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HYBRID EXPONENTIALLY WEIGHTED MOVING AVERAGE (HEWMA) CONTROL CHART BASED ON EXPONENTIAL TYPE ESTIMATOR OF MEAN

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Abstract

In this paper, we have proposed a Hybrid Exponentially Weighted Moving Average (HEWMA) control chart. The proposed control chart is based on the exponential type estimator for mean using two auxiliary variables (cf. Noor-ul-Amin and Hanif, 2012). We call it an E-HEWMA control chart because it is based on the exponential estimator of the mean. From this study, the fact is revealed that E-HEWMA control chart shows more efficient results as compared to traditional/simple EWMA chart and DS.EWMA control chart (cf. Raza and Butt, 2018). The comparison of the E-HEWMA control chart is also performed with the DS-EWMA chart. The proposed chart also outperforms the other control chartsin comparison. The E-HEWMA chart can be used for efficient monitoring of the production process in manufacturing industries.A simulated example has been used to compare the proposed and traditional/simple EWMA charts and DS.EWMA control chart. The control charts' performance is measured using the average run length-out of control (ARL_1). It is observed that the proposed chart performs better than existing EWMA control charts.

Key Words: Hybrid, EWMA, Estimator, DS.EWMA, Average Run Length.

1. Introduction

Statistical Process Control is usually referred to as SPC. It consists of methods which comprise of monitoring, controlling and refining of a process. From the industrial applications, it is observed that all the processes have some variations. But sometimes the process illustrates a great level of inconsistency and results in the occurrence of out of control signals. One of the uses of the SPC is to decrease discrepancy and achieve the best objective value (Montgomery, 2009).

The Control charts are the most important tool of SPC tool kit. It is commonly used to differentiate between the assignable and un-assignable causes. The purpose of the effective process monitoring system is to detect the presence of an assignable cause as rapidly as possible without stopping checking too often or too late. The control charts are of different types. Some are memory control charts and other are memory-less control charts. Shewhart are memory-less control charts and are being used to detect a large size shift whereas the memory type charts are used for dealing with small size shifts (Butt and Raza,2017).

The use of statistical quality control charts in industrial applications reveals that most of the control charts are structured to cater information about the quality characteristic/ studied variable. The efficiency of a control chart can be enhanced by using efficient charting statistics or estimators of the parameter under study. The estimation techniques make use of auxiliary variables highly correlated with the variable of interest to increase the precision and efficiency of estimators (Noor-ul-Amin and Hanif, 2012).

Watson (1937) proposed estimation procedures using auxiliary information. Mandel (1969) gave a regression control chart and Zhang (1984) used the additional / auxiliary information in preparation of the "cause-selecting-type control chart". Wade and Woodall (1993) proposed the prediction limits by inserting some modifications in Zhang (1984) control limits.

Many efficient estimators can be obtained/developed by the use of auxiliary variables, for example, see the researches by Kadilar and Cingi(2005), Singh and Vishwakarma (2007), Singh et al. (2008), Hanif et al.(2009), Awan and Shabbir (2014).

Riaz (2008) and Riaz and Does (2009) proposed the control chart for the variability using one auxiliary variable for Phase-I. When the additional/auxiliary information is incorporated in the construction of quality control charting attributes, the efficiency of such charts is improved. Riaz (2009 & 2011) suggested a control chart for location parameters using one auxiliary variable for Phase I.

Woodall et al (2004) and Rashid et al (2010) proposed control charts using regression estimators' incorporation of the use of one auxiliary variable. Ahmed et al (2014) also used a single auxiliary variable in designing control charts.

In this paper, we propose a new Hybrid Exponentially Weighted Moving Average (EWMA) control chart based on the Exponential type Estimator of Mean using twoauxiliaryvariables.In the later sections of the paper, we have given the expressions for control limits of the proposed E-HEWMA control charts. A simulated example is also included to compare the performance E-HEWMA control charts with Exponentially Weighted Moving Average (EWMA) control charts. The performance is measured using average run length-out of control (ARL_1). The chart showing less ARL_1 will be awarded as a more efficient chart as it will be more sensitive to the change and it will be detecting the shift in the parameter(s) more rapidly than other charts in comparison.

2. Methodology

2.1 TheE-Hybrid EWMA (E-HEWMA) Control Chart

In this section, we will discuss the construction of the proposed E-Hybrid Exponentially Weighted Moving Average (E-HEWMA) controlchart. The charting parameters are defined using the Exponential type Estimator of Mean using two Auxiliary variables. The estimator used for the control chart development (cf. Noor-ul-Amin and Hanif, 2012) is given as:

$$t_1 = \bar{y}_2 \exp\left[\frac{\bar{Z} - \bar{z}_2}{\bar{Z} + \bar{z}_2} - \frac{\bar{W} - \bar{w}_1}{\bar{W} + \bar{w}_1}\right] \tag{2.1}$$

The mean square error (MSE) and bias of the estimator is given as (2.2) and (2.3):

MSE $(t_1) \approx \sigma^2{}_1 \approx$

$$\bar{Y}^2\left[\theta_2\left(C_y^2 + \frac{C_z^2}{4} - \rho_{yz}C_zC_y\right) + \theta_1\left(\frac{C_w^2}{4} + \rho_{wy}C_wC_y - \frac{\rho_{wz}C_wC_z}{2}\right)\right] \tag{2.2}$$

$$Bias(t_1) \approx \bar{Y}\left[\frac{\theta_2 C_z}{2}\left(\frac{3C_z}{4} - \rho_{yz}C_y\right) - \frac{\theta_1 C_w}{2}\left(\frac{C_w}{4} + \frac{\rho_{wz}C_z}{2} - \rho_{yw}C_y\right)\right] \tag{2.3}$$

The DS.EWMA control chart was proposed by Raza and Butt (2018) which is based on two-phase sampling with two auxiliary variables.

The test statistics of the proposed DS.EWMA control charts is given as:

$$DS.EWMA_i = \lambda_1 t_{1i} + (1-\lambda_1)(DS.EWMA_{i-1})$$

Where "λ_1" the smoothing constant and t_{1i}isthe value of statistic $t_1$for the i[th] sample. $DS.EWMA_{i-1}$ represents the preceding information. The initial value $DS.EWMA_0$ for starting is the mean of t_1.The control limits of the proposed charts are given based on proposed estimator t_1 is given in (2.4):

$$LCL = E(t_1) - L_{DS}\sigma_{t_1}\sqrt{\frac{\lambda_1}{2-\lambda_1}}$$

$$CL = E(t_1) \tag{2.4}$$

$$UCL = E(t_1) + L_{DS}\sigma_{t_1}\sqrt{\frac{\lambda_1}{2-\lambda_1}}$$

Where $E(t_1) = \mu_0$, σ_{t1} is the variance of $t_1$estimator, λ_1 is the smoothing parameter and L_{DS} determines the width of the control limits for the DS.EWMA chart (cf. Raza and Butt, 2018).

Now we define the new sequences $HE_1, HE_2, \ldots$, as follows:

$$HE_t = \lambda_2(DS.EWMA_t) + (1-\lambda_2)HE_{t-1} \tag{2.5}$$

The HE_t statistics is the Hybrid charting statistics for proposed E-HEWMA control chart, λ_1 & λ_2 are the smoothing parameters and the control limits of the proposed chart will be derived as:

$$E(HE_t) = \mu_0 \tag{2.6}$$

And the variance of HE_t(cf. Haq, 2017):

$$V(HE_t) = \frac{\lambda_1^2 \lambda_2^2 \sigma^2_{t_1}}{(\lambda_1-\lambda_2)^2} \left[\frac{(1-\lambda_1)^2(1-(1-\lambda_1)^{2i})}{1-(1-\lambda_1)^2} + \frac{(1-\lambda_2)^2(1-(1-\lambda_2)^{2i})}{1-(1-\lambda_2)^2} - \frac{2(1-\lambda_1)(1-\lambda_2)\left\{1-(1-\lambda_1)^i(1-\lambda_2)^i\right\}}{1-(1-\lambda_1)(1-\lambda_2)} \right] \quad (2.7)$$

So control limits are:UCL= $\mu_0 + L\sqrt{Var(HE_t)}$ (2.8)

LCL= $\mu_0 - L\sqrt{Var(HE_t)}$

Where Lis the 99.73 quantile point of the distribution under study and $\sigma^2_{t_1}$ is the variance of the estimator given in equation 1.

2.2 Algorithm for E-Hybrid EWMA Control Chart

To evaluate the *E-H EWMA* control charts we need to find the corresponding UCL and LCL of the control chart.The steps to compute constant control limits are given as under:

i. Generate a population on exponentially distributed variables Y, W and Z.

ii. The population means of the variables $\bar{y}$, $\bar{w}$ and $\bar{z}$ are $\bar{Y}$, $\bar{W}$ and $\bar{Z}$ respectively.

iii. Take a first phase sample of size n_1. The first phase sample is drawn by simple random sampling (SRS) without replacement and is selected from the population of "N" units.

iv. Compute $\bar{w}_1$ which is the sample mean of variable 'W' for the first phase sample,

v. Now select the second phase sample is also selected using simple random sampling without replacement of size n_2 from the first phase sample of size n_1.

vi. Compute $\bar{y}_2$, $\bar{w}_2$ and $\bar{z}_2$ which shows the means of variables 'Y',' W ' and ' Z ' respectively for the second phase sample.

vii. Calculate the estimator t_1.

viii. Simulate steps iii-vii 2000 times.

ix. Determine the value for L for which we get the appropriate/fix in-control ARL_0.

x. Now compute the constant control limits of E-HEWMA control chart given in (2.8).

3. Results and Discussion

3.1 Illustrative Example

To study the features of the proposed E-HEWMA control chart, we have simulated the data and applied the E-HEWMA chart. The simulated data (given in Appendix A) consists of variables Y representing the study variable and W, Z are auxiliary variables.The procedure proposed in section 2 is evaluated using different choices of sample size (n), EWMA weight (λ_1) and HEWMA weight (λ_2) and correlation coefficient among variables (ρ). Among these, some results are presented here for discussion purposes. Tables 1–2 give the ARLs of the proposed E-HEWMA control and its comparison with the existing EWMA control chart. We have considered n = 20, 40 and λ_1 = 0.10and λ_2=0.08for ARL_0 = 220.00 and Tables 3 report the results forARL_0= 370.

Four cases of the correlation among variables are considered, which include $\rho_{wz} = \rho_{yz} = \rho = 0.0,\ 0.10,\ 0.30,\ and\ 0.50.$

$\rho \rightarrow$	**0**		**0.1**		**0.3**		**0.5**		
	L=2.45		**L=3.7**		**L=5.7**		**L=5.9**		**L=2.35**
Shift(δ) ↓	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**Conventional EWMA**
0	219.58 (206.52)	220.05 (206.52)	216.98 (213.2)	217.51 (213.4)	224.2 (206.7)	224.6 (207.5)	221.41 (203.9)	221.8 (204.1)	220.53 (209.06)
0.1	108.33 (103)	108.51 (104.50)	81.72 (75.14)	82.15 (75.45)	78.59 (69.18)	79.45 (70.12)	89.85 (80.5)	90.60 (81.50)	144.97 (136.78)
0.2	31.11 (23.6)	31.20 (24.05)	18.03 (17.7)	18.44 (17.99)	20.44 (19.14)	21.12 (19.45)	23.67 (19.89)	24.23 (20.50)	75.31 (60.84)
0.3	8.18 (6.88)	9.11 (7.12)	3.3 (3.19)	4.21 (3.77)	6.01 (4.61)	6.12 (4.66)	8.63 (7.07)	8.78 (7.99)	44.97 (31.87)
0.4	2.53 (2.9)	3.18 (2.99)	1.77 (1.66)	2.02 (1.88)	1.26 (1.41)	2.05 (1.95)	2.33 (2.49)	3.30 (2.87)	30.22 (19.39)
0.5	1 (0.95)	1.95 (1.71)	1 (0.91)	1.01 (0.64)	1 (0.70)	1.05 (0.80)	1.43 (1.62)	1.87 (1.79)	23.03 (13.20)

Table 1: ARLs of the proposed E-HEWMA chart and conventional EWMA chart for n = 20, ARLo = 220 and λ_1 = 0.10 and λ_2=0.08

$\rho \rightarrow$	0		0.1		0.3		0.5		
	L=2.45		L=3.7		L=5.7		L=5.9		L=2.35
Shift(δ) ↓	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	Conventional EWMA
0	217.08 (210.43)	221.65 (214.52)	217.22 (213.0)	218.09 (215.5)	223.41 (213.2)	221.6 (216.45)	221.52 (210.3)	224.14 (212.2)	219.42 (208.65)
0.1	53.63 (57.06)	58.54 (60.52)	39.85 (36.4)	39.48 (36.49)	39.66 (41.18)	42.28 (39.27)	87.33 (79.81)	87.55 (77.73)	106.54 (99.35)
0.2	9.01 (8.74)	9.4 (7.42)	5.65 (5.2)	4.46 (3.93)	5.66 (4.28)	5.15 (5.34)	18.86 (17.78)	22.16 (18)	51.04 (33.58)
0.3	2.42 (1.81)	2.42 (2.04)	1.48 (1)	1.87 (1.5)	1.09 (1.23)	1.73 (1.48)	7.98 (7.15)	8.46 (7.49)	25.15 (19.45)
0.4	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	2.19 (2.23)	2.59 (2.48)	16.69 (10.89)
0.5	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1.47)	1.21 (1.44)	12 (13.19)

Table 2: ARLs of the proposed E-HEWMA chart and conventional EWMA chart for n = 40, ARL_o= 220 and λ_1 = 0.10 and λ_2=0.08

$\rho \rightarrow$	0		0.1		0.3		0.5		
	L=2.9		L=4.2		L=6.15		L=6.30		L=2.61
Shift(δ) ↓	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	E-HEWMA	DS-EWMA by Raza and Butt (2018)	Conventional EWMA
0	368.13 (359.73)	368.8 (361.58)	368.21 (364.3)	367.06 (366.4)	368.27 (366.1)	370.13 (367.95)	369.54 (361.0)	367.13 (365.37)	368.63 (366.3)
0.1	97.16 (98.38)	101.67 (96.52)	90.84 (85.22)	92.98 (87.82)	89.16 (82.92)	87.72 (85.17)	89.21 (82.39)	92.06 (86.57)	134.68 (108.55)
0.2	14.89 (14.97)	16.51 (312.4)	11.64 (8.94)	12.96 (11.73)	11.44 (88.46)	10.82 (7.08)	11.88 (9.61)	15.28 (11.06)	60.4 (49.5)
0.3	2.93 (1.69)	4.01 (3.3)	1.14 (0.66)	3.44 (2.29)	1.25 (1.2)	2.39 (2.73)	2.13 (1.8)	2.75 (2.54)	19.75 (4.23)
0.4	1.14 (1)	1.63 (1.93)	1 (1)	1 (1)	1 (1)	1.32 (1.09)	1 (1)	1.75 (1.04)	8.9 (4.01)
0.5	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1.09 (1)	5.06 (1.95)

Table 3 :ARLs of the proposed E-HEWMA chart and conventional EWMA chart for n = 40, ARL_0 = 370 and λ_1 = 0.10 and λ_2=0.08

ρ →	0		0.1		0.3		0.5		
	L=2.65		L=3.95		L=5.72		L=5.9		L=2.5
Shift(δ) ↓	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**E-HEWMA**	**DS-EWMA by Raza and Butt (2018)**	**Conventional EWMA**
0	277 (275.7)	281.6 (264.87)	277.2 (270.1)	278 (259.5)	283.4 (258.8)	281.6 (281.06)	281.5 (263.6)	284.1 (259.5)	279.4 (256.51)
0.1	46.39 (45.98)	51.89 (45.06)	36.6 (23.43)	36.27 (17.91)	35.68 (18.77)	35.69 (32.38)	80.52 (58.43)	86.53 (73.74)	103.47 (102.25)
0.2	7.41 (6.11)	7.43 (6.79)	4.68 (4.11)	2.52 (1.84)	2.85 (2.22)	1.94 (1.54)	17.2 (15.54)	21.57 (3.37)	50.05 (48.47)
0.3	1.19 (0.15)	0.72 (1)	1 (1)	1 (1)	1 (1)	1 (1)	6.72 (5.66)	8.32 (5.64)	23.25 (22.62)
0.4	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	15.05 (14.49)
0.5	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	1 (1)	8.54 (7.69)

Table 4 :ARLs of the proposed E-HEWMA chart and conventional EWMA chart for n = 60, ARL_0 = 280 and λ_1 = 0.10 and λ_2=0.15

From Table 1-3 we observed that E-HEWMA performs better than the DS-EWMA control chart. The efficiency is evaluated using ARL_1 criterion. The efficiency of the E-HEWMA chart increases (i.e. ARL_1 values reduces) with the increase in Shift(δ) from 0 to 0.5 respectively. The E-HEWMA chart has better performance at correlation ρ = 0.1 and 0.3 than at ρ = 0.5.The ARL is the average number of sample points that is plotted on a chart before the first out-of-control signal is detected whereas the SDRL measures the spread of the run length distribution. If the SDRL values are larger then we say that there is a large varation in Run lengths. In table 1-3 the values given in parentheses are SDRL values from which we can observe that with the increase in shift sizes the SDRL values for the E-HEWMA chart decreases, which shows that there exists an indirect relation between SDRL values and Shift sizes. From table 3, the value: 1(1) for E-HEWMA chart using Shift(δ) =0.4, ρ =0.1 and L=4.2 shows that on average we detect the assignable cause on the very next sample when the shift occurs and the standard deviation of run length is found as 1 for the specified runlengths distribution.

Tables 1–4 show that:

(i) The control chart constants are directly proportional in relation to the correlation coefficient i.e.control chart constants increase as the correlation coefficient increases. However, with the increase in sample size the correlation coefficient and charting constants become stable/constant.

(ii) When the variables are correlated and the process is shifted, the ARL values decrease rapidly as compared to the case when ρ = 0.

(iii) The proposed control chart has better performance when the correlation ρ = 0.1 and 0.3 than at ρ = 0.5.
(iv) The ARL_1values decrease as sample size increases for a fixed value of ARL_0 and λ's.
(v) The standard deviation of the run length is approximately the same as the mean, which is expected as the run length for an independent sample.
(vi) For low correlation between variables i.e. from 0.1-0.3 the Control charts shows efficient performance whereas the for high correlation greater than 0.5 the control chart efficiency decreases. It is because as the MSE (given in eq. 2.2) is a function containing ρ. If the correlation increases than MSE decreases and the quantile point L value increases. Therefore, we observe better results for low correlations than the high correlations.

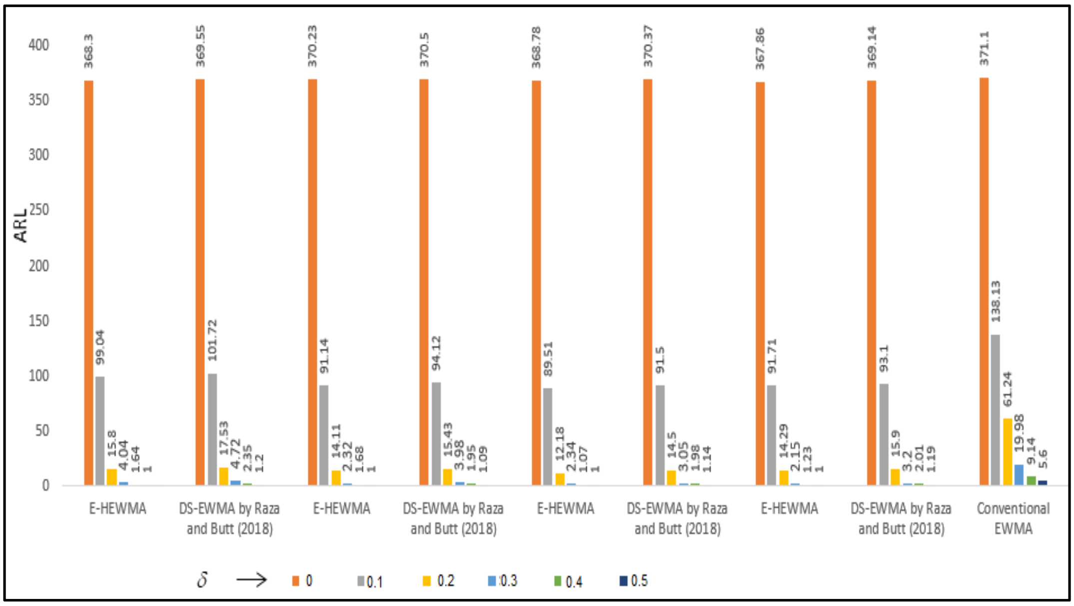

Figure 1: The ARL of the proposed E-HEWMA chart for n = 40, ARL_0 = 370 and λ_1 = 0.10 and λ_2=0.08

We use the of the out-of-control ARL_1as a performance measure to compare the efficiency of the proposed chart to detect a small shift in the mean. Figure 1 gives $ARL_1$for various choices of shifts and correlation coefficients. This figure shows that the proposed E-HEWMA chart performs better for a smaller value of correlationcoefficient (0, 0.1 and 0.3) than at ρ = 0.5.We have compared the proposed E-HEWMA chart with DS-EWMA by Raza and Butt (2018) and found that E-HEWMA chart outperforms the DS-EWMA chart.

4. Conclusions and Recommendations

This article proposed anE-HEWMA control chart using the auxiliary information for efficient monitoring of process mean. The control chart constants have been determined using Monte Carlo Simulation for various values of correlation coefficients. It is observed that the choice of correlation coefficient has a significant and directly proportional effect on the value of control chart constants. These charting constants are used in the construction of the proposed E-HEWMA control chart. The performance of the proposed control chart is evaluated using the average run length (out-of-control); ARL_1. The ARL_1 values are calculated using different values of correlation coefficient and sample sizes.

It is observed that ARL_1 values are significantly small for low correlation (i.e. between 0.1–0.3) while it is large for correlation coefficient (0.5). The performance of the proposed E-HEWMA chart is also compared with a simple EWMA chart and DS.EWMA control chart proposed by Raza and Butt (2018) for a fixed sample size and correlation coefficient. It is observed that the proposed chart is more efficient to detect a small shift in the process mean than other control charts under study. Therefore, the use of the proposed E-HEWMA chart is recommended when the study variable and auxiliary variablesare statistically correlated with each other.

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Appendix-A
Simulated Samples

Y	W	Z	Y	W	Z	Y	W	Z	Y	W	Z	Y	W	Z
2157	3	380	2049	3	345	2099	5	216	2140	5	396	2189	2	248
2174	3	398	2075	4	145	2154	2	228	2175	4	593	2178	4	238
2062	2	185	2045	2	435	2049	2	578	2159	1	585	2098	2	578
2111	3	117	2040	2	554	2159	3	461	2074	1	336	2041	5	347
2134	3	730	2145	2	462	2085	5	237	2049	1	288	2154	5	534
2185	3	382	2048	3	368	2064	3	262	2040	4	568	2161	5	519
2210	3	474	2152	1	434	2107	4	298	2073	4	375	2131	1	513
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
2267	3	431	2131	4	526	2159	3	420	2139	5	600	2180	4	242
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2047	2	139	2170	2	514	2094	3	254	2077	3	300	2160	1	315
2174	3	398	2075	4	145	2154	2	228	2175	4	593	2178	4	579
2062	2	185	2045	2	435	2049	2	578	2159	1	585	2098	2	541
2111	3	117	2040	2	554	2159	3	461	2074	1	336	2041	5	519
2134	3	730	2145	2	462	2085	5	237	2049	1	288	2154	5	513
2185	3	382	2048	3	368	2064	3	262	2040	4	568	2161	5	591
2210	3	474	2152	1	434	2107	4	298	2073	4	375	2131	1	242
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	441
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
2267	3	431	2131	4	526	2159	3	420	2139	5	600	2180	4	242
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
2267	3	431	2131	4	526	2159	3	420	2139	5	600	2180	4	242
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2047	2	139	2170	2	514	2094	3	254	2077	3	300	2160	1	315
2174	3	398	2075	4	145	2154	2	228	2175	4	593	2178	4	579
2062	2	185	2045	2	435	2049	2	578	2159	1	585	2098	2	541
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2047	2	139	2170	2	514	2094	3	254	2077	3	300	2160	1	315

2174	4	498	2144	3	214	2054	5	120	2170	3	372	2158	3	579
2067	3	239	2180	5	120	2055	5	267	2093	4	596	2139	3	541
2185	3	382	2048	3	368	2064	3	262	2040	4	568	2161	5	519
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2047	2	139	2170	2	514	2094	3	254	2077	3	300	2160	1	315
2174	4	498	2144	3	214	2054	5	120	2170	3	372	2158	3	579
2067	3	239	2180	5	120	2055	5	267	2093	4	596	2139	3	541
2185	3	382	2048	3	368	2064	3	262	2040	4	568	2161	5	519
2210	3	474	2152	1	434	2107	4	298	2073	4	375	2131	1	242
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	441
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2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2047	2	139	2170	2	514	2094	3	254	2077	3	300	2160	1	315
2174	3	398	2075	4	145	2154	2	228	2175	4	593	2178	4	579
2062	2	185	2045	2	435	2049	2	578	2159	1	585	2098	2	541
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
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2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
2267	3	431	2131	4	526	2159	3	420	2139	5	600	2180	4	242
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2210	3	474	2152	1	434	2107	4	298	2073	4	375	2131	1	513
2109	2	271	2132	3	440	2042	5	377	2119	5	124	2131	4	264
2108	3	259	2160	2	250	2134	1	240	2126	2	263	2121	3	573
2134	3	730	2145	2	462	2085	5	237	2049	1	288	2154	5	534
2185	3	382	2048	3	368	2064	3	262	2040	4	568	2161	5	519
2210	3	474	2152	1	434	2107	4	298	2073	4	375	2131	1	513
2105	2	255	2168	5	221	2066	5	236	2076	4	206	2044	1	591
2267	3	431	2131	4	526	2159	3	420	2139	5	600	2180	4	242
2205	2	373	2066	2	153	2158	4	279	2093	5	161	2118	2	441
2121	3	312	2172	3	466	2068	5	335	2099	3	181	2134	2	373