Bayes Estimation of the Reliability Function of Pareto Distribution Under Three Different Loss Functions

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Abstract

In this paper, we have proposed Bayes estimators of shape parameter of Pareto distribution as well as reliability function under SELF, QLF and APLF loss functions. For better understanding of Bayesian approach, we consider Jeffrey's prior as non-informative prior, exponential and gamma priors as informative priors. The proposed estimators have been compared with Maximum likelihood estimator (MLE) and the uniformly minimum variance unbiased estimator (UMVUE). Moreover, the current study also derives the expressions for risk function under these three loss functions. The results obtained have been illustrated with the real as well as simulated data set.

Keywords: Bayes estimator, maximum likelihood estimator, reliability function, prior, shape parameter, squared error loss function, quadratic loss function and asymmetric precautionary loss function.

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1 Introduction

Pareto distribution is used in diversified fields because of its own unique characteristics. This distribution was introduced by Wilfredo Pareto (1848–1923) especially for wealth distribution of the population of a city within a given area. Later on, it was also found suitable to analyze stock price instability, oil field sites, biomedical field, insurance risk, migration, word frequencies, business mortality, service time in queuing systems, reliability and life testing.

The pdf of Pareto distribution is given as

$$\begin{aligned} f(t,\sigma,\omega) &= \frac{\omega\sigma^{\omega}}{t^{\omega+1}} \qquad t \geq \sigma, \sigma > 0, \omega > 0 \\ &= 0, \qquad\qquad otherwise \end{aligned} \tag{1}$$

Where t is a random variable, ω is shape parameter and σ is scale parameter.

The distribution function (F(t)), reliability function (R(t)) and hazard rate function (h(t)) of (1) are given by

$$F(t) = 1 - \left(\frac{\sigma}{t}\right)^{\omega} \qquad t \geq \sigma, \sigma > 0, \omega > 0 \tag{2}$$

$$R(t) = \left(\frac{\sigma}{t}\right)^{\omega} \qquad t \geq \sigma, \omega > 0, \sigma > 0 \tag{3}$$

$$h(t) = \frac{\omega}{t} \qquad t \geq \sigma, \omega > 0, \sigma > 0 \tag{4}$$

Many researchers have also used the Pareto model in past for implementing diverse applications. Lomax (1954) applied this model to explication of business mortality. Steindle (1965) suggested this model to compute the size of the cities and firms. The major application of this model is used by Freiling (1966). He applied this model for the distributions of nuclear particles. Harris (1968) applied this model to study service time of maintenance while Davis & Feldstein (1979) employed it to study the failure time of equipment. Nadarajah and Kots (2003) used this model in reliability estimation while Shortle and Fischer (2005) used this distribution in queuing modeling. Pareto distribution is usually assumed to adequately represent the distribution of incomes. By Bayesian aspects, we can improve the adequacy of distribution of income. Here, firstly, we study the problem of estimating the shape parameter of Pareto distribution using both classical and Bayesian approaches. Later on, Bayes estimator of Reliability function is also obtained by using informative and non-informative priors for SELF, QLF and APLF loss functions. This paper is structured into six sections of which the introduction is the first. Section 2 has some description of MLE, UMVUE and complete statistic of ω. In Section 3, Bayes estimators of ω using informative and non-informative priors under SELF, QLF and APLF are derived while in Section 4, Bayes estimators of reliability function for above said priors and loss function are obtained. Section 5 is based on illustrative example using real and simulated data set and the concluded summary of the results is shown in Section 6.

2 Classical Estimators

Classical estimation approach is an important technique in statistics. Although the statistician can perform some analysis instinctively, estimation requires a specific method. In this section, we have obtained maximum likelihood estimator, unbiased estimator, sufficient statistic and uniformaly minimum variance unbiased estimator of shape parameter of Pareto distribution when scale parameter is known.

2.1 Maximum Likelihood Estimator

Let $t_i(i = 1, 2, ..., n)$ be a random sample of size n from (1), then likelihood function

$$L \;=\; L(t_i, \sigma, \omega) = \prod_{i=1}^{n} \frac{\omega\sigma^{\omega}}{t_i^{\omega+1}} = \frac{\omega^n \sigma^{n\omega}}{\prod_{i=1}^{n} t_i^{\omega+1}} = \omega^n \sigma^{n\omega} e^{-(\omega+1)\sum_{i=1}^{n}\ln t_i} \tag{5}$$

$$L \;=\; \omega^n e^{n\omega\ln\sigma} e^{-(\omega+1)\sum_{i=1}^{n}\ln t_i} \tag{6}$$

log likelihood function

$$ln\, L = n\, ln\, \omega + n\omega\, ln\, \sigma - (\omega + 1) \sum\nolimits_{i=1}^{n} ln\,(t_i)$$

Now

$$\frac{\partial}{\partial\omega} \ln\, L = 0 \quad \text{and} \quad \frac{\partial}{\partial\sigma} \ln\, L = 0$$

$$\frac{n}{\omega} + n\, ln\, \sigma - \sum\nolimits_{i=1}^{n} ln(t_i) = 0$$

Hence
$$\widehat{\omega}_{MLE}=\frac{n}{\sum\limits_{i=1}^{n}\ln\left(\frac{t_i}{\widehat{\sigma}}\right)} \quad \text{and} \quad \widehat{\sigma}_{MLE}=\min_{1\le i\le n}(t_i)$$

If σ is known then
$$\widehat{\omega}_{MLE}=\frac{n}{\sum\limits_{i=1}^{n}\ln\left(\frac{t_i}{\sigma}\right)} \tag{7}$$

By invariant property of MLE, the MLE of Reliability function can be described as
$$R(t)_M=\left(\frac{\sigma}{t}\right)^{\widehat{\omega}_{MLE}} \tag{8}$$

2.2 Exponential Family and Uniformly Minimum Variance Unbiased Estimator

The distributions whose density functions have the following general form $\mathrm{f(t,\omega)=a(\omega)b(t)\exp[c(\omega)d(t)]}$ is known as one parameter exponential family of distributions. In case of Pareto distribution, the density function (1) can be expressed as

$$f(t,\omega,\sigma)=\omega e^{\omega\ln\sigma-(\omega+1)\ln t}=\omega e^{\omega\ln\sigma-\omega\ln t-\ln t}=\omega e^{-\ln t}e^{-\omega\ln\left(\frac{t}{\sigma}\right)}$$

Where

$$a(\omega)=\omega, b(t)=e^{-\ln t}, c(\omega)=-\omega, d(t)=\ln\left(\frac{t}{\sigma}\right),$$

Hence Pareto distribution belongs to one parameter exponential family of distributions.

Theorem: Let $T_i(i = 1,2,.....,n)$ be a random sample from $\mathrm{f}(,\omega)$, $\omega \in \Theta$, where Θ is an interval (possibly infinite). If $\mathrm{f(t,\omega)} = \mathrm{a(\omega)b(t)}$ $\mathrm{\exp[c(\omega)d(t)]}$, then $\left(\sum\limits_{i=1}^{n} d(T_i)\right)$ is a complete sufficient statistic.

Using the above theorem, the complete sufficient statistic of ω is $\left(\sum\limits_{i=1}^{n}\ln\left(\frac{t_i}{\sigma}\right)\right)$.

$$\textit{If } t_i \sim \textit{Pareto}(\sigma,\omega), \quad \textit{Then } \ln\left(\frac{t_i}{\sigma}\right) \sim \exp(\omega)$$

$$\Rightarrow S = \left(\sum_{i=1}^{n} \ln\left(\frac{t_i}{\sigma}\right)\right) \sim \textit{Gamma}(n,\omega)$$

Now the probability density function of S is given as:

$$g(s,n,\omega) = \frac{\omega^n}{\overline{)n}} s^{n-1} e^{-\omega s}; \qquad s \geq 0,\ \omega > 0 \tag{9}$$

Using (9) we have

$$E\left(\frac{S}{n}\right) = \frac{1}{\omega} \quad \text{and} \quad E\left(\frac{n-1}{S}\right) = \omega$$

Since, $\frac{n-1}{S}$ is unbiased estimator for ω, and S is a complete sufficient statistic for ω. Thus, by Lehmann-Scheffe theorem

$$\widehat{\omega}_{UMBUE} = \frac{n-1}{S} \tag{10}$$

and an estimator of reliability function based on UMVUE of ω may be given by

$$R(t)_U = \left(\frac{\sigma}{t}\right)^{\widehat{\omega}_{UMVUE}} \tag{11}$$

3 Bayesian Estimation

Bayesian estimation is not always easy task as the likelihood function and the prior provide typical posterior forms which are sometimes impossible to analyze logically. Kifayat et al. (2012) obtained Bayes estimator of the parameter of power distribution by using two informative and two non-informative priors. Setiya and Kumar (2013) derived Bayes estimators of θ of Pareto distribution by using two different approaches for different priors under SELF and APLF. Asgharjadeh (2014) obtained MLE and Bayes estimators of two unknown parameters of Pareto Distribution for left censored data. A loss function shows the associated risk with an event. If loss is symmetric, we prefer to use Squares Error Loss Function (SELF) which gives equal weightage to over estimation and under estimation. If loss is not symmetric, then it is reasonable to use other loss functions like Quadratic Loss Function (QLF) or asymmetric precautionary loss function (APLF). In

Table 1 Bayes estimators of ω using different priors under SELF, QLF & APLF

Prior	Loss F	Risk Function	Bayes Estimator
Jeffrey's	SELF	$\psi = \widehat{\omega}_J^2 + \frac{n(n+1)}{S^2} - \frac{2\widehat{\omega}_J n}{S}$	$\widehat{\omega}_J = \frac{n}{S}$
	QLF	$\psi_Q = 1 + \widehat{\omega}_J^2 \left(\frac{S^2}{n(n-1)}\right) - \frac{2\widehat{\omega}_J S}{n}$	$\widehat{\omega}_{JQ} = \frac{n-1}{S}$
	APLF	$\psi_A = \widehat{\omega}_J + \frac{n(n+1)}{\widehat{\omega}_J S^2} - \frac{2n}{S}$	$\widehat{\omega}_{JA} = \frac{\sqrt{n(n+1)}}{S}$
Exponential	SELF	$\psi = \widehat{\omega}_E^2 + \frac{(n+2)(n+1)}{\left(S+\frac{1}{\beta}\right)^2} - \frac{2\widehat{\omega}_E(n+1)}{\left(S+\frac{1}{\beta}\right)}$	$\widehat{\omega}_E = \frac{n+1}{\left(S+\frac{1}{\beta}\right)}$
	QLF	$\psi_Q = 1 + \widehat{\omega}_E^2 \left(\frac{(S+(1/\beta))^2}{n(n-1)}\right) - \frac{2\widehat{\omega}_E(S+(1/\beta))}{n}$	$\widehat{\omega}_{EQ} = \frac{n}{\left(S+\frac{1}{\beta}\right)}$
	APLF	$\psi_A = \widehat{\omega}_E + \frac{(n+2)(n+1)}{\widehat{\omega}_E\left(S+\frac{1}{\beta}\right)^2} - \frac{2(n+1)}{\left(S+\frac{1}{\beta}\right)}$	$\widehat{\omega}_{EA} = \frac{\sqrt{(n+1)(n+2}}{\left(S+\frac{1}{\beta}\right)}$
Gamma	SELF	$\psi = \widehat{\omega}_G^2 + \frac{(n+\lambda)(n+\lambda+1)}{(S+a)^2} - \frac{2\widehat{\omega}_G(n+\lambda)}{(S+a)}$	$\widehat{\omega}_G = \frac{n+\lambda}{(S+a)}$
	QLF	$\psi_Q = 1 + \widehat{\omega}_G^2 \left(\frac{(S+a)^2}{(n+\lambda)(n+\lambda-1)}\right) - \frac{2\widehat{\omega}_E(S+a)}{n+\lambda}$	$\widehat{\omega}_{GQ} = \frac{n+\lambda-1}{(S+a)}$
	APLF	$\psi_A = \widehat{\omega}_G + \frac{(n+\lambda)(n+\lambda+1)}{\widehat{\omega}_G(S+a)^2} - \frac{2(n+\lambda)}{(S+a)}$	$\widehat{\omega}_{GA} = \frac{\sqrt{(n+\lambda)(n+\lambda+1)}}{(S+a)}$

this section, we have obtained Bayes estimators of shape parameter of (1) by using Jeffrey's, exponential and gamma priors under SELF, QLF and APLF which are shown in Table 1.

4 Bayesian Estimation of Reliability Function

Bayesian idea was firstly introduced in reliability and life testing by Bhattacharya (1967). Bayesian inference approach is used to estimate an unknown parameter which requires a specific prior distribution and this prior distribution is totally based on theoretical knowledge. Howlader et.al (2007) has presented HPD intervals for parameters of Pareto distribution and reliability function based on NCP. Pandey and Rao (2009) have obtained Bayes estimator of reliability function of Pareto distribution under different loss functions. In this section, we obtained Bayes estimators of reliability function of (1) by using Jeffrey's, exponential and gamma prior under SELF, QLF and APLF.

We know that $R(t) = \left(\frac{\sigma}{t}\right)^{\omega}$

$$\log_e R(t) = \omega \log_e(\sigma/t) \quad \Rightarrow \quad \omega = \frac{\log_e R(t)}{\log_e(\sigma/t)} \tag{12}$$

Hence

$$\pi(R(t)/t_1, t_2, ..., t_n) = \pi(\omega/t_1, t_2, ..., t_n)|J| \tag{13}$$

4.1 Jeffrey's Prior

The posterior distribution of reliability function given the random sample for fixed σ under Jeffrey's prior is given by

$$\pi(R(t)/t_1, t_2, ..., t_n) = \frac{S^n}{\overline{)n}} e^{-SP} P^{n-1} \frac{1}{R(t)\log_e(\sigma/t)};\, ; S>0, n>0, \sigma>0$$

where

$$P = \left(\frac{\log_e R(t)}{\log_e(\sigma/t)}\right) \tag{14}$$

(i) Risk Function under SELF

$$R = \int_0^\infty [\widehat{R}\ (t) - R(t)]^2 \pi[R(t)/t_1, t_2, ..., t_n] dR(t)$$

$$R = [\widehat{R}\ (t)]^2 + \left[\frac{S}{S - 2\log_e(\sigma/t)}\right]^n - 2\ \widehat{R}\ (t) \left[\frac{S}{S - \log_e(\sigma/t)}\right]^n$$

Solving $\frac{\partial R}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_J = \left[\frac{S}{S - \log_e(\sigma/t)}\right]^n \tag{15}$$

(ii) Risk Function under QLF

$$R_Q = \int_0^\infty \left(1 - \frac{\widehat{R}\ (t)}{R(t)}\right)^2 \pi[R(t)/t_1, t_2, ..., t_n] dR(t)$$

$$R_Q = 1 + [\widehat{R}\ (t)]^2 \left[\frac{S}{S + 2\log_e(\sigma/t)}\right]^n - 2\ \widehat{R}\ (t) \left[\frac{S}{S + \log_e(\sigma/t)}\right]^n$$

Solving $\frac{\partial R_Q}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_{JQ} = \left[\frac{S + 2\log_e(\sigma/t)}{S + \log_e(\sigma/t)}\right]^n \tag{16}$$

(iii) Risk function under APLF

$$R_A = \int_0^\infty \left(\frac{[\widehat{R}(t) - R(t)]^2}{\widehat{R}(t)} \right) \pi[R(t)/t_1, t_2, ..., t_n] dR(t)$$

$$R_A = [\widehat{R}(t)] + \frac{1}{\widehat{R}(t)} \left[\frac{S}{S - 2\log_e(\sigma/t)} \right]^n - 2 \left[\frac{S}{S - \log_e(\sigma/t)} \right]^n$$

Solving $\frac{\partial R_A}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}(t)]_{JA} = \left[\frac{S}{S - 2\log_e(\sigma/t)} \right]^{n/2} \tag{17}$$

4.2 Exponential Prior

The posterior distribution of reliability function given the random sample for fixed σ under exponential prior is given by

$$\pi(R(t)/t_1, t_2, ..., t_n) = \frac{\left(S + \frac{1}{\beta}\right)^{n+1}}{\rceil n+1} e^{-\left(S + \frac{1}{\beta}\right)P} \times P^{n+1-1} \frac{1}{R(t)\log_e(\sigma/t)}; \quad S > 0, n > 0, \beta > 0$$

where

$$P = \left(\frac{\log_e R(t)}{\log_e(\sigma/t)} \right) \tag{18}$$

(i) Risk function under SELF

$$R = \int_0^\infty [\widehat{R}(t) - R(t)]^2 \pi[R(t)/t_1, t_2, ..., t_n] dR(t)$$

$$R = [\widehat{R}(t)]^2 + \left[\frac{S + (1/\beta)}{S + (1/\beta) - 2\log_e(\sigma/t)} \right]^{n+1} - 2\widehat{R}(t) \left[\frac{S + (1/\beta)}{S + (1/\beta) - \log_e(\sigma/t)} \right]^{n+1}$$

Solving $\frac{\partial R}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_E = \left[\frac{S + (1/\beta)}{S + (1/\beta) - \log_e(\sigma/t)}\right]^{n+1} \tag{19}$$

(ii) Risk function under QLF

$$R_Q = \int_0^\infty \left(1 - \frac{\widehat{R}\ (t)}{R(t)}\right)^2 \pi[R(t)/t_1, t_2, \ldots\ldots, t_n] dR(t)$$

$$R_Q = 1 + [\widehat{R}\ (t)]^2 \left[\frac{S + (1/\beta)}{S + (1/\beta) + 2\log_e(\sigma/t)}\right]^{n+1}$$
$$- 2\ \widehat{R}\ (t) \left[\frac{S + (1/\beta)}{S + (1/\beta) + \log_e(\sigma/t)}\right]^{n+1}$$

Solving $\frac{\partial R_Q}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_{EQ} = \left[\frac{S + (1/\beta) + 2\log_e(\sigma/t)}{S + (1/\beta) + \log_e(\sigma/t)}\right]^{n+1} \tag{20}$$

(iii) Risk function under APLF

$$R_A = \int_0^\infty \left(\frac{[\widehat{R}\ (t) - R(t)]^2}{\widehat{R}\ (t)}\right) \pi[R(t)/t_1, t_2, \ldots\ldots, t_n] dR(t)$$

$$R_A = [\widehat{R}\ (t)] + \frac{1}{\widehat{R}\ (t)} \left[\frac{S + (1/\beta)}{S + (1/\beta) - 2\log_e(\sigma/t)}\right]^{n+1}$$
$$- 2 \left[\frac{S + (1/\beta)}{S + (1/\beta) - \log_e(\sigma/t)}\right]^{n+1}$$

Solving $\frac{\partial R_A}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_{EA} = \left[\frac{S + (1/\beta)}{S + (1/\beta) - 2\log_e(\sigma/t)}\right]^{(n+1)/2} \tag{21}$$

4.3 Gamma Prior

The posterior distribution of reliability function given the random sample for fixed σ under gamma prior is given by

$$\pi(R(t)/t_1, t_2, ..., t_n) = \frac{(S+a)^{n+\lambda}}{\overline{)n+\lambda}}\, e^{-(S+a)P} \times P^{n+\lambda-1} \frac{1}{R(t)\log_e(\sigma/t)}; S>0, n>0, a>0, \lambda>0$$

where

$$P = \left(\frac{\log_e R(t)}{\log_e(\sigma/t)}\right)$$

(i) Risk Function under SELF

$$R = \int_0^\infty [\widehat{R}\,(t) - R(t)]^2 \pi[R(t)/t_1, t_2, \ldots\ldots, t_n] dR(t)$$

$$R = [\widehat{R}\,(t)]^2 + \left[\frac{S+a}{S+a-2\log_e(\sigma/t)}\right]^{n+\lambda} - 2\,\widehat{R}\,(t)\left[\frac{S+a}{S+a-\log_e(\sigma/t)}\right]^{n+\lambda}$$

Solving $\frac{\partial R}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\,(t)]_G = \left[\frac{S+a}{S+a-\log_e(\sigma/t)}\right]^{n+\lambda} \tag{22}$$

(ii) Risk Function under QLF

$$R_Q = \int_0^\infty \left(1 - \frac{\widehat{R}\,(t)}{R(t)}\right)^2 \pi[R(t)/t_1, t_2, ..., t_n] dR(t)$$

$$R_Q = 1 + [\widehat{R}\,(t)]^2 \left[\frac{S+a}{S+a+2\log_e(\sigma/t)}\right]^{n+\lambda} - 2\,\widehat{R}\,(t)\left[\frac{S+a}{S+a+\log_e(\sigma/t)}\right]^{n+\lambda}$$

Solving $\frac{\partial R_Q}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_{GQ} = \left[\frac{S + a + 2\log_e(\sigma/t)}{S + a + \log_e(\sigma/t)}\right]^{n+\lambda} \tag{23}$$

(iii) Risk Function under APLF

$$R_A = \int_0^{\infty} \left(\frac{[\widehat{R}\ (t) - R(t)]^2}{\widehat{R}\ (t)}\right) \pi[R(t)/t_1, t_2, \ldots\ldots, t_n] dR(t)$$

$$R_A = [\widehat{R}\ (t)] + \frac{1}{\widehat{R}\ (t)} \left[\frac{S + a}{S + a - 2\log_e(\sigma/t)}\right]^{n+\lambda} - 2\left[\frac{S + a}{S + a - \log_e(\sigma/t)}\right]^{n+\lambda}$$

Solving $\frac{\partial R_A}{\partial \widehat{R}(t)} = 0$, we get

$$[\widehat{R}\ (t)]_{GA} = \left[\frac{S + a}{S + a - 2\log_e(\sigma/t)}\right]^{(n+\lambda)/2} \tag{24}$$

5 Illustration

5.1 Simulation Study

For simulation study, we have used $\mathrm{R} = 1000$ replications, for samples of sizes $\mathrm{n} = 20, 50$, and 100 respectively from Pareto distribution for shape parameter $\omega = 2$ and fixed $\sigma = 4$ and 4.3 respectively. We have chosen $\beta = 1, 2$ for the Exponential prior and $(\lambda, \mathrm{a}) = (1, 1), (1, 2)$ respectively for gamma prior. The simulation was done by using R program. After estimating the parameters, mean square error may be calculated by

$$MSE = \frac{1}{R}\sum_{i=1}^{R}(\widehat{\omega}_i - \omega)^2$$

The results of simulation study for estimation of shape parameter (ω) along with their MLE, UMVUE, Bayes Estimator values and MSE's for estimating ω as well as the reliability function R(t) are summarized and tabulated in Tables A.1 to A.6.

5.2 Real Data Set

A real data set of Norwegian Fire Claims is considered for illustration of the proposed methodology. This data set represents the total damage of 142 fires in Norway for the year1975 (see Rizzo, Page No. 709). This data is well fitted to Pareto distribution hence we compare different estimators for this data which are shown in Tables A.7–A.8.

6 Conclusion

In this paper, Bayes estimation of reliability function of Pareto distribution under three different loss functions viz. SELF, QLF and APLF has been observed. The results obtained from the simulation study and of real data are presented in Tables A.1–A.7. From Table (A.1–A.3), we observe that for almost all values of t and σ, Bayes estimator of Reliability function under APLF has uniformly smaller MSE than its other Bayesian complements as well as MLE and UMVUE based Reliability function. Table A.4 shows that UMVUE of ω has uniformly smaller MSE than its Bayesian counterparts as well as MLE of ω with Jeffrey's prior. From Table A.5, it may be seen that Bayes estimator of ω under QLF with exponential prior ($\beta = 1$ and $\beta = 2$) has smaller MSE for all sample sizes than its other Bayesian complements as well as MLE and UMVUE of ω. From Table A.6, it is seen that Bayes estimator of ω under QLF with gamma prior ($a = 1$ and $\lambda = 1$) has smaller MSE for all sample sizes than its MLE and UMVUE as well as Bayesian estimators while with gamma prior ($a = 2$ and $\lambda = 1$), Bayes estimator of ω under APLF is best. Table A.7 shows that Bayes estimator of ω under QLF has smaller expected loss as compared to other complements and in Table A.8, it may be seen that under SELF, Bayes estimator of reliability function has smaller expected loss.

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Appendix

Table A.1 MLE, UMVUE and Bayes estimators of reliability function under Jeffrey's prior by using SELF, QLF and APLF for $\omega = 2$

n, σ	t	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20, 4	4.5	0.790123	0.780813	0.0022587	0.790464	0.002015	0.782050	0.002187
	5	0.640000	0.627681	0.0049349	0.642277	0.004549	0.631135	0.004689
	5.5	0.528926	0.516393	0.0064791	0.533428	0.006150	0.522028	0.006099
	6	0.444444	0.432909	0.0070898	0.450998	0.006910	0.440395	0.006658
	6.5	0.378698	0.368627	0.0071252	0.386986	0.007115	0.377589	0.006709
50, 4	4.5	0.790123	0.786188	0.0007959	0.78997	0.000757	0.786649	0.000786
	5	0.640000	0.634652	0.0018289	0.640422	0.001762	0.635973	0.001791
	5.5	0.528926	0.523345	0.0024962	0.530119	0.002432	0.525541	0.002434
	6	0.444444	0.439174	0.002815	0.446397	0.002772	0.442136	0.00274
	6.5	0.378698	0.373971	0.0028966	0.381322	0.00288	0.377559	0.00282
100, 4	4.5	0.790123	0.78721	0.0003777	0.789093	0.000365	0.787436	0.000375
	5	0.640000	0.635857	0.0008736	0.638736	0.000849	0.636511	0.000863
	5.5	0.528926	0.524385	0.0011975	0.52777	0.00117	0.525478	0.001179
	6	0.444444	0.439921	0.0013541	0.443534	0.00133	0.441402	0.00133
	6.5	0.378698	0.374395	0.0013953	0.378073	0.001377	0.376194	0.001369
20, 4.3	4.5	0.913086	0.909111	0.000389	0.913443	0.000341	0.909326	0.000384
	5	0.739600	0.730226	0.002612	0.741711	0.002363	0.732071	0.002512
	5.5	0.611240	0.60012	0.004491	0.615456	0.004183	0.60406	0.004254
	6	0.513611	0.502486	0.005558	0.519801	0.005317	0.508407	0.005226
	6.5	0.437633	0.427308	0.005995	0.44551	0.005877	0.434919	0.005628
50, 4.3	4.5	0.913086	0.911574	0.000154	0.913262	0.000146	0.911653	0.000153
	5	0.739600	0.736058	0.001079	0.740568	0.001038	0.736756	0.001063
	5.5	0.611240	0.607087	0.001914	0.613146	0.001864	0.608606	0.001875
	6	0.513611	0.509526	0.002428	0.516396	0.002389	0.511842	0.002371
	6.5	0.437633	0.433928	0.002668	0.441172	0.002651	0.436938	0.002605
100,4.3	4.5	0.913086	0.912199	7.33E-05	0.913037	7.12E-05	0.912238	7.31E-05
	5	0.739600	0.737464	0.00052	0.73971	0.000509	0.737808	0.000516
	5.5	0.611240	0.608660	0.000933	0.611682	0.000918	0.609412	0.000923
	6	0.513611	0.510987	0.001193	0.514417	0.001179	0.512138	0.001178
	6.5	0.437633	0.435158	0.001319	0.438778	0.001311	0.436661	0.001302

(*Continued*)

Table A.1 Continued

n, σ	T	R(t)	R(t) Q	MSE	R(t) A	MSE
20, 4	4.5	0.790123	0.777002	0.0025051	0.783267	0.0021204
	5	0.640000	0.616857	0.0058933	0.634499	0.0044777
	5.5	0.528926	0.498570	0.0081564	0.527477	0.0058012
	6	0.444444	0.409127	0.0092632	0.447604	0.0063585
	6.5	0.378698	0.340136	0.0095499	0.386202	0.0064703
50, 4	4.5	0.790123	0.784791	0.0008295	0.787107	0.0007762
	5	0.640000	0.630625	0.0019671	0.63728	0.0017577
	5.5	0.528926	0.51662	0.0027503	0.527708	0.0023826
	6	0.444444	0.430083	0.0031592	0.445052	0.0026840
	6.5	0.378698	0.362944	0.0032963	0.381089	0.0027709
100, 4	4.5	0.790123	0.786526	0.0003869	0.787662	0.0003721
	5	0.640000	0.633879	0.0009123	0.637161	0.0008525
	5.5	0.528926	0.521073	0.00127	0.526564	0.0011621
	6	0.444444	0.43543	0.0014541	0.442871	0.0013108
	6.5	0.378698	0.368931	0.0015136	0.377979	0.0013505
20, 4.3	4.5	0.913086	0.908459	0.000406	0.90954	0.000379
	5	0.739600	0.724511	0.002978	0.73388	0.002421
	5.5	0.611240	0.587745	0.005471	0.607891	0.004058
	6	0.513611	0.483739	0.007123	0.51413	0.00498
	6.5	0.437633	0.403117	0.007982	0.442247	0.005393
50, 4.3	4.5	0.913086	0.911334	0.000157	0.911733	0.000153
	5	0.739600	0.733936	0.001133	0.73745	0.001048
	5.5	0.611240	0.602449	0.002064	0.610109	0.001842
	6	0.513611	0.502431	0.00267	0.514126	0.002327
	6.5	0.437633	0.424685	0.002981	0.439903	0.002561
100, 4.3	4.5	0.913086	0.912082	7.4E-05	0.912277	7.29E-05
	5	0.739600	0.736427	0.000534	0.738151	0.000513
	5.5	0.611240	0.606385	0.000971	0.61016	0.000914
	6	0.513611	0.507495	0.001256	0.513282	0.001165
	6.5	0.437633	0.430597	0.001402	0.438152	0.001289

Table A.2 MLE, UMVUE and Bayes estimators of reliability function under exponential prior by using SELF, QLF and APLF for $\omega = 2$

$\beta = 1, \sigma = 4$								
n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.790123	0.780813	0.0022587	0.790464	0.002015	0.792301	0.001571
	5	0.640000	0.627681	0.0049349	0.642277	0.004549	0.646125	0.003572
	5.5	0.528926	0.516393	0.0064791	0.533428	0.006150	0.539046	0.004882
	6	0.444444	0.432909	0.0070898	0.450998	0.006910	0.458053	0.005560
	6.5	0.378698	0.368627	0.0071252	0.386986	0.007115	0.395165	0.005812
50	4.5	0.790123	0.786188	0.0007959	0.78997	0.000757	0.790546	0.000691
	5	0.640000	0.634652	0.0018289	0.640422	0.001762	0.641838	0.001604
	5.5	0.528926	0.523345	0.0024962	0.530119	0.002432	0.532350	0.002217
	6	0.444444	0.439174	0.0028150	0.446397	0.002772	0.449327	0.002533
	6.5	0.378698	0.373971	0.0028966	0.381322	0.00288	0.384817	0.002642
100	4.5	0.790123	0.78721	0.0003777	0.789093	0.000365	0.789367	0.000348
	5	0.640000	0.635857	0.0008736	0.638736	0.000849	0.639442	0.000808
	5.5	0.528926	0.524385	0.0011975	0.52777	0.00117	0.528905	0.001112
	6	0.444444	0.439921	0.0013541	0.443534	0.00133	0.445042	0.001265
	6.5	0.378698	0.374395	0.0013953	0.378073	0.001377	0.379886	0.001311
n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.790123	0.78121	0.002155	0.790849	0.001926	0.78244	0.002087
	5	0.640000	0.628199	0.004717	0.642789	0.004356	0.63164	0.004482
	5.5	0.528926	0.516915	0.006198	0.533954	0.005895	0.522535	0.005835
	6	0.444444	0.433385	0.006783	0.451487	0.006626	0.440858	0.006371
	6.5	0.378698	0.369039	0.006815	0.387419	0.006821	0.377993	0.006419
50	4.5	0.790123	0.786902	0.000706	0.790674	0.000675	0.78736	0.000698
	5	0.640000	0.635671	0.001625	0.641432	0.001574	0.636983	0.001593
	5.5	0.528926	0.524465	0.002221	0.531237	0.002176	0.52665	0.002168
	6	0.444444	0.440295	0.002506	0.447521	0.002481	0.443245	0.002444
	6.5	0.378698	0.375043	0.002581	0.382402	0.00258	0.37862	0.002518
100	4.5	0.790123	0.788547	0.000359	0.790421	0.000351	0.788771	0.000357
	5	0.640000	0.637893	0.000833	0.640761	0.000821	0.63854	0.000825
	5.5	0.528926	0.526771	0.001146	0.530147	0.001135	0.527853	0.001133
	6	0.444444	0.442459	0.001330	0.446067	0.001294	0.443927	0.001284
	6.5	0.378698	0.37697	0.001343	0.380648	0.001343	0.378757	0.001326

(*Continued*)

Table A.2 Continued

n	T	R(t)	R(t) Q	MSE	R(t) A	MSE
			$\beta = 1, \sigma = 4$			
20	4.5	0.790123	0.787977	0.0017095	0.793348	0.0015451
	5	0.640000	0.633678	0.0040928	0.649073	0.0035023
	5.5	0.528926	0.518284	0.005759	0.543896	0.0048174
	6	0.444444	0.430008	0.0066421	0.464552	0.0055535
	6.5	0.378698	0.361168	0.0069463	0.403012	0.0058958
50	4.5	0.790123	0.788792	0.0007156	0.790978	0.0006855
	5	0.640000	0.636765	0.0017019	0.643079	0.0015889
	5.5	0.528926	0.523849	0.0023863	0.534417	0.0021985
	6	0.444444	0.437792	0.0027486	0.452121	0.0025223
	6.5	0.378698	0.370778	0.0028755	0.388211	0.0026465
100	4.5	0.790123	0.788482	0.0003559	0.789587	0.0003461
	5	0.640000	0.636879	0.0008399	0.640076	0.0008018
	5.5	0.528926	0.524604	0.0011702	0.529966	0.001104
	6	0.444444	0.439200	0.0013410	0.446479	0.0012566
	6.5	0.378698	0.372767	0.0013970	0.381635	0.0013054
			$\beta = 2, \sigma = 4$			
20	4.5	0.790123	0.777419	0.00239	0.78365	0.002023
	5	0.640000	0.617417	0.005638	0.634991	0.004281
	5.5	0.528926	0.499139	0.007818	0.527971	0.005553
	6	0.444444	0.409642	0.008891	0.448056	0.00609
	6.5	0.378698	0.34057	0.009175	0.386598	0.006199
50	4.5	0.790123	0.785517	0.000736	0.787814	0.000689
	5	0.640000	0.631671	0.001746	0.638282	0.001565
	5.5	0.528926	0.517776	0.002445	0.528806	0.002126
	6	0.444444	0.43124	0.002812	0.44615	0.0024
	6.5	0.378698	0.364048	0.002938	0.38214	0.002482
100	4.5	0.790123	0.787872	0.000366	0.788994	0.000355
	5	0.640000	0.635938	0.000863	0.639183	0.000818
	5.5	0.528926	0.52349	0.001202	0.528929	0.001122
	6	0.444444	0.438004	0.001375	0.445384	0.001272
	6.5	0.378698	0.371545	0.001431	0.380529	0.001317

(*Continued*)

Table A.2 Continued

$\beta = 1, \sigma = 4.3$

n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.913086	0.909111	0.000389	0.913443	0.000341	0.913924	0.000268
	5	0.739600	0.730226	0.002612	0.741711	0.002363	0.743871	0.001863
	5.5	0.611240	0.60012	0.004491	0.615456	0.004183	0.619408	0.003323
	6	0.513611	0.502486	0.005558	0.519801	0.005317	0.525359	0.004269
	6.5	0.437633	0.427308	0.005995	0.44551	0.005877	0.452411	0.00478
50	4.5	0.913086	0.911574	0.000154	0.913262	0.000146	0.913393	0.000134
	5	0.739600	0.736058	0.001079	0.740568	0.001038	0.741340	0.000946
	5.5	0.611240	0.607087	0.001914	0.613146	0.001864	0.614689	0.001699
	6	0.513611	0.509526	0.002428	0.516396	0.002389	0.518670	0.002183
	6.5	0.437633	0.433928	0.002668	0.441172	0.002651	0.444076	0.002431
100	4.5	0.913086	0.912199	7.33E-05	0.913037	7.12E-05	0.913092	6.82E-05
	5	0.739600	0.737464	0.00052	0.73971	0.000509	0.74008	0.000486
	5.5	0.611240	0.60866	0.000933	0.611682	0.000918	0.612451	0.000876
	6	0.513611	0.510987	0.001193	0.514417	0.001179	0.515571	0.001126
	6.5	0.437633	0.435158	0.001319	0.438778	0.001311	0.440267	0.001252

n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.913086	0.909081	0.00043	0.913413	0.000378	0.909695	0.000374
	5	0.739600	0.730283	0.002896	0.741756	0.002629	0.732962	0.002481
	5.5	0.611240	0.600361	0.004991	0.615671	0.004665	0.605163	0.004248
	6	0.513611	0.502927	0.006191	0.520203	0.005944	0.509575	0.005266
	6.5	0.437633	0.427934	0.006695	0.446089	0.006585	0.436082	0.005712
50	4.5	0.913086	0.9123272	0.000145	0.9140014	0.00014	0.9124505	0.0001382
	5	0.739600	0.7380519	0.0010243	0.7425353	0.0010025	0.7388306	0.0009703
	5.5	0.611240	0.609744	0.0018325	0.6157769	0.0018146	0.6113294	0.0017304
	6	0.513611	0.5125181	0.0023399	0.5193696	0.0023419	0.514869	0.0022082
	6.5	0.437633	0.4370635	0.0025873	0.4442987	0.0026148	0.4400751	0.0024449
100	4.5	0.913086	0.9122478	7.229E-05	0.9130856	7.029E-05	0.9123027	7.059E-05
	5	0.739600	0.7375924	0.0005151	0.7398373	0.0005041	0.7379713	0.0005007
	5.5	0.611240	0.6088294	0.0009272	0.6118505	0.0009126	0.6096196	0.0008987
	6	0.513611	0.5111765	0.0011893	0.5146066	0.0011767	0.5123623	0.0011511
	6.5	0.437633	0.4353566	0.0013193	0.4389769	0.0013116	0.4368868	0.0012763

(*Continued*)

Table A.2 Continued

n	T	R(t)	R(t) Q	MSE	R(t) A	MSE
			$\beta = 1, \sigma = 4.3$			
20	4.5	0.913086	0.913187	0.000277	0.914105	0.000266
	5	0.739600	0.737339	0.002048	0.745441	0.001833
	5.5	0.611240	0.605107	0.0038	0.622784	0.003278
	6	0.513611	0.503461	0.004996	0.530467	0.004252
	6.5	0.437633	0.423858	0.005654	0.459025	0.004826
50	4.5	0.913086	0.913093	0.000136	0.913467	0.000133
	5	0.739600	0.738673	0.000984	0.741996	0.000940
	5.5	0.611240	0.608841	0.001796	0.616118	0.001687
	6	0.513611	0.509694	0.002329	0.520851	0.002173
	6.5	0.437633	0.432345	0.002605	0.446918	0.002432
100	4.5	0.913086	0.912941	6.87E-05	0.913129	6.81E-05
	5	0.739600	0.738736	0.000496	0.740413	0.000484
	5.5	0.611240	0.609499	0.000903	0.61318	0.000871
	6	0.513611	0.511036	0.001169	0.516688	0.001121
	6.5	0.437633	0.434334	0.001305	0.441727	0.00125
			$\beta = 2, \sigma = 4.3$			
20	4.5	0.913086	0.908877	0.0003939	0.9098967	0.0003692
	5	0.739600	0.7258195	0.0029007	0.7346755	0.0023995
	5.5	0.611240	0.5897338	0.0053486	0.6087975	0.0040701
	6	0.513611	0.4862264	0.0069817	0.5150119	0.0050407
	6.5	0.437633	0.4059579	0.0078416	0.44305	0.0054978
50	4.5	0.913086	0.9121437	0.0001406	0.9125268	0.0001376
	5	0.739600	0.7361122	0.0010229	0.7394994	0.0009598
	5.5	0.611240	0.6053798	0.0018706	0.6127822	0.0017077
	6	0.513611	0.5057573	0.0024289	0.5170826	0.002181
	6.5	0.437633	0.428189	0.0027199	0.4429534	0.0024224
100	4.5	0.913086	0.9121488	7.135E-05	0.912341	7.041E-05
	5	0.739600	0.7366058	0.0005174	0.73831	0.0004972
	5.5	0.611240	0.6066258	0.0009445	0.6103592	0.0008903
	6	0.513611	0.5077701	0.001225	0.5134939	0.0011395
	6.5	0.437633	0.4308876	0.0013711	0.4383626	0.0012643

Table A.3 MLE, UMVUE and Bayes estimators of reliability function under Gamma prior by using SELF, QLF and APLF for $\omega = 2$

				$\lambda = 1, a = 1\ and\ \sigma = 4$				
n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.790123	0.780813	0.0022587	0.790464	0.002015	0.792301	0.001571
	5	0.640000	0.627681	0.0049349	0.642277	0.004549	0.646125	0.003572
	5.5	0.528926	0.516393	0.0064791	0.533428	0.006150	0.539046	0.004882
	6	0.444444	0.432909	0.0070898	0.450998	0.006910	0.458053	0.005560
	6.5	0.378698	0.368627	0.0071252	0.386986	0.007115	0.395165	0.005812
50	4.5	0.790123	0.786188	0.0007959	0.789970	0.000757	0.790546	0.000691
	5	0.640000	0.634652	0.0018289	0.640422	0.001762	0.641838	0.001604
	5.5	0.528926	0.523345	0.0024962	0.530119	0.002432	0.53235	0.002217
	6	0.444444	0.439174	0.002815	0.446397	0.002772	0.449327	0.002533
	6.5	0.378698	0.373971	0.0028966	0.381322	0.00288	0.384817	0.002642
100	4.5	0.790123	0.787210	0.0003777	0.789093	0.000365	0.789367	0.000348
	5	0.640000	0.635857	0.0008736	0.638736	0.000849	0.639442	0.000808
	5.5	0.528926	0.524385	0.0011975	0.527770	0.00117	0.528905	0.001112
	6	0.444444	0.439921	0.0013541	0.443534	0.00133	0.445042	0.001265
	6.5	0.378698	0.374395	0.0013953	0.378073	0.001377	0.379886	0.001311
n	T	R(t)	MLE R(t)	MSE	UMV Based R(t)	MSE	R(t) S	MSE
20	4.5	0.913086	0.909111	0.000389	0.913443	0.000341	0.913924	0.000268
	5	0.739600	0.730226	0.002612	0.741711	0.002363	0.743871	0.001863
	5.5	0.611240	0.60012	0.004491	0.615456	0.004183	0.619408	0.003323
	6	0.513611	0.502486	0.005558	0.519801	0.005317	0.525359	0.004269
	6.5	0.437633	0.427308	0.005995	0.44551	0.005877	0.452411	0.00478
50	4.5	0.913086	0.911574	0.000154	0.913262	0.000146	0.913393	0.000134
	5	0.739600	0.736058	0.001079	0.740568	0.001038	0.74134	0.000946
	5.5	0.611240	0.607087	0.001914	0.613146	0.001864	0.614689	0.001699
	6	0.513611	0.509526	0.002428	0.516396	0.002389	0.51867	0.002183
	6.5	0.437633	0.433928	0.002668	0.441172	0.002651	0.444076	0.002431
100	4.5	0.913086	0.912199	7.33E-05	0.913037	7.12E-05	0.913092	6.82E-05
	5	0.739600	0.737464	0.00052	0.73971	0.000509	0.74008	0.000486
	5.5	0.611240	0.60866	0.000933	0.611682	0.000918	0.612451	0.000876
	6	0.513611	0.510987	0.001193	0.514417	0.001179	0.515571	0.001126
	6.5	0.437633	0.435158	0.001319	0.438778	0.001311	0.440267	0.001252

(*Continued*)

Table A.3 Continued

n	T	R(t)	R(t) Q	MSE	R(t) A	MSE
			$\lambda = 1, a = 1$ *and* $\sigma = 4$			
20	4.5	0.790123	0.787977	0.0017095	0.793348	0.0015451
	5	0.640000	0.633678	0.0040928	0.649073	0.0035023
	5.5	0.528926	0.518284	0.0057590	0.543896	0.0048174
	6	0.444444	0.430008	0.0066421	0.464552	0.0055535
	6.5	0.378698	0.361168	0.0069463	0.403012	0.0058958
50	4.5	0.790123	0.788792	0.0007156	0.790978	0.0006855
	5	0.640000	0.636765	0.0017019	0.643079	0.0015889
	5.5	0.528926	0.523849	0.0023863	0.534417	0.0021985
	6	0.444444	0.437792	0.0027486	0.452121	0.0025223
	6.5	0.378698	0.370778	0.0028755	0.388211	0.0026465
100	4.5	0.790123	0.788482	0.0003559	0.789587	0.0003461
	5	0.640000	0.636879	0.0008399	0.640076	0.0008018
	5.5	0.528926	0.524604	0.0011702	0.529966	0.001104
	6	0.444444	0.439200	0.0013410	0.446479	0.0012566
	6.5	0.378698	0.372767	0.0013970	0.381635	0.0013054
			$\lambda = 1, a = 1$ *and* $\sigma = 4.3$			
20	4.5	0.913086	0.913187	0.000277	0.914105	0.000266
	5	0.739600	0.737339	0.002048	0.745441	0.001833
	5.5	0.611240	0.605107	0.0038	0.622784	0.003278
	6	0.513611	0.503461	0.004996	0.530467	0.004252
	6.5	0.437633	0.423858	0.005654	0.459025	0.004826
50	4.5	0.913086	0.913093	0.000136	0.913467	0.000133
	5	0.739600	0.738673	0.000984	0.741996	0.00094
	5.5	0.611240	0.608841	0.001796	0.616118	0.001687
	6	0.513611	0.509694	0.002329	0.520851	0.002173
	6.5	0.437633	0.432345	0.002605	0.446918	0.002432
100	4.5	0.913086	0.912941	6.87E-05	0.913129	6.81E-05
	5	0.739600	0.738736	0.000496	0.740413	0.000484
	5.5	0.611240	0.609499	0.000903	0.61318	0.000871
	6	0.513611	0.511036	0.001169	0.516688	0.001121
	6.5	0.437633	0.434334	0.001305	0.441727	0.001250

Table A.4 MLE, UMVUE and Bayes Estimators of ω under Jeffrey's Prior with their corresponding MSE's for $\omega = 2$

n	σ	ω MLE	MSE	ω UMV	MSE	ω (SELF)	MSE
20	4	2.116423	0.305778	2.010602	0.238333	2.116423	0.287769
	4.3	2.101020	0.298358	1.995969	0.232550	2.10102	0.234030
50	4	2.047815	0.094761	2.006859	0.085758	2.047815	0.094851
	4.3	2.038492	0.093847	1.997722	0.084931	2.038492	0.090791
100	4	2.033917	0.043948	2.013578	0.041807	2.033917	0.044582
	4.3	2.022352	0.043444	2.002129	0.041327	2.022352	0.042905

n	σ	ω (QLF)	MSE	ω (APLF)	MSE
20	4	2.010602	0.247591	2.168688	0.316381
	4.3	1.995969	0.202018	2.152905	0.258396
50	4	2.006859	0.088946	2.068192	0.099066
	4.3	1.997722	0.085778	2.058776	0.094550
100	4	2.013578	0.042752	2.044061	0.045807
	4.3	2.002129	0.041566	2.032439	0.043882

Table A.5 MLE, UMVUE and Bayes estimators of ω under exponential prior with their corresponding MSE's for $\omega = 2$

n	σ	β	ω MLE	MSE	ω UMV	MSE	ω (SELF)	MSE
20	4	1	2.116423	0.305778	2.010602	0.238333	1.999197	0.192850
	4.3		2.101020	0.298358	1.995969	0.232550	1.987832	0.158821
50	4		2.047815	0.094761	2.006859	0.085758	2.004920	0.081595
	4.3		2.038492	0.093847	1.997722	0.084931	1.996202	0.078556
100	4		2.033917	0.043948	2.013578	0.041807	2.012894	0.040964
	4.3		2.022352	0.043444	2.002129	0.041327	2.001684	0.039842
20	4	2	2.111372	0.303562	2.005803	0.236606	2.099988	0.238490
	4.3		2.102308	0.300259	1.997192	0.234031	2.091679	0.226319
50	4		2.039511	0.093788	1.998721	0.084877	2.03793	0.080369
	4.3		2.020216	0.092102	1.979811	0.083352	2.019004	0.081325
100	4		2.019406	0.043313	1.999212	0.041202	2.019007	0.041304
	4.3		2.021160	0.043387	2.000948	0.041273	2.020743	0.041323

(*Continued*)

Table A.5 Continued

n	σ	β	ω (QLF)	MSE	ω (APLF)	MSE
20	4	1	1.903997	0.184137	2.046244	0.204171
	4.3		1.893173	0.155333	2.034611	0.167427
50	4		1.965608	0.079586	2.024481	0.083769
	4.3		1.957061	0.077335	2.015678	0.080327
100	4		1.992965	0.040044	2.022835	0.041724
	4.3		1.981865	0.039383	2.011569	0.040368
20	4	2	1.999989	0.207249	2.149407	0.261695
	4.3		1.992075	0.197717	2.140902	0.248144
50	4		1.997971	0.075869	2.057813	0.083820
	4.3		1.979416	0.078243	2.038702	0.084049
100	4		1.999017	0.040137	2.028978	0.042188
	4.3		2.000736	0.040087	2.030722	0.042241

Table A.6 MLE, UMVUE and Bayes estimators of ω under Gamma prior with their corresponding MSE's for $\omega = 2$

n	Σ	(a, λ)	θ MLE	MSE	θ UMV	MSE	θ (SELF)	MSE
20	4	(1,1)	2.116423	0.305778	2.010602	0.238333	1.999197	0.192850
	4.3		2.101020	0.298358	1.995969	0.232550	1.987832	0.158821
50	4		2.047815	0.094761	2.006859	0.085758	2.00492	0.081595
	4.3		2.038492	0.093847	1.997722	0.084931	1.996202	0.078556
100	4		2.033917	0.043948	2.013578	0.041807	2.012894	0.040964
	4.3		2.022352	0.043444	2.002129	0.041327	2.001684	0.039842
20	4	(2,1)	2.131816	0.309242	2.025225	0.241033	1.830109	0.154855
	4.3		2.087640	0.295667	1.983258	0.230452	1.799667	0.156966
50	4		2.057113	0.095511	2.015971	0.086437	1.935904	0.070216
	4.3		2.040187	0.094018	1.999384	0.085086	1.921094	0.073778
100	4		2.018375	0.043238	1.998192	0.041131	1.958765	0.035395
	4.3		2.020434	0.043350	2.000230	0.041238	1.960644	0.037307

(*Continued*)

Table A.6 (Continued)

n	σ	(a, λ)	θ (QLF)	MSE	θ (APLF)	MSE
20	4	(1,1)	1.903997	0.184137	2.046244	0.204171
	4.3		1.893173	0.155333	2.034611	0.167427
50	4		1.965608	0.079586	2.024481	0.083769
	4.3		1.957061	0.077335	2.015678	0.080327
100	4		1.992965	0.040044	2.022835	0.041724
	4.3		1.981865	0.039383	2.011569	0.040368
20	4	(2,1)	1.742961	0.180347	1.873176	0.148075
	4.3		1.713969	0.187785	1.842018	0.147354
50	4		1.897945	0.073956	1.954791	0.069448
	4.3		1.883426	0.078518	1.939837	0.072496
100	4		1.939371	0.036706	1.968438	0.035024
	4.3		1.941232	0.038507	1.970326	0.036992

Table A.7 MLE, UMVUE and Bayes estimators of ω under Jeffrey's, exponential and Gamma priors (real data set) for $\widehat{\sigma} = 500$

	β	a, λ	MLE	UMV	(SELF)	(QLF)	(APLF)
$\widehat{\omega}$			1.217870	1.209294			
$\widehat{\omega}_J$					1.217870	1.209294	1.222151
					(0.010445)	(0.007042)	(0.008561)
$\widehat{\omega}_E$	1.5				1.219474	1.210946	1.223731
					(0.010399)	(0.006993)	(0.008512)
$\widehat{\omega}_E$	2.0				1.22121	1.212670	1.225472
					(0.010424)	(0.007021)	(0.008523)
$\widehat{\omega}_G$		2, 1			1.205764	1.197332	1.209973
					(0.010162)	(0.006993)	(0.008721)
$\widehat{\omega}_G$		1, 2			1.224521	1.216018	1.228766
					(0.010408)	(0.006944)	(0.008487)
$\widehat{\omega}_G$		1, 1			1.215727	1.207225	1.219971
					(0.010335)	(0.006993)	(0.008487)

Table A.8 MLE, UMVUE and Bayes Estimators of Reliability Function under different Priors by using SELF, QLF and APLF (Real Data Set)

	MLE	UMV	Jeffrey's			Exponential ($\beta = 1.5$)			Exponential ($\beta = 2.0$)		
T	R(t)	R(t)	R(t) S	R(t) Q	R(t) A	R(t) S	R(t) Q	R(t) A	R(t) S	R(t) Q	R(t) A
550	0.89040	0.89113	0.89045	0.89028	0.89049	0.89031	0.89014	0.89035	0.89016	0.88999	0.89020
			(0.00007508)	(0.0000949)	(0.0000843)	(0.0000747)	(0.0000946)	(0.0000839)	(0.00007492)	(0.00009484)	(0.00008416)
1550	0.25210	0.25456	0.25378	0.24702	0.25545	0.25331	0.24659	0.25497	0.25282	0.24610	0.25448
			(0.00085045)	(0.0135354)	(0.0033389)	(0.0008436)	(0.0134751)	(0.0033185)	(0.00084279)	(0.0135136)	(0.0033215)
2550	0.13749	0.13942	0.13939	0.13176	0.14128	0.13901	0.13144	0.14089	0.13863	0.13106	0.14051
			(0.00053139)	(0.0281037)	(0.0037847)	(0.0005263)	(0.0279780)	(0.0037586)	(0.00052486)	(0.0280577)	(0.0037587)
3550	0.09189	0.09344	0.09373	0.08633	0.09557	0.09343	0.08613	0.09525	0.09311	0.08582	0.09494
			(0.00034799)	(0.0406503)	(0.0036745)	(0.0003442)	(0.0404684)	(0.0036472)	(0.00034296)	(0.0405833)	(0.0036453)
4550	0.06792	0.06922	0.06965	0.06278	0.07138	0.06939	0.06258	0.07111	0.06931	0.06232	0.07085
			(0.00024418)	(0.0515298)	(0.0034605)	(0.0002413)	(0.0512993)	(0.0034334)	(0.00024024)	(0.0514443)	(0.0034301)
			Gamma (a = 2, $\lambda = 1$)			Gamma (a = 1, $\lambda = 2$)			Gamma (a = 1, $\lambda = 1$)		
T			R(t) S	R(t) Q	R(t) A	R(t) S	R(t) Q	R(t) A	R(t) S	R(t) Q	R(t) A
550			0.89147	0.88606	0.89151	0.88988	0.88971	0.88992	0.89062	0.890464	0.890673
			(0.0000733)	(0.0000925)	(0.0000821)	(0.0000748)	(0.0000947)	(0.0000840)	(0.0000744)	(0.0000941)	(0.00008348)
1550			0.25724	0.25056	0.25888	0.25187	0.24518	0.25353	0.254386	0.247679	0.256042
			(0.000851)	(0.013173)	(0.003295)	(0.000835)	(0.013492)	(0.0033041)	(0.000845)	(0.013399)	(0.0033126)
2550			0.14211	0.13454	0.14399	0.13788	0.13036	0.13975	0.139859	0.132288	0.141738
			(0.000538)	(0.027353)	(0.003757)	(0.000518)	(0.028011)	(0.0037330)	(0.000529)	(0.027821)	(0.0037582)
3550			0.09593	0.08860	0.09776	0.09251	0.08528	0.09432	0.094109	0.086805	0.095934
			0.(0.000355)	(0.039567)	(0.0036617)	(0.000338)	(0.040516)	(0.0036164)	(0.000347)	(0.04024)	(0.0036509)
4550			0.07149	0.06461	0.07322	0.06863	0.06188	0.07033	0.069964	0.063131	0.071684
			(0.000250)	(0.050161)	(0.0034583)	(0.000236)	(0.051358)	(0.0034002)	(0.000244)	(0.05101)	(0.0034397)

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