
Availability Analysis of Hybrid Systems Consisting of Main Units and Cold Standby Processors

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Received 01 September 2020; Accepted 08 December 2020;
Publication 05 January 2021

Abstract

The present paper studies availability of four hybrid systems configured as series-parallel systems. Each system or configuration consisting of main units and their corresponding processors. Configuration I consist of three processors is a 2-out-of-3 unit connected to 2-out-of-3 processors, Configuration II is a 2-out-of-3 unit connected to 2-out-of-4 processors, Configuration III is a 2-out-of-4 unit connected to 2-out-of-4 processors while Configuration IV is a 2-out-of-4 unit connected to 2-out-of-3 processors. The failure and repair times of units and their processors are assumed to be exponentially distributed. Explicit expressions for steady state availability are developed for each system using first order linear differential difference equations and validated by performing numerical experiments. Analysis of the effect of various system parameters on availability was performed. Graphical illustrations are given to highlight important results. The systems are ranked based

Journal of Reliability and Statistical Studies, Vol. 13, Issues 2–4 (2020), 429–460.

doi: 10.13052/jrss0974-8024.132411

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on their availability and found that Configuration IV is better. Sensitivity analysis on the model's outcomes are performed using partial rank correlation coefficients (PRCC) to determine the most critical parameters leading to increase (decrease) in the value of availability.

Keywords: Availability, processor, configuration, main unit, series parallel.

1 Introduction

Many engineering, industrial or manufacturing systems are hybrid in nature, consisting of components, units or subsystems arranged as parallel, series, series-parallel, parallel-series or k-out-of-n system. These manufacturing, industrial and engineering systems are hybrid in nature consisting of build-in similar and dissimilar units, external unit, etc. Such system can only work with the help of external processors/units/devices linked to the system. Examples of such systems can be found in computer systems, power plants, manufacturing systems and industrial systems such as feeding, crushing, refining, steam generation, evaporation, crystallization, fertilizer plant, crystallization unit of a sugar plant, piston manufacturing plant, etc. Failure of the entire units or external processors/units/devices leads to the failure of the system called complete failure and the system operation must cease whereas the failure of a unit or an external processor/units/devices with standby in the configuration is called partial failure. Due to their importance in production output, customer's satisfaction as well as revenue mobilisation, reliability and availability analysis of such systems is of paramount importance. Reliability and availability of systems are improved through redundancy. There are different forms of redundancy used in improving the system reliability leading to system safety, quality of the product, adequate production output, less maintenance cost, high revenue mobilisation, customer's satisfaction, etc. Such forms of redundancy can be cold, hot, warm or k-out-of-n system.

One of the forms of redundancy is the 2-out-of-3 system which has wide application in industrial setting. Moreover, the k-out-of-n works if and only if at least k of the n components work. Example of the 2-out-of-3 system can be seen in a communication system with three transmitters and the average message load may be such that at least two transmitters must be operational at all times or critical messages may be lost. Thus, the transmission subsystem functions as a 2-out-of-3: G system. Due to their importance in industries and design, the 2-out-of-3 systems have received attention from different

researchers. Bhardwaj and Chander [2] propose method of analysing reliability and cost benefit analysis of 2-out-of-3 redundant system with general distribution of repair and waiting time. Bhardwaj and Malik [3] studied MTSF and cost effectiveness of 2-out-of-3 standby system with probability of repair and inspection. Chander and Bhardwaj [4] discussed the reliability and economic analysis of 2-out-of-3 redundant system with priority to repair. Chander and Bhardwaj [5] discussed the reliability and economic analysis of 2-out-of-3 redundant system with priority and replacement. Chander and Bhardwaj [6] analyzed reliability modelling and analysis of 2-out-of-3 redundant system subject to conditional arrival time of server. Khatab et al. [11] analyzed the availability of of k-out-of-n: G systems with non-identical components subject to repair priorities. Li et al. [13] presented reliability analysis of a repairable k-out-of-n system with some components being suspended when the system is down. Yusuf and Hussaini [16] deal with evaluation of reliability and availability characteristics of 2-out-3 standby system under a perfect repair condition.

Many researchers have presented their works in the field of reliability and availability analysis of engineering, industrial and manufacturing system by evaluating the reliability characteristics of the systems under the different types of failure, preventive maintenance, repairs and replacement actions. To cite few, Aliyu et al. [1] discussed the availability and profit optimization of series-parallel system with linear consecutive cold standby unit. Garg [7] proposed methods of PSO and fuzzy to analyzed the reliability, availability and maintainability analysis of industrial systems. Garg [8] discussed an approach for analyzing the reliability of series-parallel system using credibility theory and different types of intuitionistic fuzzy numbers. Garg [9] presented an approach for analyzing the reliability of industrial system using fuzzy kolmogrov's differential equations. Niwas and Garg [14] dealt with an approach for analyzing the reliability and profit of an industrial system based on the cost-free warranty policy. Garg [10] focus on the performance of an industrial system using soft computing based hybridized technique Kumar et al. [12] dealt with assessment of reliability of a system by applying hesitant fuzzy set. Singh and Rawal [15] presented study on availability, MTTF, and cost analysis of the complex system under preemptive resume repair policy using copula distribution. Yusuf et al. [17] discussed the reliability Analysis of a linear consecutive 2-out-of-3 System in the presence of supporting device and repairable service station. Yusuf [18] dealt with reliability modeling of a parallel system with a supporting device and two types of preventive maintenance.

Although literature above and the existing ones have carried out on reliability analysis for engineering, industrial and manufacturing systems. Little is known on reliability and availability of such system working with the aid of an external processor supporting the working of the entire system. The present paper tends to develop an availability models of different configuration connected to external processor for operation to addresses the gap. The present paper studies availability of four hybrid systems configured as series-parallel systems in which each system or configuration consisting of main units and their corresponding processors. The systems or configurations have been studied by using the first order linear differential difference equations to derive the corresponding availability models for each configuration. Comparative analysis of the study among configurations been carried out to determine the optimal configuration. Surface plots and partial rank correlation have also been computed in the study. The organization of the paper is as follows. Section 2 contains a description of the configurations under study. Section 3 presents formulations of the models. The results of our numerical simulations are presented and discussed in Section 4. Finally, we make some concluding remarks in Section 5.

2 Description of the Configurations

Figure 1 depict Configuration I consisting of three processors is a 2-out-of-3 unit connected to 2-out-of-3 processors, Figure 2 displayed Configuration II having 2-out-of-3 unit connected to 2-out-of-4 processors, Figure 3 depict Configuration III consisting of 2-out-of-4 unit connected to 2-out-of-4 processors while Figure 4 is Configuration IV consisting of 2-out-of-4 unit connected to 2-out-of-3 processors. Each configuration composed of two subsystems. Subsystem I of each configuration consist of the identical cold standby processors. Subsystem II each configuration consists of identical cold standby main units. The processors in each subsystem received the signal, process it and forward to the main unit for usage. Each of the main unit and processor fails with exponential failure distribution with parameter β_1 and β_2 , exponential repair distribution with parameter α_1 and α_2 respectively. When one of the main unit (processor) failed, which occurs with failure rate $\beta_1(\beta_2)$, it is repaired with the rate $\alpha_1(\alpha_2)$ and the corresponding standby main unit (processor) then carries out the function of the failed main unit (processor). It is assumed that switching from standby to operation is perfect. System failure results from the failure of subsystem I or subsystem II or both.

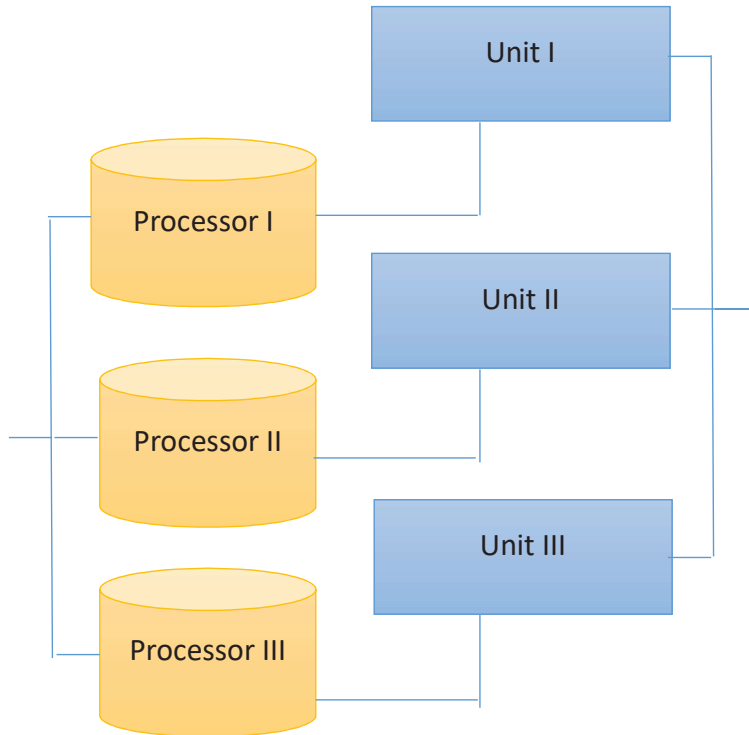


Figure 1 Reliability block diagram of configuration I.

3 Availability Models Formulation

3.1 Availability Formulation of Configuration I

Define $p_i(t)$ to be the probability that the system is in state i at time t . Thus, $p_0(t)$, $p_1(t)$, $p_2(t)$, $p_3(t)$, $p_4(t)$, $p_5(t)$ and $p_6(t)$ are probabilities that the system is in state S_0 , S_1 , S_2 , S_3 , S_4 , S_5 and S_6 . Let $P(t) = [p_1(t), p_2(t), \dots, p_{11}(t)]$ be the probability row vector with initial conditions

$$p_k(0) = \begin{cases} 1, & k = 0 \\ 0, & k = 1, 2, 3, 4, 5, 6 \end{cases}$$

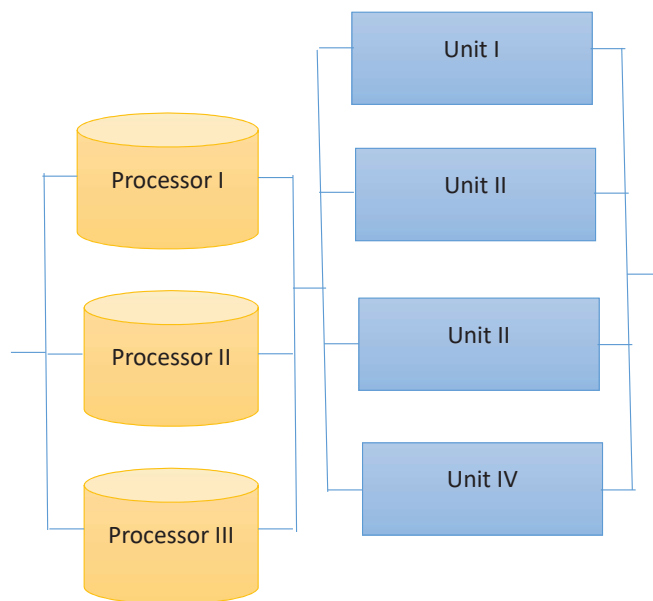


Figure 2 Reliability block diagram of configuration II.

The corresponding set of differential-difference-equations for configuration I are

$$\left. \begin{aligned}
 \frac{dp_0(t)}{dt} &= -(2\beta_1 + 2\beta_2) p_0(t) + \alpha_1 p_1(t) + \alpha_2 p_2(t) \\
 \frac{dp_1(t)}{dt} &= -(2\beta_1 + 2\beta_2 + \alpha_1) p_1(t) + 2\beta_1 p_0(t) + \alpha_1 p_3(t) + \alpha_2 p_5(t) \\
 \frac{dp_2(t)}{dt} &= -(2\beta_1 + 2\beta_2 + \alpha_2) p_2(t) + 2\beta_2 p_0(t) + \alpha_2 p_4(t) + \alpha_1 p_6(t) \\
 \frac{dp_3(t)}{dt} &= -\alpha_1 p_3(t) + 2\beta_1 p_1(t) \\
 \frac{dp_4(t)}{dt} &= -\alpha_2 p_4(t) + 2\beta_2 p_2(t) \\
 \frac{dp_5(t)}{dt} &= -\alpha_2 p_5(t) + 2\beta_2 p_1(t) \\
 \frac{dp_6(t)}{dt} &= -\alpha_1 p_6(t) + 2\beta_1 p_2(t)
 \end{aligned} \right\} (1)$$

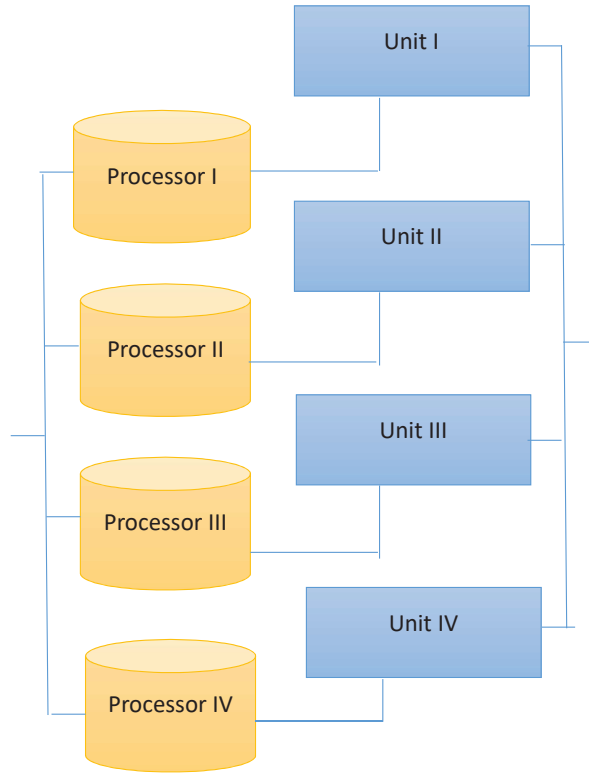


Figure 3 Reliability block diagram of configuration III.

To compute the availability of configuration I, the differential difference equation given in (1) are expressed in the matrix format

$$\begin{pmatrix} p'_0 \\ p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \end{pmatrix} = \begin{pmatrix} -(2\beta_1 + 2\beta_2) & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -(2\beta_1 & 0 & \alpha_1 & 0 & \alpha_2 & 0 \\ & +2\beta_2 & & & & & \\ & +\alpha_1) & & & & & \\ 2\beta_2 & 0 & -(2\beta_1 & 0 & \alpha_2 & 0 & \alpha_1 \\ & & +2\beta_2 & & & & \\ & & +\alpha_2) & & & & \\ 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 \\ 0 & 0 & 2\beta_1 & 0 & 0 & 0 & -\alpha_1 \end{pmatrix} \begin{pmatrix} p_0(t) \\ p_1(t) \\ p_2(t) \\ p_3(t) \\ p_4(t) \\ p_5(t) \\ p_6(t) \end{pmatrix} \tag{2}$$

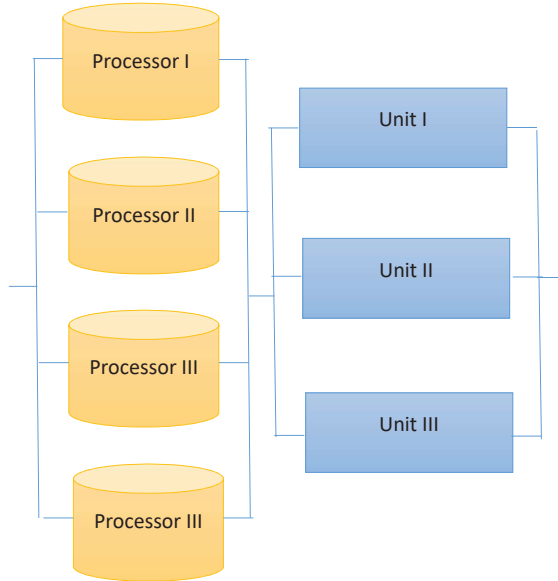


Figure 4 Reliability block diagram of configuration IV.

Setting left side of (2) to zero in steady state to give:

$$\begin{pmatrix}
 -(2\beta_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\
 +2\beta_2) & & & & & & \\
 2\beta_1 & -(2\beta_1 & 0 & \alpha_1 & 0 & \alpha_2 & 0 \\
 +2\beta_2 & & & & & & \\
 +\alpha_1) & & & & & & \\
 2\beta_2 & 0 & -(2\beta_1 & 0 & \alpha_2 & 0 & \alpha_1 \\
 & & +2\beta_2 & & & & \\
 & & +\alpha_2) & & & & \\
 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\
 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\
 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 \\
 0 & 0 & 2\beta_1 & 0 & 0 & 0 & -\alpha_1)
 \end{pmatrix}
 \begin{pmatrix}
 p_0(t) \\
 p_1(t) \\
 p_2(t) \\
 p_3(t) \\
 p_4(t) \\
 p_5(t) \\
 p_6(t)
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}
 \tag{3}$$

Combining (3) and normalizing condition

$$\sum_{n=0}^6 p_n(\infty) = 1 \tag{4}$$

to give

$$\begin{pmatrix}
 -(2\beta_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\
 +2\beta_2) & & & & & & \\
 2\beta_1 & -(2\beta_1 & 0 & \alpha_1 & 0 & \alpha_2 & 0 \\
 +2\beta_2 & & & & & & \\
 +\alpha_1) & & & & & & \\
 2\beta_2 & 0 & -(2\beta_1 & 0 & \alpha_2 & 0 & \alpha_1 \\
 +2\beta_2 & & & & & & \\
 +\alpha_2) & & & & & & \\
 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\
 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\
 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 \\
 1 & 1 & 1 & 1 & 1 & 1 & 1
 \end{pmatrix}
 \begin{pmatrix}
 p_0(t) \\
 p_1(t) \\
 p_2(t) \\
 p_3(t) \\
 p_4(t) \\
 p_5(t) \\
 p_6(t)
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 1
 \end{pmatrix}
 \tag{5}$$

To obtain the states probabilities $p_k(t)$, (5) is solved using MATLAB package.

The steady-state availability for Configuration I is given by

$$\begin{aligned}
 A_{V1}(\infty) &= p_0(\infty) + p_1(\infty) + p_2(\infty) \\
 &= \frac{\alpha_1^2 \alpha_2^2 + 2\alpha_1 \alpha_2^2 \beta_1 + 2\alpha_1^2 \alpha_2 \beta_2}{4\alpha_2^2 \beta_1^2 + 2\alpha_1 \alpha_2^2 \beta_1 + 2\alpha_1^2 \alpha_2 \beta_2 + \alpha_1^2 \alpha_2^2 + 8\alpha_1 \alpha_2 \beta_1 \beta_2 + 4\alpha_1^2 \beta_2^2}
 \end{aligned}
 \tag{6}$$

3.2 Availability Formulation of Configuration II

To compute the availability of configuration II, the differential difference equation is expressed in the form

$$\begin{pmatrix}
 p'_0 \\
 p'_1 \\
 p'_2 \\
 p'_3 \\
 p'_4 \\
 p'_5 \\
 p'_6 \\
 p'_7 \\
 p'_8 \\
 p'_9 \\
 p'_{10}
 \end{pmatrix}$$

$$= \begin{pmatrix} -h_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -h_2 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -h_2 & \alpha_2 & 0 & 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_2 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 2\beta_1 & 0 & 0 & 0 & -h_4 & \alpha_1 & \alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_1 & 0 & 2\beta_2 & 0 & -h_4 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \end{pmatrix} \tag{7}$$

Where $h_1 = (2\beta_1 + 2\beta_2)$, $h_2 = (2\beta_1 + 2\beta_2 + \alpha_1)$, $h_3 = (2\beta_1 + 2\beta_2 + \alpha_2)$, $h_4 = (2\beta_1 + 2\beta_2 + \alpha_1 + \alpha_2)$.

From similar procedure described above, left hand side of (7) is set to zero in steady state and becomes

$$\begin{pmatrix} -h_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -h_2 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -h_2 & \alpha_2 & 0 & 0 & \alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_2 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 2\beta_1 & 0 & 0 & 0 & -h_4 & \alpha_1 & \alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_1 & 0 & 2\beta_2 & 0 & -h_4 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 \end{pmatrix}$$

Solving (10) using MATLAB package to obtained $p_i(t)$, the explicit expressions for the the steady state availability for Configuration II is given by

$$\begin{aligned}
A_{V2}(\infty) &= p_0(\infty) + p_1(\infty) + p_2(\infty) + p_4(\infty) + p_6(\infty) + p_8(\infty) \\
&= \frac{N_1 + N_2 + N_3 + N_4 + N_5}{D_1 + D_2 + D_3 + D_4 + D_5 + D_6 + D_7} \quad (11) \\
N_1 &= \alpha_1^2 \alpha_2^3 (\alpha_1^2 + \alpha_1 \alpha_2 + 4\alpha_1 \beta_2 + 3\alpha_1 \beta_1 - \alpha_2 \beta_2 + \alpha_2 \beta_1 \\
&\quad + 2\beta_1^2 + 2\beta_2^2 + 4\beta_1 \beta_2) + 2\alpha_1 \alpha_2^3 (\alpha_1^2 \beta_1 + \alpha_1 \alpha_2 \beta_1 + 3\alpha_1 \beta_1^2 \\
N_2 &= 4\alpha_1 \beta_1 \beta_2 + \alpha_1 \beta_2^2 + \alpha_2 \beta_1^2 - \alpha_2 \beta_2^2 - \alpha_2 \beta_1 \beta_2 + 2\beta_1 \beta_2^2 \\
&\quad + 4\beta_1^2 \beta_2 + 2\beta_1^3) + 2\alpha_1^2 \alpha_2^2 \beta_2 (\alpha_1^2 + \alpha_1 \alpha_2 + 3\alpha_1 \beta_2 + 3\alpha_1 \beta_1 \\
N_3 &= \alpha_2 \beta_1 + 2\beta_2^2 + 4\beta_1 \beta_2 + 2\beta_1^2) + 2\alpha_1^2 \alpha_2 \beta_2^2 (\alpha_1^2 + 2\alpha_1 \alpha_2 \\
&\quad + 4\alpha_1 \beta_2 + 4\alpha_1 \beta_1 + \alpha_2^2 + 2\alpha_2 \beta_2 + 4\alpha_2 \beta_1 + 8\beta_1 \beta_2 + 4\beta_1^2 \\
&\quad + 4\beta_2^2) \\
N_4 &= 2\alpha_1 \alpha_2^2 \beta_2 (\alpha_1^2 \beta_2 + 2\alpha_1^2 \beta_1 - \alpha_1 \alpha_2 \beta_2 + 2\alpha_1 \alpha_2 \beta_1 + 2\alpha_1 \beta_2^2 \\
&\quad + 6\alpha_1 \beta_1^2 + 8\alpha_1 \beta_1 \beta_2 + 2\alpha_2 \beta_1^2 - 2\alpha_2 \beta_2^2 - 2\alpha_2 \beta_1 \beta_2 \\
N_5 &= 8\beta_1^2 \beta_2 + 4\beta_1 \beta_2^2 + 4\beta_1^3) + 2\alpha_1 \alpha_2 \beta_2^2 (\alpha_1^3 + 4\alpha_1^2 \beta_2 \\
&\quad + 6\alpha_1^2 \beta_1 - \alpha_1 \alpha_2^2 + 2\alpha_1 \alpha_2 \beta_1 - 4\alpha_1 \alpha_2 \beta_2 + 12\alpha_1 \beta_1^2 + 4\alpha_1 \beta_2^2 \\
&\quad + 16\alpha_1 \beta_1 \beta_2 - 4\alpha_2 \beta_1 \beta_2 - 4\alpha_2 \beta_2^2 + 4\alpha_2 \beta_1^2 + 8\beta_1 \beta_2^2 \\
&\quad + 8\beta_1^3 + 16\beta_1^2 \beta_2) \\
D_1 &= -4\alpha_2^4 \beta_1^2 \beta_2 + 32\alpha_1 \alpha_2^2 \beta_1^3 \beta_2 - 8\alpha_1 \alpha_2^2 \beta_1 \beta_2^3 + 4\alpha_2^4 \beta_1^3 \\
&\quad + 32\alpha_2 \beta_1^2 \beta_2^4 + 24\alpha_2^3 \beta_1^3 \beta_2 + 2\alpha_1^5 \beta_2^2 + 64\alpha_2 \beta_1^3 \beta_2^3 + 8\alpha_2^3 \beta_1^4 \\
D_2 &= 8\alpha_1^3 \beta_1^2 \beta_2^2 + 8\alpha_1^4 \beta_1 \beta_2^2 - 16\alpha_1 \alpha_2 \beta_1^5 - 4\alpha_1 \alpha_2^3 \beta_2^3 \\
&\quad - 2\alpha_1 \alpha_2^4 \beta_2^2 + 32\alpha_1 \beta_1^3 \beta_2^3 - 16\alpha_2^2 \beta_1 \beta_2^4 + 32\alpha_2 \beta_1^4 \beta_2^2 - 8\alpha_1 \alpha_2^2 \beta_2^4 \\
D_3 &= 6\alpha_1 \alpha_2^4 \beta_1^2 - 4\alpha_2^4 \beta_1 \beta_2^2 + 64\alpha_1 \beta_1^2 \beta_2^4 + 96\alpha_1^2 \beta_1 \beta_2^4 \\
&\quad - 8\alpha_2^3 \beta_1 \beta_2^3 + 32\alpha_1 \beta_1 \beta_2^5 + 10\alpha_1^3 \alpha_2^2 \beta_2^2 + 20\alpha_1^3 \alpha_2 \beta_2^3 + \alpha_1^4 \alpha_2^3 \\
&\quad + \alpha_1^3 \alpha_2^4 \\
D_4 &= 64\alpha_1 \alpha_2^2 \beta_1^2 \beta_2^2 + 64\alpha_1 \alpha_2 \beta_1^3 \beta_2^2 + 112\alpha_1 \alpha_2 \beta_1^2 \beta_2^3 \\
&\quad + 48\alpha_1^2 \alpha_2 \beta_1^2 \beta_2^2 + 36\alpha_1 \alpha_2^3 \beta_1^2 \beta_2 - 4\alpha_1 \alpha_2^3 \beta_1 \beta_2^2 + 18\alpha_1^2 \alpha_2^3 \beta_1 \beta_2 \\
&\quad + 32\alpha_1^2 \beta_2^5
\end{aligned}$$

$$\begin{aligned}
 D_5 &= 24\alpha_1^2\alpha_2^2\beta_1^2\beta_2 + 32\alpha_1^2\alpha_2^2\beta_1\beta_2^2 + 10\alpha_1^3\alpha_2^2\beta_1\beta_2 \\
 &\quad + 40\alpha_1^3\beta_2^4 + 64\alpha_1^2\beta_1^2\beta_2^3 + 56\alpha_1^3\beta_1\beta_2^3 + 3\alpha_1^2\alpha_2^4\beta_1 - \alpha_1^2\alpha_2^4\beta_2 \\
 &\quad + 16\alpha_1^4\beta_2^3 \\
 D_6 &= 16\alpha_1\alpha_2^3\beta_1^3 + 12\alpha_1^2\alpha_2^3\beta_1^2 + 5\alpha_1^3\alpha_2^3\beta_1 + 6\alpha_1^3\alpha_2^3\beta_2 \\
 &\quad + 72\alpha_1^2\alpha_2\beta_1\beta_2^3 + 2\alpha_1^4\alpha_2^2\beta_2 + 6\alpha_1^4\alpha_2\beta_2^2 + 16\alpha_1\alpha_2\beta_1\beta_2^4 \\
 &\quad + 16\alpha_2^2\beta_1^4\beta_2 + 48\alpha_2^2\beta_1^3\beta_2^2 + 24\alpha_1^3\alpha_2\beta_1\beta_2^2 - 2\alpha_1\alpha_2^4\beta_1\beta_2
 \end{aligned}$$

3.3 Availability Formulation of Configuration III

From similar argument in Section 3.1 above, the differential difference equation is expressed in the form

$$\begin{pmatrix} p'_0 \\ p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \\ p'_7 \\ p'_8 \\ p'_9 \\ p'_{10} \\ p'_{11} \\ p'_{12} \\ p'_{13} \\ p'_{14} \end{pmatrix} = \begin{pmatrix} -h_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -h_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -h_2 & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2\beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_3 & 0 & \alpha_1 & 0 & \alpha_2 & 0 \\ 0 & 0 & 2\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_2 & 0 & \alpha_1 & 0 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \\ p_{11}(\infty) \\ p_{12}(\infty) \\ p_{13}(\infty) \\ p_{14}(\infty) \end{pmatrix} \tag{12}$$

(12) is set to zero in steady state which is

$$\begin{pmatrix}
 -h_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 2\beta_1 & -h_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 \\
 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\
 0 & 2\beta_1 & 0 & -h_2 & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 2\beta_2 & 0 & -h_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 2\beta_1 & 0 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 2\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_3 & 0 & \alpha_1 & 0 & \alpha_2 & 0 \\
 0 & 0 & 2\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -h_2 & 0 & \alpha_1 & 0 & \alpha_2 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 2\beta_2 & 0 & 0 & -\alpha_2
 \end{pmatrix}
 \begin{pmatrix}
 p_0(\infty) \\
 p_1(\infty) \\
 p_2(\infty) \\
 p_3(\infty) \\
 p_4(\infty) \\
 p_5(\infty) \\
 p_6(\infty) \\
 p_7(\infty) \\
 p_8(\infty) \\
 p_9(\infty) \\
 p_{10}(\infty) \\
 p_{11}(\infty) \\
 p_{12}(\infty) \\
 p_{13}(\infty) \\
 p_{14}(\infty)
 \end{pmatrix}
 =
 \begin{pmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{pmatrix}
 \tag{13}$$

Using the following normalizing condition

$$\sum_{m=0}^{14} p_m(\infty) = 1 \tag{14}$$

Solving (13) using (14), the steady state availability for Configuration III is given by

$$A_{V3}(\infty) = p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty)$$

$$\begin{aligned}
 &+ p_4(\infty) + p_9(\infty) + p_{10}(\infty) \\
 &\frac{\alpha_1^3 \alpha_2^3 + 2\alpha_1^2 \alpha_2^3 \beta_1 + 2\alpha_1^3 \alpha_2^2 \beta_2 + 4\alpha_1 \alpha_2^3 \beta_1^2}{+4\alpha_1^3 \alpha_2 \beta_2^2 + 8\alpha_1^2 \alpha_2^2 \beta_1 \beta_2} \\
 &\frac{\alpha_1^3 \alpha_2^3 + 2\alpha_1^3 \alpha_2^2 \beta_2 + 4\alpha_1^3 \alpha_2 \beta_2^2 + 4\alpha_1 \alpha_2^3 \beta_1^2 + 8\alpha_1^3 \beta_2^3}{+8\alpha_2^3 \beta_1^3 + 2\alpha_1^2 \alpha_2^3 \beta_1 + 8\alpha_1^2 \alpha_2^2 \beta_1 \beta_2} \\
 &\frac{+24\alpha_1 \alpha_2^3 \beta_1^2 + 24\alpha_1 \alpha_2^2 \beta_1^2 \beta_2}{+24\alpha_1 \alpha_2^3 \beta_1^2 + 24\alpha_1 \alpha_2^2 \beta_1^2 \beta_2} \tag{15}
 \end{aligned}$$

3.4 Availability Formulation of Configuration IV

For the availability analysis of configuration IV, the differential difference equation is expressed in the form

$$\begin{pmatrix} p'_0 \\ p'_1 \\ p'_2 \\ p'_3 \\ p'_4 \\ p'_5 \\ p'_6 \\ p'_7 \\ p'_8 \\ p'_9 \\ p'_{10} \end{pmatrix} = \begin{pmatrix} -y_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -y_2 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -y_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -y_2 & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 2\beta_1 & 0 & 0 & 0 & -y_4 & \alpha_1 & \alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 2\beta_1 & -y_4 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \end{pmatrix} \tag{16}$$

Following above argument, in steady state (16) becomes

$$\begin{pmatrix} -y_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -y_2 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -y_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -y_2 & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 2\beta_1 & 0 & 0 & 0 & -y_4 & \alpha_1 & \alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 2\beta_1 & -y_4 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{17}$$

Using the following normalizing condition

$$\sum_{m=0}^{10} p_m(\infty) = 1 \tag{18}$$

Substituting (18) in (17) to obtain

$$\begin{pmatrix} -y_1 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2\beta_1 & -y_2 & 0 & \alpha_1 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 \\ 2\beta_2 & 0 & -y_3 & 0 & \alpha_2 & 0 & \alpha_1 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_1 & 0 & -y_2 & 0 & \alpha_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2\beta_2 & 2\beta_1 & 0 & 0 & 0 & -y_4 & \alpha_1 & \alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 2\beta_2 & 0 & 0 & 2\beta_1 & -y_4 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_2 & 0 & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\beta_1 & 0 & -\alpha_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} p_0(\infty) \\ p_1(\infty) \\ p_2(\infty) \\ p_3(\infty) \\ p_4(\infty) \\ p_5(\infty) \\ p_6(\infty) \\ p_7(\infty) \\ p_8(\infty) \\ p_9(\infty) \\ p_{10}(\infty) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{19}$$

Solving the system of equations in (19) using MATLAB package, the steady state availability of Configuration IV is given by

$$\begin{aligned} A_{V4}(\infty) &= p_0(\infty) + p_1(\infty) + p_2(\infty) + p_3(\infty) + p_6(\infty) + p_7(\infty) \\ &= \frac{\alpha_1^3\alpha_2^2 + 2\alpha_1^2\alpha_2^2\beta_1 + 2\alpha_1^3\alpha_2\beta_2 + 4\alpha_1\alpha_2^2\beta_1^2 + 4\alpha_1^2\alpha_2\beta_2\beta_1}{4\alpha_1^3\beta_2^2 + 2\alpha_1^3\alpha_2\beta_2 + \alpha_1^3\alpha_2^2 + 4\alpha_1^2\alpha_2\beta_1\beta_2 + 2\alpha_1^2\alpha_2^2\beta_1 + 8\alpha_1^2\beta_1\beta_2^2 + 4\alpha_1\alpha_2^2\beta_1^2 + 8\alpha_1\alpha_2\beta_1^2\beta_2 + 16\alpha_1\beta_1^2\beta_2^2 + 8\alpha_2^2\beta_2^3 + 16\alpha_2\beta_1^3\beta_2} \end{aligned} \tag{20}$$

4 Results and Discussion

4.1 Surface plot of the Configurations

The purpose of this section is to see the impact of failure and repair rate on availability through surface plot using MATLAB software package. The results are summarized in Tables below.

Figures 5a have shown the simulations of availability for Configuration I with respect to α_1 and β_1 . It is evident from this figure that availability increases with increase in α_1 and decreases with increase in β_1 . Similarly Figure 5b have shown the simulations of availability for Configuration I with respect to α_2 and β_2 . It is clear from this figure that availability increases with increase in α_2 and decreases with increase in β_2 . It can be observe from these figures, that availability is higher with respect to the combination of α_1 and β_1 . This called for perfect repair with respect to α_2 and major maintenance to reduce the occurrence of β_2 . Simulation in Figures 6a and 6b revealed

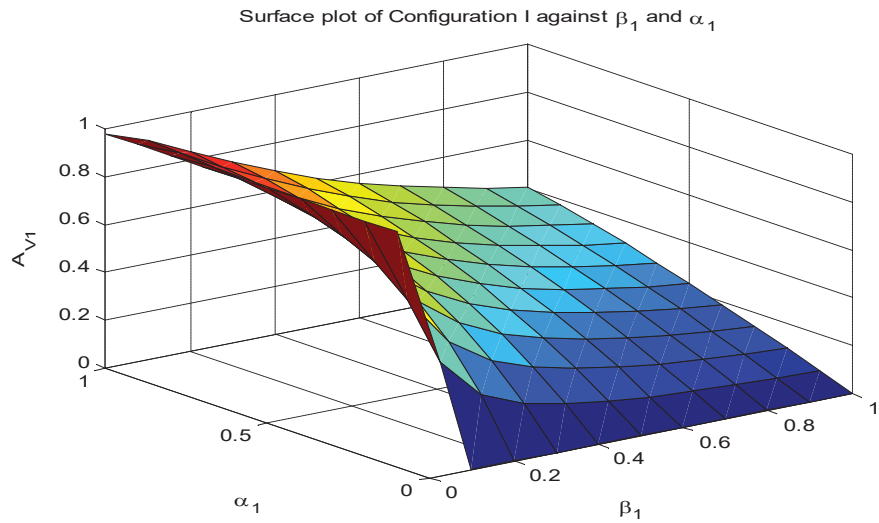


Figure 5a Availability of Configuration I against β_1 and α_1 .

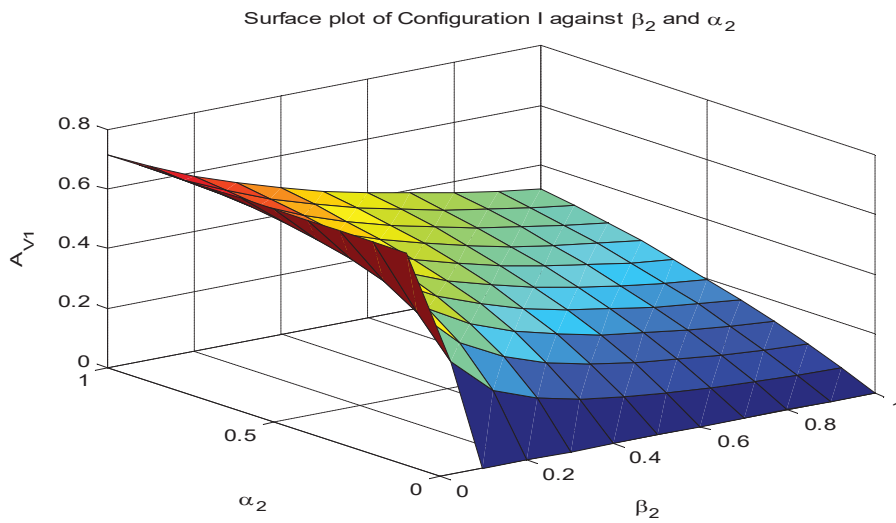


Figure 5b Availability of Configuration I against β_2 and α_2 .

the impact of α_1 and β_1 and the impact of α_2 and β_2 on availability of Configuration II respectively. It is evident from these figures that availability increase with increase in α_1 and α_2 , decreases with increase in β_1 and β_2 respectively. However, availability is higher in Figure 6a than Figure 6b. It is

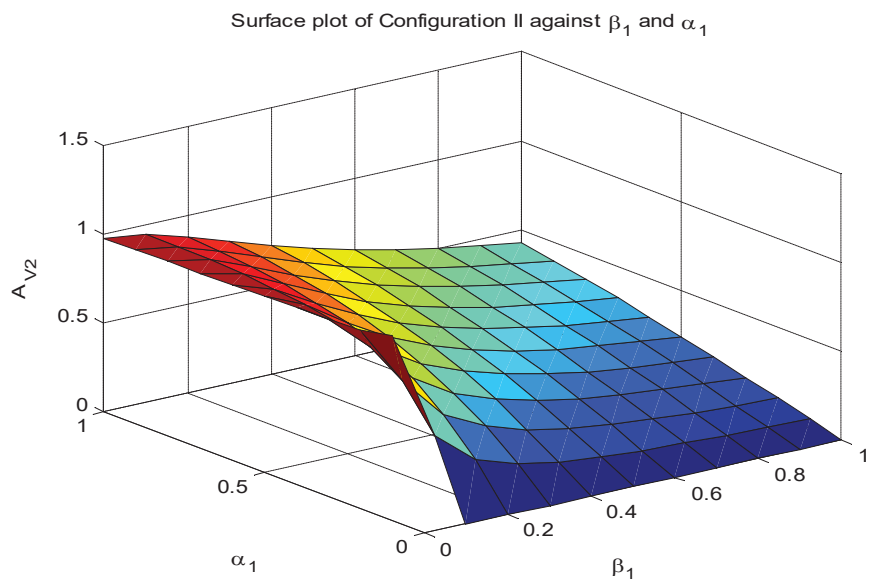


Figure 6a Availability of Configuration II against β_1 and α_1 .

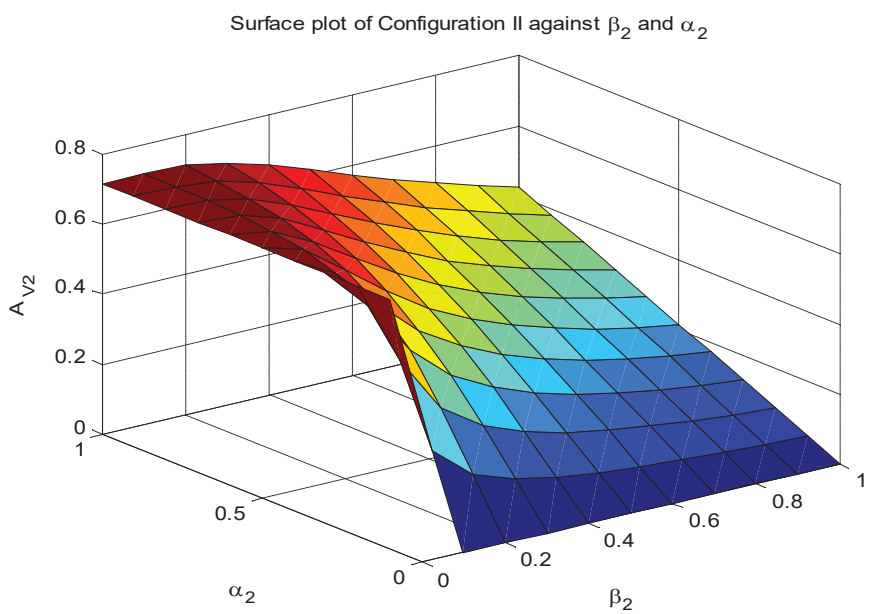


Figure 6b Availability of Configuration II against β_2 and α_2 .

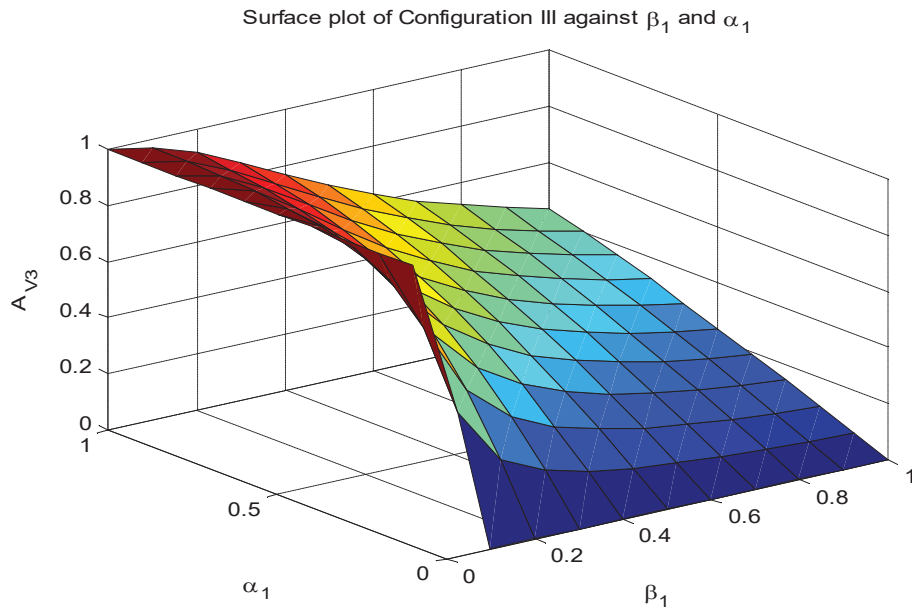


Figure 7a Availability of Configuration III against β_1 and α_1 .

clear from this observation that the preventive maintenance and perfect repair should be invoke to control system degradation, fault and subsequent failures. On the other hand, simulations in Figures 7a to Figures 8b displayed similar pattern with Figures 5a to Figure 6b with respect to α_1 and β_1 , α_2 and β_2 for Configuration III and Configuration IV.

It is evident from Figure 5a to Figure 8b, β_2 is the most critical to all configuration. Increase in β_2 decrease the values of availability for any configuration. To this effect, the configurations require adequate maintenance action in order to avoid catastrophic failure and prolong the life span of the configurations.

4.2 Ranking the Configurations based on their Availability

The purpose of this section is to rank the configurations for their availability using MATLAB software package. The results are summarized in Tables below.

Tables 1 to 4 depict the ranking of configuration base on their availability. It clear from the tables that configuration IV is the optimal configuration.

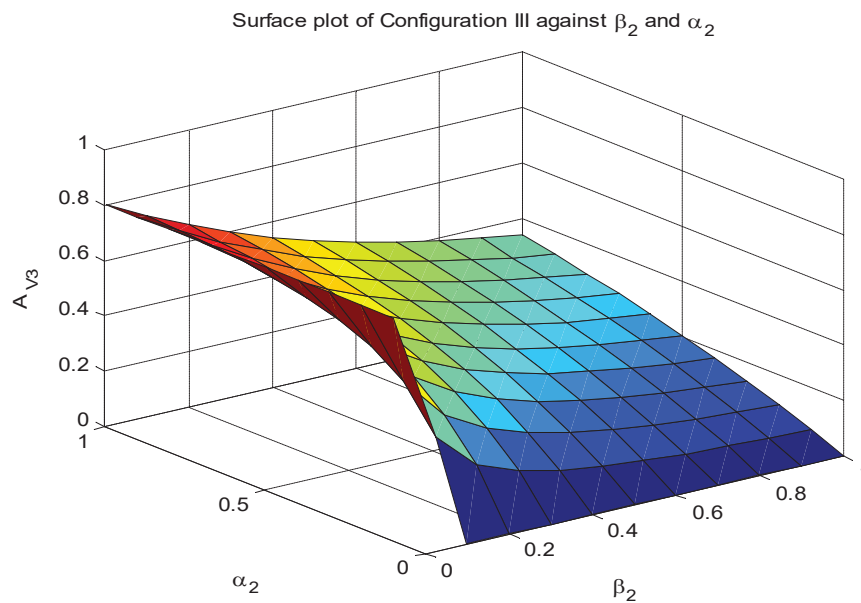


Figure 7b Availability of Configuration III against β_2 and α_2 .

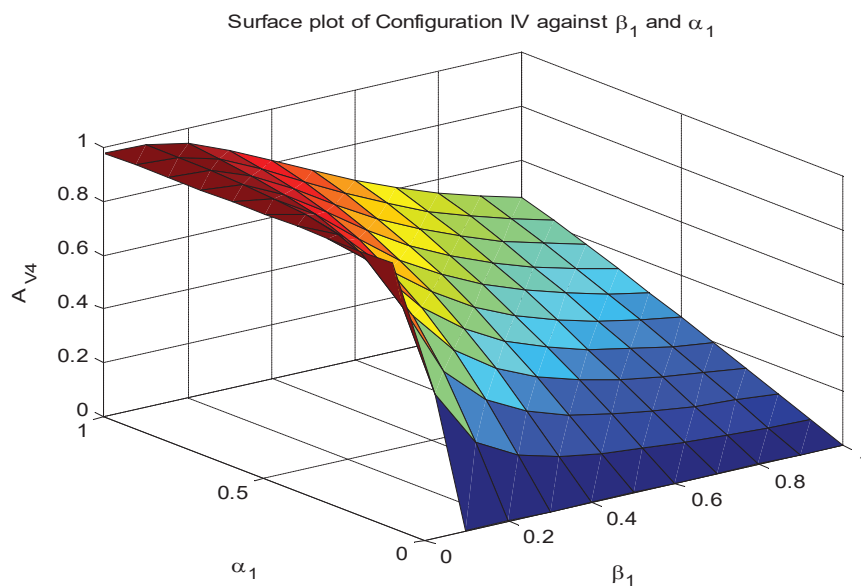


Figure 8a Availability of Configuration IV against β_1 and α_1 .

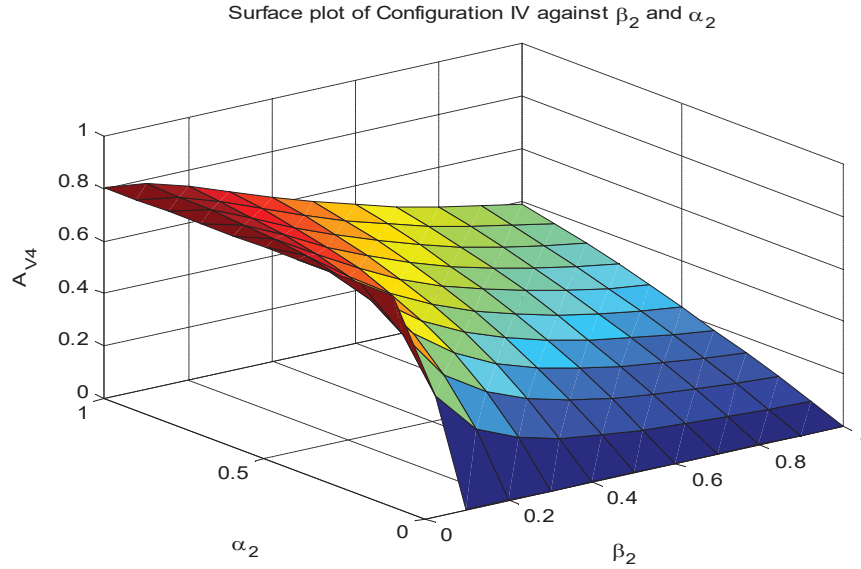


Figure 8b Availability of Configuration IV against β_2 and α_2 .

Table 1 Ranking between the configurations in terms of their availability

Case	Parameter Range	Results	Constant values
1	$0 < \alpha_1 < 0.1$	$A_{V1}(\infty) = A_{V2}(\infty) = A_{V3}(\infty) = A_{V4}(\infty)$	$\beta_1 = 0.3$
	$0.1 < \alpha_1 < 0.2$	$A_{V1}(\infty) < A_{V2}(\infty) = A_{V3}(\infty) < A_{V4}(\infty)$	$\beta_2 = 0.03$
	$0.2 < \alpha_1 < 0.3$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\alpha_2 = 0.4$
	$0.3 < \alpha_1 < 0.4$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.4 < \alpha_1 < 0.5$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.5 < \alpha_1 < 0.6$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.6 < \alpha_1 < 0.7$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.7 < \alpha_1 < 0.8$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.8 < \alpha_1 < 0.9$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.9 < \alpha_1 < 1$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	

Thus,

$$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty) \quad \text{for some } \alpha_k, \beta_k > 0, \\ k = 1, 2.$$

Table 2 Ranking between the configurations in terms of their availability

Case	Parameter Range	Results	Constant values
2	$0 < \beta_1 < 0.1$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\alpha_1 = 0.7$
	$0.1 < \beta_1 < 0.2$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\beta_2 = 0.03$
	$0.2 < \beta_1 < 0.3$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\alpha_2 = 0.4$
	$0.3 < \beta_1 < 0.4$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.4 < \beta_1 < 0.5$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.5 < \beta_1 < 0.6$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.6 < \beta_1 < 0.7$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.7 < \beta_1 < 0.8$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.8 < \beta_1 < 0.9$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.9 < \beta_1 < 1$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	

Table 3 Ranking between the configurations in terms of their availability

Case	Parameter Range	Results	Constant values
3	$0 < \alpha_2 < 0.1$	$A_{V2}(\infty) < A_{V1}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\beta_1 = 0.3$
	$0.1 < \alpha_2 < 0.2$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\beta_2 = 0.03$
	$0.2 < \alpha_2 < 0.3$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	$\alpha_1 = 0.7$
	$0.3 < \alpha_2 < 0.4$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.4 < \alpha_2 < 0.5$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.5 < \alpha_2 < 0.6$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.6 < \alpha_2 < 0.7$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.7 < \alpha_2 < 0.8$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.8 < \alpha_2 < 0.9$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	
	$0.9 < \alpha_2 < 1$	$A_{V1}(\infty) < A_{V2}(\infty) < A_{V3}(\infty) < A_{V4}(\infty)$	

4.3 Sensitivity Analysis

This section presents the sensitivity analysis of the availability of each configuration with respect to variation in the values of input parameters.

- Case 1. By fixing $\alpha_2 = 0.4$, $\beta_2 = 0.03$, $\alpha_1 = 0.2$ and $\alpha_1 = 0.3$ varying β_1 as 0, 0.1, 0.2, 0.3, ..., 0.9, results of, A_{V1} , A_{V2} , A_{V3} and A_{V4} with respect to β_1 are presented in Table 5.
- Case 2. By fixing $\alpha_2 = 0.4$, $\beta_2 = 0.03$, $\beta_1 = 0.2$ and $\beta_1 = 0.3$ varying α_1 as 0, 0.1, 0.2, 0.3, ..., 0.9, results of, A_{V1} , A_{V2} , A_{V3} and A_{V4} with respect to α_1 are presented in Table 6.

Table 4 Ranking between the configurations in terms of their availability

Case	Parameter Range	Results	Constant values
4	$0 < \beta_2 < 0.1$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	$\beta_1 = 0.3$
	$0.1 < \beta_2 < 0.2$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	$\alpha_1 = 0.7$
	$0.2 < \beta_2 < 0.3$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	$\alpha_2 = 0.4$
	$0.3 < \beta_2 < 0.4$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.4 < \beta_2 < 0.5$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.5 < \beta_2 < 0.6$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.6 < \beta_2 < 0.7$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.7 < \beta_2 < 0.8$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.8 < \beta_2 < 0.9$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	
	$0.9 < \beta_2 < 1$	$A_{V1}(\infty) < A_{V3}(\infty) < A_{V2}(\infty) < A_{V4}(\infty)$	

Table 5 Variation of availability of four configurations for different values of α_1

β_1	$\alpha_1 = 0.2$				$\alpha_1 = 0.3$			
	A_{V1}	A_{V2}	A_{V3}	A_{V4}	A_{V1}	A_{V2}	A_{V3}	A_{V4}
0	0.9808	0.9912	0.9971	0.9808	0.9808	0.9928	0.9971	0.9808
0.1	0.6192	0.6712	0.6954	0.7392	0.7315	0.7908	0.8201	0.8621
0.2	0.4053	0.4298	0.4389	0.4624	0.5302	0.5679	0.5893	0.6265
0.3	0.2949	0.3081	0.3105	0.3229	0.4053	0.4286	0.4389	0.4624
0.4	0.2302	0.2383	0.2384	0.2459	0.3248	0.3402	0.3446	0.3598
0.5	0.1882	0.1936	0.1930	0.1979	0.2698	0.2805	0.2822	0.2926
0.6	0.1590	0.1628	0.1620	0.1655	0.2302	0.2381	0.2384	0.2459
0.7	0.1375	0.1404	0.1395	0.1421	0.2005	0.2054	0.2061	0.2117
0.8	0.1211	0.1233	0.1225	0.1245	0.1774	0.1821	0.1815	0.1858
0.9	0.1081	0.1099	0.1092	0.1107	0.1590	0.1628	0.1620	0.1655

- (c) Case 3. By fixing $\alpha_1 = 0.7, \beta_1 = 0.3, \alpha_2 = 0.3$ and $\alpha_2 = 0.5$ varying β_2 as $0, 0.1, 0.2, 0.3, \dots, 0.9$, results of, A_{V1}, A_{V2}, A_{V3} and A_{V4} with respect to β_2 are presented in Table 7.
- (d) Case 4. By fixing $\alpha_1 = 0.7, \beta_1 = 0.3, \beta_2 = 0.3$ and $\beta_2 = 0.5$ varying β_1 as $0, 0.1, 0.2, 0.3, \dots, 0.9$, results of, A_{V1}, A_{V2}, A_{V3} and A_{V4} with respect to α_2 are presented in Table 8.

From sensitivity analysis presented in Table 5 for all configurations, one can observe that as β_1 increases from 0 to 0.9, the availability of all

Table 6 Variation of availability of four configurations for different values of β_1

α_1	$\beta_1 = 0.2$				$\beta_1 = 0.3$			
	A_{V1}	A_{V2}	A_{V3}	A_{V4}	A_{V1}	A_{V2}	A_{V3}	A_{V4}
0	0	0	0	0	0	0	0	0
0.1	0.2302	0.2390	0.2384	0.2459	0.1590	0.1630	0.1620	0.1655
0.2	0.4053	0.4298	0.4389	0.4624	0.2949	0.3081	0.3105	0.3229
0.3	0.5302	0.5679	0.5893	0.6265	0.4053	0.4286	0.4389	0.4624
0.4	0.6192	0.6654	0.6954	0.7392	0.4933	0.5255	0.5446	0.5780
0.5	0.6836	0.7343	0.7689	0.8134	0.5632	0.6023	0.6291	0.6692
0.6	0.7315	0.7838	0.8201	0.8621	0.6192	0.6629	0.6954	0.7392
0.7	0.7678	0.8198	0.8565	0.8946	0.6643	0.7108	0.7473	0.7921
0.8	0.7961	0.8464	0.8830	0.9166	0.7011	0.7490	0.7880	0.8319
0.9	0.8186	0.8664	0.9027	0.9319	0.7315	0.7795	0.8201	0.8621

Table 7 Variation of availability of four configurations for different values of α_2

β_2	$\alpha_2 = 0.3$				$\alpha_2 = 0.5$			
	A_{V1}	A_{V2}	A_{V3}	A_{V4}	A_{V1}	A_{V2}	A_{V3}	A_{V4}
0	0.7165	0.7165	0.8045	0.8045	0.7165	0.7165	0.8045	0.8045
0.1	0.5208	0.5952	0.5780	0.6624	0.5882	0.6835	0.6589	0.7368
0.2	0.3994	0.4411	0.4318	0.4988	0.4918	0.5978	0.5428	0.6256
0.3	0.3209	0.3416	0.3401	0.3882	0.4194	0.5072	0.4557	0.5270
0.4	0.2670	0.2776	0.2791	0.3142	0.3641	0.4318	0.3902	0.4489
0.5	0.2281	0.2336	0.2362	0.2627	0.3209	0.3276	0.3401	0.3882
0.6	0.1989	0.2017	0.2045	0.2251	0.2864	0.3263	0.3008	0.3405
0.7	0.1761	0.1775	0.1802	0.1966	0.2583	0.2896	0.2694	0.3025
0.8	0.1580	0.1585	0.1610	0.1744	0.2350	0.2601	0.2437	0.2717
0.9	0.1432	0.1432	0.1454	0.1566	0.2155	0.2359	0.2224	0.2463

configurations decreases. Examination of the availability results of all configurations shown in this table reveals that configuration III has higher value of availability compared to other configurations initially for both $\alpha_1 = 0.2$ and $\alpha_1 = 0.3$. However, as the values of β_1 increases from 0.1 to 0.9, configuration IV has higher value of availability compared to other configurations. From the analysis in Table 5, it is clear that the contributing parameter toward the system performance is α_1 . This can be seen from the availability results

Table 8 Variation of availability of four configurations for different values of β_2

α_2	$\beta_2 = 0.3$				$\beta_2 = 0.5$			
	A_{V1}	A_{V2}	A_{V3}	A_{V4}	A_{V1}	A_{V2}	A_{V3}	A_{V4}
0	0	0	0	0	0	0	0	0
0.1	0.1432	0.1187	0.1454	0.1566	0.0914	0.0796	0.0920	0.0968
0.2	0.2461	0.2350	0.2559	0.2863	0.1666	0.1578	0.1700	0.1849
0.3	0.3209	0.3416	0.3401	0.3882	0.2281	0.2336	0.2362	0.2627
0.4	0.3766	0.4330	0.4050	0.4666	0.2788	0.3056	0.2923	0.3302
0.5	0.4194	0.5072	0.4557	0.5270	0.3209	0.3726	0.3401	0.3882
0.6	0.4531	0.5655	0.4961	0.5737	0.3562	0.4336	0.3810	0.4377
0.7	0.4801	0.6103	0.5288	0.6103	0.3861	0.4883	0.4161	0.4800
0.8	0.5023	0.6445	0.5556	0.6392	0.4117	0.5367	0.4465	0.5162
0.9	0.5208	0.6705	0.5780	0.6624	0.4338	0.5791	0.4729	0.5471

of the configurations for $\alpha_1 = 0.2$ and $\alpha_1 = 0.3$. Thus, as α_1 increases, so also the availability increase.

On the other hand, sensitivity analysis presented in Table 6 for all configurations, one can observe that as α_1 increases from 0 to 0.9, the availability of all configurations increases also. Availability results of all configurations displayed in this table reveals that configuration IV has higher value of availability compared to other configurations initially for both $\beta_1 = 0.2$ and $\beta_1 = 0.3$. However, as the values of β_1 changes from 0.2 to 0.3, availability of all configurations decreases. Nonetheless, configuration IV remains the optimal configuration. β_1 can be viewed as the parameter contributing the decrease in the performance of the configurations.

Sensitivity analysis displayed in Table 7 for all configurations have shown that availability decreases with increase β_2 increases from 0 to 0.9 as α_2 changes from 0.3 to 0.5 with configuration IV having the highest value of availability compared to other configurations. From the analysis it is clear that the performance of the system can be improve whenever $\alpha_2 \geq 0.3$.

From the sensitivity analysis shown in Table 8 for all configurations, it is evident that as α_2 increases from 0 to 0.9, the availability of all configurations increases also. Availability results of all configurations displayed in this table reveals that configuration IV has higher value of availability compared to other configurations whenever $0 \leq \alpha_2 \leq 0.7$ when β_2 changes from 0.3 to 0.5. However, as the values of α_2 from 0.8 to 0.9, configuration II has higher value of availability compared to other configurations.

It evident from Tables 5 and 7 that availability decreases as β_1 and β_2 increase. The analysis has shown that Configuration IV has higher availability than the remaining configurations for each $\alpha_1 \in [0.2, 0.3]$ and $\alpha_2 \in [0.3, 0.5]$. Similarly, sensitivity analysis in Tables 6 and 8 have shown that availability increases as α_1 and α_2 increase. However, availability of Configuration IV is higher in these tables than the remaining configuration for each $\beta_1 \in [0.2, 0.3]$ and $\beta_2 \in [0.3, 0.5]$.

4.4 Partial Rank Correlation Coefficients (PRCC)

This section used the partial rank correlation coefficient of the availability of each configuration in order to determine the most critical parameter that may lead to decrease in the availability using MATLAB package. The experiments are presented below:

We conducted sensitivity analysis on β_k and α_k , $k = 1, 2$ to measure their statistical influence on A_{V1} to A_{V4} . To do that each β_k and α_k is uniformly distributed on $[0, 1]$ and we draw 1000 samples from this distribution using Latin Hypercube sampling. This gives a matrix with 1000 rows 5 columns. Each row of the matrix represents a unique parameter set. For each of these sets, we simulated the model outcomes of availability A_{V1} to A_{V4} . We then performed sensitivity analysis on the model outcomes using partial rank correlation coefficients (PRCC). Figures 5 to 8 depicts the tornado plots of the results. From the Figures 9 to 12, it is evident that α_2 is the most sensitive parameter affecting all the outcomes (A_{V1} to A_{V4}). Increase in α_2 will lead to increase in the outcomes. The PRCC results for β_1 are similar in the Figures 9 to 11. The results indicate that β_1 is the most sensitive, while β_2 is the least sensitive for all the outcomes in the Figures 9 to 12. Increasing the value of β_1 will decrease each one of the outcomes more significantly than the other. The PRCC results for β_2 depicted in Figures 12. The results in this Figure indicates that β_2 is the most sensitive, while β_1 is the least sensitive for all the outcomes in the Figures 12. Increasing the value of β_2 will decrease each one of the outcomes more significantly than the other β_1 . These means that in order to have high values of availability of the configurations I to IV, it is necessary to consider the combinations of high values of α_2 together with low values of β_1 and β_2 in the Figures 9 to 12. This means that processors should be given more care in terms of maintenance.

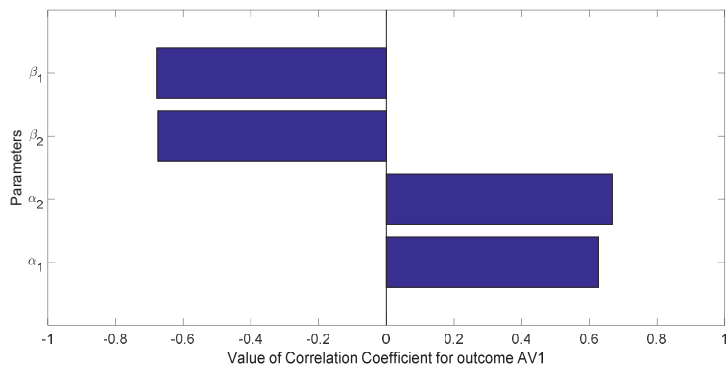


Figure 9 Tornado plot of Configuration I.

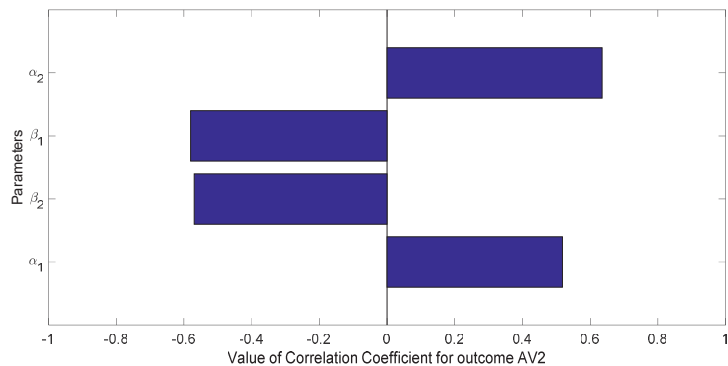


Figure 10 Tornado plot of Configuration II.

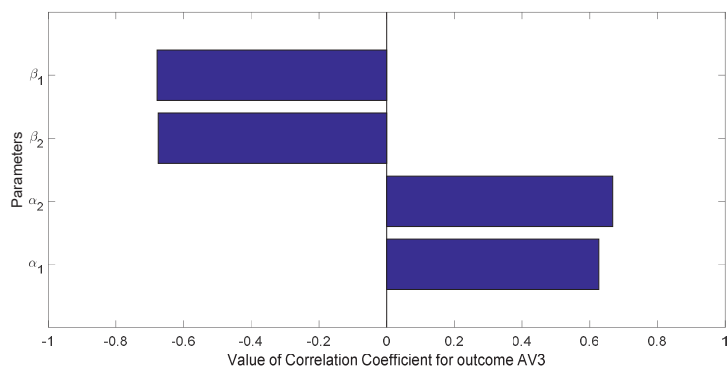


Figure 11 Tornado plot of Configuration III.

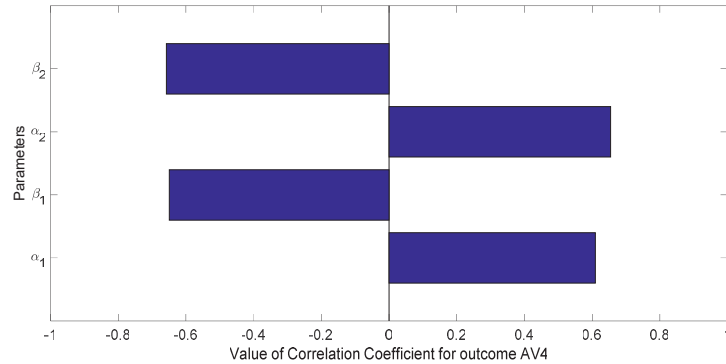


Figure 12 Tornado plot of Configuration IV.

5 Conclusion

This paper studied availability of four hybrid systems configured as series-parallel systems in which each system or configuration consisting of main units and their corresponding processors in the form of k-out-of-n system. Explicit expression for the steady-state availability is derived and validated by performing numerical experiments. Analysis of the effect of various system parameters on availability was performed through MATLAB surface plot and tornado plot. These are the main contribution of the paper. From the surface plot presented in the study, it can be concluded that maintenance strategies that will keep the system as failure free should be invoke to maximize availability, product quality, and production output and revenue generation. The present work can be extended further for a system to connect to multi standby devices. On the basis of the tornado plot obtained in Figures 9–12 and ranking in Tables 1–4, it is evident that the availability can be enhanced through high value of α_2 together with low values of β_1 and β_2 . Thus, the processors should be given priority in terms of maintenance. From the analysis, it is evident that availability can be enhancing through, proper maintenance planting to avoid the occurrence of catastrophic failure, maintaining the system availability at the highest order through high value of α_2 and low value of β_1 and β_2 , by adding fault tolerant units and processors. The present work can be extended further for a system to connect to multi-processor and solve using supplementary variable techniques.

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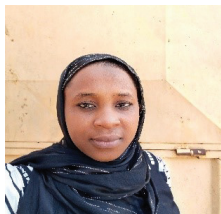
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