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AN EFFICIENT CLASS OF CHAIN-TYPE EXPONENTIAL ESTIMATORS FOR POPULATION MEAN UNDER TWO-PHASE SAMPLING SCHEME

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Abstract

This paper intends to develop an efficient estimation strategy to estimate mean of a finite population using information on two auxiliary variates, which are associated with study variable y under two-phase (double) sampling setup. The properties of the proposed estimation procedure have been examined up to first order of approximations and empirical studies are performed on seven data sets to show the dominance over some contemporary estimators. A separate numerical computation is also performed to illustrate the predominance of the suggested estimation procedures for various combinations of correlation coefficients. Recommendations are also made for survey statisticians.

Key Words: Two-Phase (Double) Sampling, Auxiliary Information, Study Variate, Bias, Mean Squared Error, Relative Efficiency

1. Introduction

In sample surveys, it is a general consensus of survey researchers that auxiliary information is frequently used in different forms to achieve more precise estimators of unknown population parameters by utilizing information on auxiliary variates, which are highly correlated with the variable of interest y. When parameters of the auxiliary variates are known, several researchers have introduced modified versions of ratio, product and regression estimators. Sometimes, there may be cases where such auxiliary information is not available (known) in advance but may be obtained easily at a comparatively low cost (in terms of time and money). In such circumstances, it is convenient (suitable) to draw a relatively large sample from a population and enumerate it for various parameters of the auxiliary variable x, and then take an independent sample from the first sample for the various parameters (such as mean, variance, etc.) of the variable y. This technique of taking samples in two-phases is known as double sampling.

Some notable contributions in this direction were made by several researchers (Neyman 1938; Murthy 1964; Mukharjee et al. 1987; Singh and Singh 1991; Singh et al. 1994; Mishra and Rout 1997; Upadhyaya and Singh 2001; Singh 2001; Pradhan 2005; Dash and Mishra 2011; Bandyopadhyay and Singh 2014; Singh and Sharma 2014, 2015) among others.

In the present investigation, motivated by the above researchers we suggest an efficient class of chain-type exponential estimators under double sampling setup to estimate the population mean $\bar{Y}$ more precisely as compare to the other existing estimators.

1.1 Two-phase sampling set up

Consider a finite population composed of N units identified by $U=(U_1,U_2,....,U_N)$. Let y_i, x_i, z_i $(i=1,2,....,N)$ be the values of variable of interest y and auxiliary variates x and z respectively for the i^{th} $(i=1,2,....,N)$ unit of the finite population. We are interested to estimate $\bar{Y}$ more precisely in the presence of two auxiliary variables x and z, when the population mean $\bar{Z}$ of the variable z is known in advance while the population mean of the auxiliary variable x is not available (known). It is also assumed that $\rho_{yx} > \rho_{yz}$. In this situation, population mean of study variate is obtained using two-phase sampling method.

Consider a double sampling scheme, where a large preliminary random sample s' of size n' $(n'<N)$ under SRSWOR scheme is drawn to observe x and z. Subsequently in the second phase, a sub-sample s of size n $(n<n')$ is selected by the same sampling procedure as discussed previous to obtain (observe) the characteristics of the study variable y.

2. Some existing estimators of population mean $\bar{Y}$

We have revisited to some existing estimators of the population mean $(\bar{Y})$ in presence of one and two auxiliary variables under two-phase sampling scheme, which have been frequently used by survey researchers. The estimators and their corresponding mean squared errors are given as follows:

2.1 Estimators consist of single auxiliary variable

The two-phase sampling versions of ratio and regression estimators with their MSEs are reproduced as:

$$t_1 = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right) \tag{2.1}$$

$$M\left(t_1\right) = \bar{Y}^2\left[f_1 C_y^2 + f_3\left(C_x^2 - 2\rho_{yx} C_y C_x\right)\right] \tag{2.2}$$

$$t_2 = \bar{y} + b_{yx}\left(n\right)\left(\bar{x}' - \bar{x}\right) \tag{2.3}$$

$$M\left(t_2\right) = \bar{Y}^2 C_y^2\left[f_1\left(1-\rho_{yx}^2\right) + f_2\rho_{yx}^2\right] \tag{2.4}$$

Singh and Vishwakarma (2007) suggested exponential ratio and product type estimators for $\bar{Y}$ as

$$t_3 = \bar{y}\exp\left(\frac{\bar{x}' - \bar{x}}{\bar{x}' + \bar{x}}\right) \tag{2.5}$$

$$t_4 = \bar{y}\exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right) \tag{2.6}$$

The MSEs of the estimators t_1 and t_2 are

$$M\left(t_3\right)=\bar{Y}^2\left\{f_1C_y^2+\frac{f_3}{4}\left(C_x^2-4\rho_{yx}C_yC_x\right)\right\} \tag{2.7}$$

$$M\left(t_4\right)=\bar{Y}^2\left[f_1C_y^2+\frac{f_3}{4}\left(C_x^2+4\rho_{yx}C_yC_x\right)\right] \tag{2.8}$$

2.2 Estimators consist of two auxiliary variables

Sometimes, it is also possible in the study of survey research that the information on more than one auxiliary variable is present at both (design and estimation) stages. Chand (1975) developed a chain-type ratio estimator as

$$t_5=\bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}'}\right) \tag{2.9}$$

$$M\left(t_5\right)=\bar{Y}^2\left\{f_1C_y^2+f_3\left(C_x^2-2\rho_{yx}C_yC_x\right)+f_2\left(C_z^2-2\rho_{yz}C_yC_z\right)\right\} \tag{2.10}$$

where

$b_{yx}\left(n\right)$: Sample regression coefficient of y on x based on s (sub-sample)

$b_{xz}\left(n'\right)$: Sample regression coefficient of x on z based on s' (sample)

$\rho_{yx},\rho_{yz},\rho_{xz}$: Population correlation coefficients between the variables shown in subscripts.

$$\bar{y}=n^{-1}\sum_{i\in s}y_i,\ \bar{x}=n^{-1}\sum_{i\in s}x_i\ \ and\ \ \bar{x}'=n'^{-1}\sum_{i\in s'}x_i,\ S_x^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2,S_y^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2,$$

$$S_z^2=\frac{1}{(N-1)}\sum_{i=1}^{N}\left(z_i-\bar{Z}\right)^2,C_x=\frac{S_x}{\bar{X}},C_y=\frac{S_y}{\bar{Y}},C_z=\frac{S_z}{\bar{Z}}\ f_1=\left(\frac{1}{n}-\frac{1}{N}\right),\ f_2=\left(\frac{1}{n'}-\frac{1}{N}\right),\ f_3=\left(f_1-f_2\right).$$

Kiregyera (1980, 1984) suggested some modifications of Chand (1975) estimator t_3 and proposed three different types of estimators (ratio to regression, regression to ratio and regression to regression) of population mean. The estimators and their respective mean squared errors are given as

$$t_6=\frac{\bar{y}}{\bar{x}}\left[\bar{x}'+b_{xz}\left(n'\right)\left(\bar{Z}-\bar{z}'\right)\right] \tag{2.11}$$

$$M\left(t_6\right)=\bar{Y}^2\left\{f_3\left(C_x^2+C_y^2-2\rho_{yx}C_yC_x\right)+f_2C_y^2+f_2\rho_{xz}C_x\left(\rho_{xz}C_x-2\rho_{yz}C_y\right)\right\} \tag{2.12}$$

$$t_7=\bar{y}+b_{yx}\left(n\right)\left(\frac{\bar{x}'}{\bar{z}'}\bar{Z}-\bar{x}\right) \tag{2.13}$$

$$M\left(t_7\right)=\bar{Y}^2C_y^2\left[f_1\{1-\rho_{yx}^2\}+f_2\left\{\rho_{yx}^2+\rho_{yx}^2\frac{C_z^2}{C_x^2}-2\rho_{yx}\rho_{yz}\frac{C_z}{C_x}\right\}\right] \tag{2.14}$$

$$t_8=\bar{y}+b_{yx}\left(n\right)\left(\bar{x}'_{ld}-\bar{x}\right);\ \bar{x}'_{ld}=\left[\bar{x}'+b_{xz}\left(n'\right)\left(\bar{Z}-\bar{z}'\right)\right] \tag{2.15}$$

$$M\left(t_8\right)=\bar{Y}^2C_y^2\left[f_3\left(1-\rho_{yx}^2\right)+f_2\left(1+\rho_{yx}^2\rho_{xz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)\right] \tag{2.16}$$

Singh and Khalid (2015) suggested the following estimators

$$t_9=\bar{y}\exp\left\{\frac{\bar{x}'(\bar{z}^*/\bar{Z})-x}{\bar{x}'(\bar{z}^*/\bar{Z})+\bar{x}}\right\} \tag{2.17}$$

$$t_{10}=\bar{y}\exp\left\{\frac{\bar{x}'+b_{xz}\left(n'\right)\left(\bar{Z}-\bar{z}'\right)-\bar{x}}{\bar{x}'+b_{xz}\left(n'\right)\left(\bar{Z}-\bar{z}'\right)+\bar{x}}\right\} \tag{2.18}$$

The MSEs of the estimators t_7 and t_8 are

$$M\left(t_9\right)=\bar{Y}^2\left[f_1C_y^2+f_2\left(\frac{k^2}{4}C_z^2-k\rho_{yz}C_yC_z\right)+\frac{f_3}{4}\left(C_x^2-4\rho_{yx}C_yC_x\right)\right] \tag{2.19}$$

$$M\left(t_{10}\right)=\bar{Y}^2\left[f_1C_y^2+\frac{f_2}{4}\left(\rho_{xz}^2C_x^2-4\rho_{yz}\rho_{xz}C_yC_x\right)+\frac{f_3}{4}\left(C_x^2-4\rho_{yx}C_yC_x\right)\right] \tag{2.20}$$

where

$$\bar{z}^*=\frac{(N\bar{Z}-n'\bar{z}')}{(N-n')};\ k=\frac{n'}{(N-n')}.$$

Singh and Choudhury (2012) developed the following exponential chain-type ratio and product estimators of $\bar{Y}$ under double sampling as

$$t_{11}=\bar{y}\exp\left\{\frac{\left(\bar{x}'/\bar{z}'\right)\bar{Z}-\bar{x}}{\left(\bar{x}'/\bar{z}'\right)\bar{Z}+\bar{x}}\right\} \tag{2.21}$$

and

$$t_{12}=\bar{y}\exp\left\{\frac{\bar{x}-\left(\bar{x}'/\bar{z}'\right)\bar{Z}}{\bar{x}+\left(\bar{x}'/\bar{z}'\right)\bar{Z}}\right\} \tag{2.22}$$

The MSEs of the estimators t_9 and t_{10} are reproduced as

$$M\left(t_{11}\right)=\bar{Y}^2\left[f_1C_y^2+\frac{1}{4}\left(f_3C_x^2+f_2C_z^2\right)-\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_z\right)\right] \tag{2.23}$$

$$M\left(t_{12}\right)=\bar{Y}^2\left[f_1C_y^2+\frac{1}{4}\left(f_3C_x^2+f_2C_z^2\right)+\left(f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_z\right)\right] \tag{2.24}$$

3. Proposed class of estimators

Following the previously discussed estimation procedures, we suggest an improved class of estimators (t_{ps}) of $\bar{Y}$ as

$$t_{ps}=\bar{y}\exp\left\{\frac{\bar{x}'_{FT}-\bar{x}}{\bar{x}'_{FT}+\bar{x}}\right\} \tag{3.1}$$

where

$\bar{x}'_{FT}$ is a factor type estimator (see, Singh and Shukla 1987) defined as

$$\bar{x}'_{FT}=\bar{x}'\left\{\frac{(A+C)\bar{Z}+fB\bar{z}'}{(A+f\ B)\bar{Z}+C\ \bar{z}'}\right\}$$

$A=(d-1)(d-2)$, $B=(d-1)(d-4)$ $C=(d-2)(d-3)(d-4)$, $(d>0)$ and motivation to construct exponential type estimator is structured originally from Bahl and Tuteja (1991).

3.1 Some particular cases

(i) For $d=1$, the estimator t_{ps} reduces to the Singh and Choudhury (2012) estimator as shown in Eq. (2.21).

(ii) For $d=2$, the estimator t_{ps} reduces to exponential chain-type ratio estimator, given as

$$t_{ps(2)}=\bar{y}\exp\left\{\frac{\bar{x}'(\bar{z}'/\bar{Z})-\bar{x}}{\bar{x}'(\bar{z}'/\bar{Z})+\bar{x}}\right\} \tag{3.2}$$

(iii) For $d=3$, the estimator t_{ps} given in Eq. (3.1) converges to the estimator described by Singh and Khalid (2015) and represented in Eq. (2.17).

(iv) For $d=4$, the estimator t_{ps} given in Eq. (3.1) reduces to the estimator developed by Singh and Vishwakarma (2007) and reproduced in Eq. (2.5).

4. Properties of the proposed class of estimators t_{ps}

To get the bias (B(.)) and MSE (M(.)) of the estimator $\mathrm{t_{ps}}$, we wright the following in terms of errors:

$$\frac{\bar{y}}{\bar{Y}}=(1+\varepsilon_0),\ \frac{\bar{x}}{\bar{X}}=(1+\varepsilon_1),\ \frac{\bar{x}'}{\bar{X}}=(1+\varepsilon_2),\ \frac{\bar{z}'}{\bar{Z}}=(1+\varepsilon_3)\ \text{ such that}$$

$$E(\varepsilon_i)=0;(i=0,1,2,3)\ \text{ and}$$

$$E(\varepsilon_0^2)=f_1C_y^2,\ E(\varepsilon_1^2)=f_1C_x^2,\ E(\varepsilon_2^2)=E(\varepsilon_1\varepsilon_2)=f_2C_x^2, E(\varepsilon_3^2)=f_2C_z^2,\ E(\varepsilon_0\varepsilon_1)=f_1\rho_{yx}C_yC_x,$$

$$E(\varepsilon_0\varepsilon_2)=f_2\rho_{yx}C_yC_x, E(\varepsilon_0\varepsilon_3)=f_2\rho_{yz}C_yC_z, E(\varepsilon_1\varepsilon_3)=E(\varepsilon_2\varepsilon_3)=f_2\rho_{xz}C_xC_z$$

Expressing Eq. (3.1) in terms of $\varepsilon_i's\,(i=0,1,.,3)$, we have

$$t_{ps}=\bar{Y}(1+\varepsilon_0)\exp\left\{\frac{\bar{X}(1+\varepsilon_2)\left\{\dfrac{(A+C)\bar{Z}+fB\bar{Z}(1+\varepsilon_3)}{(A+fB)\bar{Z}+C\,\bar{Z}(1+\varepsilon_3)}\right\}-\bar{X}(1+\varepsilon_1)}{\bar{X}(1+\varepsilon_2)\left\{\dfrac{(A+C)\bar{Z}+fB\bar{Z}(1+\varepsilon_3)}{(A+fB)\bar{Z}+C\bar{Z}(1+\varepsilon_3)}\right\}+\bar{X}(1+\varepsilon_1)}\right\} \tag{4.1}$$

Now simplifying the expression given in Eq. (4.1), up to terms of order n^{-1}, we have

$$\begin{aligned}t_{ps}-\bar{Y}=\bar{Y}\Bigg[&\varepsilon_0+\frac{1}{2}\left\{\varepsilon_2-\varepsilon_1+\varepsilon_0\varepsilon_2-\varepsilon_0\varepsilon_1+\frac{(fB-C)\varepsilon_0\varepsilon_3}{(A+C+fB)}+\frac{(fB-C)\varepsilon_3}{(A+C+fB)}+\right\}+\frac{3}{8}\varepsilon_1^2-\frac{1}{8}\varepsilon_2^2\\&+\frac{1}{4}\left\{-\varepsilon_1\varepsilon_2-\frac{C\varepsilon_2\varepsilon_3}{(A+C+fB)}+\frac{fB\varepsilon_2\varepsilon_3}{(A+C+fB)}+\frac{C\varepsilon_1\varepsilon_3}{(A+C+fB)}-\frac{fB\varepsilon_1\varepsilon_3}{(A+C+fB)}-\frac{fBC\varepsilon_3^2}{(A+C+fB)}\right\}\\&+\frac{3}{8}\frac{(C\varepsilon_3)^2}{(A+C+fB)^2}-\frac{1}{8}\frac{(fB)^2\varepsilon_3^2}{(A+C+fB)^2}\Bigg]\end{aligned} \tag{4.2}$$

Now, applying the conventional procedure, we get the expressions for bias and mean squared error of the estimator t_{ps} as

$$\mathrm{B}(t_{ps}) = \mathrm{E}(t_{ps} - \bar{\mathrm{Y}})$$

$$\mathrm{B}(t_{ps}) = \bar{Y}\left[\frac{1}{8}\left\{3f_3C_x^2 + f_2C_z^2\left\{3C^2 - \frac{fB(fB+2C)}{(A+C+fB)^2}\right\} - f_3\rho_{yx}C_yC_x + \frac{(fB-C)}{2\,(A+C+fB)}f_2\rho_{yz}C_yC_z\right\}\right] \tag{4.3}$$

and

$$M(t_{ps}) = E[t_{ps} - \bar{Y}]^2$$

$$= \bar{Y}^2\left[f_1C_y^2 + \frac{1}{4}f_3C_x^2 + \frac{1}{4}f_2P^2C_z^2 - f_3\rho_{yx}C_yC_x + f_2P\rho_{yz}C_yC_z\right] \tag{4.4}$$

where

$$P = \frac{(f\,B - C)}{(A + C + f\,B)}$$

The expression of MSE of the estimator t_{ps} in Eq. (4.4) consist the unknown scalar P and P is a function of d, hence we get $P_{opt.}$ (optimum value of P) by minimizing Eq. (4.4) with respect to d, as

$$P_{opt} = -2\rho_{yz}\frac{C_y}{C_z} \tag{4.5}$$

Putting the value of $P_{opt.}$ in Eq. (4.4), we obtain the minimum MSE of the estimator t_{ps} as

$$M\left(t_{ps}\right)_{\min} = \bar{Y}^2\left[f_1C_y^2 + \frac{1}{4}f_3C_x^2 - f_2\rho_{yz}^2C_y^2 - f_3\rho_{yx}C_yC_x\right] \tag{4.6}$$

Remark 4.1

The term P_{opt} in Eq. (4.5) consist of unknown parameters such as ρ_{yz} and C_y. Hence for practical applications, the unknown parameters can be replaced by their guess values available from the past (pilot) surveys. If such guess values are not readily available then these parameters may be estimated by their corresponding sample estimates, see (Murthy 1967; Reddy 1973; 1978) and Srivenkataramana and Tracy (1980).

5. Theoretical comparisons

The suggested estimator t_{ps} under its optimality condition is more efficient than the existing estimators $t_i\,(i = 1,2,...12)$ under the following conditions which are obtained by comparing their respective mean square errors. Hence, we have the following results

(i) By Eqs. (4.6) and (2.2), $M\left(t_{ps}\right)_{\min} < M\left(t_1\right)$ if

$$\frac{f_2}{f_3} > \frac{(\rho_{yx}C_yC_x - 3C_x^2)}{(\rho_{yz}^2C_y^2)} \tag{5.1}$$

(ii) By Eqs. (4.6) and (2.4), $M\left(t_{ps}\right)_{\min} < M\left(t_2\right)$ if

$$\frac{f_2}{f_3} > \frac{(C_x^2 - 4\rho_{yx}C_yC_x + 4\rho_{yx}^2C_y^2)}{(4\rho_{yz}^2C_y^2)} \tag{5.2}$$

(iii) By Eqs. (4.6) and (2.7), $M\left(t_{ps}\right)_{\min} < M\left(t_3\right)$ if

$$f_2\rho_{yz}^2C_y^2 > 0 \tag{5.3}$$

(iv) By Eqs. (4.6) and (2.8), $M\left(t_{ps}\right)_{\min} < M\left(t_4\right)$ if

$$\frac{f_2}{f_3} > \frac{(-2\rho_{yx}C_yC_x)}{\rho_{yz}^2C_y^2} \tag{5.4}$$

(v) By Eqs. (4.6) and (2.10), $M\left(t_{ps}\right)_{\min} < M\left(t_5\right)$ if

$$\frac{f_2}{f_3} > \frac{(4\rho_{yx}C_yC_x - 3C_x^2 - 4\rho_{yx}^2C_y^2)}{(2(\rho_{yz}C_y - C_z))^2} \tag{5.5}$$

(vi) By Eqs. (4.6) and (2.12), $M\left(t_{ps}\right)_{\min} < M\left(t_6\right)$ if

$$\frac{f_2}{f_3} > \frac{(4\rho_{yx}C_yC_x - 3C_x^2)}{4\{\rho_{xz}C_x(\rho_{xz}C_x - 2\rho_{yz}C_y) + \rho_{yz}^2C_y^2\}} \tag{5.6}$$

(vii) By Eqs. (4.6) and (2.14), $M\left(t_{ps}\right)_{\min} < M\left(t_7\right)$ if

$$\frac{f_2}{f_3} > \frac{(C_x^2 - 4\rho_{yx}C_yC_x + 4\rho_{yx}^2C_y^2)}{4\left\{\rho_{yx}^2 + \rho_{yx}^2\dfrac{C_z^2}{C_x^2} - 2\rho_{yx}\rho_{yz}\dfrac{C_z}{C_x} + (\rho_{yz}^2 + \rho_{yx}^2)C_y^2\right\}} \tag{5.7}$$

(viii) By Eqs. (4.6) and (2.16), $M\left(t_{ps}\right)_{\min} < M\left(t_8\right)$ if

$$\frac{f_2}{f_3} > \frac{(C_x^2 - 4\rho_{yx}C_yC_x + 4\rho_{yx}^2C_y^2)}{4\left\{C_y^2(\rho_{yx}^2\rho_{xz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz} + \rho_{yz}^2)\right\}} \tag{5.8}$$

(ix) By Eqs. (4.6) and (2.19), $M\left(t_{ps}\right)_{\min} < M\left(t_9\right)$ if

$$(k^2C_z^2 - 4k\rho_{yz}C_yC_z + 4\rho_{yz}^2C_y^2) > 0 \tag{5.9}$$

(x) By Eqs. (4.6) and (2.20), $M\left(t_{ps}\right)_{\min} < M\left(t_{10}\right)$ if

$$(4\rho_{yz}^2C_y^2 + \rho_{xz}^2C_x^2 - 4\rho_{yz}\rho_{xz}C_yC_x) > 0 \tag{5.10}$$

(xi) By Eqs. (4.6) and (2.23), $M\left(t_{ps}\right)_{\min} < M\left(t_{11}\right)$ if

$$f_2(C_z^2 - 4\rho_{yz}C_yC_z + 4\rho_{yz}^2C_y^2) > 0 \tag{5.11}$$

(xii) By Eqs. (4.6) and (2.24), $M\left(t_{ps}\right)_{\min} < M\left(t_{12}\right)$ if

$$\frac{f_2}{f_3} > \frac{(-8\rho_{yx}C_yC_x)}{(C_z^2 + 4\rho_{yz}C_yC_z + 4\rho_{yz}^2C_y^2)} \tag{5.12}$$

6. Empirical studies

To examine the validity of mathematical comparisons of proposed estimator and other available estimators discussed in this work, we borrowed the following numerical values of different population parameters from the four different real population datasets as given in Table 1, where the first three population datasets are same as considered by Singh and Khalid (2015).

Parameters	Population 1 (Cochran (1977))	Population 2 (Fisher (1936))	Population 3 (Shukla ((1966))	Population 4 (Sukhatme and Chand (1977))
N	34	50	50	200
n'	15	20	15	30
n	10	10	8	20
$\bar{Y}$	4.9200	2.7700	2.5800	1031.8200
ρ_{yx}	0.7326	0.5605	0.4800	0.9300
ρ_{yz}	0.6430	0.5259	0.3700	0.7700
ρ_{xz}	0.6837	0.7540	0.7300	0.8400
C_y^2	1.0248	0.0125	0.0866	2.5528
C_x^2	1.5175	0.0073	0.0116	4.0250
C_z^2	1.1492	0.0119	0.0170	2.9379

Table 1: Population Parameters of Four Different Populations

To assess the effectiveness of the suggested estimator, we have examined the performance of the suggested estimator t_{ps} over other existing estimators $t_i (i = 1, 2, ... 12)$ using the following formula and findings are shown in Table 2.

$$\text{PRE } (t_i, \text{t}_{ps}) = \left[\frac{MSE(t_i)}{MSE(t_{ps})_{\min}} \right] \times 100 : i = 1, 2, ...12 \tag{6.1}$$

7. General study

In this section, we introduced an assumption that coefficients of variation (CV) of variables x, y and z are approximately equal i.e. $C_x \cong C_y \cong C_z$, and it is also considered by several survey researchers as (Murthy 1967; Cochran 1977 and Reddy 1978).

In follow up the above assumption, the mean squared errors of the existing estimators $t_i \ (i = 1, 2, ...12)$ and proposed estimator t_p may take the following form.

$$M(t_1) = S_y^2 \left[f_1 + f_3 \left(1 - 2\rho_{yx} \right) \right] \tag{7.1}$$

$$M(t_2) = S_y^2 \left[f_1 \left(1 - \rho_{yx}^2 \right) + f_2 \rho_{yx}^2 \right] \tag{7.2}$$

$$M\left(t_3\right)=S_y^2\left[f_1+\frac{f_3}{4}\left(1-4\rho_{yx}\right)\right] \tag{7.3}$$

$$M\left(t_4\right)=S_y^2\left[f_1+\frac{f_3}{4}\left(1+4\rho_{yx}\right)\right] \tag{7.4}$$

$$M\left(t_5\right)=S_y^2\left[f_1+f_3\left(1-2\rho_{yx}\right)+f_2\left(1-2\rho_{yz}\right)\right] \tag{7.5}$$

$$M\left(t_6\right)=S_y^2\left[f_3\left(2-2\rho_{yx}\right)+f_2+f_2\rho_{xz}\left(\rho_{xz}-2\rho_{yz}\right)\right] \tag{7.6}$$

$$M\left(t_7\right)=S_y^2\left[f_1\left(1-\rho_{yx}^2\right)+f_2\left(\rho_{yx}^2+\rho_{yx}^2-2\rho_{yx}\rho_{yz}\right)\right] \tag{7.7}$$

$$M\left(t_8\right)=S_y^2\left[f_3\left(1-\rho_{yx}^2\right)+f_2\left(1+\rho_{yx}^2\rho_{xz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)\right] \quad 7.8)$$

$$M\left(t_9\right)=S_y^2\left[f_1+f_2\left(\frac{k^2}{4}-k\rho_{yz}\right)+\frac{1}{4}f_3\left(1-4\rho_{yx}\right)\right] \tag{7.9}$$

$$M\left(t_{10}\right)=S_y^2\left[f_1+\frac{1}{4}f_2\left(\rho_{xz}^2-4\rho_{yz}\rho_{xz}\right)+\frac{1}{4}f_3\left(1-4\rho_{yx}\right)\right] \tag{7.10}$$

$$M\left(t_{11}\right)=S_y^2\left[f_1+\frac{1}{4}\left(f_3+f_2\right)-\left(f_3\rho_{yx}+f_2\rho_{yz}\right)\right] \tag{7.11}$$

$$M\left(t_{12}\right)=S_y^2\left[f_1+\frac{1}{4}\left(f_3+f_2\right)+\left(f_3\rho_{yx}+f_2\rho_{yz}\right)\right] \tag{7.12}$$

and

$$M\left(t_{ps}\right)_{\min}=S_y^2\left[f_1+\frac{1}{4}f_3-f_2\rho_{yz}^2-f_3\rho_{yx}\right] \tag{7.13}$$

8. Theoretical comparisons

As discussed in Sec.4, we can obtain theoretical comparisons of the estimators under the assumption $C_x \cong C_y \cong C_z$.

(i) By Eqs. (7.13) and (7.1), $M\left(t_{ps}\right)_{\min} < M\left(t_1\right)$ if

$$\frac{f_2}{f_3} > \frac{(\rho_{yx}-3)}{\rho_{yz}^2} \tag{8.1}$$

(ii) By Eqs. (7.13) and (7.2), $M\left(t_{ps}\right)_{\min} < M\left(t_2\right)$ if

$$\frac{f_2}{f_3} > \frac{(1-4\rho_{yx}+4\rho_{yx}^2)}{4\rho_{yz}^2} \tag{8.2}$$

(iii) By Eqs. (7.13) and (7.3), $M\left(t_{ps}\right)_{\min} < M\left(t_3\right)$ if

$$f_2\rho_{yz}^2 > 0 \tag{8.3}$$

(iv) By Eqs. (7.13) and (7.4), $M\left(t_{ps}\right)_{\min} < M\left(t_4\right)$ if

$$\frac{f_2}{f_3} > \frac{(-2\rho_{yx})}{\rho_{yz}^2} \tag{8.4}$$

(v) By Eqs. (7.13) and (7.5), $M\left(t_{ps}\right)_{\min} < M\left(t_5\right)$ if

$$\frac{f_2}{f_3} > \frac{(4\rho_{yx} - 3 - 4\rho_{yx}^2 C_y^2)}{(2(\rho_{yz} - 1))^2} \tag{8.5}$$

(vi) By Eqs. (7.13) and (7.6), $M\left(t_{ps}\right)_{\min} < M\left(t_6\right)$ if

$$\frac{f_2}{f_3} > \frac{(4\rho_{yx} - 3)}{4\{\rho_{xz}(\rho_{xz} - 2\rho_{yz}) + \rho_{yz}^2\}} \tag{8.6}$$

(v). By Eqs. (7.13) and (7.7), $M\left(t_{ps}\right)_{\min} < M\left(t_7\right)$ if

$$\frac{f_2}{f_3} > \frac{(1 - 4\rho_{yx} + 4\rho_{yx}^2)}{4\left\{\rho_{yx}^2 + \rho_{yx}^2 - 2\rho_{yx}\rho_{yz} + (\rho_{yz}^2 + \rho_{yx}^2)\right\}} \tag{8.7}$$

(vi) By Eqs. (7.13) and (7.8), $M\left(t_{ps}\right)_{\min} < M\left(t_8\right)$ if

$$\frac{f_2}{f_3} > \frac{(1 - 4\rho_{yx} + 4\rho_{yx}^2)}{4(\rho_{yx}^2\rho_{xz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz} + \rho_{yz}^2)} \tag{8.8}$$

(vii) By Eqs. (7.13) and (7.9), $M\left(t_{ps}\right)_{\min} < M\left(t_9\right)$ if

$$(k^2 - 4k\rho_{yz} + 4\rho_{yz}^2) > 0 \tag{8.9}$$

(viii) By Eqs. (7.13) and (7.10), $M\left(t_{ps}\right)_{\min} < M\left(t_{10}\right)$ if

$$(4\rho_{yz}^2 + \rho_{xz}^2 - 4\rho_{yz}\rho_{xz}) > 0 \tag{8.10}$$

(xi) By Eqs. (7.13) and (7.11), $M\left(t_{ps}\right)_{\min} < M\left(t_{11}\right)$ if

$$f_2(1 - 4\rho_{yz} + 4\rho_{yz}^2) > 0 \tag{8.11}$$

(xii) By Eqs. (7.13) and (7.12), $M\left(t_{ps}\right)_{\min} < M\left(t_{12}\right)$ if

$$\frac{f_2}{f_3} > \frac{(-8\rho_{yx})}{(1 + 4\rho_{yz} + 4\rho_{yz}^2)} \tag{8.12}$$

9. Efficiency comparisons

The percentage relative efficiencies of suggested class of estimators with respect to (w. r. t.) the other existing estimators are calculated under the assumption $C_x \cong C_y \cong C_z$, we use the formula given in equation (6.1) and findings are shown in Tables 3 and 4 for different choices of ρ_{yx} and ρ_{yz} . Here, we consider two data sets as (i) N=400, $n' = 60, n = 40$ (ii) $N = 400,\ n' = 40,\ n = 20$.

Existing estimators	Data Sets			
	I	II	III	IV
t_1	160.06	120.90	138.13	211.35
t_2	139.38	118.17	106.78	199.80
t_3	140.74	120.25	107.45	209.74
t_4	297.93	176.40	183.13	464.02
t_5	136.37	115.35	130.97	118.57
t_6	122.84	100.83	143.08	116.62
t_7	*	100.51	101.22	*
t_8	100.63	*	100.00	*
t_9	104.99	101.46	104.12	184.42
t_{10}	105.08	104.14	100.15	110.90
t_{11}	101.27	100.11	101.20	110.10
t_{12}	392.65	231.29	194.73	670.17

(*) indicate no gain

Table 2: PREs of the suggested estimator t_{ps} w. r. t. the estimators t_i

$\rho_{yz} \rightarrow$	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho_{yx} \rightarrow$	0.2						
t_1	134.3570	138.8889	145.1852	153.7014	165.1500	180.6785	202.2286
t_2	100.2597	102.6077	107.2593	113.5508	122.0088	133.4808	149.4016
t_3	104.1952	107.7098	112.5926	119.1970	128.0755	140.1180	156.8304
t_4	126.1311	130.3855	136.2963	144.2911	155.0388	169.6165	189.8473
t_5	153.0025	148.5261	145.1852	143.0364	142.2312	143.0678	146.0999
t_6	134.3570	133.1066	133.0963	134.5044	137.6475	143.0678	151.7128
t_7	*	*	100.2000	102.8858	108.2575	115.9292	126.9501
t_8	*	*	101.9401	106.6399	113.2080	122.3481	135.2571
t_9	102.0900	104.6835	108.5403	113.9660	121.4437	131.7565	146.2334
t_{10}	100.0000	100.4819	102.0148	104.7992	109.1675	115.6711	125.2579
t_{11}	101.8645	100.4819	100.0000	100.5332	102.2919	105.6416	111.2258
t_{12}	151.7686	161.7063	174.0741	189.6173	209.4708	235.4351	270.5324
$\rho_{yx} \rightarrow$	0.3						
t_1	130.5483	135.2163	141.7323	150.6024	162.6310	179.1401	202.4291

t_2	102.1178	105.7692	110.8661	117.8046	127.2136	140.1274	158.3446
t_3	104.4386	108.1731	113.3858	120.4819	130.1048	143.3121	161.9433
t_4	139.2515	144.2308	151.1811	160.6426	173.4731	191.0828	215.9244
t_5	150.2756	145.4327	141.7323	139.2236	138.0557	138.5350	141.2506
t_6	130.5483	129.0865	128.8819	130.1205	133.1406	138.5350	147.3684
t_7	*	*	100.0221	102.4431	106.9389	113.7341	123.9316
t_8	*	100.0685	102.9631	107.3588	113.7217	122.8296	136.0450
t_9	102.2116	104.9650	109.0783	114.9008	122.9937	134.2848	150.3930
t_{10}	100.0000	100.5108	102.1417	105.1205	109.8301	116.9188	127.5304
t_{11}	101.9727	100.5108	100.0000	100.5689	102.4575	106.0908	112.2357
t_{12}	166.3766	177.4339	191.3386	209.0027	231.8395	262.1417	303.8686
$\rho_{yx} \rightarrow$	0.4						
t_1	126.2704	131.0742	137.8151	147.0588	159.7195	177.3356	202.6693
t_2	104.0961	108.0563	113.6134	121.2339	131.6712	146.1938	167.0786
t_3	104.7120	108.6957	114.2857	121.9512	132.4503	147.0588	168.0672
t_4	153.9883	159.8465	168.0672	179.3400	194.7799	216.2630	247.1577
t_5	147.2128	141.9437	137.8151	134.8637	133.2295	133.2180	135.4424
t_6	126.2704	124.5524	124.1008	125.1076	127.9314	133.2180	142.1651
t_7	100.9076	100.3606	100.0092	101.7217	105.1811	110.8997	120.0198
t_8	100.0725	100.7519	103.1906	107.1851	113.2341	122.1938	135.6164
t_9	102.3479	105.2825	109.6886	115.9697	124.7852	137.2507	155.3750
t_{10}	100.0000	100.5435	102.2857	105.4878	110.5960	118.3824	130.2521
t_{11}	102.0942	100.5435	100.0000	100.6098	102.6490	106.6176	113.4454
t_{12}	182.7841	195.1726	210.9244	231.1693	257.6938	293.4689	343.7963
$\rho_{yx} \rightarrow$	0.5						
t_1	121.4309	126.3661	133.3333	142.9675	156.3160	175.1894	202.9622
t_2	105.0213	109.2896	115.3153	123.6476	135.1922	151.5152	175.5348
t_3	105.0213	109.2896	115.3153	123.6476	135.1922	151.5152	175.5348
t_4	170.6597	177.5956	187.3874	200.9274	219.6874	246.2121	285.2441
t_5	143.7479	137.9781	133.3333	129.8300	127.5877	126.8939	128.3598
t_6	121.4309	119.3989	118.6306	119.3199	121.8420	126.8939	135.8201
t_7	102.2317	100.5806	100.0000	100.6569	102.8728	107.2443	114.9205
t_8	100.0000	100.5806	102.4505	105.9119	111.4913	120.1231	133.5710
t_9	102.5020	105.6433	110.3869	117.2038	126.8794	140.7782	161.4501
t_{10}	100.0000	100.5806	102.4505	105.9119	111.4913	120.1231	133.5710
t_{11}	102.2317	100.5806	100.0015	100.6569	102.8728	107.2443	114.9205

t_{12}	201.3456	215.3347	233.3333	256.7620	287.9172	330.7292	392.4849
$\rho_{yx} \rightarrow$	0.6						
t_1	115.9115	120.9677	128.1553	138.1910	152.2843	172.5941	203.3272
t_2	104.6716	109.2375	115.7282	124.7906	137.5173	155.8577	183.6106
t_3	105.3741	109.9707	116.5049	125.6281	138.4402	156.9038	184.8429
t_4	189.6733	197.9472	209.7087	226.1307	249.1924	282.4268	332.7172
t_5	139.7963	133.4311	128.1553	123.9531	120.9045	119.2469	119.5317
t_6	115.9115	113.4897	112.3107	112.5628	114.6285	119.2469	127.9113
t_7	104.6716	101.7595	100.0035	100.1625	100.0035	102.5105	108.1947
t_8	100.0000	100.3666	100.5173	103.2630	108.1458	116.1674	129.3112
t_9	102.6777	106.0570	111.1936	118.6447	129.3602	145.0437	169.0225
t_{10}	100.0000	100.6232	102.6408	106.4070	112.5519	122.2280	137.7079
t_{11}	102.3885	100.6232	100.003	100.7119	103.1380	108.0021	116.7591
t_{12}	222.5149	238.4531	259.2233	286.6415	323.7194	375.7845	453.1731
$\rho_{yx} \rightarrow$	0.7						
t_1	109.5580	114.7152	122.1053	132.5411	147.4326	169.3925	203.7948
t_2	102.7578	107.5949	114.5263	124.3144	138.2816	158.8785	191.1455
t_3	105.7801	110.7595	117.8947	127.9707	142.3488	163.5514	196.7674
t_4	211.5603	221.5190	235.7895	255.9415	284.6975	327.1028	393.5348
t_5	135.2474	128.1646	122.1053	117.0018	112.8622	109.8131	108.2221
t_6	109.5580	106.6456	104.9263	104.5704	105.9481	109.8131	117.7793
t_7	107.2535	102.8877	100.0000	*	*	*	*
t_8	*	*	*	*	102.7087	109.6659	121.9030
t_9	102.8801	106.5362	112.1362	120.3490	132.3455	150.3058	178.7235
t_{10}	100.0000	100.6725	102.8632	106.9927	113.8282	124.8248	143.0077
t_{11}	102.5689	100.6725	100.0000	100.7770	103.4570	108.9369	119.1145
t_{12}	246.8833	265.2294	289.4737	321.9835	366.8022	431.3668	530.9206
$\rho_{yx} \rightarrow$	0.8						
t_1	102.1659	107.3883	114.9425	125.7545	141.4827	165.3439	204.4154
t_2	*	103.9519	111.2644	121.7304	136.9553	160.0529	197.8741
t_3	106.2526	111.6838	119.5402	130.7847	147.1420	171.9577	212.5920
t_4	237.0249	249.1409	266.6667	291.7505	328.2400	383.5979	474.2437
t_5	129.9550	121.9931	114.9425	108.6519	102.9994	*	*
t_6	102.1659	*	*	*	*	*	104.3336
t_7	110.0123	103.9519	*	*	*	*	*
t_8	*	*	*	*	*	100.000	109.8021

t_9	103.1155	107.0977	113.2522	122.3961	136.0065	156.9600	191.5973
t_{10}	100.0000	100.7302	103.1264	107.6962	115.3933	128.1085	150.0409
t_{11}	102.7789	100.7302	100.0000	100.8551	103.8483	110.1190	122.2404
t_{12}	275.2350	296.6065	325.2874	364.4366	419.6378	501.6534	634.0965

(*) indicate no gain

Table 3: PREs of the proposed estimator t_{ps} w. r. t. the estimators t_i for the first data set.

$\rho_{yz} \rightarrow$	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\rho_{yx} \rightarrow$				0.2			
t_1	133.7614	138.4275	144.9275	153.7515	165.6726	181.9505	204.7502
t_2	100.0005	102.9900	107.8261	114.3911	123.2604	135.3712	152.3342
t_3	153.0230	148.3942	144.9275	142.6814	141.8158	142.6492	145.7821
t_4	104.3339	107.9734	113.0435	119.9262	129.2247	141.9214	159.7052
t_5	125.7357	130.1218	136.2319	144.5264	155.7323	171.0335	192.4652
t_6	133.7614	132.4474	132.4058	133.8253	137.0444	142.6492	151.6790
t_7	*	*	100.0083	103.3210	108.9463	117.0306	128.7469
t_8	*	*	102.3165	107.2177	114.0994	123.7380	137.4742
t_9	102.8774	105.9124	110.3060	116.4070	124.7699	136.3011	152.5617
t_{10}	100.0000	100.4983	102.0870	104.9815	109.5427	116.3755	126.5356
t_{11}	101.9262	100.4983	100.0000	100.5535	102.3857	105.8952	111.7936
t_{12}	152.2204	162.5138	175.3623	191.5744	212.3923	239.8108	277.2318
$\rho_{yx} \rightarrow$				0.3			
t_1	130.0170	134.8183	141.5385	150.7208	163.2363	180.5338	205.1740
t_2	102.3177	106.0961	111.3846	118.6107	128.4599	142.0722	161.4630
t_3	104.5789	108.4408	113.8462	121.2320	131.2988	145.2119	165.0312
t_4	138.4963	143.6108	150.7692	160.5505	173.8822	192.3077	218.5549
t_5	150.3674	145.3693	141.5385	138.9253	137.6863	138.1476	140.9456
t_6	130.0170	128.4877	128.2462	129.4889	132.5763	138.1476	147.3684
t_7	*	*	100.7539	102.6868	107.3811	114.5212	125.3345
t_8	*	100.2087	103.2098	107.7824	114.4329	124.0157	138.0517
t_9	103.0400	106.2590	110.9402	117.4822	126.5279	139.1505	157.2505
t_{10}	100.0000	100.5275	102.2154	105.3080	110.2200	117.6609	128.9028
t_{11}	102.0350	100.5275	100.0000	100.5898	102.5550	106.3579	112.8457
t_{12}	166.4782	177.9015	192.3077	210.6815	234.5635	266.4835	310.8831
$\rho_{yx} \rightarrow$				0.4			

t_1	125.8238	130.7597	137.7049	147.2651	160.4278	178.8756	205.6807
t_2	104.2540	108.3437	114.0984	122.0196	132.9259	148.2112	170.4212
t_3	104.8532	108.9664	114.7541	122.7209	133.6898	149.0630	171.4006
t_4	152.7861	158.7796	167.2131	178.8219	194.8052	217.2061	249.7551
t_5	147.3936	141.9676	137.7049	134.6424	132.9259	132.8790	135.1616
t_6	125.8238	124.0349	123.5410	124.5442	127.4255	132.8790	142.2135
t_7	100.0001	100.000	100.0344	101.8233	105.4240	111.4140	121.0578
t_8	100.5950	100.8120	103.3338	107.4783	113.7846	123.1891	137.4182
t_9	103.2222	106.6487	111.6576	118.7081	128.5545	142.4853	162.8578
t_{10}	100.0000	100.5604	102.3607	105.6802	111.0008	119.1652	131.7336
t_{11}	102.1570	100.5604	100.0000	100.6311	102.7502	106.8995	114.1038
t_{12}	182.4446	195.2055	211.4754	232.4684	260.1222	297.7002	351.1263
$\rho_{yx} \rightarrow$	0.5						
t_1	121.0962	126.1620	133.3333	143.2881	157.1547	176.9088	206.2975
t_2	105.1625	109.5618	115.7895	124.4344	136.4764	153.6313	179.1531
t_3	105.1625	109.5618	115.7895	124.4344	136.4764	153.6313	179.1531
t_4	168.8974	175.9628	185.9649	199.8492	219.1894	246.7412	287.7307
t_5	144.0408	138.1142	133.3333	129.7134	127.3780	126.6294	128.1216
t_6	121.0962	118.9907	118.1754	118.8537	121.4227	126.6294	135.9392
t_7	102.2945	100.5976	100.0000	100.6787	102.9777	107.5419	115.6352
t_8	100.0000	100.5976	102.5263	106.1086	111.9107	120.9497	135.1792
t_9	103.4275	107.0902	112.4756	120.1190	130.9163	146.4411	169.6827
t_{10}	100.0000	100.5976	102.5263	106.1086	111.9107	120.9497	135.1792
t_{11}	102.2945	100.5976	100.0000	100.6787	102.9777	107.5419	115.6352
t_{12}	200.4461	214.8074	233.3333	257.5415	289.9090	334.7300	400.1086
$\rho_{yx} \rightarrow$	0.6						
t_1	115.7250	120.9104	128.3019	138.6623	153.2913	174.5380	207.0646
t_2	104.8332	109.5306	116.2264	125.6117	138.8638	158.1109	187.5761
t_3	105.5140	110.2418	116.9811	126.4274	139.7656	159.1376	188.7942
t_4	187.2022	195.5903	207.5472	224.3067	247.9711	282.3409	334.9574
t_5	140.2314	133.7127	128.3019	123.9804	120.8296	119.0965	119.3666
t_6	115.7250	113.2290	112.0000	112.2349	114.3372	119.0965	128.1364
t_7	104.8332	101.8492	100.0245	100.1843	100.0098	102.6694	108.6480
t_8	100.0098	100.3912	100.5766	103.4127	108.4797	116.8624	130.7479
t_9	103.6608	107.5944	113.4172	121.7600	133.7040	151.2092	178.1703
t_{10}	100.0000	100.6401	102.7170	106.6069	112.9847	123.1006	139.4641

t_{11}	102.4506	100.6401	100.0000	100.7341	103.2462	108.3162	117.5396
t_{12}	220.8986	237.1977	258.4906	286.7047	325.0676	379.3634	461.0231
$\rho_{yx} \rightarrow$	0.7						
t_1	109.5690	114.8545	122.4490	133.2149	148.6620	171.6247	208.0444
t_2	102.9949	107.9632	115.1020	125.2220	139.7423	161.3272	195.5617
t_3	105.9167	111.0260	118.3673	128.7744	143.7066	165.9039	201.1096
t_4	208.1812	218.2236	232.6531	253.1083	282.4579	326.0870	395.2843
t_5	135.8656	128.6371	122.4490	117.2291	112.9832	109.8398	108.1831
t_6	109.5690	106.5850	104.8163	104.4405	105.8474	109.8398	118.1692
t_7	107.5968	103.1394	100.0735	*	*	*	100.0452
t_8	*	*	*	100.0373	103.0287	110.2929	123.2122
t_9	103.9283	108.1759	114.5125	123.6925	137.0444	157.0684	189.0122
t_{10}	100.0000	100.6891	102.9388	107.1936	114.2716	125.7437	144.9376
t_{11}	102.6297	100.6891	100.0000	100.7993	103.5679	109.2677	119.9723
t_{12}	244.3389	263.0168	287.7551	321.0480	367.1952	434.2105	538.8350
$\rho_{yx} \rightarrow$	0.8						
t_1	102.4429	107.7944	115.5556	126.7057	143.0143	167.9587	209.3398
t_2	100.2908	104.4776	112.0000	122.8070	138.6139	162.7907	202.8986
t_3	130.8117	122.7197	115.5556	109.1618	103.4103	*	*
t_4	106.3830	111.9403	120.0000	131.5789	148.5149	174.4186	217.3913
t_5	232.4665	244.6103	262.2222	287.5244	324.5325	381.1370	475.0403
t_6	102.4429	*	*	*	*	*	104.9919
t_7	110.6383	104.4776	100.0002	*	*	*	*
t_8	*	*	*	*	*	100.2791	111.0725
t_9	104.2378	108.8539	115.8025	126.0017	141.1197	164.4416	203.3459
t_{10}	100.0000	100.7463	103.2000	107.8947	115.8416	129.0698	152.1739
t_{11}	102.8369	100.7463	100.0000	100.8772	103.9604	110.4651	123.1884
t_{12}	271.4736	293.1177	322.2222	362.0858	418.5919	503.2300	641.7069

(*) indicate no gain

Table 4: PREs of the suggested estimator t_{ps} w. r. t. the estimators t_i for the second set.

9. Interpretations of results and conclusions

It is noticed from the Table 2, there is an effectual improvement in efficiency of the recommended estimators t_{ps} over the other discussed two phase sampling version of estimators t_i, for all the population data sets. Further, we observed that suggested estimators t_{ps} are effectively employed for various combinations of correlation coefficients over the estimators $t_i\ (i = 1, 2, ...12)$. Tables 3 and 4 indicate gain in efficiency of the suggested class of estimators t_{ps} with respect to other existing estimators $t_i\ (i = 1, 2, ...12)$ for different choices of ρ_{yx} and ρ_{yz}. From the above interpretations, it has been concluded that structure of the proposed class of estimators are justified and can be considered for various practical aspects (applications) by survey researchers.

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