Probabilistic Assessment of Computer-Based Test (CBT) Network System Consists of Four Subsystems in Series Configuration Using Copula Linguistic Approach

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Abstract

A complex repairable computer-based test (CBT) network system studied in this paper consists of three client computers, a load balancer, two database servers, with the centralized server structured in a series configuration. Subsystem 1 consists of three homogeneous clients arranged in parallel configuration, subsystem 2 comprises a load balancer, subsystem 3 is comprised of two distributed homogeneous database servers in parallel arrangement and subsystem 4 consists of a centralized database server. Through the transition diagram, the first-order differential equations are derived. The model has solved using supplementary variables, with implications of Laplace transforms. Reliability metrics of system effects such as availability, reliability,

MTTF, MTTF sensitivity, and the cost function is estimated to see the impact of failure and repair patterns on reliability evaluations. The results of this study indicated that system performance could be improved when the copula repair is employed.

Keywords: Repairable systems, availability, MTTF, sensitivity MTTF, cost analysis, general repair, copula repair.

1 Introduction

Information and communication technology (ICT) is critical in reshaping the evaluation and assessment methods in various academic domains by improving educational measures to computer-based testing (CBT). The traditional method of paper and pencil, which time-consuming, has proven to be hazardous to students or candidates in delivering examination, assessment, and seeking knowledge. As mentioned earlier, the method constitutes tedious and risky task such as inadequate examination materials, examination malpractice, delay in marking and production of results, and human error. One of the forms of ICT for assessment in the educational sector and other sectors interviewing in some developing countries is Computer-Based Testing (CBT), also known as E-examination and assessment/testing. CBT is considered a method of conducting tests or assessments in which the solutions are electronically recorded, assessed, and released. To attain the desired CBT network availability and reliability, some CBT centers must include CCTV cameras, computers (clients and a server), networking cables, hardware, software, satellite systems, and standby generators. Because of those, as mentioned above, a computer networking system with high efficiency is required. Such as computer network consists of devices ranging from hardware to software devices. The network could include multiple clients, load balancers, centralized data server, and multiple distributed database servers to improve system reliability. The load balancer communicates the request and responses between clients and servers. With the recent development of information and communication technology, CBT network reliability and availability have become an area of discussion resulting in quality assurance of service and system safety. Numerous researchers in the field of reliability have done extensive research and have proposed improving and enhancing various systems and the CBT network. Persisting some of them, Garg [1] reviewed an industrial structure using the Kolmogorov equation method with the Fuzzy concept. Gahlot et al. [2] overcame two types of repair policies using the repairable system's copula linguistics evaluation technique under

various kinds of failures in the series arrangement. A reliability assessment of the communication system thru redundant relay stations with two types of failure has been proposed by Ibrahim et al. [3]. Abdul Kareem Lado et al. [4] examined the necessary reliability measures for a repairable complex system comprising two subsystems using a supplementary variable to preclude repair. In the series configuration employing the Gumbel-Hougaard family copula distribution, Lado and Singh [5] examined the cost function and other reliability measures of complex repairable systems containing two subsystems different failure rates. Malik and Tewari [6] demonstrated a repairable system's effectiveness by demonstrating repair priorities in a coal-based thermal power plant's water flow network. The MTBF and other reliability parameters were evaluated by Mortazavi et al. [7] for 2-out-of-3: G redundant system with common cause failures, taking fuzzy failure and repair rates: thru the case study of motor water pumping system. Ram Niwas and Harish Garg [8] proposed a new approach focused on a cost-free maintenance strategy during the warranty period to enhance an industrial system's reliability and effectiveness. Ram Nawas et al. [9] assessed the reliability of railway systems with probabilistic ramification. Singh et al. [10] addressed a complex system's performance analysis under different failures and two repair discipline using copula's concept. Singh and Ayagi [11] investigated system performance using the preemptive resume repair policy and Gumbel-Hougaard family copula repair approach. The availability, MTTF, and cost analysis of the complex system using copula distribution under the preemptive resume repair policy were studied by Singh and Rawal [12].

A comparative study of four different solar panel configurations was proposed by Suleiman et al. [13]. Yusuf, I. Yusuf. [14], examined the availability of a parallel unit supporting system under preventive maintenance. Throughout the comparative study of three-unit redundant systems, Yusuf and Hussaini [15] dealt with three kinds of failures and constant repair using the stochastic methodology. Taj et al. [16] review a subsystem with the probabilistic approach and regenerative point's technique approach. Damedii and Noureddine [17] evaluated the reliability of a computer network under effective maintenance. Elyasi et al. [18] analyzed computer network reliability and critical features of system performances. The software tool was developed to estimate the cooperate computer network (CCN) critical failure probability by constructing the criticality matrix using the FME(C)A-technique. T Komari [19] performed an availability assessment of network systems using the Markov model. Liu [20] computed reliability with of optimization design of computer network based on the genetic algorithm. Lin et al. [21] dealt with the method for network reliability evaluation in

ad-hoc networks. Rahman [22] has studied the stationary availability factor with arbitrary topology for two-level computer networks. Singh et al. [23] have premeditated three computer labs' reliability measures associated with a server under 2-out-3: G configuration. Singh and Rawal [24] analyzed a system comprising two subsystems in series arrangement with k-out-of-n: G, scheme, and controllers. Kumar and Gupta [25] analyzed a single unit system with a supporting unit in which the main unit has two modes, and the supporting unit has three modes. Kumar and Sirohi [26] studied a k-out-of-n: G type of system with repair and without repair using the linear differential equation and regenerative point technique. Kumar and Sirohi [27] conferred on the reliability evaluation of a system having two units cold standby system for the delay in repairing the partially failed unit was truckled under regenerative point technique. Singh and Poonia [28] analyzed two units in a parallel configuration with correlated lifetime under inspection and regenerative point technique. Singh et al. [29] analyzed a system that comprised the two subsystems in a series arrangement with a human failure controller. The configuration consisting of the first subsystem working policy k-out-of-n: G policy and the second subsystems containing three identical units with uniform failure rates. Raghav et al. [30] analyzed a system comprising two subsystems under the k-out-of-n: G strategy with two types of repair with copula repair approach. Kumar and Singh [31] computed reliability indices of a repairable system with deliberate failure and reboot delay employing supplementary variables. Rawal et al. [32] studied the reliability metrics for a local area network with two different network topologies (Star topology and Bus topology) by incorporating different failure and repair using the copula methodology. M. Ram and Monika Manglik [33] studied reliability and other probabilistic measures of common cause failure. Kumar et al. [34] studied reliability indices, including availability and cost function systems involving two subsystems in the series configuration using supplementary variables and Laplace transformation. Singhal et al. [35] analyzed network reliability using different binary decision diagrams. Xin et al. [36] analyzed a model associated with the network reliability analysis. The significance of the network services in prominent fields like communication and aviation industry, network transmission, hardware equipment, software services, and human factors were presented. Zhang [37] dealt with a performance assessment on a computer network based on intelligent cloud computing methods through the reliability concept. Nivedita Gupta et al. [38] evaluated the generator's steam turbine power plant's reliability and operational availability by employing the Kolmogorov equation and Markova process.

As mentioned earlier, the literature presented a considerable contribution in enhancing the systems' performance and proclaimed the validity of their models. However, few paid attention to the practical model of the Machine-Base Test like (CBT) network system in the literature. The CBT system comprises four subsystems: Client computer, Distributed database server, Load Balancer, and Unified Database server. To make up for the difference between previous studies and recent development in this paper, we proposed a new mathematical model CBT for probabilistic assessment. Composing the programmed name, the same device reliability, availability, MTTF, and cost function expressions are obtained. Some numerical calculations on reliability metric, availability, MTTF, and cost function have been conducted to observe the impact of both failure and repair rates on system performance. Copula repair is observed from quantitative simulation to make the structure more reliable for better performance.

2 Notations, Assumptions, and Description of the System

2.1 Notations

t/s:	Time variable/ Laplace Transformation variable.
$\lambda_1/\lambda_2/\lambda_3/\lambda_4$:	Failure rate for subsystems 1/subsystem failure 2/ subsystem failure 3/subsystem 4.
$\varphi(x)/\varphi(y)$:	Repair rate for subsystem 1/subsystem 2.
$\varphi(z)/\varphi(m)$:	Repair rate for the subsystem 3/ subsystem 4.
$\mu_0(x)/\mu_0(y)$:	Repair rates for totally failed states.
$\mu_0(z)/\mu_0(m)$:	Repair rates for completely failed states.
$p_i(t)/\overline{Pi}(s)$:	The state transition probability of state Si/Laplace Transform state Si, i = 0 to 10.
$P_i(x,t)$:	The probability of a system to be in the state with Si for i = 1 … 10, the system. The system under repair with pass repair time (x,t), x, and t are repair and time variables.
$K_1/K_2/E_p(t)$:	Revenue generation/ Service cost per unit item/ Expected profit in the interval $[0,t)$.
$\mu_0(x)$:	An expression for Gumbel Hougaard family copula distribution.

$\mu_0(x) = c_\theta(u_1, u_2(x)) = exp(x^\theta + \{log\phi(x)^\theta\}^{\frac{1}{\theta}})$, $1 \leq \theta < \infty$, $\mu_1 = \phi(x) = x$, and $u_2 = e^x$. For $\theta = 1, x = 1, \emptyset = 1$ the Gumbel-Hougaard copula approach to 2.7183 approx.

2.2 Assumptions

In the study of the model, we took the necessary assumptions:

(i) Initially, both subsystems work perfectly.

(ii) Subsystem 1 is the client's PC, subsystem 2 is a load balancer, subsystem 3 is a set of database servers consisting of two DDS, and subsystem 4 is a centralized server. The subsystem will be inoperative if any of the subsystems completely fails.

(iii) The system is repaired when it is in degraded mode due to its partial failure in the complete failed state.

(iv) All failure rates are fixed, and they are presumed to follow a negative exponential distribution.

(v) Repair of partially failed states has been done by general distribution, and the total failed state is restored by using the Gumbel-Hougaard family copula distribution.

(vi) The repaired subsystems are assumed to perform as a new, and no damage occurs during the repair process.

2.3 Description of the System

The system consists of four subsystems in a series configuration. Subsystem 1: has three identical client's PC, subsystem 2: a load balancer (LB), subsystem 3: has two distributed database servers (DDS), and subsystem 4: Centralized database server (CDS). Initially, the system is in the perfect operational state, and failing one client PC; the system start works with reduced efficiency. Immediately one client's PC fails, the system is operative, and the failed client's PC is immediately sent for repair. Whenever the second client's PC fails, the standby PC automatically switches on to operational mode, LB, DDS, and CDS are operating, the system is operative. The second failed client's PC was assigned for repair. However, if DDS I fail, two clients' PC, LB, and CDS are okay, then If any subsystem fails, the system would've been in idle mode. Partially / degraded states have been repaired by general distribution, and a comprehensively failed state is repaired using the Gumbel-Hougaard family's copula distribution. DS II automatically switches on to functioning mode, the system is operational and failed DDS I is assigned for repairs.
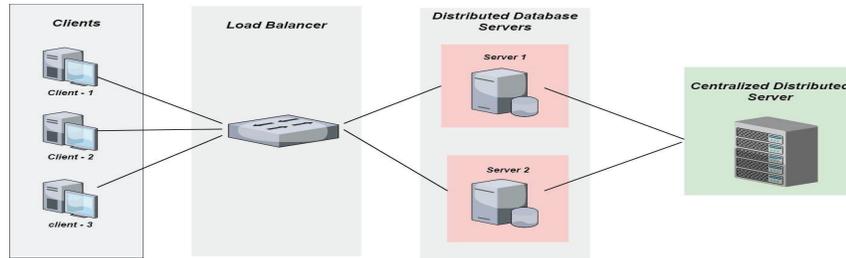
Figure 1a   Reliability block diagram of the CBT network.
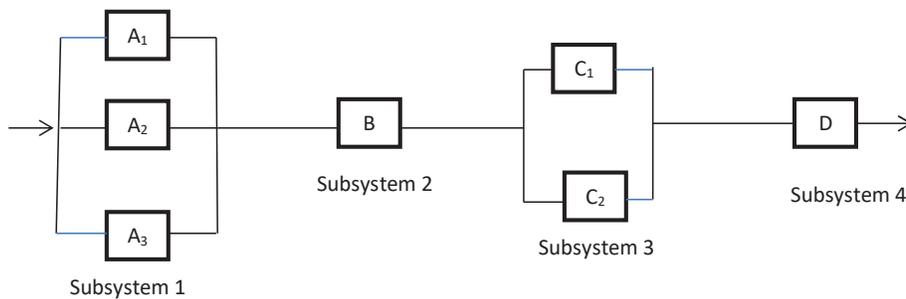
Figure 1n   Block diagram of the CBT network as a series-parallel system.

2.4 State Description

S_o: Perfect state, all subsystems are okay.

S_1/S_9:   The client's PC failed, and it was assigned to repair the system is operative.

S_2/S_8:   DDS I failed and is immediately sent for repairs; the system is operative.

S_3:   Complete fail state due to failure of the load balancer.

S_4:   Complete fail state due to the failure of CDS.

S_5/S_{10}:   Partially fail state due to the failure of the second client's PC and is assigned for repair, the system is operative.

S_6:   Complete fail state due to the complete failure of subsystem 1.

S_7:   Complete fail state due to the complete failure of subsystem 3.

2.5 Formulation of Mathematical Model

The following sets of differential equations are obtained from the transition diagram is shown in figure 2 above, probability of considerations, and
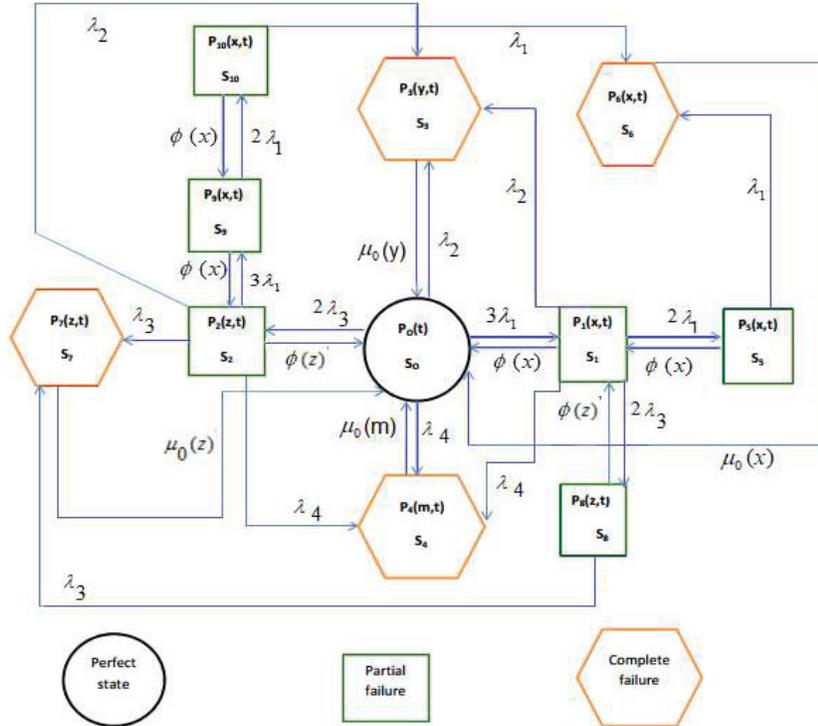
Figure 2 State transition diagram of the mathematical model.

persistence of arguments.

$$\left(\frac{\partial}{\partial t} + 3\lambda_1 + 2\lambda_3 + \lambda_4\right) p_o(t)$$

$$= \int_o^\infty \phi(x)p_1(x,t)dx + \int_o^\infty \phi(z)p_2(z,t)dz + \int_0^\infty \mu_0(x)p_6(x,t)dx$$

$$+ \int_0^\infty \mu_0(y)p_3(y,t)dy + \int_0^\infty \mu_0(z)p_7(z,t)dz$$

$$+ \int_0^\infty \mu_0(m)p_4(m,t)dm \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4 + \phi(x)\right) p_1(x,t) = 0 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \phi(z) \right) p_2(z, t) = 0 \tag{3}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_0(y) \right) p_3(y, t) = 0 \tag{4}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial m} + \mu_0(m) \right) p_4(m, t) = 0 \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \phi(x) \right) p_5(x, t) = 0 \tag{6}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu_0(x) \right) p_6(x, t) = 0 \tag{7}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \mu_0(z) \right) p_7(z, t) = 0 \tag{8}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial z} + \lambda_3 + \phi(z) \right) p_8(z, t) = 0 \tag{9}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + 2\lambda_1 + \phi(x) \right) p_9(x, t) = 0 \tag{10}$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \lambda_1 + \phi(x) \right) p_{10}(x, t) = 0 \tag{11}$$

Boundary conditions: During the operational mode the repair facility is not available than the relation of two consecutive state transition probabilities can be obtain with help of boundary conditions. i.e. $p_{j+1}(0, t) = \sum_j \lambda_j p_j(0, t)$ where j represents the any state. From state transition diagram one can easily have the following relations;

$$p_1(0, t) = 3\lambda_1 p_0(t), p_2(0, t) = 2\lambda_3 p_0(t),$$

$$p_3(0, t) = \lambda_2 \left(p_0(t) + p_1(0, t) + p_2(o, t) \right)$$

$$p_4(0, t) = \lambda_4 \left(p_0(t) + p_1(0, t) + p_2(0, t) \right), p_5(0, t) = 2\lambda_1 p_1(0, t),$$

$$p_6(0, t) = \lambda_1 \left(p_5(0, t) + p_{10}(0, t) \right), p_7(0, t)$$

$$= \lambda_3 \left(p_2(0, t) + p_8(0, t) \right), p_8(0, t) = 2\lambda_3 p_1(0, t), p_9(0, t)$$

$$= 3\lambda_1 p_2(0, t), p_{10}(0, t) = 2\lambda_1 p_9(0, t) \tag{11}$$

Initials Condition

$p_0(0) = 1$, Other state transition probabilities are zero at $t = 0$.   (12)

2.6 Solution of the Model

Taking Laplace transformations of the equations (1) to (11) with help of equation (12) one obtains

$$(s + 3\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4)\,\overline{p}(s)$$

$$= 1 + \int_0^\infty \phi(x)\overline{p}_1(x, s) + \int_0^\infty \phi(x)\overline{p}_2(z, s)dz$$

$$+ \int_0^\infty \mu_0(x)\overline{p}_6(x, s)dx + \int_0^\infty \mu_0(y)\overline{p}_3(y, s)dy$$

$$+ \int_0^\infty \mu_0(z)\overline{p}_7(z, s)dz + \int_0^\infty \mu_0(m)\overline{p}_4(m, s)dm \qquad (13)$$

$$\left(s + \frac{\partial}{\partial x} + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4 + \phi(x)\right)\overline{p}_1(x, s) = 0 \qquad (14)$$

$$\left(s + \frac{\partial}{\partial z} + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \phi(z)\right)\overline{p}_2(z, s) = 0 \qquad (15)$$

$$\left(s + \frac{\partial}{\partial y} + \mu_0(y)\right)\overline{p}_3(y, s) = 0 \qquad (16)$$

$$\left(s + \frac{\partial}{\partial m} + \mu_0(m)\right)\overline{p}_4(m, s) = 0 \qquad (17)$$

$$\left(s + \frac{\partial}{\partial x} + \lambda_1 + \phi(x)\right)\overline{p}_5(x, y) = 0 \qquad (18)$$

$$\left(s + \frac{\partial}{\partial x} + \mu_0(x)\right)\overline{p}_6(x, s) = 0 \qquad (19)$$

$$\left(s + \frac{\partial}{\partial z} + \mu_0(z)\right)\overline{p}_7(z, s) = 0 \qquad (20)$$

$$\left(s + \frac{\partial}{\partial z} + \lambda_3 + \phi(z)\right)\overline{p}_8(z, s) = 0 \qquad (21)$$

$$\left(s + \frac{\partial}{\partial x} + 2\lambda_1 + \phi(x)\right)\overline{p}_9(x, s) = 0 \tag{22}$$

$$\left(s + \frac{\partial}{\partial x} + \lambda_1 + \phi(x)\right)\overline{p}_{10}(x, s) = 0 \tag{23}$$

Laplace transform of boundary conditions

$$\overline{p}_1(0, s) = 3\lambda_1\overline{p}_0(s), \overline{p}_2(0, s) = 2\lambda_3\overline{p}_0(s), \overline{p}_3(0, s)$$
$$= \lambda_2\left(\overline{p}_0(s) + \overline{p}_1(0, s) + \overline{p}_2(0, s)\right)$$
$$\overline{p}_4(0, s) = \lambda_4\left(\overline{p}_0(s) + \overline{p}_1(0, s) + \overline{p}_2(0, s)\right), \overline{p}_5(0, s) = 2\lambda_1\overline{p}_1(0, s),$$
$$\overline{p}_6(0, s) = \lambda_1\left(\overline{p}_5(0, s) + \overline{p}_{10}(0, s)\right), \overline{p}_7(0, s) = \lambda_3\left(\overline{p}_2(0, s) + \overline{p}_8(0, s)\right),$$
$$\overline{p}_8(0, s) = 2\lambda_3\overline{p}_1(0, s), \overline{p}_9(0, s) = 3\lambda_1\overline{p}_2(0, s), \overline{p}_{10}(0, s) = 2\lambda_1\overline{p}_9(0, s) \tag{24}$$

Solving Equations (13) to (23) with the help of Equation (24), one may get,

$$\overline{p}_0(s) = \frac{1}{D(s)} \tag{25}$$

$$\overline{p}_1(s) = \frac{3\lambda_1}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right)}{s + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4}\right] \tag{26}$$

$$\overline{p}_2(s) = \frac{2\lambda_2}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right)}{s + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}\right] \tag{27}$$

$$\overline{p}_3(s) = \frac{(\lambda_2 + 3\lambda_1\lambda_2 + 2\lambda_2\lambda_3)}{D(s)}\left[\frac{1 - \overline{S}_{\mu_0}(s)}{s}\right] \tag{28}$$

$$\overline{p}_4(s) = \frac{(\lambda_4 + 3\lambda_1\lambda_4 + 2\lambda_3\lambda_4)}{D(s)}\left[\frac{1 - \overline{S}_{\mu_0}(s)}{s}\right] \tag{29}$$

$$\overline{p}_5(s) = \frac{6\lambda_1^2}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + \lambda_1\right)}{s + \lambda_1}\right] \tag{30}$$

$$\overline{p}_6(s) = \frac{\left(6\lambda_1^3 + 12\lambda_1^3\lambda_2\right)}{D(s)}\left[\frac{1 - \overline{S}_{\mu_0}(s)}{s}\right] \tag{31}$$

$$\overline{p}_7(s) = \frac{\left(2\lambda_2\lambda_3 + 6\lambda_1\lambda_3^2\right)}{D(s)}\left[\frac{1 - \overline{S}_{\mu_0}(s)}{s}\right] \tag{32}$$

$$\overline{p}_8(s) = \frac{6\lambda_1\lambda_3}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + \lambda_3\right)}{s + \lambda_3}\right] \tag{33}$$

$$\overline{p}_9(s) = \frac{6\lambda_1\lambda_2}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + 2\lambda_1\right)}{s + 2\lambda_1}\right] \tag{34}$$

$$\overline{p}_{10}(s) = \frac{12\lambda_1^2\lambda_2}{D(s)}\left[\frac{1 - \overline{S}_\phi\left(s + \lambda_1\right)}{s + \lambda_1}\right] \tag{35}$$

Where

$$D(s) = (s + 3\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4)$$
$$- \left\{\begin{array}{l} 3\lambda_1\left(\overline{S}_\phi\left(s + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right)\right) \\ +2\lambda_2\left(\overline{S}_\phi\left(s + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right)\right) \\ +\left(\left(2\lambda_2\lambda_3 + 6\lambda_1\lambda_3^2\right) + \left(\lambda_4 + 3\lambda_1\lambda_4 + 2\lambda_3\lambda_4\right)\right)\overline{S}_{\mu_0}(s)\end{array}\right\}$$

The sum of Laplace transformations of the state transition probabilities that the system is in perfect and partially failed state (S$_0$, S$_1$, S$_2$, S$_5$, S$_8$, S$_9$, S$_{10}$) at any time are as follows:

$$\overline{p}_{up}(s) = \overline{p}_0(s) + \overline{p}_1(s) + \overline{p}_2(s) + \overline{p}_5(s) + \overline{p}_8(s) + \overline{p}_9(s) + \overline{p}_{10}(s) \tag{36}$$

$$\overline{p}_{up}(s) = \frac{1}{D(s)}\left[1 + 3\lambda_1\left\{\frac{1 - \overline{S}_\phi\left(s + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4\right)}{s + 2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4}\right\}\right.$$
$$+ 2\lambda_2\left\{\frac{1 - \overline{S}_\phi\left(s + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4\right)}{s + 3\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}\right\}$$
$$+ 6\lambda_1^2\left\{\frac{1 - \overline{S}_\phi\left(s + \lambda_1\right)}{s + \lambda_1}\right\} + 6\lambda_1\lambda_3\left\{\frac{1 - \overline{S}_\phi\left(s + \lambda_3\right)}{s + \lambda_3}\right\}$$
$$\left.+ 6\lambda_1\lambda_2\left\{\frac{1 - \overline{S}_\phi\left(s + 2\lambda_1\right)}{s + 2\lambda_1}\right\} + 12\lambda_1^2\lambda_2\left\{\frac{1 - \overline{\overline{S}}_\phi\left(s + \lambda_1\right)}{s + \lambda_1}\right\}\right] \tag{37}$$

$$\overline{p}_{down}(s) = 1 - \overline{p}_{up}(s) \tag{38}$$

3 Analytical Study of the Model as Particular Cases

3.1 Availability Analysis

When a regular repair is employed in the repairable system, the performance of the system is named as availability. Let the repair facility be available and follow below; if the repair follows two types of distribution, i.e. General distribution and copula distribution of the Gumbel-Hougaard family.

Setting, $\overline{S}_{\mu_0}(s) = \hat{S}_{exp[\,x^\theta + \{log\varphi\,(x)\}^\theta]^{1/\theta}}(s) = \frac{exp[\,x^\theta + \{log\varphi\,(x)\}^\theta]^{1/\theta}}{s + exp[\,x^\theta + \{log\varphi\,(x)\}^\theta]^{1/\theta}}$, $\hat{S}_\phi(s) = \frac{\phi}{s+\phi}$, and taking the values of different values of failure and repair rates parameters as λ_1=0.01, λ_2=0.02, $\lambda_3 = 0.03$ and $\lambda_2 = 0.04, \theta = 1, \phi = 1, x = y = z = m = 1$ in Equation (37), and then taking inverse Laplace transform, one can obtain

$$P_{up}(t) = -0.001302e^{-1.03000t} - 0.000651e^{-1.02000t}$$
$$- 0.002314e^{-1.04000t} - 0.000179e^{-1.01000t} + 0.024246e^{-2.78968t}$$
$$- 0.049629e^{-1.42766t} - 0.000142e^{-1.21861t} - 0.000141e^{-1.19470t}$$
$$- 0.006092e^{-1.12230t} + 1.036206e^{-0.02533t} \tag{39a}$$

If the repair follows general repair distribution then, i.e, $\theta = 1$, $\phi = \mu_0 = 1$

$$P_{up}(t) = -0.001333e^{-1.03000t} - 0.000668e^{-1.02000t} - 0.002361e^{-1.04000t}$$
$$- 0.037995e^{-1.44977t} - 0.000073e^{-1.21868t}$$
$$- 0.000080e^{-1.19476t} - 0.004985e^{-1.12344t} + 1.047681e^{-0.07333t}$$
$$- 0.000184e^{-1.01000t} \tag{39b}$$

For different time variable values $t = 0, 1, 2, 3, \ldots, 10$, time units, one would get different Pup(t) values employing in expressions (39a) and (39b) as shown in Table 1.

3.2 Reliability Analysis

Treating all repair rates, ϕ, μ_0, in Equation (37) to zero for certain values of failure rates as $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03$ and $\lambda_2 = 0.04$ and then taking inverse Laplace transformation, one can get the expression of reliability as

Table 1 Variation of availability with respect to time copula distribution and general distribution

Time t	Availability Copula Repair	Availability General Repair
0	1.0000	1.0000
1	0.9648	0.9614
2	0.9469	0.9015
3	0.9290	0.8399
4	0.9111	0.7810
5	0.8934	0.7260
6	0.8760	0.6747
7	0.8589	0.6270
8	0.8421	0.5826
9	0.8257	0.5414
10	0.8095	0.5032

Figure 3   Availability as a function of time.

$$R(t) = 0.18750e^{-0.19000t} + 0.428571e^{-0.12000t} + 0.750000e^{-0.23000t}$$
$$+ 0.001764e^{-0.01000t} + 0.051555e^{-0.08000t}$$
$$+ 0.007272e^{-0.02000t} - 0.943461e^{-.35000t}0.003592e^{-0.04000t}$$
$$+ 0.016875e^{-0.03000t} \tag{40}$$

Table 2 Reliability computing for the different time values (t)

Time (t)	Reliability
0	1.0000
1	0.9249
2	0.8529
3	0.7843
4	0.7196
5	0.6589
6	0.6021
7	0.5493
8	0.5005
9	0.4554
10	0.4139
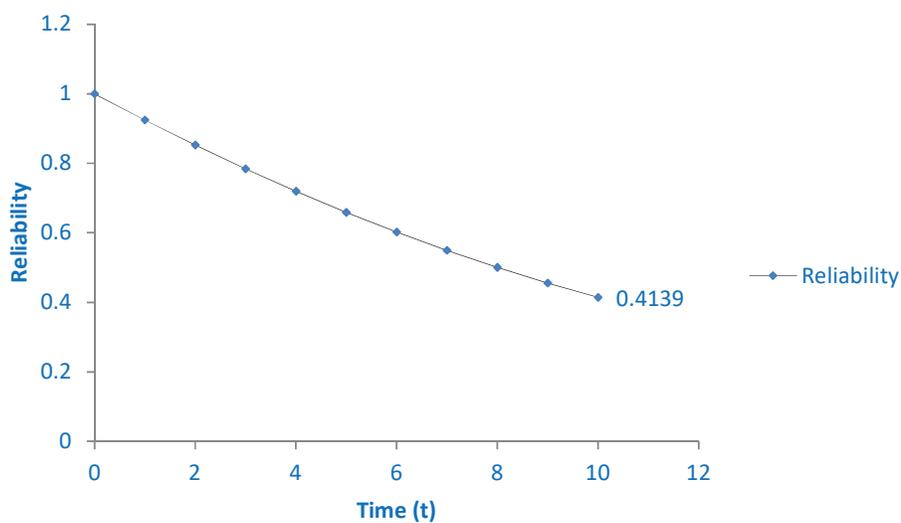
Figure 4 Reliability as a function of time (t).

3.3 Mean Time to Failure (MTTF) Analysis

The system MTTF can be calculated using the relation, MTTF $= \int_0^\infty R(t)dt = \lim_{s \to 0} R(s)$.

Table 3   Variation of MTTF corresponding to the failure rates

Failure Rates	MTTFλ_1	MTTFλ_2	MTTFλ_3	MTTFλ_4
0.01	12.1334	11.8127	20.5017	28.4050
0.02	11.8137	12.1334	16.2039	23.8600
0.03	11.5210	12.2821	13.3343	20.5017
0.04	11.2503	12.3160	11.2858	17.9309
0.05	11.0017	12.2729	9.7586	15.9063
0.06	10.7751	12.1782	8.5800	14.2748
0.07	10.5691	12.0491	7.6451	12.9347
0.08	10.3820	11.8975	6.8868	11.8160
0.09	10.2120	11.7319	6.2603	10.8691

Assigning all maintenances to zero in the equation in (57) and then $\lim_{s \to 0} R(s)$, one can obtain the expression for MTTF as;

$$MTTF = \lim_{s \to 0} \bar{p}_{up}(s) = \frac{1}{3\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4}$$

$$\times \left\{ 1 + \frac{3\lambda_1}{2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4} \right.$$

$$\left. + \frac{3\lambda_1}{2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4} \frac{3\lambda_1}{2\lambda_1 + \lambda_2 + 2\lambda_3 + \lambda_4} \right\} \quad (41)$$

Setting $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_3 = 0.03$ and $\lambda_2 = 0.04$ and varying λ_1, λ_2, λ_3, and λ_4 one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in Equation (41), one may obtain the variation of MTTF with respect to failure rates as shown in Table 3.

3.4 Sensitivity Analysis

The failure effect is a natural phenomenon in the system, and MTTF is a vital reliability measure to observe which component is profoundly affected during a repairable system's operations. Hence the analysis of MTTF's sensitivity is vital in controlling the impact of failure. The sensitivity analysis of MTTF is calculated by the relative variation in MTTF regarding the system's failure rates through the MTTF's partial derivative. Fixing the set of parameters as $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_3 = 0.03$ and $\lambda_2 = 0.04$ constant and the partial
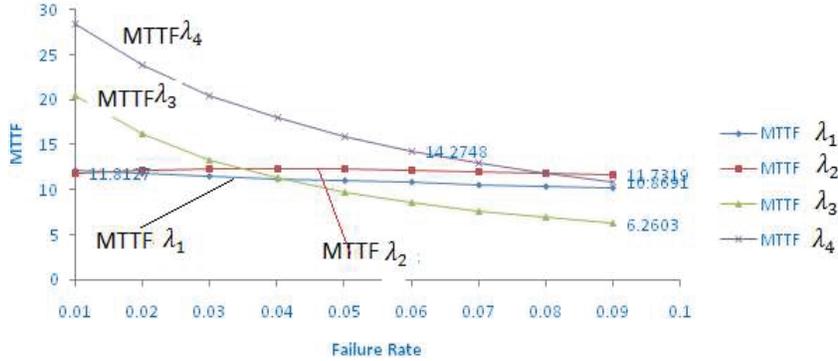
Figure 5   MTTF against failure rates.

Table 4   Sensitivity MTTF in respect of failure rates

Failure Rate	$\frac{\partial(MTTF)}{\lambda_1}$	$\frac{\partial(MTTF)}{\lambda_2}$	$\frac{\partial(MTTF)}{\lambda_3}$	$\frac{\partial(MTTF)}{\lambda_4}$
0.01	−34.1757	43.2929	−533.4923	−533.6339
0.02	−30.3996	22.2987	−344.8122	−386.6388
0.03	−28.1703	8.3679	−239.4623	−291.4375
0.04	−25.9668	−0.9637	−175.1006	−226.6446
0.05	−23.7433	−7.2343	−133.1130	−180.7623
0.06	−21.6052	−11.4298	−104.3147	−147.1998
0.07	−19.6224	−14.1986	−83.7660	−121.9766
0.08	−17.8232	−15.9748	−68.6264	−102.5828
0.09	−16.2106	−17.0547	−57.1723	−87.3767

differentiation of MTTF, one can compute the MTTF sensitivity as shown in Table 4 below and depicted below in Figure 6.

3.5 Cost Analysis with Copula Distribution

If the service facility is always available, then the expected profit can be computed from the formula

$$E_p(t) = K_1 \int_0^t P_{up}(t) - K_2 t$$

The constants (K1 & K2) represent revenue generation and service cost per unit item, respectively, the interval [0, t].

$$E_p(t) = K_1 \{ 0.00126 e^{-0.300t} + 0.00063 e^{-0.0200t} + 0.00222 e^{-1.0400t}$$
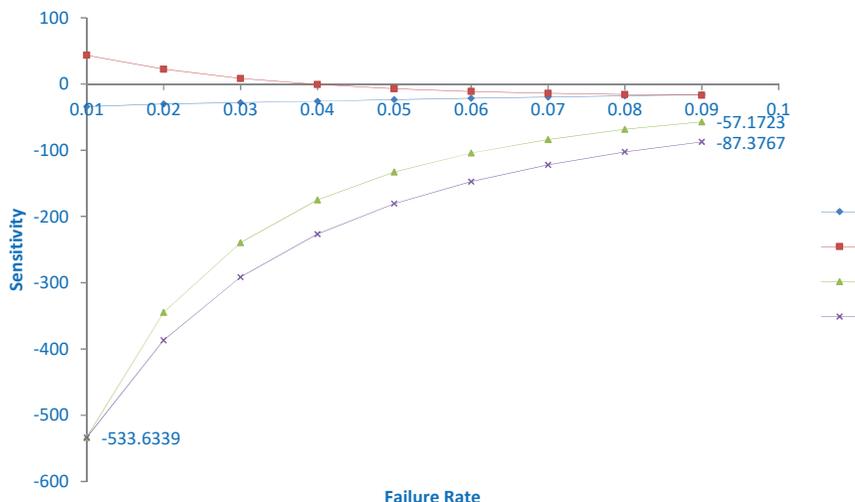
Figure 6 MTTF sensitivity against a failure rate.

$$
\begin{aligned}
&+ 0.00017e^{-0.0100t} - 0.00869e^{-2.7897t} + 0.03476e^{-1.4277t} \\
&+ 0.00011e^{-1.2187t} + 0.00011e^{-1.1948t} + 0.00542e^{-1.1223t} \\
&- 40.90479e^{-0.0253t} + 0.00542e^{-1.1223t} \\
&- 40.90479e^{-0.0253t} + 40.686\} - K_2(t)
\end{aligned} \tag{42}
$$

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 respectively and varying $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$. Units of time, the results for expected profit can be obtain as shown in Table 5.

3.6 Cost Analysis with General Distribution

The predicted gain over interval $[0, t]$ is calculated by using the formula for service facilities K1 and K2 for the cost of the service per unit time and over interval $[0, t)$, $E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t$.

For same values of failure rates in Equation (37), one can obtain Equation (43) as

$$
\begin{aligned}
E_p(t) = K_1\{&0.00129e^{-0.300t} + 0.00065e^{-0.0200t} + 0.00227e^{-1.0400t} \\
&+ 0.0262e^{-1.4497t} + 0.00006e^{-1.2186t} + 0.00006e^{-1.1947t} \\
&+ 0.00443e^{-1.1234t} - 14.28652e^{-0.0733t} + 0.00018e^{-1.0100t} \\
&+ 14.251\} - K_2(t)
\end{aligned} \tag{43}
$$

Table 5 Expected profit in $[0, t)t = 0, 1, 2 \ldots 10$. When the repair follows copula distribution

Time(t)	$E_p(t)$ K = 0.6	$E_p(t)$ K = 0.5	$E_p(t)$ K = 0.4	$E_p(t)$ K = 0.3	$E_p(t)$ K = 0.2	$E_p(t)$ K = 0.1
0	0	0	0	0	0	0
1	0.3781	0.4781	0.5781	0.6781	0.7781	0.8781
2	0.7339	0.9339	1.1339	1.3339	1.5339	1.7339
3	1.0719	1.3719	1.6719	1.9719	2.2719	2.5719
4	1.3920	1.7920	2.1920	2.5920	2.9920	3.3920
5	1.6943	2.1943	2.6943	3.1943	3.6943	4.1943
6	1.9790	2.5790	3.1790	3.7790	4.3790	4.9790
7	2.2464	2.9464	3.6464	4.3464	5.0464	5.7464
8	2.4969	3.2969	4.0969	4.8969	5.6969	6.4969
9	2.7308	3.6308	4.5308	5.4308	6.3308	7.2308
10	2.9484	3.9484	4.9484	5.9484	6.9484	7.9484
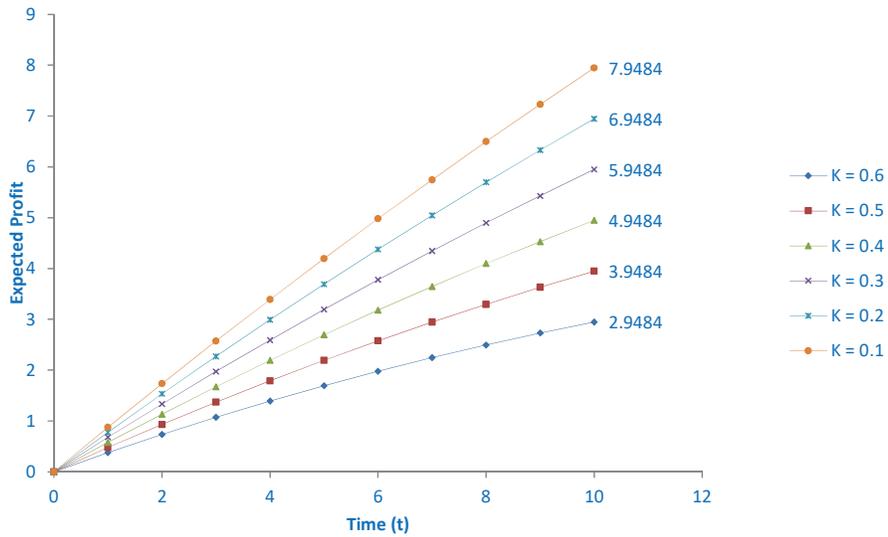
Figure 7   Time vs. expected profit with different service cost when the repair follows copula distribution.

Table 6 Expected profit in $[0, t)$ $t = 0, 1, 2 \ldots 10$. When the repair follows the general distribution

Time(t)	$E_p(t)$ $K_2 = 0.6$	$E_p(t)$ $K_2 = 0.5$	$E_p(t)$ $K_2 = 0.4$	$E_p(t)$ $K_2 = 0.3$	$E_p(t)$ $K_2 = 0.2$	$E_p(t)$ $K_2 = 0.1$
0	0	0	0	0	0	0
1	0.3838	0.4838	0.5838	0.6838	0.7838	0.8838
2	0.7158	0.9158	1.1158	1.3158	1.5158	1.7158
3	0.9864	1.2864	1.5864	1.8864	2.1864	2.4864
4	1.1966	1.5966	1.9966	2.3966	2.7966	3.1966
5	1.3498	1.8498	2.3998	2.8498	3.3498	3.8498
6	1.4499	2.0499	2.6499	3.2499	3.8499	4.4499
7	1.5005	2.2005	2.9005	3.6005	4.3005	5.0005
8	1.5051	2.3051	3.1051	3.9051	4.7051	5.5051
9	1.4669	2.3669	3.2669	4.1669	5.0669	5.9669
10	1.3890	2.3890	3.3890	4.3890	5.3890	6.3890
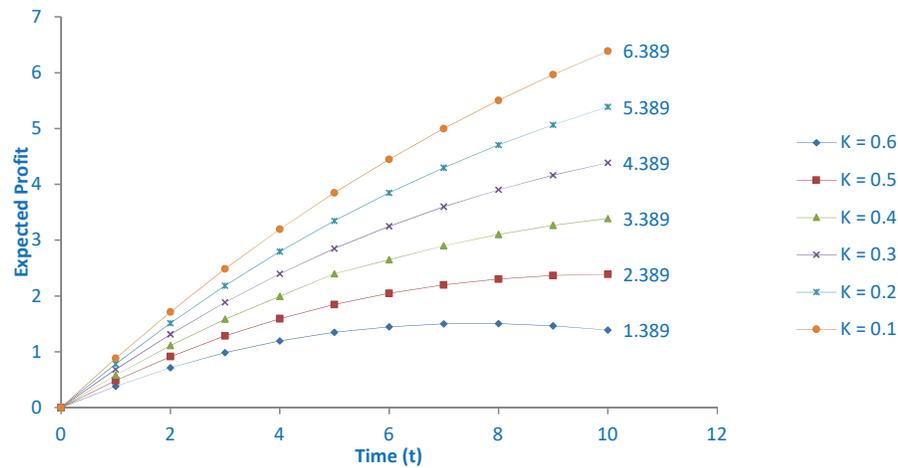
Figure 8 Time vs. expected profit with different service cost when the repair follows the general distribution.

Setting $K_1 = 1$ and $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2$ and 0.1 and varying t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 respectively. The results for the anticipated benefit can be obtained units of time as shown in Table 6.

4 Conclusion

Table 1 and Figure 3 provide details on how the system's availability varies when the failure rates are set at various values. When failure rates, $\lambda_1 = 0.01$, $\lambda_2 = 0.02$, $\lambda_3 = 0.03$, $\lambda_4 = 0.04$, are fixed at lower values, the system availability decreases gradually with varying t, and it becomes steady to the value zero after a sufficiently long interval of time. Table 1 and the correspondent Figure 3 result reveals the system availability when the repair follows the repair of the copula distribution and the general repair distribution, respectively. It can be seen from Table 1, and Figure 3 that repair through copula distribution shows better performance over the general repair. Long-term, system availability values are lower when the repair follows the general distribution. Conclusively, copula repair is a more efficient repair policy for improving repairable device performance. Table 2 and Figure 4 provide information on system reliability for the different values of system failure rates. Table 3 and Figure 5 yield the MTTF of the system with reverence to variation in λ_1, λ_2, λ_3, and λ_4, respectively, when other parameters are kept constant. Consequently, for any given set of parametric values, a complex system's future behaviors can be predicted at any time, as is evident from the model's graphical consideration. The variation in MTTF corresponding to various failure rates shows that incremental change in parameter values decreases the system MTTF. Curiously, the MTTF of the system smoothly decreases with each failure rate in this case. It is important to note that, with each failure rate, the system's MTTF decreases effortlessly in this predicament. Figure 4 shows that MTTF compares to λ 1 and λ 2 failure rates, so λ 3 and λ 4 are similar. However, MTTF corresponds to λ_3 and λ_4 are almost the same λ_1 and λ_2. Therefore, λ_1 and λ_2 are more responsible for the better operation of the system. The variations of sensitivity to change in λ_2, λ_3, and $\lambda4$ can be seen in Table 4 and Figure 6. From Table 5 and Figures 7, it is critically observed that MTTF sensitivity increases with a rise in failure rate values. And also, MTTF is impervious to failure stages. If sales cost per unit time K_1 is set at 1, service cost $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$, profit is estimated, and the results are shown in graphs. A massively critical analysis from Table 6 and Figure 8 shows that the estimated profit increases when the minimum value of the service cost K_1 are provided. The table's

approximately expected profit shows that the maximum is 0.1 for $K_2 =$ and the minimum is 0.6 for K_2. Eventually, it is found that profit rises as service costs reduce. In general, the expected profit for low service costs is high compared to the high cost of service. Table 6 and corresponding Figure 8 present the system's expected profit when the repair follows the general distribution. However, the expected profit of the system is less when the repair follows the general distribution. It shows that copula is a more efficient repair policy for better repairable systems efficiency. A network infrastructure with the highest reliability measures should achieve high quality, reliability, and higher production performance and expected benefit.

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