

ON TESTING POVERTY INDICES AMONG SAMPLED CULTIVATING HOUSEHOLDS

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Abstract

Poverty gap ratio has been estimated by adopting Arora and Bagai' approach, Takayama's censored Gini Ratio and Greer and Thorbecke's approach under two different profiles. Further asymptotic tests of significance have been obtained. The sampling distribution of Gini ratio has also been provided. The study on the basis of poverty gap ratio further affirms that there is significant difference in the incidence of poverty in the two village profiles.

Key Words: Poverty Gap Ratio, Gini –Coefficient, Censored Distribution.

1. Introduction

Enhancing economy is a necessary but not a sufficient condition for improving social welfare. To improve social welfare the sufficient condition is that the benefits of economic growth are distributed in a manner that lift the economically weaker population of poverty. Measurement of poverty has, therefore, important policy implications. In India, the first rigorous analysis of the concept and measurement of poverty can be attributed to Sen (1976). Sen conceived the measure of poverty as a weighted sum of the shortfalls of incomes of the poor from the poverty line by using the rank order weighting scheme. Kakwani (1980) modified Sen's axiom of ordinal rank weights to provide a more general structure than Sen's axiom. In the literature, there are a number of studies on the measurement of poverty (Sen, 1981; Manna, 2012; Townsend, 1954; Arora, *et al.*, 1989; Lewis and Ulph, 1989; Takayama, 1979; Sharma and Sharma, 2004; Sharma, 2008 and Arora *et. al.* 1991). Consequently studies on poverty outnumber the studies on any other aspect of socio-economic development. More than three-fourths of the population in the world does not have access to minimum requirements essential for survival. These less fortunate, popularly, called poor, have drawn worldwide attention. India is home to most poor in the world. Indian's economy has grown tremendously rendering the most dynamic economy in the world. The benefits of growth, however, remain concentrated among the rich, and reached only to a small proportion of low income population. Poverty estimates are, thus, vital input to designing, monitoring and implementing anti-poverty programs and policies. It is equally important to analyze poverty profiles by regions and socio-economic groups for effective targeting of efforts and investments. Precise estimates of poverty are neither easy nor universally acceptable. In the measurement of poverty, there are two distinct problems (i) identifying the poor and (ii) constructing an index of poverty using the available information. The former involves the choice of a criterion of poverty (e.g., the selection of a "poverty line" in terms of real income per capita), and

then ascertaining those who satisfy that criterion (e.g., fall below the "poverty line") and those who do not. In the literature, significant contributions have been made in tackling this problem (see Weisbrod, 1965 Townsend, 1954, Vaidyanathan, 1971 and Atkinson 1970), but relatively little work has been done on construction of poverty indices.

Interestingly, there are various indices of poverty (Sen,1981; Julka,1986; Lewis and Ulph,1989 and Maiti and Pal,1988). Arora et. al. (1991) has given the test of significance for indices of poverty. Some of the measures are alternative to one another and some are claimed to be superior than others. Significant work has been done in developing the alternative measurements, but not much attention has been paid to problem of estimation of these indices. In the present study, an attempt has been made to test significance of some poverty indices based on sampled observations.

2. Methodology

In the present paper, an attempt has been made to estimate the depth of poverty by choosing the poverty norm as defined by Government of India at 2014-15 prices. The test of significance of poverty gap ratio is based on two independent strata of households falling in different blocks. The Mann-Whitney U-statistics is used to test the difference in location of two samples. The significant difference in the incidence of poverty is presented through poverty gap ratio based on Takayama's censored Gini ratio.

2.1 Poverty gap ratio

Let $y_1, y_2, y_3, \dots, y_n$ be the incomes of n units drawn from a population (Y). Let φ be the chosen poverty norms so that the units are designated as poor or non-poor as per the inequalities defined below :

$$\begin{aligned} y_i &< \varphi & i \in P \\ y_i &\geq \varphi & i \in NP \end{aligned}$$

where P stands for the set of poor and NP belongs to the set of non-poor.

Let p out of these n -units be poor. We may arrange them in ascending order such that

$$y_1 \leq y_2 \leq y_3 \leq \dots \leq y_p \leq y_{p+1} \leq \dots \leq y_n$$

The total poverty gap, g , for the poor may be defined as :

$$g = \sum_{i=1}^p g_i = \sum_{i=1}^p (\varphi - y_i)$$

and the average poverty gap is

$$\bar{g} = g/p \tag{1}$$

The poverty gap ratio is defined as

$$PG = \frac{1}{n} \sum_{i=1}^p \left(\frac{\varphi - y_i}{\varphi} \right) \tag{2}$$

and the income gap ratio is

$$IG = \bar{g} / \mu \tag{3}$$

Where, μ is the mean income of the whole population

Squared poverty gap (SPG) is defined as follows:

$$SPG = \frac{1}{n} \sum_{i=1}^p \left(\frac{\varphi - y_i}{\varphi} \right)^2 \tag{4}$$

Where, p is the number of poor people, n is the total population.

To test the significance of poverty gap ratio, let n_1 be the sample of observations from an income distribution with IG_1 be the value of poverty gap ratio. Further, let IG_2 be the value of poverty gap ratio based on another random sample of size n_2 drawn from another income distribution. Let us assume that both samples are drawn independently of each other.

Let

p_1 = number of poor in the first sample

p_2 = number of poor in the second sample

For test of significance, we set up a null hypothesis of non-significance as;

$$H_0 : I_1 = I_2 \tag{5}$$

Here, I_1 and I_2 are population parametric value of poverty gap ratio from which two random samples of size n_1 and n_2 are drawn.

The test statistic is

$$\frac{1}{p_1} \sum_{i=1}^{p_1} \left(\frac{\varphi - y_i}{\varphi} \right) = \frac{1}{p_2} \sum_{i=1}^{p_2} \left(\frac{\varphi - y_i}{\varphi} \right) \tag{6}$$

$$\varphi - \sum_{i=1}^{p_1} \frac{y_i}{p_1} = \varphi - \sum_{i=1}^{p_2} \frac{y_i}{p_2} \tag{7}$$

Since population is unknown for which the inference has to be drawn, but p_1 and p_2 and the samples are independent of each other. It is feasible to apply the non-parametric “Mann-Whitney U-statistic” test under the assumption of non-normality of parent population.

The test statistic is given by

$$U = p_1 p_2 + \frac{p_1(p_1 + 1)}{2} - R_1$$

Where, R_1 denotes the ranks of observations from samples of size p_1 , when ranking is done for sample of size $(p_1 + p_2) = p$ (say).

When sample is large, i.e. if p is large, the appropriate test-statistic is given by (under H_0)

$$Z = \frac{U - E(U)}{\sqrt{Var(U)}} \approx N(0, 1)$$

$$\text{Where, } E(u) = \frac{p_1 p_2}{2} \quad \text{and} \quad \text{Var}(U) = \frac{p_1 p_2 (p + 1)}{12} \quad (8)$$

We reject H_0 in favor of one-sided or two sided alternative at $\alpha\%$ or $\alpha/2\%$ level of significance.

2.2 Takayama’s censored Gini ratio

We use the Hamda and Takayama (1978) index, which is the translation of usual Gini ratio of inequality to the censored income distribution known as Takayama’s poverty index. The censored income distribution is obtained from the actual distribution by replacing all the incomes above poverty line by the incomes exactly equal to the poverty line, i.e.

$$y_i = \varphi \quad \forall i > p \quad \text{where } i=1,2,\dots,n.$$

$$\hat{T} = \frac{\frac{1}{n(n-1)} \sum_i^n \sum_j^n |y_i^* - y_j^*|}{2 \sum_{i=1}^n y_i^* / n} \quad (9)$$

Where

$$y_i^* = \begin{cases} y_i & \text{if } y_i \leq \varphi \\ \varphi & \text{otherwise} \end{cases}$$

Considering y_1^*, \dots, y_n^* as the random sample of size n from the censored income distribution, the sample estimate of Takayama’s censored Gini ratio is obtained like estimate of Gini index of inequality.

The sampling distribution of \hat{T} being similar to that of \hat{G} can be obtained using the following procedure.

Let the incomes in the censored population be ordered in ascending order as follows:

$$y^*_1 \leq y^*_2 \leq y^*_3 \leq \dots \leq y^*_n$$

Then, \hat{T} can be expressed as

$$\hat{T} = \frac{\hat{\Delta}}{2\bar{x}} \quad (10)$$

Where $\hat{\Delta}$ = Mean difference in the sample with observations $y^*_1, y^*_2, \dots, y^*_n$.

and \bar{x} = sample mean for $y^*_i, i= 1,2,\dots,n$.

The censored Gini ratio in the population is

$$T = \frac{\Delta}{2\mu}$$

Where Δ is the population mean difference and μ is the population mean.

Now, considering $y^*_1, y^*_2, \dots, y^*_n$ as a random sample of size n , the sampling distribution of T can be obtained in the same way as that of G (Ramakrishnan, 1984 ; Arora et. al, 1989).

σ^2_t is estimated by the derivation as given in Arora and Bagai (1991)

$$\hat{\sigma}_t^2 = \frac{1}{(y^*)^2} [\text{var } \hat{\Delta}_i - 2\hat{T}\text{Cov}(T_i, y_i^*) + \hat{T}^2 \text{Var}(y_i^*)] \tag{11}$$

Where,

$$\text{Var}(\hat{\Delta}_i) = \frac{1}{n} \sum_{i=1}^n \hat{\Delta}_i^2 - (\hat{\Delta})^2$$

$$\hat{\Delta}_i = [\frac{(2i - n - 1)y_i^* + t_i}{(n - 1)}]$$

and

$$t_i = \sum_{j>i}^n y_j^* - \sum_{j<i}^n y_j^* ; \quad \hat{\Delta} = \frac{1}{n} \sum_{i=1}^n \hat{\Delta}_i$$

$$\text{Cov}(\hat{\Delta}_i, y_i^*) = \frac{1}{n} \sum_{i=1}^n \hat{\Delta}_i y_i^* - \bar{y}^* ; \quad \bar{y}^* = \frac{1}{n} \sum_{i=1}^n y_i^*$$

$$\text{Var}(y_i^*) = \frac{1}{n} \sum_{i=1}^n (y_i^* - \bar{y}^*)^2$$

We can test significance of Takayama’s censored Gini ratio following the procedure as outlined by Julka 1986 and Arora *et al.*, 1991.

Let \hat{T}_1 and \hat{T}_2 be two estimates of modified Takayama’s censored Gini ratio based on two independent samples of sizes n_1 and n_2 respectively.

Let T_1 and T_2 be two population values of estimated \hat{T}_1 and \hat{T}_2 respectively and

$$\frac{\hat{\sigma}_{t_1}^2}{n_1} = \text{asymptotic variance of } \hat{T}_1 .$$

$$\frac{\hat{\sigma}_{t_2}^2}{n_2} = \text{asymptotic variance of } \hat{T}_2 .$$

Then under null hypothesis $H_0 : T_1 = T_2$, the appropriate test statistic is given by

$$Z = \frac{\hat{T}_1 - \hat{T}_2}{\left(\frac{\hat{\sigma}_{t1}^2}{n_1} + \frac{\hat{\sigma}_{t2}^2}{n_2}\right)^{1/2}} \approx N(0,1) \text{ for large } n,$$

Since n_1 and n_2 are sufficiently large sample size, the test statistic Z can be assumed to follow asymptotically normally distributed.

3. Results and discussions

We estimate poverty gap ratio and Takayama’s censored Gini ratio using survey data on farm households from Meerut district of Uttar Pradesh. The data pertain to 2014. A multi stage stratified random sampling methodology was adapted to two villages, one each from Rohta and sardhana blocks out of twelve blocks in Meerut district. From the selected villages, list of households was enumerated and then 72 households from Rohta and 68 households from Sardhana were selected for surveys. Primry data were collected by survey method from the sample households in pretested schedules and questionnaires. Farm income profiles of sampled households were obtained using the raw data flowing from the two selected villages . These income profiles were assumed to be statistically independent. The poverty norm used in this study is taken Rs 942/ per capita /month from Government of India (2014) .

To examine the null hypothesis of no difference in the indices of poverty measured through poverty gap ratio and Takayama’s censored Gini ratio .

The estimated poverty gap ratio, U –statistic and Z-statistic are exhibited in Table 1.

	Poverty gap ratio (I ₁)	Number of poor (p)	U-statistic	Z-statistic
Village -1	0.8274	72	2016	-3.8969*
Village-2	0.6479	68		

Table 1: Poverty gap ratio in two villages

*Significant at 5% level of significance

The results indicate that there is a significant difference in the incidence of poverty between the two blocks . The incidence of poverty is significantly higher in Rohta than in Sardhana.

	\hat{T}	Asy. Var(\hat{T})	Z-statistic
Village -1	0.7986	0.0247	4.8977*
Village-2	0.3216	0.0026	

Table 2: Poverty gap ratio based on Takayama's censored Gini Ratio

*Significant at 5%

	\hat{T}	$\hat{Asy. Var}(\hat{T})$	Z-statistic
Village -1	0.7756	0.01986	3.9827*
Village-2	0.3953	0.0031	

Table 3: Poverty Gap ratio based on Greer and Thorbecke' Gini Ratio

The results based on Takayama's ratio and Greer and Thorbecke's ratio also confirm the findings of significance difference in the incidence of poverty between the two blocks. Both the indices confirm that the village in Rohta block has a significantly higher incidence of poverty than the village in Sardhana block which is evident from z-statistic.

Conclusions

It is concluded that there is a wide spread of inequalities in incomes by operating households of two villages in Meerut district. The study on the basis of poverty gap ratio further affirms that there is significant difference in the incidence of poverty in the two village profiles. The findings conclude that inequality is higher in villages which need further extensive research by focusing study on various clusters of villages. The measurement of inequality and poverty should extensively be devoted more attention to the purpose of measurement which may take us further towards a sound theoretical interpretation for poverty gap ratio. It may be used to examine the significant difference in the incidence of poverty.

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