GENERALIZED SYNTHETIC ESTIMATOR FOR DOMAIN MEAN IN TWO PHASE SAMPLING USING SINGLE AUXILIARY CHARACTER

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Abstract

In this paper, we have proposed a two phase sampling estimator for domain mean using auxiliary character with unknown $\overline{X}_a$ domain mean. Also discussed properties of the proposed estimator for domain mean $T_{ps,\gamma,a}$ using auxiliary character. Simulation study of the proposed estimator $T_{ps,\gamma,a}$ has been made with conventional ratio synthetic estimator for domain mean $T_{ps,-1,a}$ using auxiliary character in terms of simulated relative standard error (SRSE) and absolute relative bias (ARB). Simulation study shows that under synthetic assumption proposed estimator is more efficient than conventional ratio synthetic estimator for domain mean using auxiliary character.

Key Words: Two Phase Sampling, Auxiliary Character, Domain Estimation, Absolute Relative Bias, Simulated Relative Standard Error.

1. Introduction

Sample surveys are usually conducted for a specific need for the estimation of population parameters, but when we are interested in the estimation of parameters of subpopulation (domain), it is required to have sufficient number of sampling units in the population. Due to the increasing use of domain mean estimation in the government and private sectors, the use of auxiliary character with unknown mean of auxiliary is used in proposed estimator for domain mean. Two phase sampling which is an important and effective method of estimation which was first discussed by Neyman (1938). Several works using two phase sampling design have been discussed in the small area field of estimation problem by Sarndal (1992), Sarndal and Swensson (1987), Choudhary and Roy (1994), Hidiroglou and Sarndal (1995, 1998), Wu and Luan (2003), Udofia (2012) and Sud et al. (2014).

The population mean of the auxiliary character is known but number of units in the domain is too small. Due to small sampling units, domain estimator has large standard error. We use other method of estimation such as synthetic estimator for domain mean estimation. The main advantage of synthetic estimation is that we use sample units from population instead of domain. The synthetic estimator gives better

than the direct estimator when sample size in the domain is small which have been discussed by Gonzalez (1973). In recent years, due to growing different types of demand by policy makers the difficulties arises during for program implementation in the estimation of domain parameters. We use other methods of synthetic estimation for the domain mean using auxiliary character which have been discussed by Tikkiwal and Ghiya (2000), Tikkiwal et al. (2013), Singh and Seth (2014, 2015), Chandra et al. (2014) and Khare and Ashutosh (2017, 2018) using simulation study.

In this paper, we have proposed a conventional generalized estimator for domain mean using auxiliary character and discussed its members also. A simulation study of the proposed estimator has been made with the conventional ratio estimator for domain mean using auxiliary character in support of the problem.

2. Formulation of Problem and Notations

Let us suppose a finite population U: $(1,2,3…….…N)$ is classified into P non-overlapping subpopulation (domain) U_a of size N_a $(a=1,2,3………P)$. Here y and x represent the study character and auxiliary character. We select a sample s of size n from the population S of size N by using simple random sampling without replacement (SRSWOR) method of sampling and we select the sample s_a' of size n_a' at first phase from sample S_a of domain U_a of size N_a. We denote

$$\sum_{a=1}^{P} N_a = N \quad \text{and} \quad \sum_{a=1}^{P} n_a = n .$$

(2.1)

Notations used in the present context are given as follows:

$\overline{Y}$: Population mean of study character y based on N observations.

$\overline{Y}_a$: a^{th} domain mean of study character y based on N_a observations.

$\overline{y}$: Sample mean of study character y based on n observations.

$\overline{X}$: Population mean of auxiliary character x based on N observations.

$\overline{X}_a$: a^{th} domain mean of auxiliary character x based on N_a observations.

$\overline{x}$: Sample mean of auxiliary character x based on n observations.

$\overline{x}_a'$: Sample mean of auxiliary character x of ath domain based on n_a' observations.

The population mean square, coefficient of variation between y and x for population U are given as follows:

$$S_Y^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (Y_i - \overline{Y})^2 , S_X^2 = \frac{1}{(N-1)} \sum_{i=1}^{N} (X_i - \overline{X})^2 ,$$

$$S_{YX} = \frac{1}{(N-1)} \sum_{i=1}^{N} (Y_i - \overline{Y})(X_i - \overline{X}), C_Y = \frac{S_Y}{\overline{Y}}, C_X = \frac{S_X}{\overline{X}} \quad \text{and} \quad C_{XY} = \frac{S_{YX}}{\overline{X}\overline{Y}} .$$

(2.2)

The domain mean square, coefficient of variation between y and x of domain U_a are given as follows:

$$S_{Y_a}^2 = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} (Y_{a_i} - \overline{Y}_a)^2, \qquad S_{X_a}^2 = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} (X_{a_i} - \overline{X}_a)^2,$$

$$S_{Y_a X_a} = \frac{1}{(N_a - 1)} \sum_{i=1}^{N_a} (Y_{a_i} - \overline{Y}_a)(X_{a_i} - \overline{X}_a), \quad C_{Y_a} = \frac{S_{Y_a}}{\overline{Y}_a}, \quad C_{X_a} = \frac{S_{X_a}}{\overline{X}_a}$$

and

$$C_{X_a Y_a} = \frac{S_{Y_a X_a}}{\overline{X}_a \overline{Y}_a} \tag{2.3}$$

3. Synthetic estimators for domain mean using auxiliary character

(i) Ratio Synthetic Estimator for Domain Mean ($T_{RS,a}$)

$$T_{RS,a} = \frac{\overline{y}}{\overline{x}} \overline{X}_a \qquad \text{Rao (2003)}$$

$$\tag{3.1}$$

$$Bias(T_{RS,a}) = \left(\frac{\overline{Y}}{\overline{X}} \overline{X}_a - \overline{Y}_a \right) - \frac{N-n}{Nn} \frac{\overline{Y}}{\overline{X}} \overline{X}_a \left(C_X^2 - C_{YX} \right) \tag{3.2}$$

$$MSE(T_{RS,a}) = \left(\frac{\overline{Y}}{\overline{X}} \overline{X}_a - \overline{Y}_a \right)^2 + \frac{N-n}{Nn} \frac{\overline{Y}}{\overline{X}} \overline{X}_a \left\{ \frac{\overline{Y}}{\overline{X}} \overline{X}_a \left(3C_X^2 + C_Y^2 - 4C_{YX} \right) - 2\overline{Y}_a \left(C_X^2 - C_{YX} \right) \right\} \tag{3.3}$$

(ii) Generalized synthetic estimator for domain mean ($T_{GS,\beta,a}$)

$$T_{GS,\beta,a} = \overline{y} \left(\frac{\overline{x}}{\overline{X}_a} \right)^\beta \qquad \text{Tikkiwal and Ghiya (2000)} \tag{3.4}$$

$$Bias(T_{GS,\beta,a}) = \overline{Y} \left(\frac{\overline{X}}{\overline{X}_a} \right)^\beta \left[1 + \frac{N-n}{Nn} \left\{ \frac{\beta(\beta-1)}{2} C_X^2 + \beta C_{YX} \right\} \right] - \overline{Y}_a \tag{3.5}$$

$$MSE(T_{GS,\beta,a}) = \overline{Y}^2 \left(\frac{\overline{X}}{\overline{X}_a} \right)^{2\beta} \left[1 + \frac{N-n}{Nn} \left\{ \left(2\beta^2 - \beta \right) C_X^2 + C_Y^2 + 4\beta C_{YX} \right\} \right] + \overline{Y}_a^2$$
$$- 2\overline{Y}_a \overline{Y} \left(\frac{\overline{X}}{\overline{X}_a} \right)^\beta \left[1 + \frac{N-n}{Nn} \left\{ \frac{\beta(\beta-1)}{2} C_X^2 + \beta C_{YX} \right\} \right] \tag{3.6}$$

4. Proposed Estimator for Domain Mean ($T_{ps,\gamma,a}$)

We have proposed a conventional generalized estimator for domain mean using single auxiliary character which is given as follows:

$$T_{ps,\gamma,a} = \bar{y}\left(\frac{\bar{x}}{\bar{x}'_a}\right)^{\gamma} \tag{4.1}$$

The members of the estimator are given as follows:

(i) $T_{ps,0,a} = \bar{y}$, if $\gamma = 0$ \hfill (4.2)

(ii) $T_{ps,-1,a} = \dfrac{\bar{y}}{\bar{x}}\bar{x}'_a$, \quad if $\gamma = -1$ \hfill (4.3)

(iii) $T_{ps,1,,a} = \bar{y}\dfrac{\bar{x}}{\bar{x}'_a}$, \quad if $\gamma = 1$ \hfill (4.4)

For large sample approximations, we assume that

$$\bar{y} = \bar{Y}(1+\varepsilon_0), \bar{x} = \bar{X}(1+\varepsilon_1) \text{ and } \bar{x}'_a = \bar{X}_a(1+\varepsilon_2), \text{ such that } E(\varepsilon_0) = 0,$$
$$E(\varepsilon_1) = 0,$$

$$E(\varepsilon_2) = 0, E(\varepsilon_0\varepsilon_2) = 0, E(\varepsilon_1\varepsilon_2) = 0 \text{ and } E(\varepsilon_0^2) = \frac{(N-n)}{Nn}C_Y^2,$$

$$E(\varepsilon_1^2) = \frac{(N-n)}{Nn}C_X^2, E(\varepsilon_2^2) = \frac{(N_a-n'_a)}{N_a n'_a}C_{X_a}^2 \text{ and } E(\varepsilon_0\varepsilon_1) = \frac{(N-n)}{Nn}C_{YX}.$$
$$\tag{4.5}$$

Bias and Mean square error of the conventional ratio estimator ($T_{ps,-1,a}$) and conventional generalized estimator ($T_{ps,\gamma,a}$)

$$Bias(T_{ps,-1,a}) = \left(\frac{\bar{Y}}{\bar{X}}\bar{X}_a - \bar{Y}_a\right) - \left(\frac{1}{n} - \frac{1}{N}\right)\frac{\bar{Y}}{\bar{X}}\bar{X}_a\left(C_X^2 - C_{YX}\right) \tag{4.6}$$

Under ratio synthetic assumption $\dfrac{\bar{Y}_a}{\bar{Y}} = \dfrac{\bar{X}_a}{\bar{X}}$ \hfill (4.7)

the equation (4.6) is reduced to

$$Bias(T_{ps,-1,a}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}_a\left(C_{YX} - C_X^2\right) \tag{4.8}$$

and

$$MSE(T_{ps,-1,a}) = \left(\frac{\overline{\overline{Y}}}{\overline{\overline{X}}}\overline{X}_a - \overline{Y}_a\right)^2 + \left(\frac{1}{n} - \frac{1}{N}\right)\frac{\overline{\overline{Y}}}{\overline{\overline{X}}}\overline{X}_a\left\{\frac{\overline{\overline{Y}}}{\overline{\overline{X}}}\overline{X}_a\left(C_X^2 + C_Y^2 - 2C_{YX}\right) - 2\overline{Y}_a\left(C_X^2 - C_{YX}\right)\right\}$$

$$+ \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)\left(\frac{\overline{\overline{Y}}}{\overline{\overline{X}}}\overline{X}_a\right)^2 C_{X_a}^2 \qquad (4.9)$$

Put assumption (4.7) in equation (4.9) then we have

$$MSE(T_{ps,-1,a}) = \overline{Y}_a^2\left\{\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_Y^2 - C_X^2\right) + \left(\frac{1}{n_a^{'}} - \frac{1}{N_a}\right)C_{X_a}^2\right\} \qquad (4.10)$$

and

$$Bias(T_{ps,\gamma,a}) = \overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{\gamma(\gamma-1)}{2}C_X^2 + \gamma C_{YX}\right)\right] - \overline{Y}_a$$

$$\qquad (4.11)$$

Partially differentiate $Bias(T_{ps,\gamma,a})$ w.r.to γ and equate to zero, we have

$$\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\left[\log\left(\frac{\overline{X}}{\overline{X}_a}\right)\left\{1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{\gamma(\gamma-1)}{2}C_X^2 + \gamma C_{YX}\right)\right\}\right.$$

$$\left. + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{(2\gamma-1)}{2}C_X^2 + C_{YX}\right)\right] = 0 \qquad (4.12)$$

The equation (4.11) does not in close form, so, we can't obtain value of γ. Now we take the synthetic assumption $\dfrac{\overline{Y}_a}{\overline{Y}} = \left(\dfrac{\overline{X}}{\overline{X}_a}\right)^{\gamma}$ $\qquad (4.13)$

$Bias(T_{ps,\gamma,a})$ is reduced to the expression which is given as

$$Bias(T_{ps,\gamma,a}) = \overline{Y}_a\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left(\frac{\gamma(\gamma-1)}{2}C_X^2 + \gamma C_{YX}\right)\right] \qquad (4.14)$$

Partially differentiate w.r.to γ and equating (4.14) to zero, then we have

$$\gamma_{opt} = \frac{1}{2} - \frac{C_{YX}}{C_X^2} \qquad (4.15)$$

After substitute value of γ_{opt} in equation (4.14) the obtained $Bias(T_{ps,\gamma,a})_{opt}$. And

$$MSE(T_{ps,\gamma,a}) = \left(\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\right)^2 \left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\gamma(2\gamma-1)C_X^2 + C_Y^2 + 4\gamma C_{YX}\right\}\right.$$

$$\left. + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\gamma(2\gamma+1)C_{X_a}^2\right] + \overline{Y}_a^2 - 2\overline{Y}_a\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}$$

$$\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\gamma C_{YX} - \frac{\gamma}{2}C_X^2\right) + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\frac{\gamma}{2}C_{X_a}^2\right] \qquad (4.16)$$

For obtaining the optimum value of γ, we partially differentiate $MSE(T_{ps,\gamma,a})$ w.r.to γ and equating to zero, we have

$$2\left(\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\right)^2 \log\left(\frac{\overline{X}}{\overline{X}_a}\right)\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\gamma(2\gamma-1)C_X^2 + C_Y^2 + 4\gamma C_{YX}\right\}\right.$$

$$\left. + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\gamma(2\gamma+1)C_{X_a}^2\right] + \left(\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\right)^2\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left\{(4\gamma-1)C_X^2 + 4C_{YX}\right\} + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)(4\gamma+1)C_{X_a}^2\right]$$

$$-2\overline{Y}_a\left(\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\right)\log\left(\frac{\overline{X}}{\overline{X}_a}\right)\left[1 + \left(\frac{1}{n} - \frac{1}{N}\right)\left(\gamma C_{YX} - \frac{\gamma}{2}C_X^2\right) + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\frac{\gamma}{2}C_{X_a}^2\right]$$

$$-2\overline{Y}_a\overline{Y}\left(\frac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left(C_{YX} - \frac{1}{2}C_X^2\right) + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\frac{1}{2}C_{X_a}^2\right] = 0$$

$$(4.17)$$

The equation (4.17) does not come in the closed form for value of γ. For minimizing the *MSE* ($T_{ps,\gamma,a}$) using synthetic assumption then the expression (4.16) is reduced to the expression which is given as follows:

$$MSE(T_{ps,\gamma,a}) = \overline{Y}_a^2\left[\left(\frac{1}{n} - \frac{1}{N}\right)\left\{\gamma(2\gamma-1)C_X^2 + C_Y^2 + 4\gamma C_{XY}\right\} + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\gamma(2\gamma+1)C_{X_a}^2\right.$$

$$\left. -2\left(\frac{1}{n} - \frac{1}{N}\right)\left\{\gamma C_{YX} - \frac{\gamma}{2}C_X^2\right\} + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)\frac{\gamma}{2}C_{X_a}^2\right] \qquad (4.18)$$

Now we solve $\dfrac{\partial MSE(T_{ps,\gamma,a})}{\partial \gamma} = 0$. The optimum value of γ is given as follows:

$$\gamma_{opt} = -\frac{\left(\frac{1}{n} - \frac{1}{N}\right)C_{XY}}{\left\{\left(\frac{1}{n} - \frac{1}{N}\right)C_X^2 + \left(\frac{1}{n_a'} - \frac{1}{N_a}\right)C_{X_a}^2\right\}} \tag{4.19}$$

Now, substituting the value of γ_{opt} in the equation (4.18), we can obtain the minimum value of the $MSE(T_{ps,\gamma,a})$.

5. Simulation Study

The empirical study is based on simulation procedure in which we have taken data from Sarndal et al. ((1992), Appendix B, pages: 652-659). In the present study, we consider five geographical regions have sizes (48, 32, 38, 56 and 29) out of eight domains have sizes (25, 48, 32, 38, 56, 41, 15 and 29). The performance of the proposed estimator $T_{ps,\gamma,a}$ is compared with the conventional ratio estimator $T_{ps,-1,a}$ in terms of simulate relative standard error (SRSE) and absolute relative bias (ARB) which is given as follows:

$$ARB(T_{ps,\gamma,a}) = \frac{abs\left(\sum\limits_{s=1}^{5000} T_{ps,\gamma,a}^s - \overline{Y}_a\right)}{\overline{Y}_a} \times 100 \tag{5.1}$$

$$SRSE(T_{ps,\gamma,a}) = \frac{\sqrt{SMSE(T_{ps,\gamma,a})}}{\overline{Y}_a} \times 100 \tag{5.2}$$

where, $SMSE(T_{ps,\gamma,a}) = \frac{1}{5000}\sum\limits_{s=1}^{5000}\left(T_{ps,\gamma,a}^s - \overline{Y}_a\right)^2 \tag{5.3}$

and $T_{ps,\gamma,a}^s$ represents, the specific conventional estimator for s^{th} sample and $T_{ps,\gamma,a}$ represents the specific conventional estimator for domains $a=1,2,\dots P$.

For the purpose of simulation study we have considered two populations 1 and 2. The information about domains about population 1 and population 2 are given in Table 1 and Table 2 respectively, also population parameters values of populations 1 and 2 are given as follows:

Population 1

We consider study variable y and auxiliary variable x which are given as follows:

y: Real estate values according to 1984 assessment (in millions of Kronor).

x: Number of municipal employees in 1984.

Here we are giving the technique used for two phase sample procedure for the selection of sample which is used in simulation. We select a sample of approximately 10% units (20 out of 203) from population by study character and auxiliary character and from each domain, and we select a sufficient sample of approximately 50% units at first

phase through simple random sampling without replacement (SRSWOR). The same process repeated by 5000 times.

The population of size 203 is classified in to five domains have sizes 48, 32, 38, 56, and 29 respectively. The value of the parameters of population and domains are given as follows.

N =203, $\overline{Y}$ =2806.842, $\overline{X}$ =1651.458, S_Y^2 =13722352, S_X^2 =15234672, S_{YX} =13335313, ρ_{YX} =0.922.

Domain Values	Domain				
	1	2	3	4	5
N_a	48	32	38	56	29
$\overline{Y}_a$	2970.596	2498.75	2915.53	3046.95	2269.103
$\overline{X}_a$	1658.71	1316.94	1937.71	1950.39	1056.241
$S_{Y_a}^2$	11118969	4164522	9575690	27861139	1883429
$S_{X_a}^2$	4601899	1989177	15986129	38786393	7755964
$S_{Y_a X_a}$	6920194	2681882	11697914	31770431	3118230
$\rho_{Y_a X_a}$	0.967	0.932	0.945	0.966	0.816

Table 1. The values of the domains parameters for all domains (1, 2, 3, 4 and 5)

Population 2

We consider another population which is consisting of the 284 municipalities is referred to as the MU284 population in Sarndal et al. (1992) in appendix B, pages: 652-659. The study character y and auxiliary character x are given as follows:
y: Real estate values according to 1984 assessment (in millions of Kronor).
x: 1975 population (in thousands).

Further, two phase sampling method is used in this paper for simulation procedure. In this case, we select a sample each have approximately 10% units (20 out of 203) from population and from each domain, a sample is selected approximately 50% units at first phase by using simple random sampling without replacement (SRSWOR). This process repeat 5000 times.

For the study purpose, we use population .2 in which the population have size 203 divided into five domains have sizes 48, 32, 38, 56 and 29 respectively. The values of the parameters of population are given (Sarndal et al. (1992) in appendix B).

N =203, $\overline{Y}$ =2806.842, $\overline{X}$ =26.773, S_Y^2 =13722352, S_X^2 =1714.899, S_{YX} =147046.8, ρ_{YX} =0.959.

Domain Values	Domain				
	1	2	3	4	5
N_a	48	32	38	56	29
$\overline{Y}_a$	2970.596	2498.75	2915.53	3046.95	2269.103
$\overline{X}_a$	29.17	23.94	30.63	28.71	17.138
$S_{Y_a}^2$	11118969	4164522	9575690	27861139	7755964
$S_{X_a}^2$	1228.23	437.16	1721.16	3565.12	405.9089
$S_{Y_a X_a}$	112977.5	40499.05	126072.1	308468.1	45304.06
$\rho_{Y_a X_a}$	0.967	0.949	0.982	0.979	0.807

Table 2. The information about domain parameters values of all domains (1, 2, 3, 4 and 5)

The synthetic assumption of conventional ratio synthetic estimator $T_{ps,-1,a}$ and conventional generalized synthetic estimator $T_{ps,\gamma,a}$ of population 1 for domains (1, 2, 3, 4 and 5) are given as:

Domain	$\dfrac{\overline{Y}_a}{\overline{Y}}$	$\dfrac{\overline{X}_a}{\overline{X}}$	$abs\left(\dfrac{\overline{Y}_a}{\overline{Y}} - \dfrac{\overline{X}_a}{\overline{X}}\right)$
1	1.05847	1.00439	0.05407
2	0.89023	0.79744	0.09279
3	1.038721	1.173333	0.13436
4	1.085542	1.181013	0.09547
5	0.8084186	0.639581	0.16883

Table 3. The absolute difference under synthetic assumption of the $T_{ps,\gamma,a}$ for different domains (1, 2, 3, 4 and 5) (Population 1)

Domain	$\dfrac{\overline{Y}_a}{\overline{\overline{Y}}}$	$\left(\dfrac{\overline{X}}{\overline{X}_a}\right)^{\gamma}$	$abs\left(\dfrac{\overline{Y}_a}{\overline{Y}}-\left(\dfrac{\overline{X}}{\overline{X}_a}\right)^{\gamma}\right)$
1	1.05847	1.00198	0.05648
2	0.89024	0.90298	**0.01275**
3	1.03872	1.05862	**0.01991**
4	1.08554	1.05098	**0.03455**
5	0.80842	0.82752	**0.01909**

Table 4: The absolute difference under synthetic assumption of $T_{ps,\gamma,a}$ for different domains (1, 2, 3, 4 and 5) (Population 1)

From the Table 3 and Table 4, it is observed that the value of absolute difference of the synthetic assumption of conventional generalized estimator $T_{ps,\gamma,a}$ is lower than the value of absolute difference of synthetic assumption of the conventional ratio estimator $T_{ps,-1,a}$ for domains (2, 3, 4 and 5) and approximately near for domain 1. Hence, the proposed estimator for domain mean using auxiliary character is more preferred than the conventional ratio estimator for domain mean using auxiliary character.

Similarly, the synthetic assumption of conventional ratio estimator $T_{ps,-1,a}$ and conventional generalized estimator $T_{ps,\gamma,a}$ of population 2 for different domains (1, 2, 3, 4 and 5) are given as:

Domain	$\dfrac{\overline{Y}_a}{\overline{Y}}$	$\dfrac{\overline{X}_a}{\overline{X}}$	$abs\left(\dfrac{\overline{Y}_a}{\overline{Y}}-\dfrac{\overline{X}_a}{\overline{X}}\right)$
1	1.05847	1.08939	0.03092
2	0.89024	0.89407	0.00384
3	1.03872	1.14410	0.10538
4	1.08554	1.03392	0.05162
5	0.80841	0.64011	0.16831

Table 5. The absolute difference under synthetic assumption of the $T_{ps,-1,a}$ for all domains (1, 2, 3, 4 and 5) (Population 2)

Domain	$\dfrac{\overline{\overline{Y}}_a}{\overline{\overline{Y}}}$	$\left(\dfrac{\overline{\overline{X}}}{\overline{\overline{X}}_a}\right)^{\gamma}$	$abs\left(\dfrac{\overline{\overline{Y}}_a}{\overline{\overline{Y}}}-\left(\dfrac{\overline{\overline{X}}}{\overline{\overline{X}}_a}\right)^{\gamma}\right)$
1	1.05847	1.05628	**0.00219**
2	0.89024	0.92775	0.03751
3	1.03872	1.07903	**0.04031**
4	1.08554	1.07249	**0.01304**
5	0.80841	0.77234	**0.03608**

Table 6. The absolute difference under synthetic assumption of $T_{ps,\gamma,a}$ for different domains (1, 2, 3, 4 and 5) (Population 2)

It is seen that from Table 5 and Table 6 the amount of absolute difference under synthetic assumption of $T_{ps,\gamma,a}$ is less than the amount of absolute difference of synthetic assumption of $T_{ps,-1,a}$ for domains (1, 3, 4 and 5) and approximately near for domain 2.

Population	Estimator	Domain				
		1	2	3	4	5
1	$T_{ps,-1,a}$	60.519 13.840*	56.922 6.423	89.528 36.700	92.488 30.512	46.570 0.162
	$(T_{ps,\gamma,a})_{\min}$	**34.856** **0.586*** -0.4524**	**38.076** **6.0509** -0.5978	**36.761** **5.970** -0.3564	**33.344** **1.323** -0.2989	**25.034** **0.011** -0.2950
2	$T_{ps,-1,a}$	53.026 12.625*	49.608 10.889	59.990 20.925	54.222 7.077	43.206 12.609
	$(T_{ps,\gamma,a})_{\min}$	**38.484** **4.2656*** -0.6425**	**40.907** **10.196** -0.6998	**38.474** **7.914** -0.5697	**33.101** **3.191** -0.4766	**35.749** **0.343** -0.5791

Table 7. SRSE and ARB of $T_{ps,-1,a}$ and $T_{ps,\gamma,a}$ for different domains (1, 2, 3, 4 and 5) of populations 1 and 2

*represent Absolute Relative Bias; ** represent γ include in the proposed estimator $(T_{ps,\gamma,a})_{\min}$.

It is seen from Table 7 that the value of SRSE of the conventional generalized estimator for domain mean $(T_{ps,\gamma,a})_{\min}$ is lower than the conventional ratio estimator for domain mean $T_{ps,-1,a}$ for all domains (1, 2, 3, 4 and 5) of population 1 and population 2. However, ARB of the conventional generalized estimator for domain mean $(T_{ps,\gamma,a})_{\min}$ is lower than the conventional ratio estimator for domain mean $T_{ps,-1,a}$ for domains (2, 3, 4 and 5) and (1, 3, 4 and 5) for population 1 and population 2 respectively. Hence, conventional generalized estimator for domain mean using auxiliary character $(T_{ps,\gamma,a})_{\min}$ is more efficient than conventional ratio estimator for domain mean using auxiliary character $T_{ps,-1,a}$. It is also found that conventional generalized estimator $(T_{ps,\gamma,a})_{\min}$ is more efficient than the conventional ratio estimator $T_{ps,-1,a}$ in case of population 2 than population 1.

6. Conclusion

It is evident from the Table 7, that the value of SRSE of the proposed estimator for domain mean $T_{ps,\gamma,a}$ has less than the value of SRSE of the conventional ratio synthetic estimator for domain mean $T_{ps,-1,a}$ for which domain where absolute difference of synthetic assumption of the proposed estimator meets closely in the population 1 (See Table 3 and Table 4) and for population 2 (See Table 5 and Table 6). The proposed estimator performed better than the conventional ratio synthetic estimator for both the populations 1 and 2 (See Table 7), Also it is seen that proposed estimator is more efficient than the conventional ratio synthetic estimator in population 1 than population 2.

So, it is preferred to use conventional generalized synthetic estimator for domain mean using auxiliary character $T_{ps,\gamma,a}$ over conventional ratio synthetic estimator for domain mean $T_{ps,-1,a}$ using auxiliary character.

7. Applications

The proposed estimator may be used to estimate the domain mean when the number of units in interest domain is small and domain mean of auxiliary character is unknown. It is also applied in the ecological problem such as the estimation of pollution level in the city (domain) of the State (population).

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