

## EFFICIENCY EVALUATION OF RATIO ESTIMATORS IN SIMPLE RANDOM SAMPLING

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### Abstract

In literature, several ratio type estimators of population mean were proposed by statisticians but none of them made pair wise comparison of these estimators. In this paper an attempt has been made for pair wise efficiency comparison of the same and find out the different conditions on which one estimator performed better than the other. Depending on the structure of data used, the efficiency comparison of these estimators is varied in certain circumstances. In this study we have revealed the efficiency conditions of the existing ratio estimators, through pair wise comparisons and examine the relative performance of ratio estimators in terms of efficiency and unbiasedness empirically.

**Key Words:** Ratio Type Estimators, Efficiency Comparison, Bias, Mean Squared Error, Percentage Relative Bias.

### 1. Introduction

As we know that when a survey is performed, additional information other than study variable can be made available and there is always gain in precision of an estimator when we obtained additional information other than study variable. In that cases a numbers of estimators has been developed like ratio, product, regression estimators and their generalizations. Cochran (1940) was the first to use the auxiliary information by introducing the concept of ratio estimators. Ratio estimator was used when study variable and auxiliary variable are positively correlated and line of regression passing through origin. Several estimators for population parameters of study variable have been discussed in literature when the population mean of an auxiliary variable is known. Most of the survey statisticians like Smith (1976), Singh et al.(2015), Rashid et al. (2015), Sharma et al. (2015), Kumar et al. (2016), Yasmeen et al. (2016) and Subzar et al. (2018) have compared their proposed estimators with the existing ones. They established the different conditions when their proposed estimators are better than the existing estimators. It was observed that pair wise comparison of these estimators under realistic conditions and both efficiency and unbiasedness has not been established. Keeping in view the above facts the present study has been planned. In this an attempt has been made for the pair wise comparisons for different ratio estimators and established the different conditions under which one estimator performed better than the others.

## 2. Materials and Methods

As it is discussed earlier ratio estimator was used when there is positive correlation between study and auxiliary variable and line of regression passes through origin. When line of regression does not pass through origin in that case we should not use the concept of ratio estimator.

Let a random sample of size  $n$  is drawn from a population of size  $N$  and observations on auxiliary variable  $X$  and study variables  $Y$  are obtained. Further, the sample mean and sample variance are unbiased estimators of population mean and population variance respectively. Similarly, let  $s_{xy}$  be an unbiased estimator of population covariance  $\sigma_{xy}$ . We follow the convention that the lower case letters  $y_i$  and  $x_i$  stand for  $i^{\text{th}}$  unit in the sample ( $i=1, 2, \dots, n$ ) and upper case letters  $Y_i$  and  $X_i$  stand for the  $i^{\text{th}}$  unit in the population ( $i=1, 2, \dots, N$ ).

## 3. Notations

The following common notations have been used for the comparison of bias and efficiency conditions of the estimators:

$N$  = Population size

$n$  = Sample size

$f = n/N$ , Sampling fraction

$Y$  = Study variable

$X$  = Auxiliary variable

$\bar{Y}, \bar{X}$  = Population means  $\bar{Y} = \frac{1}{N} \sum_{i=1}^N Y_i$

$\bar{X} = \frac{1}{N} \sum_{i=1}^N X_i$

$\bar{y}, \bar{x}$  = Sample means  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$

$\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$

$S_y, S_x$  = Population standard deviations

$$S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (Y_i - \bar{Y})^2$$

$$S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2$$

$\sigma_{XY}$  – Population covariance between  $X$  and  $Y$

$$\sigma_{XY} = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{X})(y_i - \bar{Y})$$

$C_y, C_x$  = Co-efficient of variations of  $x$  and  $y$

$$C_y = \frac{S_y}{\bar{Y}}$$

$$C_x = \frac{S_x}{\bar{X}}$$

$\rho$  = Co-efficient of correlation between  $X$  and  $Y$

$$\rho = \frac{C_{11}}{(C_{02})^{1/2}(C_{20})^{1/2}}$$

$\beta_1$  = Co-efficient of skewness of the auxiliary variable

$$\beta_1(x) = \frac{\mu_3}{\mu_2^3}$$

$$\beta_2(x) = \frac{\mu_4}{\mu_2^2}$$

Relative bias = It is the ratio of bias divided by the mean of study variable

$$W = \rho \frac{C_y}{C_x}$$

$$\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}$$

$$\theta_2 = \frac{\bar{X}}{\bar{X} + \beta_2(x)}$$

$$\theta_3 = \frac{\bar{X}}{\bar{X} + \rho}$$

$$\theta_4 = \frac{\beta_2 \bar{X}}{\bar{X} \beta_2 + \beta_1(x)}$$

$$\theta_5 = \frac{\bar{X}}{\bar{X} + Q_r}$$

$$\theta_6 = \frac{\bar{X}}{\bar{X} + Q_a}$$

$$\theta_7 = \frac{\bar{X} \beta_2(x)}{\bar{X} \beta_2(x) + C_x}$$

Table 1 shows the various ratio estimators of population mean existing in the literature along with the expressions of their biases and mean squared errors.

<b>Classical Ratio Estimator</b>	$\bar{y}_r = \frac{\bar{y}}{\bar{X}} \bar{X}$
Bias	$\frac{(1-f)}{n} \left( \frac{RS_x^2}{\bar{X}} - \frac{W\bar{Y}S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2WR^2S_x^2 + R^2S_x^2)$
<b>Sisodia and Dwivedi (1981) Estimator</b>	$\bar{y}_1 = \bar{y} \left( \frac{\bar{X} + C_x}{\bar{X} + C_x} \right)$
Bias	$\frac{(1-f)}{n} \left( \frac{R\theta_1^2 S_x^2}{\bar{X}} - \frac{W\theta_1 \bar{Y} S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2\theta_1 WR^2 S_x^2 + R^2 \theta_1^2 S_x^2)$
<b>Bahl and Tuteja (1991) Estimator</b>	$\bar{y}_2 = \bar{y} \exp \left[ \frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right]$
Bias	$\frac{(1-f)}{8n} \left( \frac{3RS_x^2}{\bar{X}} - \frac{4W\bar{Y}S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} \left( \frac{4S_y^2 + R^2 S_x^2 - 4R^2 S_x^2 W}{4} \right)$
<b>Singh et al. (2004) Estimator</b>	$\bar{y}_3 = \bar{y} \frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)}$
Bias	$\frac{(1-f)}{n} \left( \frac{R\theta_2^2 S_x^2}{\bar{X}} - \frac{W\theta_2 \bar{Y} S_x^2}{\bar{X}^2} \right)$

Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2\theta_2 WR^2 S_x^2 + R^2 \theta_2^2 S_x^2)$
<b>Upadhyaya and Singh (1999) Estimator</b>	$\bar{y}_4 = \bar{y} \left[ \frac{\beta_2(x)\bar{X} + C_x}{\beta_2(x)\bar{x} + C_x} \right]$
Bias	$\frac{(1-f)}{n} \left( \frac{R\theta_2^2 S_x^2}{\bar{X}} - \frac{W\theta_2 \bar{Y} S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} \left( S_y^2 - \frac{2\theta_7 W S_x \bar{Y} S_y S_x R}{S_y \bar{X}} + R^2 \theta_7^2 S_x^2 \right)$
<b>Yan and Tian (2010) Estimator</b>	$\bar{y}_5 = \bar{y} \left[ \frac{\beta_2(x)\bar{X} + \beta_1(x)}{\beta_2(x)\bar{x} + \beta_1(x)} \right]$
Bias	$(\bar{y}_d) = \frac{(1-f)}{n} \left( \frac{R\theta_4^2 S_x^2}{\bar{X}} - \frac{W\theta_4 \bar{Y} S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2\theta_4 WR^2 S_x^2 + R^2 \theta_4^2 S_x^2)$
<b>Singh and Tailor (2003) Estimator</b>	$\bar{y}_6 = \bar{y} \frac{\bar{X} + \rho}{\bar{x} + \rho}$
Bias	$\frac{(1-f)}{n} \left( \frac{R\theta_3^2 S_x^2}{\bar{X}} - \frac{W\theta_3 \bar{Y} S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2\theta_3 WR^2 S_x^2 + R^2 \theta_3^2 S_x^2)$
<b>Kadilar and Cingi (2004) Estimator</b>	$\bar{y}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X}$
Bias	$\frac{(1-f)}{n} \theta_5^2 \frac{S_x^2}{\bar{Y}}$
Mean Squared Error	$\frac{(1-f)}{n} [\theta_5^2 S_x^2 + S_y^2 (1 - \rho^2)]$
<b>Subramani and Kumarapandiyam (2012) Estimator</b>	$\bar{y}_8 = \bar{y} \frac{\bar{X} + Q_a}{\bar{x} + Q_a}$
Bias	$\frac{(1-f)}{n} \left( \frac{R\theta_{13}^2 S_x^2}{\bar{X}} - \frac{W\theta_{13} \bar{Y} S_x^2}{\bar{X}^2} \right)$
Mean Squared Error	$\frac{(1-f)}{n} (S_y^2 - 2\theta_{13} WR^2 S_x^2 + R^2 \theta_{13}^2 S_x^2)$

**Table 1: Ratio Estimators of population mean along with bias and MSE**

Table 2 shows the pair-wise efficiency comparison of all the above said estimators with respect to mean squared errors. The condition under which one estimator is efficient than the other estimators have been worked out and are given in the last column of Table 2.

Estimator	Efficiency level	Estimator	Mean squared condition
Classical ratio estimator	Better than	Sisodia and Dwivedi (1981)	$\rho > \frac{(1 + \theta_1)C_x}{2C_y}$
	Better than	Bahl and Tuteja (1991)	$\rho > \frac{3C_x}{4C_y}$
	Better than	Singh <i>et al.</i> (2004)	$\rho > \frac{(1 + \theta_2)C_x}{2C_y}$
	Better than	Upadhyaya and Singh (1999)	$\rho > \frac{(1 + \theta_7)C_x}{2C_y}$
	Better than	Singh and Tailor(2003)	$\rho > \frac{(1 + \theta_4)C_x}{2C_y}$
	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{S_x^2(\theta_5^2 - R^2 + 2WR^2)}{S_y^2}$
	Better than	Yan and Tian (2010)	$\rho > \frac{(1 + \theta_4)C_x}{2C_y}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > \frac{(1 + \theta_6)C_x}{2C_y}$
Sisodia, and Dwivedi (1981)	Better than	Bahl and Tuteja (1991)	$\rho > \frac{4\theta_1^2 C_x - C_x}{4C_y(2\theta_1 - 1)}$
	Better than	Singh <i>et al.</i> (2004)	$\rho > (\theta_1 + \theta_2) \frac{C_x}{2C_y}$
	Better than	Upadhyaya and Singh (1999)	$\rho > (\theta_1 + \theta_7) \frac{C_x}{2C_y}$
	Better than	Singh and Tailor(2003)	$\rho > (\theta_1 + \theta_3) \frac{C_x}{2C_y}$
	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{2\theta_1 WR^2 S_x^2 + \theta_5^2 S_x^2 - \theta_1^2 R^2 S_x^2}{S_y^2}$
	Better than	Yan and Tian (2010)	$\rho > (\theta_1 + \theta_4) \frac{C_x}{2C_y}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > (\theta_1 + \theta_6) \frac{C_x}{2C_y}$
Bahl and Tuteja (1991)	Better than	Singh <i>et al.</i> (2004)	$\rho > \frac{(1 - 4\theta_2^2)C_x}{4C_x C_y (1 - 2\theta_2)}$
	Better than	Upadhyaya and Singh (1999)	$\rho > \frac{(1 - 4\theta_7^2)C_x}{4C_x C_y (1 - 2\theta_a)}$
	Better than	Singh and Tailor(2003)	$\rho > \frac{(1 - 4\theta_3^2)C_x}{4C_x C_y (1 - 2\theta_3)}$

	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{4\theta_5^2 S_x^2 - R^2 S_x^2 + 4R^2 S_x^2 W}{4S_y^2}$
	Better than	Yan and Tian (2010)	$\rho > \frac{(1 - 4\theta_4^2)C_x}{4C_x C_y (1 - 2\theta_b)}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > \frac{(1 - 4\theta_1^2)C_x}{4C_x C_y (1 - 2\theta_i)}$
Singh <i>et al.</i> (2004)	Better than	Upadhyaya and Singh (1999)	$\rho > (\theta_2 + \theta_7) \frac{C_x}{2C_y}$
	Better than	Singh and Tailor(2003)	$\rho > (\theta_2 + \theta_3) \frac{C_x}{2C_y}$
	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{2\theta_2 W R^2 S_x^2 + \theta_5^2 S_x^2 - \theta_2^2 R^2 S_x^2}{S_y^2}$
	Better than	Yan and Tian (2010)	$\rho > (\theta_2 + \theta_4) \frac{C_x}{2C_y}$
Upadhyaya and Singh (1999)	Better than	Singh and Tailor(2003)	$\rho > (\theta_7 + \theta_3) \frac{C_x}{2C_y}$
	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{2\theta_7 W R^2 S_x^2 + \theta_5^2 S_x^2 - \theta_7^2 R^2 S_x^2}{S_y^2}$
	Better than	Yan and Tian (2010)	$\rho > (\theta_7 + \theta_4) \frac{C_x}{2C_y}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > (\theta_2 + \theta_6) \frac{C_x}{2C_y}$
Singh and Tailor (2003)	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{2\theta_3 W R^2 S_x^2 + \theta_5^2 S_x^2 - \theta_3^2 R^2 S_x^2}{S_y^2}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > (\theta_3 + \theta_6) \frac{C_x}{2C_y}$
Yan and Tian (2010)	Better than	Singh and Tailor(2003)	$\rho > (\theta_4 + \theta_3) \frac{C_x}{2C_y}$
	Better than	Kadilar and Cingi (2004)	$\rho^2 < \frac{2\theta_4 W R^2 S_x^2 + \theta_5^2 S_x^2 - \theta_4^2 R^2 S_x^2}{S_y^2}$
	Better than	Subramani and Kumarapandiyan (2012)	$\rho > (\theta_4 + \theta_6) \frac{C_x}{2C_y}$

**Table 2: Pair wise Efficiency Comparisons of the Estimators over MSE Values**

### 3. Results and Discussion

List of studied ratio estimators along with their bias and mean square error have been shown in Table .1 whereas in Table 2 pair wise comparison of different ratio estimators has been made and different conditions were obtained. For the empirical comparison of various proposed estimators the data have been taken from Singh and Chaudhary (1986) page no. 177. The data consist of a sample of 20 villages selected from a population of 34 villages. The data are related to area under wheat (in acres) in the year 1971 and 1973. Table.3 shows the description of data along with constant of different estimators mentioned in notations. In Table 4, conditions over the correlation coefficient on the bias value are explained for different estimators. It has been observed which estimator when compared with another estimator satisfies condition over bias value. When compared with the estimator proposed by Kadilar and Cingi (2004, It was observed that estimator proposed by Cochran (1940), Sisodia and Dwivedi (1981), Bahl and Tuteja (1991), Singh et al. (2004), Upadhyaya and Singh (1999), Singh and Tailor (2003) satisfied the condition. But the remaining existing ratio estimators of population mean did not satisfy the conditions when compared with each other. This implied that all the estimators were better than the Kadilar and Cingi (2004) especially for the studied numerical illustration. Table 5 showed the bias, mean squared error and percentage relative bias of the existing different ratio estimators of population mean based on empirical data. It was observed that estimator  $\bar{y}_8$ (0.056) has lowest bias followed by estimator  $\bar{y}_2$  (0.990). It was also observed that estimator  $\bar{y}_7$  (8.539) has highest bias followed by estimator  $\bar{y}_r$  (4.269). The same trend followed in case of mean squared error estimator as well percent relative bias,  $\bar{y}_8$  (8834.94) has lowest value followed by estimator  $\bar{y}_2$  (8842.80) whereas  $\bar{y}_7$  (16146.61) has highest mean squared error, in case of percent relative bias  $\bar{y}_8$  (0.0065) has lowest value followed by estimator  $\bar{y}_2$  (0.1156) whereas  $\bar{y}_7$  (0.9971) has highest percent relative bias. The best estimator of the area under wheat production for the selected data set is “Subramani and Kumarapandiyam (2012)” in parallel with the theoretical findings according to the MSE criterion. Figure 1 shows the estimated values, Bias, Mean squared error and percent relative bias of the existing different ratio estimators of population mean.

Population (N=34)			Constant of different estimators	
	X(Area of wheat in 1973)	Y (Area of wheat in 1971)		
Mean	208.88	856.41	$\theta_1$	0.9965
Skewness ( $\beta_1$ )	0.97	2.95	$\theta_2$	0.9995
First quartile (Q1)	94.25	402.5	$\theta_3$	0.997
Third quartile (Q3)	254.75	1049	$\theta_4$	0.9541
Standard Deviation ( $\sigma$ )	150.50	733.14	$\theta_5$	0.5654
Sample Variance	22652.05	537495.27	$\theta_6$	0.5448
Kurtosis ( $\beta_2$ )	0.09	12.269	$\theta_7$	0.9658
Sample size (n)	20			
Correlation coefficient ( $\rho$ )	0.449			
Coefficient of variation	0.72	0.856		

**Table.3. Descriptive statistics of the empirical data**

Estimator	Name	Bias conditions over the correlation coefficient	Whether satisfied or not
Classical ratio estimators	Sisodia and Dwivedi (1981)	1.68	Not satisfied
	Bahl and Tuteja (1991)	1.052	Not satisfied
	Singh et al. (2004)	1.682	Not satisfied
	Upadhyaya and Singh (1999)	1.654	Not satisfied
	Singh and Tailor (2003)	1.681	Not satisfied
	Kadilar and Cingi (2004)	0.056	<b>Satisfied</b>
	Yan and Tian (2010)	1.644	Not satisfied
	Subramani and Kumarapandiyan (2012)	1.30	Not satisfied
Sisodia and Dwivedi (1981)	Bahl and Tuteja (1991)	1.033	Not satisfied
	Singh et al. (2004)	1.68	Not satisfied
	Upadhyaya and Singh (1999)	1.651	Not satisfied
	Singh and Tailor(2003)	1.678	Not satisfied
	Kadilar and Cingi (2004)	0.0509	<b>Satisfied</b>
	Yan and Tian (2010)	1.641	Not satisfied
	Subramani and Kumarapandiyan (2012)	1.297	Not satisfied
Bahl and Tuteja(1991)	Singh et al. (2004)	1.049	Not satisfied
	Upadhyaya and Singh (1999)	0.874	Not satisfied
	Singh and Tailor(2003)	1.04	Not satisfied
	Kadilar and Cingi (2004)	-0.938	<b>Satisfied</b>
	Yan and Tian (2010)	0.818	Not satisfied
	Subramani and Kumarapandiyan (2012)	-0.011	<b>Satisfied</b>
Singh et al. (2004)	Upadhyaya and Singh (1999)	1.654	Not satisfied
	Singh and Tailor(2003)	1.681	Not satisfied
	Kadilar and Cingi (2004)	0.055	<b>Satisfied</b>
	Yan and Tian (2010)	1.644	Not satisfied
	Subramani and Kumarapandiyan (2012)	1.299	Not satisfied



Upadhyaya and Singh (1999)	Singh and Tailor(2003)	1.652	Not satisfied
	Kadilar and Cingi (2004)	$-1.1 \times 10^{-16}$	<b>Satisfied</b>
	Yan and Tian (2010)	1.615	Not satisfied
	Subramani and Kumarapandiyan (2012)	1.271	Not satisfied
Singh and Tailor(2003)	Kadilar and Cingi (2004)	0.053	<b>Satisfied</b>
	Subramani and Kumarapandiyan (2012)	1.298	Not satisfied
Kadilar and Cingi (2004)	Subramani and Kumarapandiyan (2012)	-0.599	Not satisfied
Yan and Tian (2010)	Singh and Tailor(2003)	1.642	Not satisfied
	Kadilar and Cingi (2004)	-0.234	<b>Satisfied</b>
	Subramani and Kumarapandiyan (2012)	1.261	Not satisfied

**Table 4. Comparisons of conditions for the different ratio estimators of population mean over bias value empirically**

<b>Estimators</b>	<b>Bias</b>	<b>Mean Squared error</b>	<b> % <i>Relative bias</i>  </b>
$\bar{y}_8$	0.056	8834.94	0.0065
$\bar{y}_2$	0.990	8842.80	0.1156
$\bar{y}_5$	3.673	10220.47	0.4289
$\bar{y}_4$	3.821	10298.44	0.4462
$\bar{y}_1$	4.223	10514.23	0.4931
$\bar{y}_6$	4.263	10523.62	0.4978
$\bar{y}_3$	4.263	10535.86	0.4978
$\bar{y}_r$	4.269	10539.27	0.4985
$\bar{y}_7$	8.539	16146.61	0.9971

**Table 5. The Bias, Mean squared error and percent relative bias of the existing different ratio estimators of population mean**



**Figure 1. The estimated values, Bias, Mean squared error and percent relative bias of the existing different ratio estimators of population mean**

**Conclusion**

In this study, nine ratio type mean estimators in the existing literature are examined and the efficiency conditions were computed over correlation coefficient.. The estimator proposed by Subramani and Kumarapandiyan (2012) was found to be the best estimator when it is compared empirically amongst the all other existing ratio estimator of population mean.

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