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AN OPTIMIZATION OF FEOQ MODEL FOR WEIBULL DETERIORATING ITEMS WITH INFLATIONARY CONDITION

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Abstract

The economic instability is a haunting situation in the present era for the economy. There is a monetary depreciation due to the tremendous increase in the price of the commodities. Present study basically deals with the investigation of the inventory system considering the case of perishable products under inflationary environment under condition of partial backorder. The rate of demand is a function which exponentially increases with increase in inflation and deterioration is taken as a Weibull distributed two parameter function. In the state of shortage when the inventory system runs out of stock an assumption is made that the demands are backlogged or lost. There is a variation in the rate of backlogging which shows this variation as a consequence of lead time required for the arrival of the next replenishment. The deterioration of commodities begins after a fixed interval of time. Present model aims at minimizing the average total cost considering both the crisp and fuzzy environments considering both inflationary effect as well as the time value of money. For the fuzzy economic order quantity (FEOQ) model of the inventory system fuzzification has been done by the use of fuzzy number of the trapezoidal nature. Defuzzification has been done by the use of centroid method. The primary motive of this work is to determine and compare the cost in total of the system of inventory in both crisp and fuzzy environments. The main inference that we can draw from this study is that fuzzy economic order quantity (FEOQ) model has more accuracy as in this model the total cost has been reduced as compared to the crisp model. Thus fuzzy economic order quantity (FEOQ) model is highly beneficial to any sort of inventory system and this result can be further generalized to any sector for enhancing the total profit of the system. Appropriate example has been provided for the illustration of both the models.

Key Words: Fuzzy Economic Order Quantity (FEOQ) Model, Deterioration, Weibull Distribution Function, Inflation, Time Value of Money.

1. Introduction

Considering the financial aspects of an organization, the inventory basically represents the stock of commodities or goods that are preserved in a proper manner for ensuring efficient and smooth running of any organization. Inventory system is the basically the representation of the capital investment of any organization and this in turn must compete with other assets possessed by an organization. Inventory management is the most important aspect which must be taken care of as proper inventory management facilitates in increasing the profit of an organization. The most common problem faced in managing the inventories is control and maintenance of the goods that undergo decay as the time passes by. Deterioration is primarily the decay or the spoilage which the products undergo due to the adverse conditions. This in turn results in the decrease in the utility of the product. Products like eatables, vegetables, oil, gasoline, alcohol, radioactive chemicals, medicines etc undergo remarkable deterioration with the passage of time.

Whitin (1957) considered a model of the inventory system of the fashion goods undergoing deterioration when the prescribed shortage period comes to an end. Ghare and Scharder (1963) proposed a model of the inventory system consisting of the goods that undergo deterioration with an exponential rate and also have a demand rate which is of deterministic nature. Covert and Philip (1973) provided an extension to the existing models of the inventory system with weibull deterioration. An order level model of the inventory system having rate of deterioration of the constant nature was developed by Shah and Jaiswal (1977). Pal and Datta (1988) proposed a model of the inventory system of the order level nature, in which the rate of demand has a specific pattern and also the rate of deterioration is a Weibull distributed function. Wee (1995) proposed a model of inventory having an exponential pattern of demand which undergoes deterioration. Considering the scenario of the last two decades there is a tremendous change on the economic condition of the majority of countries as the purchasing power of money has witnessed a tremendous decline presently. Thus the concept of value of the money with regards to time cannot be ignored at all. Data and Pal (1991) developed a system of the inventory taking into consideration the effect of time on the value of money and demand is a linear function and dependent on time. Bose et al. (1995) proposed a model of the inventory system having a demand rate that is a function of the linear nature with respect to time and the effect of time on the value of money is also considered. An economic order inventory model for the goods that undergo deterioration was studied by Singh and Sharma (2003). Sana (2003) developed a model of the inventory system incorporating time for the value of money and inflation thereby giving a new dimension to the existing research work. Wee and Law (1999) proposed a model of the inventory system having rate of replenishment for the items undergoing deterioration of finite nature and also considering the time value of money. Chang (2004) proposed a model of the system of inventory considering the items which undergo deterioration considering the inflationary condition in which the purchaser is provided a delay in the payment upto a permissible level by the supplier if the purchaser places an order which is large. Jaggi et al. (2006) proposed a model of the inventory system for the items undergoing deterioration considering the effect of inflation. Also the demand in this case is fully backlogged. Thangam et al. (2010) developed a system of inventory for goods undergoing deterioration considering the effect of demand occurring due to inflation and also backorders which are exponential in nature.

Bansal (2013) proposed a model of system of inventory for products that are perishable under the inflationary condition. Chauhan and Singh (2015) proposed a model of the system of inventory having products undergoing deterioration with demand rate which is of Verhulst's Model type. Sayal et al. (2018) supply chain aspect for crisp as well as fuzzy models of the systems of inventory. Anu Sayal et al. (2018) developed a system of the inventory considering shortages for the crisp as well as fuzzy environments. Anu Sayal et al. (2018) developed a model for products undergoing deterioration having demand as function which ramp type is considering the case of shortages.

Considering real life scenario a fraction of the demand function which is insatiable from the existing system of inventory, leaves the entire inventory in a phase of stock out. There are two conditions which are primarily taken into consideration in such systems. These are: the customers wait till the next order arrives which is the situation of full backorder or the customers leave which represents the condition of lost sale. Considering the scenario of the real systems of the inventory there are a few customers who wait for the arrival of the next replenishment for satisfying the demand when there is a period of stock out, while there are other customers who are not ready to wait and approach other vendors to fulfil their demands. Thus it is a condition of partial backordering which has been represented by the mathematical formulas for the system of inventory. We consider the system taking into consideration the case of partial backorder such that a part of demand is backlogged and it is a negative function which is of exponential nature having the lead time of the items.

In this research paper a system of the inventory considering case of partial backorders for items undergoing deterioration with increasing time has been developed. Deterioration rate is assumed to be two parameter Weibull density function in nature. Rate of demand also increases exponentially as an effect of inflation considering a finite time period. Shortages are considered and when a situation of stock out occurs, the demand at that instant is supposed to be lost or backlogged. Rate of backlogging of the commodities varies. It depends on lead time taking into consideration the next replenishment. Model proposed here is considered in both crisp as well as fuzzy environment. A solution of the optimal nature of the proposed fuzzy economic order quantity (FEOQ) model here is obtained by considering suitable numerical example and also considering time value of money as well as inflationary condition for each of the cost parameters.

2. Assumptions and Notations

Following assumptions and notations are for the crisp environment:

Assumptions for the proposed model in crisp system and fuzzy economic order quantity (FEOQ) system are given below:

- Single inventory system has been considered.
- The given inventory system allows shortages.
- The rate of demand increases exponentially and is given by $X(t) = a_0 e^{it}$,

Where a_0 represents the initial rate of demand.

- There is a partial backlogging of the shortages.

- Backlogging rate is varies and depends on lead time required for next replenishment. Rate of backlogging is given by: $\frac{1}{1+\alpha(T-t)}$, where α represents parameter of backlogging which is non-negative.
- Lead time is equivalent to zero.
- The size of replenishment is finite.
- The rate of replenishment is infinite.
- Time period is considered to be finite.
- Deteriorated items do not undergo any repair.
- Deterioration takes place for items in stock.
- The time value of money and inflationary effects are taken into consideration.

We have made following notations for the proposed model in crisp system:

- H(t) level of system of the inventory at the time t, where $t \geq 0$.
- $X(t) = a_0 e^{it}$ rate of demand at the time t.
- $\delta : \delta = \gamma\lambda t^{\lambda-1}$ Weibull distribution of two parameter rate of deterioration per unit per unit time, where γ represents scale parameter and λ represents shape parameter.
- Z the total replenishment as the cycle begins.
- I the inventory level at the time t = 0.
- T span of a complete cycle.
- μ_L item's life time.
- i_r rate of inflation per unit of time.
- r_d rate of discount which represents the value of money with respect of time.
- P_c cost of purchasing per unit of item.
- D_c the cost of deterioration per unit of item.
- H_c cost of holding items in the inventory.
- O_c opportunity cost occurring due to sale lost per unit of item.
- S_c cost of shortage per unit of item.
- TC total average cost of the system of inventory.

Following notations are considered for the model proposed in fuzzy economic order quantity (FEOQ) system:

- $\tilde{P}_c$ fuzzy cost of purchasing per unit item.
- $\tilde{D}_c$ fuzzy cost of deterioration per unit item.
- $\tilde{H}_c$ the fuzzy cost of holding goods in inventory.
- $\tilde{O}_c$ fuzzy opportunity cost occurring due to sale lost per unit item.

- $\tilde{S}_c$ the fuzzy cost of shortage per unit item.
- $\widetilde{TC}$ the average cost in total of fuzzy system of inventory.

3. Model Formulation in Crisp System

We assume that Z is the total replenishment at the beginning of each of the cycle. When all the back orders have been fulfilled the initial level of the inventory system is supposed to be I. Primary motive of proposed model is to minimise optimal total cost of this inventory system. As there is a shortage of commodities in the system, determination of optimality for the shortage of goods in the system is carried out. There is a decrease in the level of the inventory system due to demand of the commodities in the market in the period (0, μ_L). In the period (μ_L , t_1) there is a decrease in the level of the inventory system due to the demand of the commodity in the market as well as due to deterioration of the commodities. When time becomes equal to t_1, the inventory level becomes equivalent to zero. Further in the interval of time [t_1, T] shortages accumulate in the system. There is partial backlogging of the shortages and partially occur as the sales that are lost in the due course. The next replenishment replaces the backlogged goods. Figure 1 represents this inventory system.

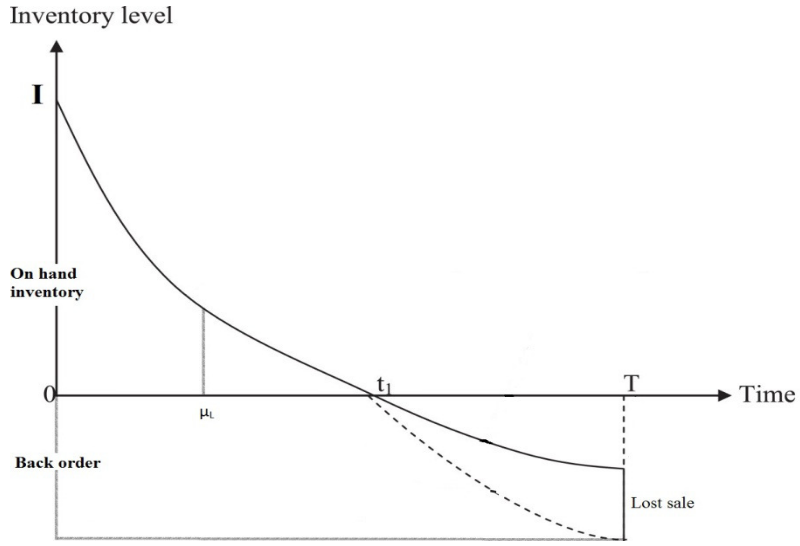

Figure 1: Graphical representation of inventory model

H(t) represents the inventory level when the time $t \geq 0$.

When time is $t + \Delta t$, i.e. in interval [0, μ_L]the level of the inventory system is given by: $H(t+\Delta t) = H(t) - X(t)\Delta(t)$

Dividing the above equation by Δt and by taking limit as $\Delta t \to 0$ we have:

$$\frac{dH}{dt} = -a_0 e^{it} \quad ; \quad 0 \le t \le \mu_L \tag{1}$$

There is a collective effect of the demand and deterioration of commodity in market for time interval [μ_L , t_1], which is given by:

$$H(t+\Delta t) = H(t) - \delta(t)H(t)\Delta(t) - X(t)\Delta(t)$$

Dividing the above equation by Δt and by taking the limit as $\Delta t \to 0$ we have:

$$\frac{dH(t)}{dt} + \gamma\lambda t^{\lambda-1}H(t) = -a_0 e^{it} \quad ; \quad \mu_L \le t \le t_1 \tag{2}$$

The shortage of items occurs in the interval [t_1, T] which is given by:

$$H(t+\Delta t) = H(t) - \frac{X(t)}{1+\alpha(T-t)}\Delta t$$

Dividing the above equation by Δt and taking the limit as $\Delta t \to 0$ we get:

$$\frac{dH(t)}{dt} = -\frac{a_0 e^{it}}{1+\alpha(T-t)} \quad ; \quad t_1 \le t \le T \tag{3}$$

The boundary conditions applied for obtaining the solution of the above equations are:
H(0) = I ; $H(t_1) = 0$.

The solution of equation (1) using the above boundary conditions is given by:

$$H(t) = I + \frac{a_0}{i}(1-e^{it}) \quad ; \quad 0 \le t \le \mu_L \tag{4}$$

The solution of equation (2) using the above boundary conditions is given by:

$$H(t) = a_0 e^{-\gamma t^{\lambda}}[t_1 - t + \frac{\gamma}{\lambda+1}\{t_1^{\lambda+1} - t^{\lambda+1}\} + \frac{i}{2}\{t_1^2 - t^2\}] \; ; \; \mu \le t \le t_1 \tag{5}$$

The solution of equation (3) using the above boundary conditions is given by:

$$H(t) = a_0[t_1 - t - \alpha T\{t_1 - t\} + \frac{\alpha + i}{2}\{t_1^2 - t^2\}] \quad ; \quad t_1 \le t \le T \tag{6}$$

The initial level of the inventory system is given by:

$$I = a_0 e^{-\gamma\mu_L^{\lambda}}[(t_1 - \mu_L) + \frac{\gamma}{\lambda+1}(t_1^{\lambda+1} - \mu_L^{\lambda+1}) + \frac{i}{2}(t_1^2 - \mu_L^2) + \frac{a_0(e^{i\mu_L} - 1)}{i} \tag{7}$$

Considering the inflationary effect and value of the money with respect to time, the cost in total is made up of the following:

Cost of purchase per cycle

$$P_c I \int_0^T e^{-(r_d - i)t} dt = \frac{P_c I}{i - r_d}[e^{-(r_d - i)T} - 1] \tag{8}$$

Cost of holding per cycle

$$H_c \int_0^{\mu_L} H(t) e^{-(r_d - i)t} dt + H_c \int_{\mu_L}^{t_1} H(t) e^{-(r_d - i)t} dt \tag{9}$$

$$= H_c \left[I\mu_L + \frac{I(i - r_d)}{2}\mu_L^2 - \frac{a_0 \mu_L^2}{2} - \frac{a_0 (i - r_d)\mu_L^3}{3} \right] +$$

$$H_c a_0 \begin{bmatrix} t_1 \left\{ t_1 + \dfrac{\gamma t_1^{\lambda+1}}{\lambda+1} - \dfrac{i t_1^2}{2} \right\} - \dfrac{t_1^2}{2} - \dfrac{\gamma}{(\lambda+1)(\lambda+2)} t_1^{\lambda+2} \\ -\dfrac{i}{6} t_1^3 + \dfrac{(i - r_d)}{6} t_1^3 - \dfrac{\gamma t_1^{\lambda+1}}{\lambda+1} + \dfrac{\gamma t_1^{\lambda+2}}{\lambda+2} \\ -\mu_L \left\{ t_1 + \dfrac{\gamma t_1^{\lambda+1}}{\lambda+1} - \dfrac{i t_1^2}{2} \right\} + \dfrac{\mu_L^2}{2} + \dfrac{\gamma}{(\lambda+1)(\lambda+2)} \mu^{\lambda+2} \\ +\dfrac{i}{6}\mu_L^3 - \dfrac{(i - r_d) t_1 \mu_L^2}{2} + \dfrac{(i - r_d)\mu_L^3}{3} + \dfrac{r_d t_1 \mu_L^{\lambda+1}}{\lambda+1} - \dfrac{r_d \mu_L^{\lambda+2}}{\lambda+2} \end{bmatrix}$$

Cost of deterioration per cycle

$$D_c \int_{\mu_L}^{t_1} \gamma \lambda t^{\lambda-1} H(t) e^{-(r_d - i)t} dt \tag{10}$$

$$= D_c \gamma \lambda a_0 \left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)} - \frac{t_1 \mu_L^{\lambda}}{\lambda} + \frac{\mu_L^{\lambda+1}}{\lambda+1} \right]$$

Cost due to shortage per cycle

$$S_c \int_{t_1}^{T} -H(t) e^{-(r_d - i)} dt \tag{11}$$

$$= -S_c a_0 \begin{bmatrix} \frac{(i+\alpha)t_1^2 T}{2} + t_1 T - \alpha t_1 T^2 - \frac{T^2}{2} + \frac{\alpha T^3}{2} - \frac{(i+\alpha)T^3}{6} \\ +(i-r_d)\left\{\frac{t_1 T}{2} - \frac{t_1 T^3 \alpha}{2} + \frac{(i+\alpha)t_1 T^2}{4} - \frac{T^3}{3} + \frac{\alpha T^4}{3} - \frac{(i+\alpha)T^4}{8}\right\} \\ -t_1^2 + \frac{t_1 \alpha T}{2} - \frac{(i+\alpha)t_1^3}{3} + \frac{t_1^2}{2} - (i-r_d)\left\{\frac{t_1^3}{6} - \frac{t_1^3 T\alpha}{6} + \frac{(i+\alpha)t_1^4}{8}\right\} \end{bmatrix}$$

Opportunity cost per cycle due to loss of sales

$$O_c \int_{t_1}^{T} X(t)\left[1 - \frac{1}{1+\alpha(T-t)}\right] e^{-(r_d - i)t} dt \tag{12}$$

$$= O_c a_0 \alpha \left[\frac{T^2}{2} + \frac{t_1^2}{2} - Tt_1 + \frac{(2i - r_d)T^3}{6} - (2i - r_d)\left\{\frac{Tt_1^2}{2} - \frac{t_1^3}{3}\right\}\right]$$

The total cost of the inventory system per unit time is given by:

TC = $\frac{1}{T}$ [Cost of purchase + Cost of holding + Cost of deterioration + Cost due to shortage + Opportunity cost]

$$= \frac{1}{T} \begin{bmatrix} \frac{P_c I}{i - r_d}\left[e^{-(r_d - i)T} - 1\right] + D_c \gamma \lambda a_0 \left[\frac{t_1^{\lambda+1}}{\lambda(\lambda+1)} - \frac{t_1 \mu_L^{\lambda}}{\lambda} + \frac{\mu_L^{\lambda+1}}{\lambda+1}\right] \\ +H_c \left[I\mu_L + \frac{I(i-r_d)}{2}\mu_L^2 - \frac{a_0 \mu_L^2}{2} - \frac{a_0 (i-r_d)\mu_L^3}{3}\right] + \end{bmatrix}$$

$$+ \frac{1}{T} \left[H_c a_0 \begin{bmatrix} t_1\left\{t_1 + \frac{\gamma t_1^{\lambda+1}}{\lambda+1} - \frac{i t_1^2}{2}\right\} - \frac{t_1^2}{2} - \frac{\gamma}{(\lambda+1)(\lambda+2)} t_1^{\lambda+2} - \\ \frac{i}{6}t_1^3 + \frac{(i-r_d)}{6}t_1^3 - \frac{\gamma t_1^{\lambda+2}}{\lambda+1} + \frac{\gamma t_1^{\lambda+2}}{\lambda+2} - \mu_L \left\{ \begin{matrix} t_1 + \frac{\gamma t_1^{\lambda+1}}{\lambda+1} - \\ \frac{i t_1^2}{2} \end{matrix} \right\} \\ + \frac{\mu_L^2}{2} + \frac{\gamma}{(\lambda+1)(\lambda+2)}\mu_L^{\lambda+2} + \frac{i}{6}\mu_L^3 - \frac{(i-r_d)t_1 \mu_L^2}{2} + \\ \frac{(i-r_d)\mu_L^3}{3} + \frac{\gamma t_1 \mu_L^{\lambda+1}}{\lambda+1} - \frac{\gamma \mu_L^{\lambda+2}}{\lambda+2} \end{bmatrix} \right] +$$

$$-\frac{1}{T}\left[\begin{array}{l} C_s a_0\left[\begin{array}{l}\frac{(i+\alpha)t_1^2T}{2}+t_1T-\alpha t_1T^2-\frac{T^2}{2}+\frac{\alpha T^3}{2}-\frac{(i-\alpha)}{6}T^3+ \\ (i-r_d)\left\{\begin{array}{l}\frac{t_1T^2}{2}-\frac{t_1T^3\alpha}{2}+\frac{(i+\alpha)t_1^2T^2}{4}-\frac{T^3}{3}+\frac{\alpha T^4}{3}- \\ \frac{(i+\alpha)T^4}{8}\end{array}\right\}\end{array}\right]- \\ O_c a_o \alpha\left[\frac{T^2}{2}+\frac{t_1^2}{2}-Tt_1+\frac{(2i-r_d)T^3}{6}-(2i-r_d)\left\{\frac{Tt_1^2}{2}-\frac{t_1^3}{3}\right\}\right]\end{array}\right] \tag{13}$$

4. Model Formulation in Fuzzy Economic Order Quantity (FEOQ) System

Fuzzy theory is extensively used in all fields as it deals with situations involving some sort of uncertainty. This uncertainty could be due to any reason which may be ambiguity, vagueness, chance or it be even incomplete knowledge. Fuzzy theory was given by Lofti A. Zadeh. It was proposed in the year 1965. It is a generalized form of the set theory which is classical theory. Just as the classical set theory known as the crisp theory deals with the logic known as symbolic logic, fuzzy set theory provides a new dimension to a concept known as fuzzy logic.

Casting a glance at the concepts related to the classical set theory we find that if we consider a particular element it will either be present in the given set or it does not exist in the set under consideration, such sets are called crisp sets. On the other hand considering the case of fuzzy sets we find that numerous degrees of membership ranging from 0 to 1 are given. A function of the membership values denoted by $\mu_A(x)$ which is actually associated with a fuzzy set A actually relates or maps each element of the set X to the given interval [0,1]. The fuzzy sets considers a broader perspective by expressing a gradual transitions in values by relating them to the membership function from $(0 < \mu_A(x) \leq 1)$ to a sort of non- membership $(\mu_A(x) = 0)$ and vice- versa. It has a very wide utility in real world. This concept not only helps in providing a meaningful and in actual sense a representation which is very powerful of the measurements of the uncertainties but it also provides a kind of representation which is very meaningful as the concepts which are actually vague and this is represented in a natural language.

As the costs like holding cost, deterioration cost etc are not fixed in nature and their values depend on specific conditions therefore it is difficult to obtain the appropriate estimate of all values in such uncertain situation. Fuzzy logic provides a convenient framework for dealing with uncertain parameters. This helps in improving the accuracy and the computational efficiency for the inventory system under consideration. Therefore in this paper the cost functions are represented by trapezoidal fuzzy numbers to improve the accuracy of the calculated results.

4.1 Trapezoidal Fuzzifier

Fuzzification is a technique of of changing a real scalar value into a value that is a fuzzy value. In case of the variable which is real valued fuzzification is done by the process of intuition, or even by experience and analysis of the rules which have been proposed and conditions associated with the variables which are regarded as input data variables. The procedure for fuzzification is not at all fixed, it is a random process.

4.2 Trapezoidal function

The membership function for a fuzzy number of trapezoidal nature for a fuzzy set *A*= (a, b, c, d) is given below, where a ≤ b ≤ c ≤ d

$$\mu_A(x) = \begin{cases} 0, & for\, x < a \\ \dfrac{x-a}{b-a}, & for\, a \le x \le b \\ 1, & for\, b \le x \le c \\ \dfrac{d-x}{d-c}, & for\, c \le x \le d \\ 0, & for\, x > d \end{cases}$$

4.3 Operations on Trapezoidal fuzzy number

Let $\tilde{P}$ = (p_1, p_2, p_3, p_4) and $\tilde{Q}$ = (q_1, q_2, q_3, q_4) are the trapezoidal type fuzzy numbers. The operations on these numbers are as given below:

(i) $\tilde{P} \oplus \tilde{Q} = (p_1 + q_1, p_2 + q_2, p_3 + q_3, p_4 + q_4)$
(ii) $\tilde{P} \otimes \tilde{Q} = (p_1q_1, p_2q_2, p_3q_3, p_4q_4)$
(iii) $\tilde{P} \ominus \tilde{Q} = (p_1 - q_1, p_2 - q_2, p_3 - q_3, p_4 - q_4)$
(iv) $\tilde{P} \, ø \, \tilde{Q} = \left(\frac{p_1}{q_4}, \frac{p_2}{q_3}, \frac{p_3}{q_2}, \frac{p_4}{q_1}\right)$

4.4 Defuzzification

It is a process of converting fuzzy variables in into crisp values for a given fuzzy set under consideration. It actually converts the membership function related to fuzzy set into numerical values related to crisp set. There are different methods used for defuzzification. In this paper we have used the centroid method for defuzzification. It is defined as below:

For the membership function of discrete nature the value in defuzzified form of variable x^* is as below:

$$x^* = \frac{\sum_{i=1}^{n} x_i \mu\,(x_i)}{\sum_{i=1}^{n} (x_i)}$$

where x_i denoted the element of the given sample, $\mu\,(x_i)$ denotes the membership function and n denotes the number of elements in the given sample.

For continuous membership function the defuzzified value of variable x^* is given by:

$$x^* = \frac{\int x\, \mu_A(x)}{\int \mu_A(x)} dx$$

where x denoted the element of the given sample, $\mu_A(x)$ denotes the membership function of the given sample.

In this fuzzy economic order quantity system (FEOQ) model we consider the cost functions as fuzzy numbers of trapezoidal nature. The values are defuzzified the use of centroid method. Here we assume that Z is the total replenishment at the beginning of each of the cycle. When all the back orders have been fulfilled the initial level of the inventory system is assumed to be I. The primary motive of this inventory system is to determine the optimal total cost of the system. As there is a shortage of commodities in the system the optimality for this inventory system is determined for the shortage of goods in the inventory system. There is a decrease in the level of the inventory system due to demand of the commodities in the market in the period $(0,\mu_L)$. In the period (μ_L, t_1) there is a decrease in the level of the inventory system due to the demand of the commodity in the market as well as due to deterioration of the commodities. When time becomes equal to t_1 the inventory system level reaches zero. In interval of time [t_1, T] shortages accumulate in the system. There is partial backlogging of the shortages and a part of it occurs as sales which are lost. The next replenishment replaces the backlogged goods. This inventory system is shown above in figure 1.

The total cost of the inventory system per unit time for the fuzzy system is given by:

$$\widetilde{TC} = \frac{1}{T}\text{[Purchasing cost + Holding cost + Deterioration cost + Cost due to shortage + Opportunity cost]}$$

$$\frac{1}{T}\left[\begin{array}{l}\dfrac{\tilde{P}_c I}{i-r_d}\left[e^{-(r_d-i)T}-1\right]+\tilde{D}_c\gamma\lambda a_0\left[\dfrac{t_1^{\lambda+1}}{\lambda(\lambda+1)}-\dfrac{t_1\mu_L^{\lambda}}{\lambda}+\dfrac{\mu_L^{\lambda+1}}{\lambda+1}\right]\\ +\tilde{H}_c\left[I\mu_L+\dfrac{I(i-r_d)}{2}\mu_L^2-\dfrac{a_0\mu_L^2}{2}-\dfrac{a_0(i-r_d)\mu_L^3}{3}\right]+\end{array}\right]+$$

$$\frac{1}{T}\left[\tilde{H}_c a_0\left[\begin{array}{l}t_1\left\{t_1+\dfrac{\gamma t_1^{\lambda+1}}{\lambda+1}-\dfrac{it_1^2}{2}\right\}-\dfrac{t_1^2}{2}-\dfrac{\gamma}{(\lambda+1)(\lambda+2)}t_1^{\lambda+2}-\\ \dfrac{i}{6}t_1^3+\dfrac{(i-r_d)}{6}t_1^3-\dfrac{\gamma t_1^{\lambda+2}}{\lambda+1}+\dfrac{\gamma t_1^{\lambda+2}}{\lambda+2}-\mu_L\left\{\begin{array}{l}t_1+\dfrac{\gamma t_1^{\lambda+1}}{\lambda+1}-\\ \dfrac{it_1^2}{2}\end{array}\right\}\\ +\dfrac{\mu_L^2}{2}+\dfrac{\gamma}{(\lambda+1)(\lambda+2)}\mu_L^{\lambda+2}+\dfrac{i}{6}\mu_L^3-\dfrac{(i-r_d)t_1\mu_L^2}{2}+\\ \dfrac{(i-r_d)\mu_L^3}{3}+\dfrac{\gamma t_1\mu_L^{\lambda+1}}{\lambda+1}-\dfrac{\gamma\mu_L^{\lambda+2}}{\lambda+2}\end{array}\right]\right]$$

$$+\frac{1}{T}\left[\begin{array}{l}-\tilde{C}_s a_0\left[\begin{array}{l}\frac{(i+\alpha)t_1^2T}{2}+t_1T-\alpha t_1T^2-\frac{T^2}{2}+\frac{\alpha T^3}{2}-\frac{(i-\alpha)}{6}T^3+\\(i-r_d)\left\{\begin{array}{l}\frac{t_1T^2}{2}-\frac{t_1T^3\alpha}{2}+\frac{(i+\alpha)t_1^2T^2}{4}-\frac{T^3}{3}+\frac{\alpha T^4}{3}-\\\frac{(i+\alpha)T^4}{8}\end{array}\right\}\end{array}\right]\\+\tilde{O}_c a_o\alpha\left[\frac{T^2}{2}+\frac{t_1^2}{2}-Tt_1+\frac{(2i-r_d)T^3}{6}-(2i-r_d)\left\{\frac{Tt_1^2}{2}-\frac{t_1^3}{3}\right\}\right]\end{array}\right] \quad (14)$$

Now the main task is to minimize equations numbered (13) and (14). This is done by using Mathematica software. Also the cost of given inventory system in total for both crisp and fuzzy environments are calculated.

5. Optimality Criteria for Crisp System

In order to get the optimal value of the Total cost TC the following method is applied:

Step 1: First start with TC

Step 2: Take the first derivative of TC with respect to t_1 and equate the result to zero

i.e. $$\frac{\partial TC}{\partial t_1}=0 \quad (15)$$

and find the critical points.

Step 3: Evaluate TC with the help of t_1 which is found in step 2.

Step 4: Repeat the above step 2 and 3 until we get $\frac{\partial TC}{\partial t_1}>0$ for b. Thus we get the optimal number t_1 for which the total cost function TC is convex.

6. Optimality Criteria for Fuzzy System

In order to get the optimal value of the Fuzzy Total cost $\widetilde{TC}$ the following method is applied:

Step 1: First start with $\widetilde{TC}$

Step 2: Take its first derivative of with respect to t_1 and equate the result to zero i.e.

$$\frac{\partial \tilde{T}C}{\partial t_1}=0 \quad (16)$$

and find the critical points.

Step 3: Evaluate $\widetilde{TC}$ with the help of t_1 which is found in step 2.

Step 4: Repeat the above step 2 and 3 until we get $\frac{\partial \tilde{T}C}{\partial t_1} > 0$ for b. Thus we get the optimal number t_1 for which the total cost function $\widetilde{TC}$ is convex.

7. Illustrative Example

For proper explanation we have considered the following example of a bakery in both crisp and fuzzy environment. We consider the data of a bakery having products like cake, pastries, biscuits, etc. These products undergo deterioration due to different factors with the passage of time. The products are manufactured by the bakery keeping in view the demand of these commodities in the market and also their deterioration with time. Sometimes there is shortage of these products also due to unavailability of raw materials.

We have taken the example of a bakery product. The purchasing cost of 50 units of raw material is Rs. 4. Sometimes as there is unavailability of raw material therefore the manufacturer has to bear a shortage cost which is equivalent to Rs. 12 per unit. In order to retain such perishable products in adverse weather conditions the manufacturer incurs a holding cost of Rs. 3 per unit. These perishable items undergo deterioration and cost of deterioration is Rs. 8. Due to shortage of item there is an inflation which is equivalent to a rate 1.2 . Sometimes the manufacturer also offers a discount on the products to clear off a lot at a rate equivalent to 0.5. We obtain the total cost estimates in both crisp and fuzzy sense for this system.

The following estimates are obtained:
Purchasing cost P_c = Rs. 4
μ_L = life time of items = 0.3
i_r= the rate of inflation per unit of item = 1.2
r_d = rate of discount = 0.5
Cost of shortage per unit per unit time, S_c = Rs 12
Holding cost, H_c = Rs. 3
Cost of deterioration, D_c = Rs. 8
Opportunity cost = O_c = Rs. 5
Deterioration rate constant, γ (scale parameter) = 0.001
Deterioration rate constant, λ (shape parameter) = 2
Rate of demand = d_0 = 50
Parameter of backlogging = α = 0.1

For crisp model

By using Mathematica software we find the optimum solution of t_1 and S from equation numbered (13). The values are, t_1 = 1.315 and I = 19782.5 Substituting these values of t_1 and T in the equation numbered (13) we find the cost in total of the system of inventory is TC = Rs. 38043.2

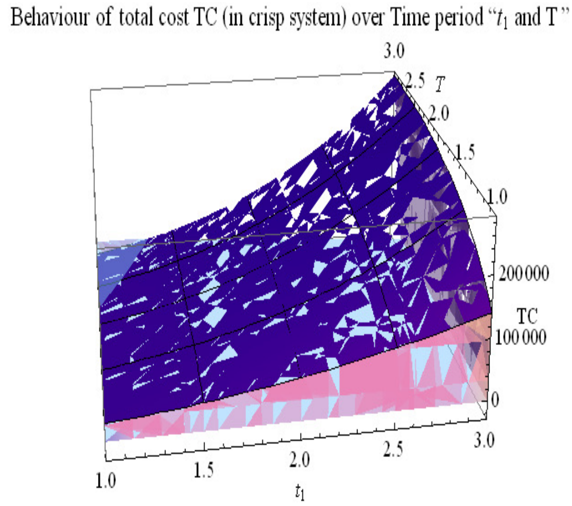

Figure 2. Behaviour of total cost TC (in crisp system) over time period t_1 and T

The behaviour of total cost over time period T in crisp system is shown in the above Figure 2. From the above figure we find that with an increase in the time period the cost in total of the crisp system also shows an increase. The upward trend of the graph shows that total cost increases with time.

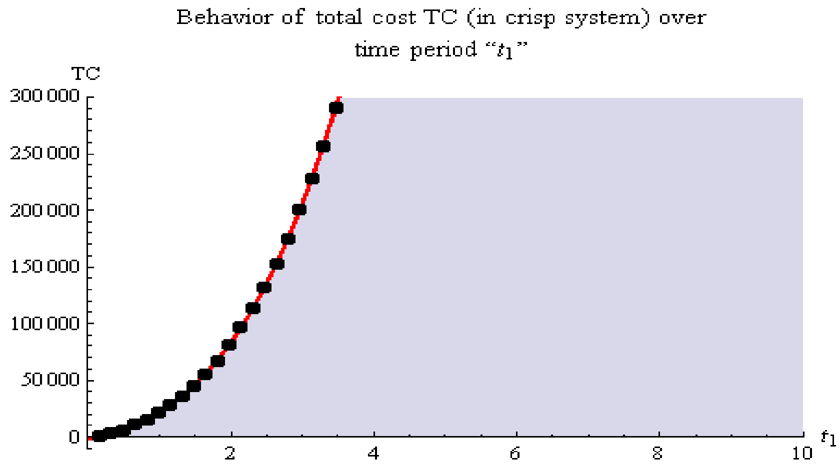

Figure 3. Behaviour of total cost TC (in crisp) over time period T

The behaviour of total cost over time period T in crisp system is shown in the above Figure 3. From the above figure we find that with the increase in the time period the cost in total for the crisp system also shows an increase. The upward trend of the graph shows that total cost increase with time.

For fuzzy economic order quantity (FEOQ) model: We have the following values:

Fuzzy Purchasing cost $\tilde{P}_c$ = (1,3,5,7) = Rs. 6.67
μ_L = life time of items = 0.3
i_r= the rate of inflation per unit of item = 1.2
r_d = rate of discount = 0.5
Fuzzy Cost of shortage per unit per unit time, $\tilde{S}_c$ = (9,11,13,15) = Rs 3.33
Fuzzy Holding cost, $\tilde{H}_c$ = (1,3,5,7) = Rs. 6.67
Fuzzy Cost of deterioration, $\tilde{D}_c$ = (5,7,9,11) = Rs. 2
Fuzzy Opportunity cost = $\tilde{O}_c$ = (2,4,6,8) = Rs. 1
Deterioration rate constant, γ (scale parameter) = 0.001
Deterioration rate constant, λ (shape parameter) = 2
Rate of demand = d_0 = 50
Parameter of backlogging = α = 0.1

By using Mathematica software we find the optimum solution of t_1 and S from equation numbered (14). The values are, t_1 = 2.196 and I = 3878.4 Substituting these values of t_1 and T in the equation numbered (14) we find the cost in total for the inventory system $\widetilde{TC}$ = Rs. 13821.2

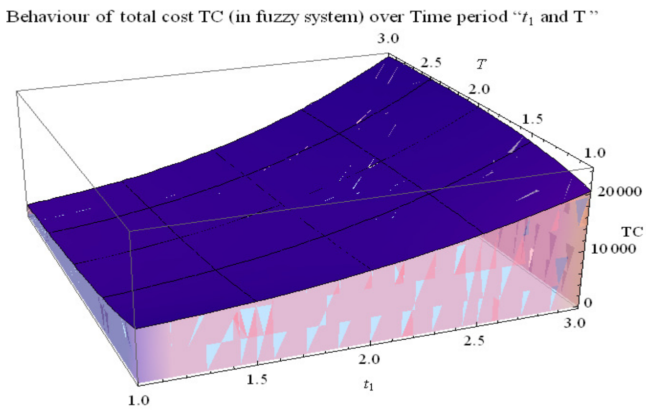

Figure 4. Behaviour of total cost TC (in fuzzy system) over time period t_1 and T

The behaviour of total cost over time period t_1 and T in fuzzy system is shown in Figure 4. As the time span increases from t_1 to T, the total cost increases.

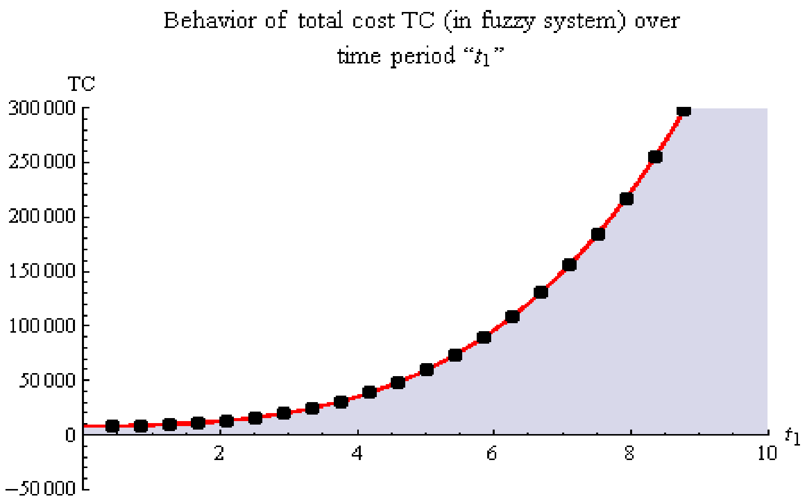

Figure 5. Behaviour of total cost TC (in fuzzy) over time period T

The behaviour of total cost over time period T in fuzzy system is shown in the above Figure 5. From the above figure we find that with the increase in the time period the cost of the inventory system in total for the fuzzy model also increases. The upward trend of the graph indicates that the total cost increase with time.

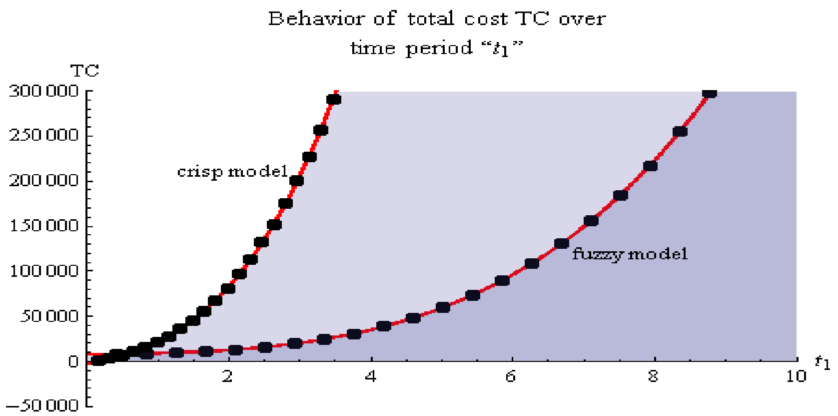

Figure 6. Behaviour of total cost TC (in crisp and fuzzy) over time period T

The behaviour of total cost over time period T in crisp and fuzzy system is shown in the above Figure 6. From the above figure we find that when time period increases the cost in total in both crisp and fuzzy system also increases. But the total cost is more for crisp system as compared to the fuzzy system. Thus by conversion of crisp system into fuzzy system the total cost of the inventory model has been considerably reduced.

Thus from the above example of bakery we conclude that the total cost for bakery product (pastry) is less in case of fuzzy system as compared to crisp system. Therefore by the application of fuzzy model there is a reduction in the cost of the item which in turn increases the total profit. Hence fuzzy economic order quantity (FEOQ) model is really beneficial for any sort of inventory system.

8. Conclusion

This study represents an inventory model which can be directly applied to any business undertaking considering the fact that all perishable items undergo deterioration with passage of time. The total cost for this model has been calculated for both crisp and fuzzy system. For the fuzzy economic order quantity (FEOQ) model fuzzification has been done by the use of trapezoidal fuzzy number. We have applied the centroid method for defuzzification. Comparing the results obtained by both the crisp and fuzzy model we observe that in case of fuzzy economic order quantity (FEOQ) model the total cost has been considerably reduced by 63.67%. Thus the main inference that we can draw from this study is that fuzzy model is more accurate as in this model the total cost has been reduced as compared to the crisp model. Thus fuzzy economic order quantity (FEOQ) model is highly beneficial to any sort of inventory system and this result can be further generalized to any sector.

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