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ESTIMATION OF POPULATION VARIANCE IN LOG – PRODUCT TYPE ESTIMATORS UNDER DOUBLE SAMPLING SCHEME

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Abstract

It is experienced that auxiliary information when suitably incorporated yields more efficient and precise estimates. Mishra et al. (2017) have introduced a log type estimator for estimating unknown population mean using ancillary information in simple random sampling. Here we propose an improved log-product type estimator for population variance under double sampling. Properties of the estimators are studied both mathematically and numerically.

Key Words: Bias, Mean Square Error, Auxiliary information, Double Sampling, Unbiased Estimator.

1. Introduction

Estimating population variance in finite sampling is an important issue in survey sampling. Also, it is known and established actuality that appropriate use of supplementary or auxiliary information leads to considerable increase in the efficiency of estimates of the parameters of an estimator. Ratio and regression methods are commonly employed in improving efficiency of estimates. In order to obtain more precise estimates researchers have utilized distinguishing forms of auxiliary information. To measure variability within y values (study variable) the problem of estimation of finite population variance has seized considerable importance in survey sampling. Several authors including Das and Tripathi (1978), Srivastava and Jhajj (1980), Misra (2016), Isaki(1983), Singh et.al (2001), Kadilar and Cingi (2006), Singh and Malik (2014), Singh et al. (2014), Sharma and Singh (2014), Mishra and Singh (2016), Sharma et al. (2018), Adichwal et al. (2016), Bandopadhaya and Singh (2015) and others have suggested improved variance estimator using auxiliary information. Consider a situation where no prior information on auxiliary variable is available, in such situations initially we select a large sample from population for obtaining auxiliary information only and then select a second sample from the selected large sample in which variable of interest(y) is observed in addition to auxiliary information. Above procedure of drawing sample is referred to as double sampling scheme. In double sampling Singh and Singh (2003), Ahmed et al. (2003), Jhajj et al. (2005),Jhajj et al. (2011), Grover (2010, 2011), Jararha et al. (2002), Singh (1991), Giancarlo et al. (2004) had proposed estimators for population variance. Here, motivated by Mishra et al. (2017), we have suggested an improved estimator for variance and studied its properties.

1.1 Notations

Let U be a finite population of size N from which we draw a sample of size n using SRSWOR. Let Y_i and X_i denote the respective values of variable y and x on the i^{th} (i=1,2,….,N) unit of the population. Denoting,

$$S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2,\ S_X^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2,$$

$$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2,\ s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2, s_x^{'2} = \frac{1}{n'-1}\sum_{i=1}^{n'}(x_i - \overline{x})^2,$$

$$\lambda = \left(\frac{1}{n} - \frac{1}{N}\right),\ \lambda' = \left(\frac{1}{n'} - \frac{1}{N}\right).$$

where, s_y^2 is an unbiased estimator of S_Y^2 and denotes the sample variance of variable y based on sample of size n and s_x^2 *and* $s_x^{'2}$ are unbiased estimator of S_X^2 and denote the sample variance of variable x based on second phase sample of size n and first phase sample of size n' respectively.

Let, $\mu_{pq} = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \overline{Y}\right)^p\left(x_i - \overline{X}\right)^q$ where, $(\overline{Y}, \overline{X})$ denote the population means of (y, x).

let $\lambda_{pq} = \dfrac{\mu_{pq}}{\mu_{20}^{p/2}\mu_{02}^{q/2}}$

and taking

$$\beta_{2y} = \lambda_{40}, \beta_{2x} = \lambda_{04},\ \beta_{2y}^* = \lambda_{40} - 1, \beta_{2x}^* = \lambda_{04} - 1, \lambda_{22}^* = \lambda_{22} - 1$$

Let ρ_{yx} be the population correlation coefficient between y and x.

Defining,

$$e_0 = \frac{s_y^2}{S_Y^2} - 1,\ e_1 = \frac{s_x^2}{S_X^2} - 1,\ e_2 = \frac{s_x^{'2}}{S_X^2} - 1$$

we assume that, $E(e_0) = E(e_1) = E(e_2) = 0$ and

$$E(e_0^{\,2}) = \lambda\beta_{2y}^*,\quad E(e_1^2) = \lambda\beta_{2x}^*,\quad E(e_2^2) = \lambda'\beta_{2x}^*,\ E(e_0e_1) = \lambda\lambda_{22}^*,$$
$$E(e_0e_2) = \lambda'\lambda_{22}^*,\quad E(e_1e_2) = \lambda'\beta_{2x}^*.$$

2. Estimators in Literature

S. No.	Estimators	MSE
1	$t_0 = s_y^2$	$Var(s_y^2) = \lambda S_y^4 \beta_{2y}^*$
2	$t_1 = s_y^2 \frac{s_x^{'2}}{s_x^2}$, Isaki (1983)	$MSE(t_1) = S_Y^4(\lambda\beta_{2y}^* + (\lambda-\lambda')(\beta_{2x}^* - 2\lambda_{22}^*))$
3	$t_2 = s_y^2 + b\left(s_x^{'2} - s_x^2\right)$, Isaki (1983)	$\min MSE(t_2) = S_y^4\lambda\beta_{2y}^* - \frac{S_y^4(\lambda-\lambda')\lambda_{22}^{*2}}{\beta_{2x}^*}$
4	$t_3 = k_1 s_y^2 + k_2 (s_x^{'2} - s_x^2)$	$\min. MSE(t_3) = \frac{\min MSE(t_2)}{1 + \frac{\min MSE(t_2)}{S_Y^4}}$
5	$t_4 = \left[k_1 s_y^2 + k_2(s_x^{'2} - s_x^2)\right].\exp\left(\frac{s_x^{'2} - s_x^2}{s_x^{'2} + s_x^2}\right)$, *Shabbir* and Gupta (2007)	$\min MSE(t_4) = \frac{\min MSE(t_2)}{1 + \frac{\min MSE(t_2)}{S_Y^4}} - \frac{(\lambda-\lambda')\beta_{2x}^*\left[\min MSE(t_2) + \frac{(\lambda-\lambda')S_Y^4\beta_{2x}^*}{16}\right]}{4\left[1 + \frac{\min MSE(t_2)}{S_Y^4}\right]}$

Table 1: Estimators considered in this paper along with respective minimum MSE's

3. Proposed Estimator

The use of auxiliary information in increasing precision of estimates is implied in sampling survey. Different transformations based on auxiliary information are also used like linear transformation, use of exponential transformation by Behl and Tuteja (1991), transformed estimator by Sahai and Ray (1980).Recently, Mishra et al. (2017) introduced estimators using log type transformation which was found to be more efficient than usual mean and ratio estimator. Motivated by Mishra et al. (2017), we propose estimators for estimating population variance under double sampling scheme.

(1) $$Pl_1 = s_y^2 + w_0 \log\left(\frac{s_x^2}{s_x^{'2}}\right) \quad (3.1)$$

Rewriting estimator Pl_1 in terms of relative error terms, we have

$$Pl_1 = S_y^2(1 + e_0) + w_0\left\{e_1 - e_2 - \frac{e_1^2}{2} + \frac{e_2^2}{2}\right\}$$

For the estimator Pl_1, we have

$$\text{Bias}(Pl_1) = \frac{w_0}{2}\left\{\lambda'\beta^*_{2x} - \lambda\beta^*_{2x}\right\} \tag{3.2}$$

$$\text{MSE}(Pl_1) = S_Y^4\lambda\beta^*_{2y} + w_0^{\,2}\left\{\lambda\beta^*_{2x} + \lambda'\beta^*_{2x} - 2\lambda'\beta^*_{2x}\right\} + 2w_0S_Y^2\left\{\lambda\lambda^*_{22} - \lambda'\lambda^*_{22}\right\} \tag{3.3}$$

At optimum value of w_0 in Eq. (3.3), expression for min MSE of Pl_1 is given by Eq. (3.4).

$$w_0^* = \frac{-S_Y^2\lambda^*_{22}(\lambda-\lambda')}{(\lambda-\lambda')\beta^*_{2x}} = \left(\frac{-S_Y^2\lambda^*_{22}}{\beta^*_{2x}}\right)$$

$$\min \text{MSE}(Pl_1) = S_Y^4\left[\lambda\beta^*_{2y} - (\lambda-\lambda')\frac{\lambda^{*2}_{22}}{\beta^*_{2x}}\right] \tag{3.4}$$

$$(2)\ \ Pl_2 = s_y^2(w_1+1) + w_2\log\left(\frac{s_x^2}{s_x^{'2}}\right) \tag{3.5}$$

Expressing the estimator Pl_2 in terms of e's, we have

$$Pl_2 - S_Y^2 = S_Y^2e_0 + w_1S_Y^2(1+e_0) + w_2(e_1-e_2)$$

Expression for Bias and MSE of Pl_2 is given by Eq. (3.6) and Eq. (3.7) respectively,

$$\text{Bias}(Pl_2) = w_1S_Y^2 \quad \text{and} \tag{3.6}$$

$$\text{MSE}(Pl_2) = S_Y^4\lambda\beta^*_{2y} + w_1^2(S_Y^4(1+\lambda\beta^*_{2y})) + w_2^2\left\{(\lambda-\lambda')\beta^*_{2x}\right\} + 2w_1(S_Y^4\lambda\beta^*_{2y}) + 2w_2(S_Y^2(\lambda-\lambda')\lambda^*_{22}) + 2w_1w_2(S_Y^2(\lambda-\lambda')\lambda^*_{22}) \tag{3.7}$$

Partially differentiating Eq. (3.7) with respect to w_1 and w_2, we get

$$w_1^* = \frac{(A-C)D}{D^2 - AB}$$

$$w_2^* = \frac{CB - D^2}{D^2 - AB}$$

Using optimum values of w_1 and w_2 in equation (3.7), we get

$$\min \text{MSE}(Pl_2) = C + \frac{BC^2 + (A-2C)D^2}{D^2 - AB} \tag{3.8}$$

where, $A = S_Y^4(1+\lambda\beta^*_{2y})$, $B = (\lambda-\lambda')\beta^*_{2x}$, $C = S_Y^4\lambda\beta^*_{2y}$, $D = S_Y^2(\lambda-\lambda')\lambda^*_{22}$

Now, we propose another estimator Pl_3 given by Eq. (3.9)

$$(3)\ Pl_3 = \left[s_y^2(1+w_3) + w_4\log\left(\frac{s_x^2}{s_x^{'2}}\right)\right]\exp\left(\frac{s_x^{'2} - s_x^2}{s_x^{'2} + s_x^2}\right) \tag{3.9}$$

Expressing equation (3.9) in terms of e's, we have

$$Pl_3 = S_Y^2(1+w_3)\left(1+e_0-\frac{e_1}{2}+\frac{e_2}{2}+\frac{3}{8}e_1^2-\frac{e_2^2}{8}-\frac{e_0e_1}{2}+\frac{e_0e_2}{2}-\frac{e_1e_2}{4}\right)$$
$$+w_4\left(e_1-e_2-\frac{e_1^2}{2}-\frac{e_2^2}{2}+e_1e_2\right)$$

The expression for the bias and MSE are given in Eq. (3.10) and Eq. (3.11).

$$\text{Bias}(Pl_3) = S_Y^4\left(\frac{3}{8}\lambda\beta_{2x}^*-\frac{1}{8}\lambda'\beta_{2x}^*-\frac{1}{2}\lambda\lambda_{22}^*+\frac{1}{2}\lambda'\lambda_{22}^*-\frac{1}{4}\lambda'\beta_{2x}^*\right)+$$
$$S_Y^4 w_3\left(1+\frac{3}{8}\lambda\beta_{2x}^*-\frac{1}{8}\lambda'\beta_{2x}^*-\frac{1}{2}\lambda\lambda_{22}^*+\frac{1}{2}\lambda'\lambda_{22}^*-\frac{1}{4}\lambda'\beta_{2x}^*\right)$$
$$+w_4\left(\lambda'\beta_{2x}^*-\frac{1}{2}\lambda\beta_{2x}^*-\frac{1}{2}\lambda'\beta_{2x}^*\right) \qquad (3.10)$$

$$MSE(Pl_3) = F+w_3^2A_1+w_4^2B_1+2w_3C_1+2w_4D_1+2w_3w_4E_1 \qquad (3.11)$$

Partially differentiating Eq. (3.11) with respect to w_1 and w_2, we get

$w_3^* = \dfrac{C_1B_1-D_1E_1}{E_1^2-A_1B_1}$ and

$w_4^* = \dfrac{A_1D_1-C_1E_1}{E_1^2-A_1B_1}$

Substituting the optimum values of w_3 and w_4 in Eq. (3.11), we obtain expression for min. MSE given by Eq. (3.12):

$$\min\ MSE\ (Pl_3) = F+\frac{B_1C_1^2+A_1D_1^2-2C_1D_1E_1}{E_1^2-A_1B_1} \qquad (3.12)$$

Where

$$D_1 = S_Y^2\ (\lambda\text{-}\lambda')\left(\lambda_{22}^*-\frac{\beta_{2x}^*}{2}\right)\ ,\ E_1 = S_Y^2(\lambda-\lambda')(\lambda_{22}^*-\beta_{2x}^*)$$

$$F_1 = S_Y^4\left[\lambda\beta_{2y}^*\ +\ (\lambda\text{-}\lambda')\left(\frac{\beta_{2x}^*}{4}-\lambda_{22}^*\right)\right]$$

We define another estimator Pl_4 as follows:

$$(4)\ Pl_4 = s_y^2(w_5+1)+w_6\log\left(\frac{s_x^2}{s_x'^2}\right)\exp\left(\frac{s_x'^2-s_x^2}{s_x'^2+s_x^2}\right) \qquad (3.13)$$

Expressing equation (3.13) in terms of e's, we have

$$Pl_4 = S_y^2+S_y^2w_5(1+e_0)+w_6\left(e_1-e_2-\frac{e_1^2}{2}-\frac{e_2^2}{2}+e_1e_2\right)$$

Expressions for Bias and MSE of estimator Pl_4 is given by Eq. (3.14) and Eq. (3.15) respectively,

$$\text{Bias }(Pl_4) = S_y^2 w_5 + w_6\left(\lambda'\beta^*_{2x} - \frac{1}{2}\lambda\beta_{2x} - \frac{1}{2}\lambda'\beta^*_{2x}\right) \quad (3.14)$$

$$\text{MSE }(Pl_4) = C_3 + w_5^2 A_3 + w_6^2 B_3 + 2w_5 C_3 + 2w_6 D_3 + 2w_5 w_6 E_3 \quad (3.15)$$

partially differentiating Eq. (3.15) w. r. to w_5 and w_6, we get optimum values given by:

$w_5^* = \dfrac{C_3 B_3 - D_3 E_3}{E_3^2 - A_3 B_3}$ and

$w_6^* = \dfrac{A_3 D_3 - C_3 E_3}{E_3^2 - A_3 B_3}$

Using optimum values of w_5 and w_6 in equation (3.15), we have-

$$\min \text{ MSE }(Pl_4) = C_3 + \frac{B_3 C_3^2 + A_3 D_3^2 - 2C_3 D_3 E_3}{E_3^2 - A_3 B_3} \quad (3.16)$$

where,

$$A_3 = S_Y^4(1+\lambda\beta^*_{2y}),\ B_3 = \beta^*_{2x}(\lambda-\lambda'),\ C_3 = S_Y^4\lambda\beta^*_{2y},\ D_3 = S_Y^2(\lambda-\lambda')\lambda^*_{22}$$

$$E_3 = S_Y^2\left\{\lambda'\beta^*_{2x} + \lambda\lambda^*_{22} - \lambda'\lambda^*_{22} - \frac{\lambda\beta^*_{2x}}{2} - \frac{\lambda'\beta^*_{2x}}{2}\right\} \text{or}$$

$$= S_Y^2(\lambda-\lambda')\left\{\lambda^*_{22} - \frac{\beta^*_{2x}}{2}\right\}$$

4. Efficiency Comparison

(i) $\min \text{ MSE }(Pl_1) \le \text{var}(s_y^2)$ Or

$\text{var}(s_y^2) - \min \text{ MSE }(Pl_1) \ge 0$ if

$$S_y^4(\lambda-\lambda')\frac{\lambda^{*2}_{22}}{\beta^*_{2x}} \ge 0$$

(ii) $\min \text{ MSE }(Pl_2) \le \text{var}(s_y^2)$ Or

$\text{var}(s_y^2) - \min \text{ MSE }(Pl_2) \ge 0$ if

$$\frac{BC^2 + (A-2C)D^2}{D^2 - AB} \le 0$$

(iii) $\min \text{ MSE }(Pl_3) \le \text{var}(s_y^2)$ Or

$\text{var}(s_y^2) - \min \text{ MSE }(Pl_3) \ge 0$ if

$$\left(\lambda^*_{22} - \frac{\beta^*_{2x}}{4}\right)(\lambda-\lambda') - \frac{B_1 C_1^2 + A_1 D_1^2 - 2C_1 D_1 E_1}{E_1^2 - A_1 B_1} \ge 0$$

(v) $\min \text{ MSE }(Pl_1) \le \text{MSE }(t_1)$ Or

$\text{MSE }(t_1) - \min \text{ MSE }(Pl_1) \ge 0$ if

$$S_y^4(\lambda-\lambda')\frac{[\beta^*_{2x} - \lambda^{*2}_{22}]}{\beta^*_{2x}} \ge 0$$

(vi) $\min \text{ MSE }(Pl_2) \le \text{MSE }(t_1)$ Or

$\text{MSE }(t_1) - \min \text{ MSE }(Pl_2) \ge 0$ if

$$S_y^4(\lambda-\lambda')(\beta^*_{2x} - \lambda^*_{22})^2 - \frac{BC^2 + (A-2C)D^2}{D^2 - AB} \ge 0$$

(vii) $\min MSE(Pl_3) \le MSE(t_1)$ Or
$MSE(t_1) - \min MSE(Pl_3) \ge 0$ if

$$S_y^4(\lambda - \lambda')\left(\frac{3\beta^*_{2x}}{4} - \lambda^*_{22}\right) \ge 0$$

(viii) $\min MSE(Pl_4) \le MSE(t_1)$ Or
$MSE(t_1) - \min MSE(Pl_4) \ge 0$ if

$$S_y^4(\lambda - \lambda')\left(\beta^*_{2x} - 2\lambda^*_{22}\right) - \frac{B_3C_3^2 + A_3D_3^2 - 2C_3D_3E_3}{E_3^2 - A_3B_3} \ge 0$$

(ix) $\min MSE(Pl_2) \le MSE(t_2)$ Or
$MSE(t_2) - \min MSE(Pl_2) \ge 0$ if

$$S_y^4(\lambda - \lambda')\frac{\lambda^{*2}_{22}}{\beta^*_{2x}} + \frac{BC^2 + (A - 2C)D^2}{D^2 - AB} \le 0$$

(x) $\min MSE(Pl_3) \le MSE(t_2)$ Or
$MSE(t_2) - \min MSE(Pl_3) \ge 0$ if

$$S_y^4(\lambda - \lambda')\frac{(\beta^*_{2x} - 2\lambda^{*2}_{22})}{4\beta^*_{2x}} + \frac{B_1C_1^2 + A_1D_1^2 - 2C_1D_1E_1}{D^2 - AB} \le 0$$

(xi) $\min MSE(Pl_4) \le MSE(t_2)$ Or
$MSE(t_2) - \min MSE(Pl_4) \ge 0$ if

$$S_y^4(\lambda - \lambda')\frac{\lambda^{*2}_{22}}{\beta^*_{2x}} + \frac{B_3C_{31}^2 + A_3D_3^2 - 2C_3D_3E_3}{E_3^2 - A_3B_3} \le 0$$

(xii) $\min MSE(Pl_4) \le MSE(t_3)$ Or
$MSE(t_3) - \min MSE(Pl_4) \ge 0$ if

$$\frac{S_Y^4(\lambda - \lambda')\lambda^*_{22}}{\beta^*_{2x}}\left(\lambda\beta^*_{2y} - 1\right) - S_Y^4\lambda^2\beta^{*2}_{2Y} - \left(1 + \lambda\beta^*_{2y} - (\lambda - \lambda')\frac{\lambda^{*2}_{22}}{\beta^*_{2x}}\right)$$
$$\left(\frac{B_3C_3^2 + A_3D_3^2 - 2C_3D_3E_3}{E_3^2 - A_3C_3}\right) \ge 0$$

(xiii) $\min MSE(Pl_4) \le MSE(t_4)$ Or
$MSE(t_4) - \min MSE(Pl_4) \ge 0$ if

$$\left[\frac{\min .MSE(t_2)}{1 + \frac{\min .MSE(t_2)}{S_Y^4}}\right] - \left[\frac{(\lambda - \lambda')\beta^*_{2x}(\min .MSE(t_2) + (\lambda - \lambda')S_Y^4\beta^*_{2x}}{4\left[1 + \frac{\min .MSE(t_2)}{S_Y^4}\right]}\right]$$
$$- C_3 - \left(\frac{B_3C_3^2 + A_3D_3^2 - 2C_3D_3E_3}{E_3^2 - A_3B_3}\right) \ge 0$$

(xiv) $\min .MSE(Pl_3) \le \min .MSE(Pl_1)$ Or
$\min .MSE(Pl_1) - \min MSE(Pl_3) \ge 0$ if

$$\frac{S_Y^4(\lambda - \lambda')(\beta^*_{2x} - 2\lambda^*_{22})^2}{4\beta^*_{2x}} + \left(\frac{B_1C_1^2 + A_1D_1^2 - 2C_1D_1E_1}{E_1^2 - A_1B_1}\right) \le 0$$

(xv) $\min .MSE(Pl_2) \le \min .MSE(Pl_1)$ Or
$\min .MSE(Pl_1) - \min MSE(Pl_2) \ge 0$ if

$$\frac{S_Y^4(\lambda - \lambda')\lambda^{*\,2}_{22}}{\beta^*_{2x}} + \left(\frac{BC^2 + (A - 2C)D^2}{D^2 - AB}\right) \le 0$$

(xvi) $\min.MSE(Pl_4) \le \min.MSE(Pl_1)$ Or
$\min.MSE(Pl_1) - \min MSE(Pl_4) \ge 0$ if

$$\frac{S_Y^4(\lambda-\lambda')\lambda_{22}^{*\,2}}{\beta_{2x}^*} + \left(\frac{B_3C_3^2 + A_3D_3^2 - 2C_3D_3E_3}{E_3^2 - A_3B_3}\right) \le 0$$

(xvii) $\min.MSE(Pl_2) \le \min.MSE(Pl_3)$ Or
$\min.MSE(Pl_3) - \min MSE(Pl_2) \ge 0$ if

$$S_Y^4(\lambda-\lambda')\left(\frac{\beta_{2x}^*}{4} - \lambda_{22}^*\right) + \left(\frac{B_1C_1^2 + A_1D_1^2 - 2C_1D_1E_1}{E_1^2 - A_1B_1}\right) - \left(\frac{BC^2 + (A-2C)D^2}{D^2 - AB}\right) \ge 0$$

(xviii) $\min.MSE(Pl_2) \le \min.MSE(Pl_3)$ Or
$\min.MSE(Pl_3) - \min MSE(Pl_2) \ge 0$ if

$$S_Y^4(\lambda-\lambda')\left(\frac{\beta_{2x}^*}{4} - \lambda_{22}^*\right) + \left(\frac{B_1C_1^2 + A_1D_1^2 - 2C_1D_1E_1}{E_1^2 - A_1B_1}\right) - \left(\frac{BC^2 + (A-2C)D^2}{D^2 - AB}\right) \ge 0$$

5. Data description and Numerical Calculation

In this section, we consider four Populations for numerical comparison of Percent Relative Efficiency of proposed estimators with relevant existing estimators. Four real data sets are used for numerical illustration:

Population 1: Cochran (1977, page 325)
N=100, n=10, n'=85, S_y^2=214.69, S_x^2=56.76, λ_{40}=2.2387, λ_{04}=2.2523, λ_{22}=1.5432.

Population 2: (Cochran 1977, page 152)
N=196, n=49, n'=158, S_y^2=1515558.83, S_x^2=10900.42, λ_{40}=8.5362, λ_{04}=7.3617, λ_{22}=7.8780.

Population 3: (Cochran 1977, page 203)
N=200, n=29, n'=159, S_y^2=99.18, S_x^2=85.09, λ_{40}=1.9249, λ_{04}=2.5932, λ_{22}=2.1149.

Population 4: (Sukhatme and Sukhatme 1970, page 185)
N=170, n=10, n'=139, S_y^2=26456.89, S_x^2=22355.76, λ_{40}=3.1842, λ_{04}=2.2030, λ_{22}=2.5597.

Population	1	2	3	4
Estimators				
t_0	100.00	100.00	100.00	100.00
t_1	88.394	1025.763	292.561	736.2863

t_2	122.923	1082.550	517.205	1134.776
t_3	134.072	1094.090	519.932	1155.333
t_4	138.180	1132.780	529.036	1201.728
Pl_1	122.923	1082.550	517.205	1134.780
Pl_2	134.072	1094.090	519.932	1155.333
Pl_3	138.939	1084.503	521.422	1145.068
Pl_4	141.368	1550.532	563.085	1883.728

Table 5.1: Percent Relative Efficiencies of Estimators

From the results of Table 5.1, it is evident that the proposed estimators Pl_1, Pl_2, Pl_3 and Pl_4 are more efficient than t_0 and t_1.It can be seen from Table 5.1 that t_2 and t_3 are equally efficient to Pl_1 and Pl_2 respectively and Pl_3 and Pl_4 are more efficient than all the estimators considered in Table 1. Among the proposed estimators, Pl_3 and Pl_4 are uniformly more efficient than Pl_1 and Pl_2 respectively.

6. Conclusion

Based on theoretical and numerical results obtained it turns out that percent relative efficiencies of estimators Pl_1, Pl_2, and Pl_3 are found to be more than existing estimators in literature (as defined in paper) under certain specified conditions. Estimator Pl_4 is found to be uniformly more efficient than other existing as well as proposed estimators. It is therefore, suggested to use proposed estimators for estimating population variance more efficiently when double sampling is used.

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