The Distribution of Service Time of Patients

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Received 24 May 2019; Accepted 05 April 2020;
Publication 13 October 2020

Abstract

The aim of this research was to study the distribution of service time of patients at the University of Ilorin Health Services Clinic. The distributions of patients' arrival and their service time were studied. The system was treated as single, time-independent arrivals with multiple service points. Based on the histograms obtained on the arrival process and the service process, appropriate distributions were fitted and tested using Chi-squared goodness of fit test. It revealed that the arrival process follows a Poisson distribution with an arrival rate of 0.8 patient per 2 minutes while the service process follows a gamma distribution with shape and scale parameters equal to 6 and 0.1 respectively. The mean service time was also found to be around 63 minutes. The results of this study may be useful to understand the magnitude of the problem of queue, relationship between resources and waiting times and to find a solution for alleviating the problem.

Keywords: Queuing, arrival time, service time, patients, Chi-squared goodness of fit test.

Journal of Reliability and Statistical Studies, Vol. 13, Issue 1 (2020), 61–72.
doi: 10.13052/jrss0974-8024.1313

1 Introduction

Arrival events and service time are two things which are probabilistic in nature. In other words, there is a measure of uncertainty in their individual distributions. This idea of uncertainty confirms the need of probability and statistics in analyzing queuing circumstances or procedures. A queuing process is given birth when the arrival and the service processes are pooled.

The arrival process studies how events or individuals emanate to a point, usually a service facility. It is assumed to have a Poisson distribution. The service process on the other hand has to do with waiting in time while receiving a service. Service time is expected to have an exponential distribution or rather, modified form of it which may be the Gamma and Erlang distributions, depending on the service mechanism.

The term "patients" as used in this study strictly refers to out-patients and more precisely student out-patients. Also the terms "clients" and "customers" are used interchangeably, generally refer to persons who come to a service point to receive some service(s). According to Iversen (1993) waiting lists indicate loss of efficiency when the hospital's resources are moved away from medical work. Cochran et al. (2009) developed an open queuing network model of an emergency department (ED) design purported to enhance the capacity of an ED to treat patients. The methodology is useful for detecting hospital-specific differences in patient acuity mix, arrival patterns, volumes and efficiencies of processes in a single common computational model. Bahadori et al. (2014) used a descriptive-analytical study carried out in a military hospital in Iran for a sample of 220 patients. The results showed that the average numbers of patients in the pharmacy was 19.21 in the morning and 14.66 in evening, whereas the average times spent in the system by clients in the morning and evening were 39 minutes and 35 minutes respectively. The respective system utilization in the morning and evening were 25% and 21%. Armony et al. (2015) used exploratory data analysis (EDA) in a large Israeli hospital exhibiting important features which are not readily explained by the existing models. Green (2002) examined data from New York State and estimated bed unavailability in intensive care units (ICUs) and obstetrics units by means of queuing analysis. He identified units which appear to have insufficient capacity using various patient delay standards. It was concluded that approximately 40% of all obstetrics units and 90% of ICUs do not have sufficient capacity to procure an appropriate bed whenever it is needed.

1.1 General Queue Problem

Queuing theory is the scientific study of waiting in lines or queues. It is a requisite phase in the field of operations research. The theory started with research by Agner Krarup Erlang in 1908 who worked on the size of a telephone exchange that is needed to keep to a sensible value of the number of telephone calls not connected because of the busy system.

Queues come up as a result of limited resources available to serve a large mass of clients. A queue may be difficult to the client because they may feel their time is being wasted by having to wait for a particular duration before getting served. On the part of the organization, it makes economic sense to have queues since no organization would want to employ too many waiters who might turn out to be idle or might not have enough work to do to justify the emolument they receive.

The notion of queues is applied in the design of banks, hospitals, petrol stations, post offices, restaurants, supermarkets, telephone exchange rooms, public transport system and so on.

1.2 Classification of Queuing Systems

Classification of types of queuing systems are:

(a) Single or Simple Queue System
This refers to a system where customers join a single waiting line on a single service point. Example is a withdrawal point in a bank.

$$\text{Arrival} \rightarrow \text{Queue} \rightarrow \text{Service point} \rightarrow \text{Exit}$$

(b) Parallel Queue System
This is a queuing system in which there are several waiting lines each having its own server. This can be found in a in a post office.

(c) Series Queue System
A system in which queues are in stages. It can also be described as a system with a single queue and several servers. This system is applicable in supermarkets, hospitals, restaurants etc.

1.3 Description of the University of Ilorin Health Services Clinic

The University Health Services Clinic is located within the school premises. It is one of the significant units in the University because it attends to the basic health complaints of students and staff of the University.

At the time of this research, there exists up to seven service points in the clinic. These do not include the wards. The service points are described below:

1. Records Section: This is where patients drop their clinic cards especially at commencement of treatment. The details on the cards will be used to trace patient's clinic files. Afterwards each patient will be directed to next required service channel; typically one of the doctors.
2. Consulting Rooms (Doctors): There are three checking rooms. Inside each of them is a doctor who interviews the patient, diagnoses his/her ailment(s) and also directs them to the next required service node by recommending drugs and/or injection.
3. Injection Rooms: In these rooms, injection is given to patients. Injection rooms are two in number.
4. Pharmacy: This is the room where drugs are given to patients according to what is written on prescription forms by the doctors who must have been consulted earlier.

2 Objectives of the Study

1. To show that service time is not always exponential.
2. To fit a suitable probability distribution to the arrival of patients at the Clinic.
3. To fit a suitable probability distribution to the service time at the Clinic.

2.1 Scope of the Analysis

This research was based on the service time of patients that arrive between 12 noon and 03: 00 pm at the University of Ilorin Health Services Clinic. This is the time that is officially devoted to students and as such the analysis shall be particularly concerned with students.

2.2 Chi Squared Goodness of Fit Test

The Chi Square test as a goodness of fit test is a peculiar method for testing whether the observed sample distribution data are representative of a specific distribution. The real frequencies are compared with the frequencies that hypothetically would be expected to happen if the data followed the exact distribution of interest.

The test statistic is given as

$$X^2 = \frac{\sum_{i=1}^{n} (O_i - E_i)^2}{E_i} \sim \chi^2_{k-p-1} \text{ degrees of freedom.}$$

Where

O_i = Observed frequency
E_i = Expected frequency
P = Number parameters estimated
K = Number of groups or categories

Decision Rule: Reject H_0 if $X^2 > \chi^2_{k-p-1,\alpha}$ or if P-value $< \alpha$ otherwise do not reject.

2.3 Source of Data

The data used for this research were collected from the University of Ilorin Health Services Clinic.

3 Results and Discussion

Table 1 exhibits the data of service time of 35 patients of the University of Ilorin Health Service clinic observed on 7 days.

The histogram is skewed to the right which suggests that the service time of patients has a distribution that belongs to the gamma family.

Let S be a random variable representing the service time (in minutes) of patients. An exponential distribution is fitted to the data as follows:

3.1 Exponential Distribution

H_0 : $S \sim \exp(\mu)$ that is service time follows exponential distribution.
H_1 : S does not $\sim \exp(\mu)$ that is service time does not follow exponential distribution.

The probability density function for the exponential distribution is

$$f(s) = \mu e^{-\mu s}, \mu > 0.$$

where

μ = mean service time $= 62.6687$

Table 1 Data presentation for the service time of patients in minutes

S/N	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
1	95	55	92	49	53	61	48
2	115	54	89	53	56	66	51
3	136	30	100	46	51	70	48
4	125	62	88	50	62	65	40
5	93	65	70	41	52	179	40
6	133	52	52	39	53	64	45
7	132	49	92	38	55	74	43
8	92	133	86	44	80	76	38
9	106	57	109	43	60	59	36
10	105	88	103	51	59	69	49
11	84	83	108	41	58	55	48
12	98	85	97	30	60	62	59
13	95	89	101	119	74	79	56
14	68	52	67	40	82	56	36
15	75	147	94	37	67	64	52
16	75	51	98	32	172	56	48
17	82	83	87	41	61	63	37
18	81	56	78	32	64	54	46
19	81	62	75	45	49	65	34
20	72	60	91	40	17	57	43
21	98	59	41	27	43	70	52
22	96	58	82	36	46	168	34
23	97	62	83	36	33	185	46
24	36	76	70	82	42	55	59
25	55	78	74	91	37	51	68
26	54	71	80	70	33	39	56
27	16	65	76	83	40	45	68
28	51	58	81	82	67	33	53
29	29	56	66	79	20	42	50
30	24	82	66	69	28	32	61
31	42	114	65	76	29	30	66
32	38	72	71	84	33	40	49
33	73	79	79	73	47	46	49
34	75	94	63	61	65	49	111
35	73	12	61	74	48	61	83
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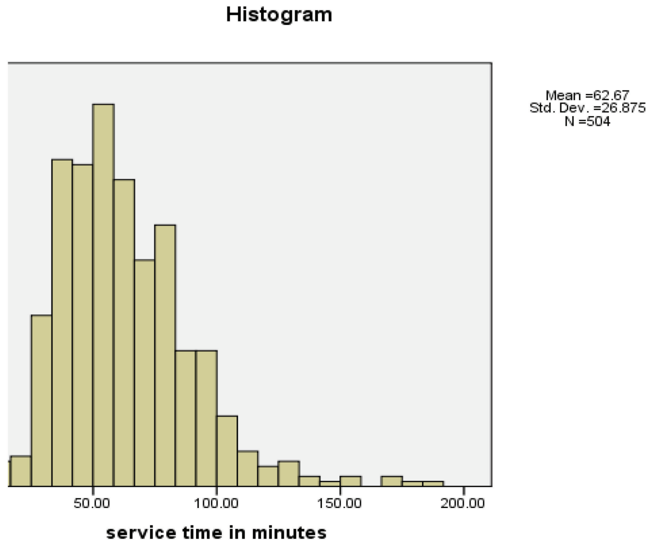

Figure 1 Distribution of service time of patients.

The mean of exponential distribution is $\frac{1}{\mu}$, therefore

$$\frac{1}{\mu} = 62.6687$$

$$\mu = 0.016$$

$$P(a < s < b) = \int_a^b 0.016e^{-0.016s}$$

$$= e^{-0.016a} - e^{-0.016b}$$

$$\chi^2\mathrm{K} - \mathrm{p} - 1, \alpha = \chi^2 8 - 1 - 1, 0.05 = \chi^2 6, 0.05 = 12.59$$

Conclusion: Since $X^2 = 401.8904 > \chi^2 6, 0.05 = 12.59$, we reject H_0 and conclude that the service time of patients does not have an exponential distribution.

3.2 Gamma Distribution

H_0: S is $\sim$ gamma $(\alpha \neq 1, \beta)$ i.e. service time has a gamma distribution

Vs.

H_1: S is not $\sim$ gamma $(\alpha \neq 1, \beta)$ i.e. service time does not have a gamma distribution $\alpha = 0.05$ level of significance

The probability density function is given as

$$f(\mathrm{s}, \alpha, \beta) = \frac{\beta^{\alpha} s^{\alpha-1} e^{-\beta s}}{\Gamma\alpha}$$

where α and β are the shape and scale parameters respectively.

$$\text{mean service time} = \mathrm{E(S)} = \frac{\alpha}{\beta}$$

$$\text{variance} = \mathrm{V(S)} = \frac{\alpha}{\beta^2}$$

The evaluation of α and β

From Table 2, the mean service time $= 62.6687$ and the variance is 722.270

$$\mathrm{E(s)} = \frac{\alpha}{\beta} = 62.6687 \tag{1}$$

$$\mathrm{Var(s)} = \frac{\alpha}{\beta^2} = 722.270 \tag{2}$$

From Equation (1) $\alpha = 62.6687\beta$

Substituting in Equation (2)

$$\frac{62.6687\beta}{\beta^2} = 722.270 => \beta^2 = \frac{62.6687}{722.270} = 0.0868 => \beta \approx 0.1$$

From Equation (2)

$$\alpha = 62.6687\,(0.1) = 6.26687$$

$$\alpha \approx 6$$

This implies that S $\sim$ gamma $(6, 0.1)$

Table 2 Summary statistics of the data

N	504
Mean	62.6688
Median	58
Mode	49
Std. Deviation	26.87510
Variance	722.270
Minimum	2
Maximum	185

Table 3 Computation of expected frequencies for exponential distribution

Interval of Service Time in Minutes	Class Mark (s)	Observed Frequency (O_i)	Probability f(s)	Expected Frequency (E_i)	$X^2 = \frac{\sum_{i=1}^{n}(O_i - E_i)^2}{E_i}$
$0 < t \leq 20$	10	13	0.2753	138.7512	113.9692
$20 < t \leq 40$	30	92	0.1995	100.5534	0.7276
$40 < t \leq 60$	50	161	0.1446	72.8784	106.5531
$60 < t \leq 80$	70	121	0.1048	52.8197	88.0100
$80 < t \leq 100$	90	80	0.0759	38.2536	45.5581
$100 < t \leq 120$	110	20	0.0550	27.7200	2.1500
$120 < t \leq 140$	130	10	0.0399	20.1096	5.0823
> 140	150	7	0.1050	52.9141	39.8401
	Total	**504**		**504.0000**	**401.8904**

Table 4 Computation of expected frequencies for gamma distribution

Interval of Service Time in Minutes	Class Mark (s)	Observed Frequency (O_i)	Probability f(s)	Expected Frequency (E_i)	$X^2 = \frac{\sum_{i=1}^{n}(O_i - E_i)^2}{E_i}$
$0 < t \leq 20$	10	13	0.0165	8.3160	2.6383
$20 < t \leq 40$	30	92	0.1975	99.5400	0.5711
$40 < t \leq 60$	50	161	0.3381	170.4024	0.5188
$60 < t \leq 80$	70	121	0.2534	127.7136	0.3529
$80 < t \leq 100$	90	80	0.1237	62.3448	4.9997
$100 < t \leq 120$	110	20	0.0465	23.4360	0.5038
$120 < t \leq 140$	130	10	0.0148	7.4592	0.8655
> 140	150	7	0.0095	4.7880	1.0219
	Total	**504**		**504.0000**	**11.4720**

$$\text{That is } f\,(\mathrm{s}, 6, 0.1) = \frac{(0.1)^6 s^{6-1} e^{-0.1s}}{\Gamma 6}$$

$$= \frac{(0.1)^6 s^5 e^{-0.1s}}{(6-1)!}$$

$$= 8.3 \times 10^{-9} [s^5 e^{-0.1s}]$$

$$\mathrm{P}\,(\mathrm{a} < \mathrm{s} < \mathrm{b}) = 8.3 \times 10^{-9} \int_a^b s^5 e^{-0.1s}$$

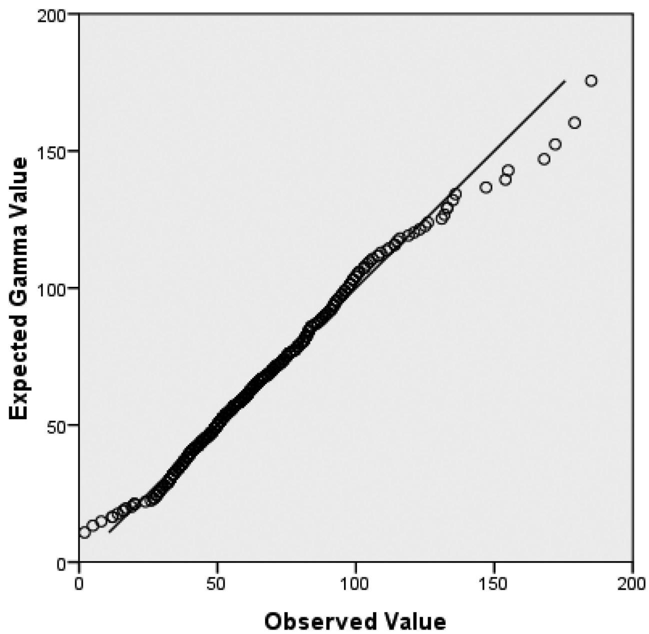

Figure 2 Q-Q Plot of service time in minutes

Table 5 Estimated distribution parameters (Large sample was used)

Gamma Distribution	Service Time in Minutes
Shape	6.000
Scale	0.100

Since P-value $= 0.12 > \alpha = 0.05$, we do not reject H_0 and we therefore conclude that the service time of patients has a gamma distribution.

4 Summary

Analysis has shown that the service time at the University of Ilorin Health Services Clinic has a gamma distribution with $\alpha = 6$ minutes and $\beta = 0.1$ minutes.

5 Conclusion

This study has shown that the distribution of service time is not always exponential.

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Biographies

Ayotunde Samson Oladimeji is a young researcher and studied at the University of Ilorin Nigeria where he obtained B.Sc. Statistics in 2013. He later proceeded to University of Ibadan Nigeria for his M.Sc. in Statistics. He has good research experience and has a good knowledge of different statistical packages. He is a data scientist and a trained statistician with passion for quality research.

Olayemi Joshua Ibidoja is currently a Ph.D. student at the Universiti Sains Malaysia. He attended the University of Ilorin Nigeria where he obtained B.Sc. Statistics in 2013 and MSc. Statistics in 2016 respectively. He served his country at Obafemi Awolowo University Nigeria and currently an academic staff at the Department of Mathematics, Federal University Gusau Nigeria. He has published articles in reputable journals and in addition to a book titled "Introductory Statistics".