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ON USE OF GAMMA DISTRIBUTION FOR EVALUATION OF RELIABILITY AND AVAILABILITY OF A SINGLE UNIT SYSTEM SUBJECT TO ARRIVAL TIME OF THE SERVER

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Abstract

The preference to the use of single unit systems over the redundant systems has been given due to their intrinsic reliability and affordability. And, stochastic modeling of repairable systems of one or more unit has been done by assuming negative exponential distribution for failure and repair times. In fact, the repairable systems may or may not have constant failure and repair rates. In such situations some other distributions possessing monotonic nature of the random variables associated with different time points may be considered. Gamma distribution is one of the distributions that may offer a good fit to some set of failure data. Also, negative exponential distribution is a special case of this distribution. Hence, in this paper reliability and availability of a single unit system by considering Gamma distribution for the random variables associated with failure and repair times of the system have been evaluated. A single server is employed to carry out the repair activities. The server is allowed to take some time to arrive at the system (called arrival time). The system has all the transit points as regenerative and so regenerative point has been used to derive the expressions for reliability measures. The values of reliability and availability are obtained for particular situations of the parameters. The behavior of these measures has been observed for the arbitrary values of the parameters.

Key Words: Single Unit System, Arrival Time, Reliability Measures, Gamma Distribution.

1. Introduction

Over the years, researchers have reported several studies on determining the best possible technique for enhancing performance and thus reliability of repairable systems. And, as a result of which system designers and engineers have succeeded in making the systems more reliable to use with less hindrances. The provision of spare unit in operating systems has been suggested as one of the best means not only to enhance availability of the systems but also to share the working load. Barak and Malik (2013) analyzed reliability measures of a cold standby system with preventive maintenance and repair. Barak et al. (2014) have obtained reliability measures of a standby system by giving priority to repair over corrective maintenance. But, there are many systems in which a spare unit cannot be considered as suitable either may because of its high cost or to avoid bulkiness in the system. Thus, in such a situation, a single unit system may be used that can provide required services with affordability and intrinsic reliability. Several authors including Malik and Bansal (2005), Chander (2007) and Kumar et al. (2016) have analyzed reliability measures of single unit systems. However, in most of these studies, it is assumed that service facility may be made available immediately to carry out the repair activities. In fact, this assumption seems to be unrealistic in case server is engaged in his pre assigned jobs. So, in these circumstances, the server may be allowed to take some time to reach at the system. Several authors including Nandal and Malik (2016) have studied a cold standby system with arrival time of the server. Kumar et al. (2017) analyzed performance of an industrial system under multistate failures with standby mode.

It is a matter of fact that negative exponential distribution has been frequently used in reliability theory may because of its memory less property. But, this distribution is a particular case of gamma distribution. And, gamma distribution is one of the distributions which consider the case of monotonic nature of the random variables associated with failure and repair times. Recently, Nandal et al. (2017) evaluated reliability measures of a single unit system by considering Gamma failure laws.

Hence, the present study is confined on the evaluation of reliability measures of a single unit system with gamma distribution for failure and repair times. There is a single server who is called to carry out repair activities as per requirement. However, server is allowed to take some time (called arrival time) to reach at the system. The repair activities are perfect. The expressions for some important reliability characteristics are derived in steady states by using Markov process and regenerative point technique. The behavior of mean time to system failure (MTSF), reliability and availability of the system has been observed for arbitrary values of the parameters. The results are shown graphically and numerically.

Gamma Distribution

For fitting of failure data in a more precise way, the gamma distribution has been considered as an appropriate distribution. The gamma distribution has more applications in Bayesian reliability. If $x \sim \Gamma(\lambda, k)$; then the failure density function for gamma distribution [System Software Reliability (2015)] is given by

$$f(x, \lambda, k) = \frac{\lambda^k}{\Gamma\, k} e^{-\lambda x} x^{n-1};\ \lambda, k > 0, x \geq 0$$

Where, $\lambda = \text{scale paprameter}, \text{k} = \text{Shape parameter}$

Then the reliability function is also defined as:

$$R(t) = e^{-\lambda t} \sum_{z=0}^{k-1} \frac{\lambda t^z}{z!}$$

Thus, hazard rate is given by $h(t) = \frac{\lambda^k t^{k-1}}{\Gamma\, k\ \sum_{z=0}^{k-1} \frac{\lambda t^z}{z!}}$

2. System Description

Here, we discuss a reliability model for a single unit system with arrival time of the server. The block diagram of the system model is shown in Fig.:1

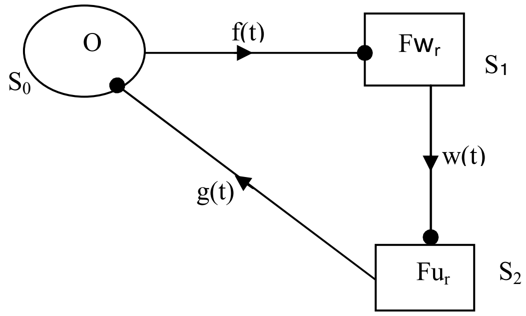

Fig. 1: State Transition Diagram

Where, • Regenerative point O Operative state □ Failed state

3. Notations

O : The unit is operative and in normal mode
Fu_r : The system is failed and under repair
Fw_r : The system is failed and waiting for repair
S_0 : The initial state in which the system is good and operative
S_1 : The second state in which system is failed and waiting for repair due to non availability of the server
S_2 : The last state in which system is failed and under repair of the server
g(t) : Probability Density Function (pdf) of repair time
f(t) : Probability Density Function (pdf) of failure rate
w(t) : Probability Density Function (pdf) of arrival time of the server

4. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements $p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)dt$ as

$$dQ_{01}(t) = q_{01}(t)dt = \left(\frac{\lambda}{(k-1)!}(\lambda t)^{k-1}e^{-\lambda t}\right)dt \quad (1)$$

$$dQ_{12}(t) = q_{12}(t)dt = w(t)dt \quad (2)$$

$$dQ_{20}(t) = q_{20}(t)dt = g(t)dt \quad (3)$$

Taking Laplace Stieltjes Transform, we have

$$Q_{01}^{**}(s) = \int_0^\infty e^{-st}\, d[Q_{01}(t)] = \int_0^\infty e^{-st}\left(\frac{\lambda}{(k-1)!}(\lambda t)^{k-1}e^{-\lambda t}\right)dt = \frac{(\lambda)^k}{(\lambda+s)^k} \quad (4)$$

$$Q_{12}^{**}(s) = w^*(s) \quad (5)$$

$$Q_{20}^{**}(s) = g^*(s) \quad (6)$$

Taking $s \to 0$, we get the following transition probabilities:

$p_{01} = 1,\ p_{12} = w^*(0) = 1,\ p_{20} = g^*(0) = 1$

Mean Sojourn Times

The mean sojourn time in a state is the expected time taken by the system in that state before transiting in to any other state. If T_i be the sojourn time in the state i, then the mean sojourn time in the state I is

$\mu_i = \int_0^\infty \Pr\,(T_i > t)$ or $\mu_i = \sum_j m_{ij}$ (i = 0,1)

But $m_{ij} = -\frac{d}{ds}[Q_{01}^{**}(s)]_{s=0}$

We have, $m_{01} = -\frac{d}{ds}[\frac{(\lambda)^k}{(\lambda+s)^k}]_{s=0} = \frac{k}{\lambda}$ (7)

$m_{12} = -\frac{d}{ds}[w^*(s)]_{s=0} = -w^{*'}(0)$ (8)

$m_{20} = -\frac{d}{ds}[g^*(s)]_{s=0} = -g^{*'}(0)$ (9)

Now, $\mu_0 = m_{01} = \frac{k}{\lambda}$, (10)

$\mu_1 = m_{12} = -w^{*'}(0)$ $\mu_2 = m_{20} = -g^{*'}(0)$

5. Reliability Measures

The following reliability measures have been evaluated for the system model:

5.1 Mean Time to System Failure (MTSF)

The cumulative distribution function of first passage time from a regenerative state s_i to a failed state is known as the MTSF. It is denoted by $\emptyset_i$(t). we have

$\emptyset_0\,(t) = \emptyset_{01}(t)$ (11)

Taking Laplace Stieltjes Transform of (11), we get

$\emptyset_0^{**}(s) = Q_{01}^{**}(s) = \frac{(\lambda)^k}{(\lambda+s)^k}$

Now, MTSF = $\lim_{S\to 0}\frac{1-\emptyset_0^{**}(s)}{s} = \lim_{S\to 0}\frac{1-\frac{(\lambda)^k}{(\lambda+s)^k}}{s} = \left(\frac{0}{0}\right)$ Form (12)

So, by Applying L' Hospital Rule, we get

MTSF = $Q_{01}^{**\prime}(0) = \mu_0 = m_{01} = \frac{k}{\lambda}$ (13)

5.2 Reliability

Generally, the probability of no failure is called reliability. Thus, reliability of the system can be obtained in terms of $\emptyset_i$(t) as

$R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s} = \frac{1-Q_{01}^{**}(s)}{s} = \frac{(\lambda+s)^k-\lambda^k}{s(\lambda+s)^k}$ (14)

The reliability of the system model can be obtained by taking Laplace Inverse of $R^*(s)$, we get

R(t) = $L^{-1}\left(\frac{(\lambda+s)^k-\lambda^k}{s(\lambda+s)^k}\right)$ (15)

5.3 Availability

In fact, availability of system is the probability that system is available for use at a specific time 't'. We have the following expression for availability ($A_i(t)$) in different states of the system as

$A_0(t)= M_0(t) + q_{01}(t)©A_1(t)$
$A_1(t) = q_{10}(t)©A_0(t)$
Taking Laplace transform of above equations, we have
A_0^* (s) = M_0^*(s) + q_{01}^*(s). A_1^*(s)

A_1^*(s) = q_{12}^*(s). A_2^*(s), A_2^*(s) = q_{20}^*(s). A_0^*(s)

Using Cramer's Rule for solving the above equations to obtain A_0^* (s), we get

$$A_0^*(s) = \frac{M_0^*(s)}{1-q_{01}^*(s)q_{12}^*(s)q_{20}^*(s)}$$

The steady state availability is given by

$$A(\infty) = \lim_{t\to\infty} A(t) = \lim_{s\to 0} sA_0^*(s)$$

$$= \lim_{s\to 0} s\left[\frac{M_0^*(s)}{1-q_{01}^*(s)q_{12}^*(s)q_{20}^*(s)}\right] \quad \left(\frac{0}{0}\ form\right)$$

Using L' Hospital Rule, we get

$$A(\infty) = \lim_{s\to 0} s\left[\frac{M_0^*(s)}{1-q_{01}^*(s)q_{12}^*(s)q_{20}^*(s)}\right]$$

$$= \lim_{s\to 0} \frac{M_0^*(s) + sM_0^{*'}(s)}{0-[q_{01}^{*'}(s)q_{12}^*(s)q_{20}^*(s) + q_{01}^*(s)q_{12}^{*'}(s)q_{20}^*(s) + q_{01}^*(s)q_{20}^{*'}(s)q_{12}^*(s)]}$$

$$= \frac{M_0^*(0)}{-[q_{01}^{*'}(0)q_{12}^*(0)q_{20}^*(0) + q_{01}^*(0)q_{12}^{*'}(0)q_{20}^*(0) + q_{01}^*(0)q_{20}^{*'}(0)q_{12}^*(0)]}$$

$$= \frac{1}{1-\frac{\lambda}{k}g^{*'}(0)-\frac{\lambda}{k}w^{*'}(0)} \qquad (16)$$

If repair and arrival times of the server follow Gamma distribution, then we can take
$g(t) = \left(\frac{\alpha}{(x-1)!}(\alpha t)^{x-1}e^{-\alpha t}\right)$, $w(t) = \left(\frac{\beta}{(z-1)!}(\beta t)^{z-1}e^{-\beta t}\right)$

Taking Laplace inverse transform of the above expressions, we have

$g^*(s) = \int_0^\infty e^{-st}\, g(t)dt = \frac{(\alpha)^x}{(\alpha+s)^x}$ and $g^{*'}(s) = -x\alpha^x(\alpha+s)^{-(x+1)}$

$w^*(s) = \int_0^\infty e^{-st}\, w(t)dt = \frac{(\beta)^z}{(\beta+s)^z}$ and $w^{*'}(s) = -\, z\beta^z(\beta+s)^{-(z+1)}$

Taking limit s→ 0 , we have
$g^*(0) = 1$ and $g^{*'}(0) = -\frac{x}{\alpha}$ $w^*(0) = 1$ and $w^{*'}(0) = -\frac{z}{\beta}$
Hence, $A(\infty) = \frac{k\alpha\beta}{k\alpha\beta + x\lambda\beta + z\lambda\beta}$

6. Numerical and Graphical Representation of MTSF, Reliability and Availability

Scale parameter λ	MTSF				
	K=1	K=2	K=3	K=4	K=5
0.01	100	200	300	400	500
0.02	50	100	150	200	250
0.03	33.33	66.66	99.99	133.33	166.65
0.04	25	50	75	100	125
0.05	20	40	60	80	100
0.06	16.66	33.33	49.98	66.66	83.33
0.07	14.28	28.56	42.84	57.12	71.42
0.08	12.5	25	37.5	50	62.5
0.09	11.11	22.22	33.33	44.44	55.55
0.1	10	20	30	40	50

Table 1: MTSF Vs Scale Parameter

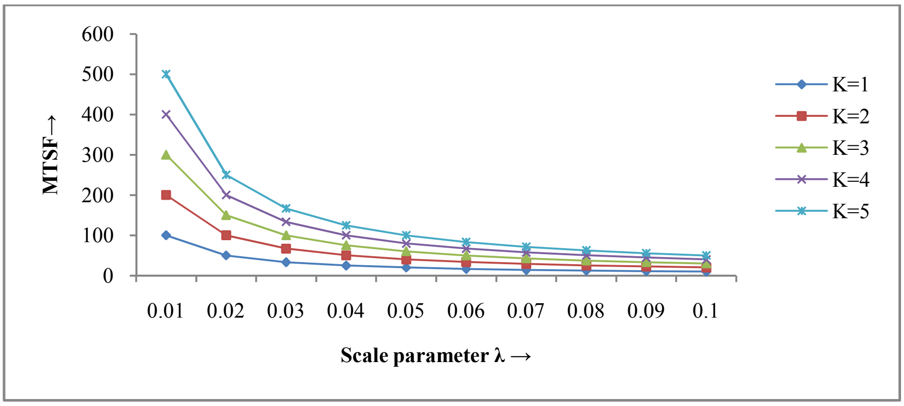

Fig.2: MTSF Vs Scale parameter

Scale Parameter	Reliability					
λ	k=1,t=10	k=2,t=10	k=3,t=10	k=1, t=15	k=2, t=15	k=3, t=15
0.01	0.90483	0.99532	0.99984	0.86070	0.98981	0.99949
0.02	0.81873	0.98247	0.99885	0.74081	0.96306	0.99640
0.03	0.74081	0.96306	0.99640	0.63762	0.92456	0.98912
0.04	0.67032	0.93844	0.99207	0.54881	0.87809	0.97688
0.05	0.60653	0.90979	0.98561	0.47236	0.82664	0.95949
0.06	0.54881	0.87809	0.97688	0.40656	0.77248	0.93714
0.07	0.49658	0.84419	0.96585	0.34993	0.71737	0.91027
0.08	0.44932	0.80879	0.95257	0.30119	0.66262	0.87948
0.09	0.40656	0.77248	0.93714	0.25924	0.60921	0.84544
0.1	0.36787	0.73575	0.91969	0.22313	0.55782	0.80884

Table 2: Reliability Vs Scale parameter

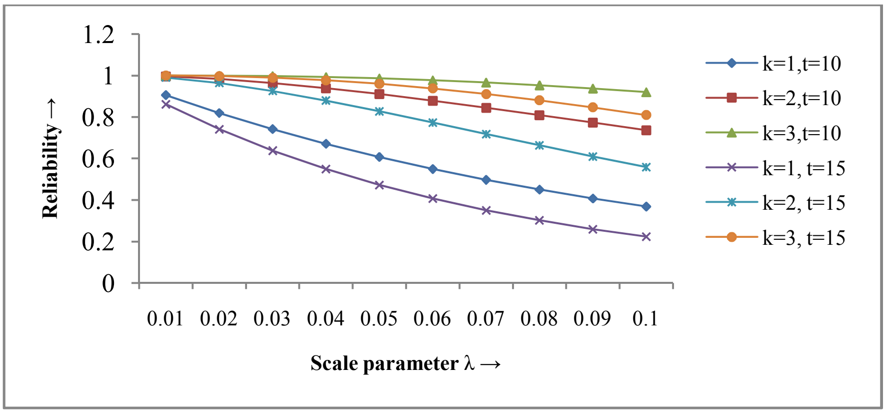

Fig.3: Reliability Vs Scale parameter

Scale Parameter λ	**Availability**					
	β=.3,α=.7 **x,z,k=1**	β=.3,α=.7 **k=3**,x,z=2	β=.3,α=.7 **x=3**,k,z=2	β=.3,α=.7 **z=3**,x,k=2	**β=.5**,α=.7 x,z,k=2	β=.3,**α=.9** x,z,k=2
0.01	0.95454	0.96923	0.94808	0.93959	0.96685	0.95744
0.02	0.91304	0.94029	0.90128	0.88607	0.93582	0.91836
0.03	0.87500	0.91304	0.85889	0.83832	0.90673	0.88235
0.04	0.84000	0.88732	0.82031	0.79545	0.87939	0.84905
0.05	0.80769	0.86301	0.78504	0.75675	0.85365	0.81818
0.06	0.77777	0.84000	0.75268	0.72164	0.82938	0.78947
0.07	0.75000	0.81818	0.72289	0.68965	0.80645	0.76271
0.08	0.72413	0.79746	0.69536	0.66037	0.78475	0.73770
0.09	0.70000	0.77777	0.66985	0.63348	0.76419	0.71428
0.1	0.67741	0.75903	0.64615	0.60869	0.74468	0.69230

Table 3: Availability Vs Scale parameter

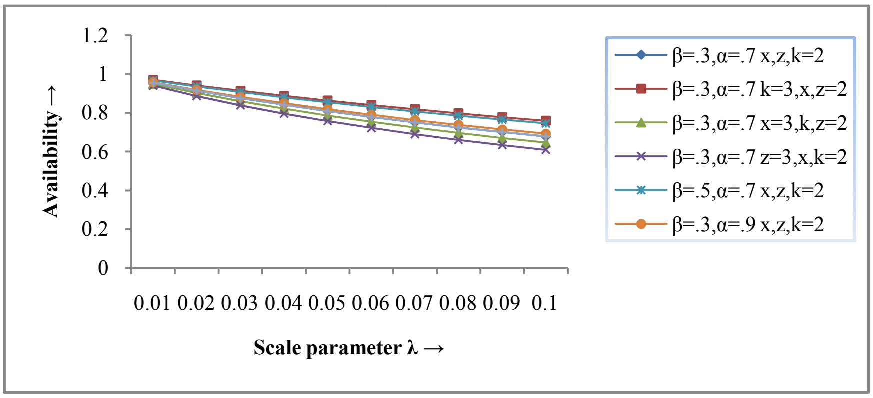

Fig. 4: Availability Vs Scale parameter

7. Conclusion and Discussion

The behavior of mean time to system failure (MTSF), reliability and availability of a single unit system has been examined for arbitrary values of failure and repair rates as shown in figures 2, 3 & 4 respectively. We found that MTSF steeply declines with the increase of scale parameter 'λ' while it increases with the increasing of shape parameter (k). And, reliability keeps on decreasing with the increasing of scale parameter 'λ' & operating time (t). However, reliability increases with the increase of the value of shape parameter (k). On the other hand, the availability of the system goes on decreasing with the increasing of failure rate but it increases with the increasing of repair rate and scale parameter (β). And, it is also observed that availability decreases with the increasing of arrival time of the server. Hence, the research findings of the study indicates that a single unit system with arrival time of the server can be made more reliable and available to use by increasing repair rate in proportionate to the shape parameter ($k > 1$) and also by diminishing the arrival time of the server. The reliability and availability of the system for $k=1$ will be same as in case failure and repair times follow negative exponential distribution. The results for these reliability measures are also given in the respective Tables 1, 2 and 3.

Future Scope of the Work

Actually, most of the stochastic models have been analyzed under a common assumption that failure and repair times are constant and thus follow negative exponential distribution. Here, the reliability measures of a single unit repairable system have been obtained by considering Gamma distribution for different epochs. The work may be extended to stochastic models for redundant systems by considering series or parallel working of the units.

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