulinum* 213 b at temperatures below 121.1 c. *Journal of Applied Microbiology* 80(3): 283–290.
- [4] Ashkar F. and Mahdi S. (2006). Fitting the log-logistic distribution by generalized moments. *Journal of Hydrology*, 328: 694–703.
- [5] Bahrami G.K., Price, J.W.H. and Mathew (2000). The constant interval replacement Model for preventive replacement maintenance: A new perspective. *International Journal of Quality and Reliability Management*, 17(8): 822–838.
- [6] Byon, E., Ntaimo L. and Ding Y. (2010); Optimal maintenance strategies of wind turbine systems under stochastic weather conditions. *IEEE Transactions on Reliability*; 59: 393–404.
- [7] Cassady, C.R., E.A. Pohl and W.P. Murdock (2003); Selective Maintenance Modeling for Industrial Systems. *Journal of Quality in Maintenance Engineering*, 7(2): 104–117.
- [8] Chouhan, R., Gaury, M. and and Tripathi, R. (2013). A Survey of Preventive Maintenance Planning Models, Techniques and Policies for

- an Ageing and Deteriorating Production Systems. *HCTL Open Int. J. of Technology Innovations and Research*, 3(May): 1–19.
- [9] Clark, D. E. and El-Taha, M. (2015). Some useful properties of Log-Logistic Random Variables for Health Care Simulations. *International Journal of Statistics in Medical Research*; 4, 79–86.
- [10] Fisk, P.R. (1961). The graduation of income distribution. *Econometrica*, 29: 171–185.
- [11] Gupta, R. D., Ahman O. and Lvin, S. (1999). A study of log-logistic model in survival analysis. *Biometrical Journal*, 41: 431–443.
- [12] Huynh, K. T., Castro, I. T. and Berenguer, C. (2012). Modeling age-based maintenance strategies with minimal repairs for systems subject to competing failure models due to degradation and shocks. *European Journal of Operations Research*, 218: 140–151.
- [13] Jibril, Y. and Ekundayo, K. R. (2015). Reliability Assessment of 33 kV Kaduna Electricity Distribution Feeders, Northern Region, Nigeria. World Congress on Engineering and Computer Science, San Francisco, USA: 1–5.
- [14] Kurt, M. and Maillart, L. M. (2009). Structured replacement policies for a Markov-modulated shock model. *Operations Research Letters*; 37: 280–284.
- [15] Lamberson, L. R. (2013). Reliability in Engineering Design, New York, USA: John Wiley and Sons, Inc.
- [16] Mahdavi, M. and Mahdavi, M. (2009). Optimisation of age replacement policy using reliability based heuristic model. *Journal of scientific & Industrial Research*, 68: 668–673.
- [17] Mahmoud, M. R. and Mohammed, S. M. (2013). Estimation of Parameters of the Marshall-Olkin Extended Log-Logistic Distribution from Progressively Censored Samples. *European Journal of Statistics and Probability*, 2(3):1–13.
- [18] Nakagawa, T. (1984). Periodic inspection policy with preventive maintenance. *Naval Research Logistic Quarterly*, 31:33–40.
- [19] Segawa, Y., Ohnishi, M. and Ibaraki, T. (1992); Optimal minimal-repair and replacement problem with age dependent cost structure. *Computers & Mathematics with Applications*, 24: 91–101.
- [20] Shoukri M. M., Mian I. U. H. and Tracy D. S. (1988); Sampling properties of estimators of the log-logistic distribution with application to Canadian precipitation data. *Canadian Journal of Statistics* 16: 223–236.

- [21] Staye, R. (2014). Reliability Analysis of Electrical Gas cooker. Master of Science Thesis, Faculty of Electrical and Electronic Engineering, University Tun Hussein Onn Malaysia.
- [22] Tahara, A. and Nishida, T. (1975). Optimal replacement policy for minimal repair model. *Operations Research Society of Japan*; 18: 113–124.
- [23] Tahir, M. H., Mansoor, M., Zubair, M. and Hamedani, G. G. (2014). McDonald Log-Logistic distribution. *Journal of Statistical Theory and Applications*, 13: 65–82.
- [24] Verhulst, P. F. (1838). Notice sur la loi que la population suit dans son accroissement. *Correspondence in Mathematical Physics*, 10: 113–121.
- [25] Wang, H. and Pham, H. (1996). Optimal age-dependent preventive maintenance policies with imperfect maintenance. *International Journal of Reliability, Quality and Safety Engineering*, 3: 119–135.

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