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COST EFFECTIVENESS ANALYSIS OF A SYSTEM COMPOSED OF TWO SUB- SYSTEMS

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Abstract

A complex system consisting of two sub-systems connecting in parallel mode has been considered in this paper. The first sub-system has two identical units with one as operative and other in cold standby mode while second sub-system has only one unit which is in operative mode. The failed units of sub-systems are subjected to repair. There is a single repair facility available with the systems which repair all the failed units with full satisfaction. There is also a inspection policy for units of first sub-system to check whether the repair is perfect or not. In case the repair is not perfect it requires post repair. The system will break down if both the sub-systems failed. On failure of either of any sub system, system will work partially but with low efficiency. Therefore, system has three modes namely operating mode, partially operating mode and failure mode. The priority in repair is given to first sub-system if both the units of first sub-system are failed while priority is given to second sub system if any of the unit of first sub-system is operative. The Failure and repair time distributions are taken to be Rayleigh distribution but with different rates. Expressions for various measures of reliability have been obtained which help in studying of effectiveness of the system such as transition probabilities, mean time to system failure, availability, busy period of repairman and expected profit incurred.

Key Words: Inspection, Sub-System, Repair, Standby, Rayleigh Distribution.

1.1 Introduction

Reliability modelling is the process of predicting or understanding the reliability of a component or system prior to its implementation. Reliability design begins with the development of a model. Reliability and availability models use block diagram to provide a graphical means of evaluating the relationships between different parts of the system. These models may incorporate predictions based on failure rates taken from historical data. Also the predictions are often not accurate in an absolute sense; they are valuable to assess relative differences in design alternatives. The most important fundamental initiating causes and failure mechanism are to be identified and analyzed with engineering tools. Reliability is vital for proper utilization and maintenance of any system. To improve the reliability of the system the technique of standby is widely used to increase systems effectiveness by reducing failure frequency of the system. The study of complex systems such as sub-system models has always attracted the researchers to design and devise the complex systems and study their functional behaviour to achieve the high reliability. Goel, Agnihotri and Gupta [5] put forth concept of a single- server two unit warm standby system with n failure modes, fault detection and inspection and Gupta and Bansal [6] worked on the analysis of a complex system composed of two sub-system with their standbys and Shrivastava and Goel [4] gave the concept of comparison of the reliability characteristics of two(double component) systems with bivariate exponential life times But some of the authors also used the concept of inspection like Agnihotri and Satsangi [1] has studied two non-identical unit system with priority based repair and inspection however, Gupta and Kumar [8] put forth the concept of analysis of two unit series subsystem with standby system model. Further, Gupta, Mahi and Sharma [7] discussed two component two unit standby system with correlated failure and repair times and Chib, Joorel and Sharma [3] has worked on MTSF and profit analysis of a two unit warm standby system with inspection. Later on Bashir, Joorel and Kour [2] worked on Cost benefit analysis of a two unit standby model with repair, inspection, post repair and random appearance and disappearance of repairman in the system and Taha and Taha [9] investigated on reliability estimation for the Rayleigh distribution based on Monte Carlo Simulation.

Rayleigh distribution is widely used in reliability of industrial equipments, clinical trials related to life and also in life testing. That is Rayleigh distribution commonly used in the reliability modelling whereas, the components have higher failure rate. Since in Rayleigh distribution failure hazard function is an increasing function of time i.e. as system continues to work its tendency to follow Rayleigh distribution increases, in ageing equipment the failure rate also increase i.e. failure rate becomes function of time and in such cases, Rayleigh distribution is to be used as it is an increasing function of time. However, most of the authors have considered either exponential distribution or general distribution. Therefore, keep in view the applicability of Rayleigh distribution in reliability modelling and life testing a complex system consisting of two sub-systems connecting in parallel mode with Raleigh as the failure and repair time distribution has been considered in this paper. The first sub-system has two identical units with one as operative and other in cold standby mode while second sub-system has only one unit which is in operative mode. The failed units of sub-systems are subjected to repair. There is also a inspection policy for units of first sub-system to check whether the repair is perfect or not. In case the repair is not perfect it requires post repair. The system will break down if both the sub-systems failed. On failure of either of any sub system, system will work partially but with low efficiency. Therefore, system has three modes namely operating mode, partially operating mode and failure mode. The priority in repair is given to first sub-system if both the units of first sub-system are failed while priority is given to second sub system if any of the unit of first sub-system is operative. There is a single repair facility available with the systems which repair all the failed units with full satisfaction. Expressions for various measures of reliability have been obtained which help in studying of effectiveness of the system such as transition probabilities, mean time to system failure, availability, busy period of repairman and expected profit incurred.

1.2 Notations

α	failure rate of units of sub-system A
β	failure rate of unit of sub-system B
α_1, λ	repair and post repair rate of units of sub-system A
μ	inspection rate
q	probability that the unit requires post repair after repair

p	probability that repair is perfect such that $p + q = 1$
θ	repair rate of unit of sub-system B
$\pi_i(\cdot)$	c.d.f of time to system failure when starting from state S_0
$A_i(t)$	P [system is up at epoch t]
$B_i(t)$	P [Repairman is busy in repair at an epoch t]
$V_i(t)$	expected number of visits by repairman in (0,t]
μ_i	mean sojourn time in state S_i
ⓢ	symbol for Laplace-stieltjes convolution
©	symbol for Laplace convolution
$M_i(t)$	probability that the system starting in upstate $S_i \in E$ is up at time't' with-out passing through any regenerative state or returning to itself
$q_{ij}(\cdot), Q_{ij}(\cdot)$	pdf and cdf of transition time from state S_i to S_j
p_{ij}	transition probabilities in steady state i.e. $\lim_{t\to\infty} Q_{ij}(i)$

Rayleigh Distribution

$$\text{pdf} = \frac{u}{\alpha^2} e^{-\frac{u^2}{2\alpha^2}}, \text{cdf} = 1 - e^{-\frac{u^2}{2\alpha^2}}$$

To explain the transition diagram, following are the symbols used:

A_0/B_0	units of sub-system A/ B are operative
A_s	unit of sub-system A is in standby mode
A_r/B_r	units of sub-system A/B are under repair
A_I/A_{PR}	units of sub-system A are under inspection and post repair
A_{wr}/B_{wr}	units of sub-system A/B are waiting for repair.
A_{wI}/A_{wPR}	units of sub-system A are waiting for inspection/ post repair

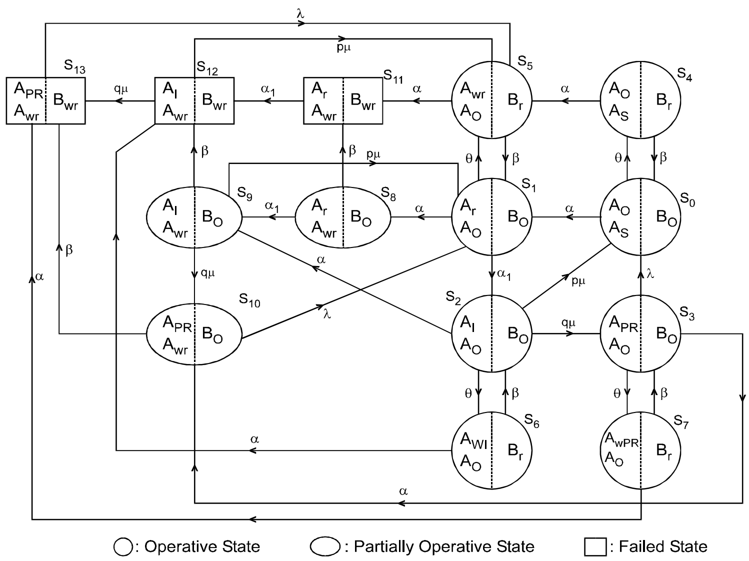

Fig. 1: Various transitions in the system are described in the figure

1.3 Transition Probabilities and Mean Sojourn Time

Transition probability is the probability of the occurrence of a transition from one state to another state during a time interval and various transition probabilities for the proposed model can be obtained as follows:

$$p_{01} = p_{40} = p_{51} = p_{62} = p_{73} = \frac{\beta^2}{\alpha^2+\beta^2}$$

$$p_{04} = p_{45} = p_{511} = p_{612} = p_{713} = \frac{\alpha^2}{\alpha^2+\beta^2}$$

$$p_{12} = \frac{\alpha^2\beta^2}{(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)}$$

$$p_{15} = \frac{\alpha^2{\alpha_1}^2}{(\alpha^2\beta^2+\alpha_1^2\beta_2+\alpha_1^2\alpha^2)}$$

$$p_{18} = \frac{{\alpha_1}^2\beta^2}{(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)}$$

$$p_{20} = \frac{p\beta^2\alpha^2}{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}$$

$$p_{23} = \frac{q\beta^2\alpha^2}{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}$$

$$p_{26} = \frac{\mu^2\alpha^2}{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}$$

$$p_{29} = \frac{\beta^2\mu^2}{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}$$

$$p_{30} = \frac{\alpha^2\beta^2}{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}$$

$$p_{37} = \frac{\alpha^2\lambda^2}{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}$$

$$p_{310} = \frac{\lambda^2\beta^2}{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}$$

$$p_{89} = \frac{{\alpha_1}^2}{\beta^2+{\alpha_1}^2}$$

$$p_{811} = \frac{\beta^2}{\beta^2+{\alpha_1}^2}$$

$$p_{91} = \frac{p\mu^2}{(\mu^2+\beta^2)}$$

$$p_{910} = \frac{q\mu^2}{(\mu^2+\beta^2)}$$

$$p_{912} = \frac{\beta^2}{(\mu^2+\beta^2)}$$

$$p_{101} = \frac{\beta^2}{\beta^2+\lambda^2}$$

$$p_{1013} = \frac{\lambda^2}{\beta^2+\lambda^2}$$

$$p_{1112} = p_{135} = 1$$

$$p_{125} = p$$

$$p_{1213} = q \qquad (1)$$

From the above steady state probabilities, it can be verified that

$$\sum_j p_{ij} = 1 \qquad (2)$$

Mean Sojourn Time

Mean sojourn time is defined as the time of stay in a particular state before transiting to any other state. Expressions for mean sojourn time can be obtained as follow:

$$\mu_0 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \frac{\alpha\beta}{\sqrt{(\alpha^2+\beta^2)}}\sqrt{\frac{\pi}{2}}$$

$$\mu_1 = \frac{\alpha_1\alpha\beta}{\sqrt{(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)}}\sqrt{\frac{\pi}{2}}$$

$$\mu_2 = \frac{\alpha\beta\mu}{\sqrt{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}}\sqrt{\frac{\pi}{2}}$$

$$\mu_3 = \frac{\alpha\beta\lambda}{\sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}}\sqrt{\frac{\pi}{2}}$$

$$\mu_8 = \frac{\alpha_1\beta}{\sqrt{\beta^2+{\alpha_1}^2}}\sqrt{\frac{\pi}{2}}$$

$$\mu_9 = \frac{\beta\mu}{\sqrt{(\mu^2+\beta^2)}}\sqrt{\frac{\pi}{2}}$$

$$\mu_{10} = \frac{\beta\lambda}{\sqrt{\beta^2+\lambda^2}}\sqrt{\frac{\pi}{2}}$$

$$\mu_{11} = \alpha_1\sqrt{\frac{\pi}{2}}$$

$$\mu_{12} = \mu\sqrt{\frac{\pi}{2}}$$

$$\mu_{13} = \lambda\sqrt{\frac{\pi}{2}} \qquad (3)$$

1.4 Mean Time to System Failure

The time taken by the system to reach in the failed state for the first time is known as time to system failure and its expected value is known as the mean time to system failure. By using simple probabilistic concepts, the following recurrence relations among $\pi_i\,(t)'s$ can be obtained:

$$\pi_0(t) = Q_{01}(t) \circledast \pi_1(t) + Q_{04}(t) \circledast \pi_4(t)$$
$$\pi_1(t) = Q_{12}(t) \circledast \pi_2(t) + Q_{15}(t) \circledast \pi_5(t) + Q_{18}(t)$$
$$\pi_2(t) = Q_{20}(t) \circledast \pi_0(t) + Q_{23}(t) \circledast \pi_3(t) + Q_{26}(t) \circledast \pi_6(t) + Q_{29}(t)$$
$$\pi_3(t) = Q_{30}(t) \circledast \pi_0(t) + Q_{37}(t) \circledast \pi_7(t) + Q_{310}(t)$$
$$\pi_4(t) = Q_{40}(t) \circledast \pi_0(t) + Q_{45}(t) \circledast \pi_5(t)$$
$$\pi_5(t) = Q_{51}(t) \circledast \pi_1(t) + Q_{511}(t)$$
$$\pi_6(t) = Q_{62}(t) \circledast \pi_2(t) + Q_{612}(t)$$
$$\pi_7(t) = Q_{73}(t) \circledast \pi_3(t) + Q_{713}(t)$$

Taking the Laplace-Stieltjes transformation of above equations and solving, the mean time to system failure is obtained as:

$$MTSF = \frac{\begin{gathered}[\theta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\alpha^2(\lambda^2\beta^2+\alpha^2\beta^2)][\theta^2(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)+\alpha^2(\beta^2\mu^2+\alpha^2\beta^2)][\theta^2(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2+ \\ \alpha^2(\alpha^2\beta^2+\alpha_1^2\beta^2)][\alpha\beta\sqrt{(\alpha^2+\beta^2)\frac{\pi}{2}}+\alpha^3\theta\sqrt{\frac{\pi}{2}}(\alpha^2+\beta^2)(\theta^2+\alpha^2)+\alpha^3\theta^3\sqrt{(\alpha^2+\beta^2)\frac{\pi}{2}}][\sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)} \\ \sqrt{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}\ \sqrt{(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)}]+\alpha^4\theta^2+\beta^2(\theta^2+\alpha^2)^2\{\theta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\alpha^2(\lambda^2\beta^2+\alpha^2\beta^2) \\ [\alpha_1\alpha\beta\sqrt{\frac{\pi}{2}}(\theta^2(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)+\alpha^2(\mu^2\beta^2+\alpha^2\beta^2)(\sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}\sqrt{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}\ (\alpha^2\beta^2+ \\ \beta^2\alpha_1^2+\alpha^2\alpha_1^2))(\theta^2+\alpha^2)^{\frac{3}{2}}(\beta^2+\alpha^2)^{\frac{3}{2}}+\alpha^3\beta^2\mu\sqrt{\frac{\pi}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)\ \sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}\sqrt{(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)} \\ (\theta^2+\alpha^2)^{\frac{3}{2}}(\beta^2+\alpha^2)^{\frac{3}{2}}+\alpha^5\beta^2\theta\mu^2[\sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}\sqrt{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}\sqrt{(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)}(\beta^2+\alpha^2)^{\frac{3}{2}} \\ +\alpha^4\beta^4 q(\alpha\beta\lambda(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)(\theta^2+\alpha^2)\sqrt{\frac{\pi}{2}}\sqrt{(\beta^2\mu^2+\mu^2\alpha^2}\sqrt{\alpha^2\beta^2)}\sqrt{(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)}(\theta^2+\alpha^2)^4(\beta^2+\alpha^2)^{\frac{3}{2}} \\ +\alpha^3\theta\lambda\sqrt{\frac{\pi}{2}}(\theta^2+\alpha^2)^2\Big[\sqrt{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)}\sqrt{(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)}\sqrt{(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)}\Big](\beta^2+\alpha^2)^{\frac{3}{2}}\end{gathered}}{\begin{gathered}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{3}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{3}{2}}(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)^{\frac{3}{2}}(\theta^2+\alpha^2)^{\frac{1}{2}}(\beta^2+\alpha^2)^{\frac{1}{2}}[\{(\alpha^2\theta^2+ \\ \alpha^2\beta^2+\theta^2\beta^2)\big(\theta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\alpha^2(\lambda^2\beta^2+\alpha^2\beta^2)\big)\big(\theta^2(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)\big)+\alpha^2(\beta^2\mu^2+ \\ \alpha^2\beta^2(\theta^2(\alpha^2\beta^2+\beta^2\alpha_1^2+\alpha^2\alpha_1^2)+(\beta^2{\alpha_1}^2+\alpha^2\beta^2))\}-\{(\alpha^4\theta^2+\beta^2(\alpha^2+\theta^2)^2)(q\alpha^6\beta^6+p\alpha^4\beta^4(\theta(\lambda^2 \\ \beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\alpha^2(\lambda^2\beta^2+\alpha^2\beta^2))\}]\end{gathered}} \tag{4}$$

1.5 Availability Analysis

Availability is the probability that the system is in its uptime and available to use when required. It is not undergoing any kind of failure or repair. Recurrence relations to obtain the expression for availability are as follows:

$$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{04}(t)©A_4(t)$$
$$A_1(t) = M_1(t) + q_{12}(t)©A_2(t) + q_{15}(t)©A_5(t) + q_{18}(t)©A_8(t)$$
$$A_2(t) = M_2(t) + q_{20}(t)©A_0(t) + q_{23}(t)©A_3(t) + q_{26}(t)©A_6(t) + q_{29}(t)©A_9(t)$$
$$A_3(t) = M_3(t) + q_{30}(t)©A_0(t) + q_{37}(t)©A_7(t) + q_{310}(t)©A_{10}(t)$$
$$A_4(t) = M_4(t) + q_{40}(t)©A_0(t) + q_{45}(t)©A_5(t)$$
$$A_5(t) = M_5(t) + q_{51}(t)©A_1(t) + q_{511}(t)©A_{11}(t)$$
$$A_6(t) = M_6(t) + q_{62}(t)©A_2(t) + q_{612}(t)©A_{12}(t)$$
$$A_7(t) = M_7(t) + q_{73}(t)©A_3(t) + q_{713}(t)©A_{13}(t)$$

$A_8(t) = M_8(t) + q_{89}(t)©A_9(t) + q_{811}(t)©A_{11}(t)$
$A_9(t) = M_9(t) + q_{40}(t)©A_0(t) + q_{45}(t)©A_5(t)$
$A_{10}(t) = M_{10}(t) + q_{101}(t)©A_1(t) + q_{1013}(t)©A_{13}(t)$
$A_{11}(t) = q_{1112}(t)©A_{12}(t)$
$A_{12}(t) = q_{125}(t)©A_5(t) + q_{1213}(t)©A_{13}(t)$
$A_{13}(t) = q_{135}(t)©A_5(t)$

Taking the Laplace transformation in above equations and on solving, the solution for availability is given as:

$$A_0 = \frac{N_1}{D_1} \quad (5)$$

where,

$$N_1 = \alpha^4\beta^2[\{(\theta^2+\lambda^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\alpha^4\mu^2\}\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{((\theta^2+\alpha^2)\beta^2\mu^2+\mu^2\alpha^4)\}(\beta^2+\alpha_1^2)(\mu^2+\beta^2)^{\frac{3}{2}}(\beta^2+\alpha^2)^{\frac{1}{2}}q\beta^2\alpha^2-\{(\theta^2+\alpha^2)\lambda^2\beta^2-\lambda^2\alpha^2\theta^2\}\{\theta\alpha^3\beta^2\sqrt{\frac{\pi}{2}}+\alpha^3\beta^3\sqrt{\frac{\pi}{2}}\}]+\alpha^2\{(\theta^2+\alpha^2)(\beta^2+\alpha^2)-\alpha^4\}\{(\theta^2+\alpha^2)(\lambda^2\beta^2\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^2\}[\alpha_1\alpha\beta\sqrt{\frac{\pi}{2}}(\beta^2+\alpha_1^2)(\theta^2+\alpha^2)^{\frac{1}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}+\alpha_1^3\beta^2\theta\sqrt{\frac{\pi}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}(\beta^2+\alpha_1^2)+\alpha_1^2\beta^5\sqrt{\frac{\pi}{2}}\mu+\lambda\beta^7 q\alpha_1^2\sqrt{\frac{\pi}{2}}(\theta^2+\alpha^2)^{\frac{1}{2}}](\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{3}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{3}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{3}{2}}+\alpha^4\beta^2\{(\theta^2+\alpha^2)^{\frac{3}{2}}(\lambda^2\beta^2++\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{\alpha\beta\sqrt{\frac{\pi}{2}}(\theta^2+\alpha^2)^{\frac{3}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}(\lambda^2+\beta^2)^{\frac{1}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{3}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{3}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{3}{2}}+\alpha\theta\mu\sqrt{\frac{\pi}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}(\lambda^2+\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{3}{2}}+\alpha\theta\mu\sqrt{\frac{\pi}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}(\lambda^2+\beta^2)^{\frac{1}{2}}+\beta^4\mu^3\lambda(\mu^2+\beta^2)(\theta^2+\lambda^2)^{\frac{1}{2}}+\beta^5\mu^2\lambda\sqrt{\frac{\pi}{2}}q(\theta^2+\lambda^2)^{\frac{1}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}+\alpha^7\beta^5 q\lambda\sqrt{\frac{\pi}{2}}(\mu^2+\beta^2)^{\frac{3}{2}}(\theta^2+\lambda^2)^{\frac{1}{2}}(\lambda^2+\beta^2)^{\frac{1}{2}}+\alpha^3\beta^2\theta\sqrt{\frac{\pi}{2}}(\lambda^2+\beta^2)^{\frac{1}{2}}(\theta^2+\lambda^2)(\mu^2+\beta^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)(\alpha^2\beta^2+\alpha_1^2\beta^2\alpha_1^2\alpha^2)+\sqrt{\frac{\pi}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)\lambda^3\beta^3+\alpha\theta\sqrt{\frac{\pi}{2}}(\theta^2+\lambda^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}[\mu^2\alpha^2\theta^2(\mu^2+\beta^2)(\lambda^2+\beta^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\mu^4\beta^2(\theta^2+\alpha^2)(\lambda^2+\beta^2)+\beta\mu^2\beta^2\lambda^2(\theta^2+\alpha^2)+q\beta^2\alpha^2(\alpha^2\lambda^2(\lambda^2+\beta^2)+\lambda^4\beta^2(\theta^2+\alpha^2))\}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\mu^2+\beta^2)^{\frac{1}{2}}(\theta^2+\alpha^2)(\beta^2+\alpha^2)^{1/2}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{3}{2}} \quad (6)$$

And

$$D_1 = \alpha^5\beta^3\sqrt{\frac{\pi}{2}}(\alpha^2+\beta^2)(\theta^2+\alpha^2)^{\frac{3}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\alpha^2(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^2))+\alpha^3\alpha_1\beta$$
$$\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)-\alpha^4)\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+$$
$$\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}\alpha^2(\theta^2+\alpha^2)^{\frac{1}{2}}(\alpha^2+\beta^2)^{\frac{1}{2}}+\alpha^3\beta\mu\sqrt{\frac{\pi}{2}}((\theta^2$$
$$\alpha^2)\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\alpha^4\beta^2(\theta^2+\alpha^2)^{\frac{1}{2}}(\alpha^2+\beta^2)^{\frac{1}{2}}$$
$$(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)(\mu^2+\beta^2)$$
$$(\beta^2+\alpha_1^2)+\alpha^7\beta^3\lambda\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)-\alpha^4)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)(\theta^2+\alpha^2)^{\frac{1}{2}}(\alpha^2+\beta^2)^{\frac{1}{2}}(\mu^2+\beta^2)$$
$$(\beta^2+\alpha_1^2)+\alpha^5\theta\sqrt{\frac{\pi}{2}}\beta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\theta^2+\alpha^2)$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\mu^2+\beta^2)(\beta^2+\alpha_1^2)(\beta^2+\alpha^2)^{\frac{3}{2}}(\beta^2\alpha^2+\{(\theta^2+\alpha^2)$$
$$(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}+q\alpha^4\beta^4)\}+\alpha^3\theta\sqrt{\frac{\Pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)$$
$$-\alpha^4)\mu^2\alpha^4\beta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{3}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\theta^2+{\alpha_1}^2)^{\frac{1}{2}}$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\mu^2+\beta^2)(\alpha^2+\beta^2)^{\frac{1}{2}}+\alpha^3\theta\sqrt{\frac{\pi}{2}}[((\theta^2+\alpha^2)(\alpha^2+\beta^2)$$
$$-\alpha^4)\alpha^8\beta^4q\lambda^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\mu^2+\beta^2)$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\beta^2+\alpha^2)^{\frac{3}{2}}(\beta^2+\alpha_1^2)(\theta^2+{\alpha_1}^2)^{\frac{1}{2}}+\alpha_1^3\beta^2\theta\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)$$
$$(\alpha^2+\beta^2)-\alpha^4)\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\mu^2\beta^2$$
$$\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}\alpha^2+\beta\mu\alpha^3\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)-\alpha^4)\{(\theta^2+\alpha^2)$$
$$\{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}$$
$$\alpha_1^2\beta^4+\alpha^2\beta^4\mu^2](\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\mu^2+\beta^2)$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\theta^2+{\alpha_1}^2)(\beta^2+\alpha_1^2)+\beta\lambda\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)$$
$$-\alpha^4)\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+$$
$$\alpha^2\beta^2))-\mu^2\alpha^4\}\alpha_1^2\beta^6\alpha^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\lambda^2+\beta^2)$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\alpha^2+\beta^2)^{\frac{1}{2}}(\theta^2+{\alpha_1}^2)^{\frac{1}{2}}-\alpha^2\beta^2(q\beta^4\lambda^2\alpha^2+\beta^4\mu^2q$$
$$(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}(\theta^2+\alpha^2)$$
$$(\mu^2+\beta^2)(\lambda^2+\beta^2)(\alpha^2+\beta^2)^{\frac{3}{2}}(\theta^2+{\alpha_1}^2)^{\frac{1}{2}}+\alpha_1\theta^2\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)(\alpha^2+\beta^2)$$
$$-\alpha^4)\{\alpha^6\beta^6q+\alpha^4\beta^4p(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)\}\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+$$
$$\alpha^2\lambda^2)-\alpha^4\lambda^2\}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2+\beta^2)^{\frac{3}{2}}$$
$$(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\theta^2+\alpha^2)^{\frac{1}{2}}(\mu^2+\beta^2)(\beta^2+\alpha_1^2)+\mu\theta^2\sqrt{\frac{\pi}{2}}((\theta^2+\alpha^2)$$
$$(\alpha^2+\beta^2)-\alpha^4)[\alpha^4\beta^4p\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}+\alpha^6\beta^6p]$$

$(\lambda^2\beta^2 + \alpha^2\beta^2 + \alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2 + \mu^2\alpha^2 + \alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2 + \alpha_1^2\beta^2 + \alpha_1^2\alpha^2)^{\frac{1}{2}}$
$(\theta^2 + \alpha^2)^{\frac{1}{2}}(\mu^2 + \beta^2)(\alpha^2 + \beta^2)^{\frac{3}{2}} + \theta^2\lambda q\sqrt{\frac{\pi}{2}}((\theta^2 + \alpha^2)(\alpha^2 + \beta^2) - \alpha^4)[\alpha^2$
$\beta^2 p\{(\theta^2 + \alpha^2)(\lambda^2\beta^2 + \alpha^2\beta^2 + \alpha^2\lambda^2) - \alpha^4\lambda^2\} + \alpha^6\beta^6 q](\alpha^2 + \beta^2)^{\frac{3}{2}}(\mu^2 + \beta^2)$
$(\lambda^2\beta^2 + \alpha^2\beta^2 + \alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2 + \mu^2\alpha^2 + \alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2 + \alpha_1^2\beta^2 + \alpha_1^2\alpha^2)^{\frac{1}{2}}$
$(\theta^2 + \alpha^2)^{\frac{1}{2}}(\beta^2 + \alpha_1^2)$
(7)

1.6 Busy Period Analysis

Probability that the repairman is busy in repairing the system at an instant of time and these probabilities are determined as:

$B_0(t) = q_{01}(t)©B_1(t) + q_{04}(t)©B_4(t)$
$B_1(t) = M_1(t) + q_{12}(t)©B_2(t) + q_{15}(t)©B_5(t) + q_{18}(t)©B_8(t)$
$B_2(t) = M_2(t) + q_{20}(t)©B_0(t) + q_{23}(t)©B_3(t) + q_{26}(t)©B_6(t) + q_{29}(t)©B_9(t)$
$B_3(t) = M_3(t) + q_{30}(t)©B_0(t) + q_{37}(t)©B_7(t) + q_{310}(t)©B_{10}(t)$
$B_4(t) = M_4(t) + q_{40}(t)©B_0(t) + q_{45}(t)©B_5(t)$
$B_5(t) = M_5(t) + q_{51}(t)©B_1(t) + q_{511}(t)©B_{11}(t)$
$B_6(t) = M_6(t) + q_{62}(t)©B_2(t) + q_{612}(t)©B_{12}(t)$
$B_7(t) = M_7(t) + q_{73}(t)©B_3(t) + q_{713}(t)©B_{13}(t)$
$B_8(t) = M_8(t) + q_{89}(t)©B_9(t) + q_{811}(t)©B_{11}(t)$
$B_9(t) = M_9(t) + q_{40}(t)©B_0(t) + q_{45}(t)©B_5(t)$
$B_{10}(t) = M_{10}(t) + q_{101}(t)©B_1(t) + q_{1013}(t)©B_{13}(t)$
$B_{11}(t) = M_{11}(t) + q_{1112}(t)©B_{12}(t)$
$B_{12}(t) = M_{12}(t) + q_{125}(t)©B_5(t) + q_{1213}(t)©B_{13}(t)$
$B_{13}(t) = M_{13}(t) + q_{135}(t)©B_5(t)$

Taking Laplace transformation of above equations and solving, the fraction of time for which system is under repair is given by:

$B_0 = \frac{N_2}{D_2}$ (8)

Where,

$N_2 = \alpha^5\beta^3\mu\sqrt{\frac{\pi}{2}}(\beta^2\mu^2 + \mu^2\alpha^2 + \alpha^2\beta^2)(\theta^2 + \alpha^2)^{\frac{1}{2}}(\alpha^2 + \beta^2)\{(\theta^2 + \alpha^2)(\lambda^2\beta^2 + \alpha^2\beta^2 +$
$\alpha^2\lambda^2) - \alpha^4\lambda^2\} + \alpha^7\beta^5\lambda\sqrt{\frac{\pi}{2}} + q(\alpha^2 + \beta^2)(\theta^2 + \alpha^2)^{\frac{1}{2}}(\lambda^2\beta^2 + \alpha^2\beta^2 + \alpha^2\lambda^2)^{\frac{1}{2}}$
$(\beta^2\mu^2 + \mu^2\alpha^2 + \alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2 + \alpha_1^2\beta^2 + \alpha_1^2\alpha^2)^{\frac{1}{2}} + \theta\alpha\sqrt{\frac{\pi}{2}}[\theta^2\{\alpha^4\beta^6\{(\lambda^2\beta^2 +$
$(\theta^2 + \alpha^2)\alpha^2\beta^2 + \alpha^2\lambda^2) - \alpha^4\lambda^2\}\{(\theta^2 + \alpha^2)(\beta^2\mu^2 + \mu^2\alpha^2 + \alpha^2\beta^2) - \mu^2\alpha^4\} -$
$(\alpha^2\beta^4\mu^2(\lambda^2 + \beta^2)p\beta^2 + q\beta^2\lambda^2) + \alpha^4\lambda^4 q((\theta^2 + \alpha^2)(\alpha^2 + \beta^2) + \lambda^2\alpha^2\theta^2) + \alpha^4\beta^4 q$
$(\beta^2 + \lambda^2)\alpha^2\beta^4\theta + \alpha^4\beta^2(\theta^2 + \alpha^2)] + \alpha\theta\sqrt{\frac{\pi}{2}}\beta\mu(\alpha^4\beta^4\mu^2(\mu^2 + \beta^2)^{\frac{3}{2}}\{(\lambda^2\beta^2 +$
$\alpha^2\beta^2(\theta^2 + \alpha^2) + \alpha^2\lambda^2) - +\beta\lambda\sqrt{\frac{\pi}{2}}(\alpha^4\beta^2[\{(\theta^2 + \alpha^2)(\lambda^2\beta^2 + \alpha^2\beta^2 + \alpha^2\lambda^2) -$
$\alpha^4\lambda^2\}\alpha^4\lambda^2\}\beta^4\mu^2 q + \alpha^6\beta^6\lambda^2 q(\mu^2 + \beta^2)^{\frac{3}{2}}](\beta^2 + \alpha_1^2)(\theta^2 + {\alpha_1}^2)^{\frac{1}{2}} + \alpha_1\sqrt{\pi/2}$

$$
\begin{aligned}
&(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\theta^2+\alpha_1{}^2)^{\frac{1}{2}}(\mu^2+\beta^2)^{\frac{1}{2}}(\alpha^6\beta^2\\
&[\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}]\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\\
&\mu^2\alpha^4\}-(\alpha^6\beta^2[\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{\alpha^2\beta^4\mu^2(\beta^2+q\beta^4)\}+\alpha^4\\
&\beta^4q(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\{\alpha^2\beta^2(\theta^2+\alpha^2)+\alpha^4\theta^2\}[\alpha^2\{(\theta^2+\alpha^2)\\
&(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}]\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}\alpha^4\beta^2\\
&+\theta^2\beta^4\alpha^4\{\alpha^4\lambda^2(\mu^2+\beta^2)+\lambda^4\beta^2(\theta^2+\alpha^2)\}+\mu\sqrt{\frac{\pi}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}\\
&(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\theta^2+\alpha_1{}^2)^{\frac{1}{2}}(\alpha^4\beta^2[\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\\
&\alpha^4\lambda^2\}(\beta^2+\mu^2)^{\frac{1}{2}}\theta^4\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}-(\alpha^4\beta^2\\
&\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\})(\alpha^2\beta^4\mu^2(p\beta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)\\
&(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)(\theta^2+\alpha^2)(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)))+q^2\beta^6\lambda^2\alpha^2(\alpha^2\beta^2+\\
&\alpha_1^2\beta^2+\alpha_1^2\alpha^2)(\theta^2+\alpha^2)+\alpha^4\beta^4q\theta^2(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)(\mu^2+\beta^2)+\alpha^4\beta^4q\\
&(\theta^2\alpha^2\lambda^2(\beta^2+\lambda^4)+\lambda^4\beta^2(\theta^2+\alpha^2)))+((\theta^2+\alpha^2)\beta^2\alpha^2+\alpha^4\theta^2)(\alpha^2\{(\theta^2+\alpha^2)\\
&(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}(\alpha_1^2\beta^2\\
&(\alpha_1^2(\beta^2+\alpha_1^2+\beta^2)+\alpha^4\beta^2\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\beta^2\mu^2+q\theta^2\\
&\alpha^4\beta^4(\lambda^2\alpha^2\theta^2(\beta^2+\lambda^2)+\lambda^4\beta^2(\theta^2+\alpha^2))\mu\sqrt{\frac{\pi}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\theta^2+\alpha_1{}^2)^{\frac{1}{2}}\\
&(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}(\beta^2+\mu^2)^{\frac{1}{2}}\theta^2q(\alpha^4\beta^2\{(\theta^2+\alpha^2)\\
&(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^2\lambda^2\theta^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}-\alpha^2\\
&\beta^4\mu^2(\beta^2p(\lambda^2+\beta^2)+q\beta^2\lambda^2)(\beta^2+\alpha_1^2)(\theta^2+\alpha_1^2))+\alpha^4\beta^4q(\lambda^2\beta^2(\theta^2+\alpha^2)-\alpha^2\\
&\lambda^2\theta^2)+\alpha^4\beta^4q(\alpha^2\lambda^2\theta^2+\lambda^4\beta^2)(\beta^2+\alpha_1^2)[(\theta^2+\alpha^2)\beta^2\alpha^2+\alpha^4\theta^2]\alpha^2\{(\theta^2+\alpha^2)\\
&(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)+\alpha^2\lambda^2\theta^2\}\\
&\{\alpha_1^2\beta^2(\alpha_1^2q(\mu^2+\beta^2)+q\mu^2\beta^2)\}+\alpha^4\beta^2\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\\
&\alpha^2\lambda^2\theta^2\}\{\mu^2\alpha^2\theta^2q+\beta^2\mu^2(q\beta^2\lambda^2+(\beta^2+\lambda^2)\mu q)\}+\alpha^2\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\\
&\alpha^2\lambda^2)-\alpha^2\lambda^2\theta^2\}\{\mu^2\alpha^4\theta^2+\beta^2\mu^2(q\beta^2\lambda^2+q\mu^2(\lambda^2+\beta^2)\}\{\alpha^6\beta^4q(\alpha^2\lambda^2\beta^2(\lambda^2+\\
&\beta^2)+\lambda^4\beta^2((\theta^2+\alpha^2)+\alpha^4\beta^4q^2\theta^2(\mu^2+\beta^2)(\beta^2+\alpha_1^2)(\theta^2+\alpha^2)(\beta^2+\alpha_1^2)\}\\
&+[\alpha^4\beta^2\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^2\lambda^2\theta^2\}\{\theta^2\mu^2\alpha^2(\beta^2+\mu^2)+\beta^2\mu^2\\
&(q\beta^2\lambda^2+\mu^2)+\{(\theta^2+\alpha^2)(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^2\lambda^2\theta^2\}\{(\theta^2+\alpha^2)(\beta^2\mu^2+\\
&\mu^2\alpha^2+\alpha^2\beta^2)-\mu^2\alpha^4\}(\alpha^2\alpha_1^2(\mu^2+\beta^2)(\beta^2+\alpha_1^2)+\alpha_1^2\beta^2(\mu^2+\beta^2)(\alpha_1^2+q\beta^4\lambda^2+\\
&p\beta^2\mu^2)\}
\end{aligned}
\qquad (9)
$$

And the value of D_2 is similar as given by equation (7)

1.7 Expected Number of Visits by Repairman

As we have defined $V_i(t)$ is the expected number of visits by the repairman on (0,t] given that the system initially starts from regenerative state S_i. By probabilistic reasoning the following recurrence relations for $V_i(t)$ are obtained.

$V_0(t) = q_{01}(t)©[1 + V_1(t)] + q_{04}(t)©[1 + V_4(t)]$

$V_1(t) = q_{12}(t)©V_2(t) + q_{15}(t)©V_5(t) + q_{18}(t)©V_8(t)$

$V_2(t) = q_{20}(t)©V_0(t) + q_{23}(t)©V_3(t) + q_{26}(t)©V_6(t) + q_{29}(t)©V_9(t)$

$V_3(t) = q_{30}(t)©V_0(t) + q_{37}(t)©V_7(t) + q_{310}(t)©V_{10}(t)$

$V_4(t) = q_{40}(t)©V_0(t) + q_{45}(t)©V_5(t)$

$V_5(t) = q_{51}(t)©V_1(t) + q_{511}(t)©V_{11}(t)$

$V_6(t) = q_{62}(t)©V_2(t) + q_{612}(t)©V_{12}(t)$
$V_7(t) = q_{73}(t)©V_3(t) + q_{713}(t)©V_{13}(t)$
$V_8(t) = q_{89}(t)©V_9(t) + q_{811}(t)©V_{11}(t)$
$V_9(t) = q_{40}(t)©V_0(t) + q_{45}(t)©V_5(t)$
$V_{10}(t) = q_{101}(t)©V_1(t) + q_{1013}(t)©V_{13}(t)$
$V_{11}(t) = q_{1112}(t)©V_{12}(t)$
$V_{12}(t) = q_{125}(t)©V_5(t) + q_{1213}(t)©V_{13}(t)$
$V_{13}(t) = q_{135}(t)©V_5(t)$

Taking Laplace transformation of above equation and solving, the number of times that the repairman becomes available is given by:

$$V_0 = \frac{N_3}{D_3} \quad (10)$$

where,

$$N_3 = [(\theta^2+\alpha^2)^{\frac{3}{2}}(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)^{\frac{1}{2}}(\beta^2\mu^2+\mu^2\alpha^2+\alpha^2\beta^2)^{\frac{1}{2}}(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)^{\frac{1}{2}}\ (\alpha^2+\beta^2)(\mu^2+\beta^2)(\beta^2+\lambda^2)^{\frac{1}{2}}(\beta^2+\alpha_1^2)^{\frac{1}{2}}]\alpha^2\ [\alpha^4\beta^2)\{(\theta^2+\alpha^2)\{(\lambda^2\beta^2+\alpha^2\beta^2+\alpha^2\lambda^2)-\alpha^4\lambda^2\}-q\beta^4\alpha^2(\alpha^2\beta^2+\alpha_1^2\beta^2+\alpha_1^2\alpha^2)$$

and the value of D_3 is similar as given by equation (7)

1.8 Profit Analysis

The profit in steady state generated by proposed model may be obtained as follows:

The expected profits incurred in (0,t] = expected total revenue in (0,t] – expected total repair in (0,t] –expected cost of visit by repairman in (0,t]

Therefore, profit analysis of the system can be written as:

$$P_1 = K_0A_0 - K_1B_0 - K_2V_0 \quad (11)$$

where,

$K_0 =$ revenue per unit up time of the system,
$K_1 =$ Cost per unit time for which the repair is busy
$K_2 =$ Cost per unit visits by the repairman

The expressions for A_0, B_0 and V_0 are given by equations (5),(8) and (10) respectively.

1.9 Graphical Study of the System Model

The model proposed above in fig; 1.1 can also be analysed graphically by analysing the behaviour of characteristics like MTSF, availability and profit function. For that Firstly values are obtained for these characteristics by using C++ language and then we graphed those values in STATISTICA.

First of all we plot the graph for MTSF, Availability and Profit with respect to failure rate **α** for different values of repair rates (μ=λ=θ=5.0,7.0,9.0 &11.0) keeping all other parameters constant as β=0.54, $\alpha_1 = 0.73$, $k_0 = 1000, k_1 = 750, k_2 = 300.$

α	MTSF			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	7.2668	9.823	12.4004	14.9565
0.2	6.5951	8.9207	11.2634	13.6132
0.3	5.9091	7.9961	10.0967	12.2029
0.4	5.2026	7.0422	8.8895	10.7427
0.5	4.4663	6.0422	7.6261	9.2133
0.6	3.6874	4.9832	6.2848	7.589
0.7	2.8522	3.8459	4.8435	5.8429
0.8	1.9613	2.6322	3.30056	3.9801
0.9	1.0692	1.4197	1.7714	2.1236
1.0	0.3388	0.4358	0.5331	0.6305

Table 1: The values of MTSF with respect to failure rate α for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0) keeping all other parameters constant as β=0.54, $\alpha_1 = 0.73, k_0 = 1000, k_1 = 750, k_2 = 300$

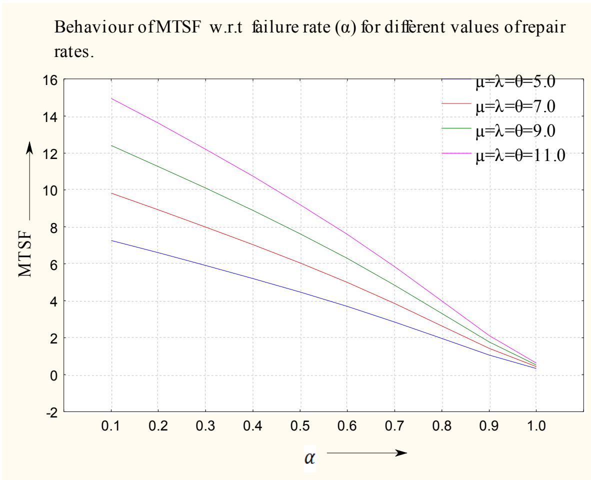

Fig. 2

α	AVAILABILITY			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	91.5395	184.932	311.466	470.726
0.2	83.4969	168289	282.602	426.461
0.3	75.7779	152.15	255.107	384.677
0.4	68.2311	136.831	229.336	354.772
0.5	61.1161	122.689	205.766	310.369
0.6	54.7337	110.226	185.173	279.598

0.7	49.522	100.309	169.033	255.715
0.8	46.5635	97.7081	160.534	240.764
0.9	46.4323	94.3884	157.432	231.621
1.0	45.3262	93.738	154.293	224.897

Table 2: The values of Availability with respect to failure rate α for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0) keeping all other parameters constant as β=0.54, $\alpha_1 = 0.73, k_0 = 1000, k_1 = 750, k_2 = 300$

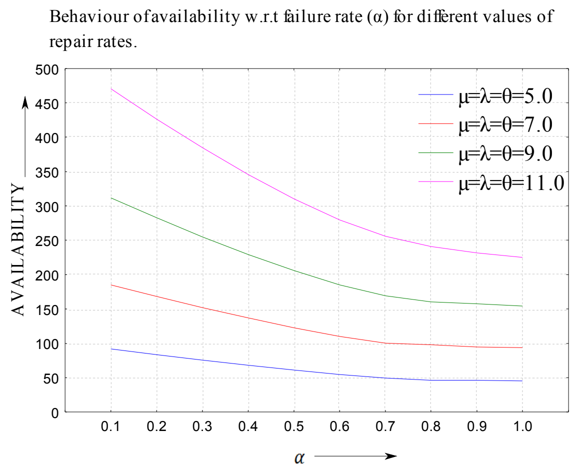

Fig. 3

α	PROFIT			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	61809.1	133118	230638	354311
0.2	55254.4	118494	205040	314883
0.3	48496.7	104109	180381	277323
0.4	41672.5	90129.1	156843	241836
0.5	34859.6	76670.7	134607	208692
0.6	28035.1	63777.8	113855	178291
0.7	20934.6	51280.8	94683.8	151173
0.8	12478.1	38101.5	76435.9	127529
0.9	-2147.83	18096.8	52810.8	102158
1.0	-51006.6	-47579.8	-39473.2	-635.895

Table 3: The values of Profit with respect to failure rate α for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0) keeping all other parameters constant as β=0.54, $\alpha_1 = 0.73, k_0 = 1000, k_1 = 750, k_2 = 300$

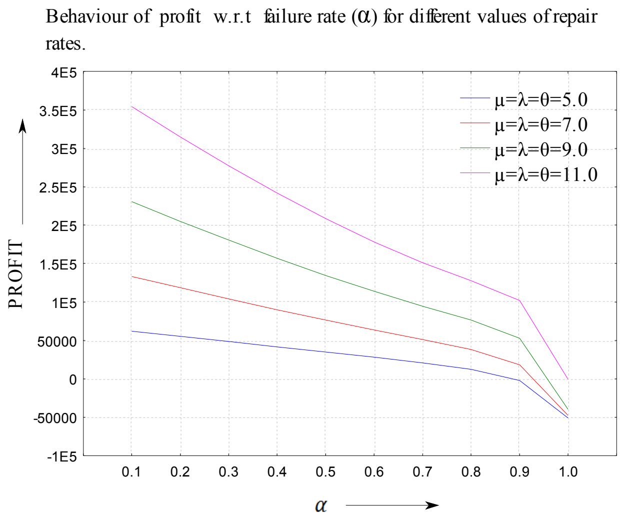

Fig. 4

We observe from Fig. 2, Fig.3 and Fig. 4 that MTSF, Availability and net Profit decrease with the increase of failure rate and as we increase the value of repair rate these characteristics also show an increase. So in order to increase the reliability of the system, minimize the failure rate and maximize the repair rate.

β	MTSF			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	3.204	3.7951	4.5303	5.4912
0.2	2.7854	3.5178	4.3976	5.4234
0.3	2.5462	3.3695	4.2803	5.274
0.4	2.4225	3.2313	4.0446	5.0099
0.5	2.295	3.0804	3.8688	4.6586
0.6	2.1985	2.9193	3.6425	4.3668
0.7	2.1402	2.81	3.4819	4.1547
0.8	2.1168	2.7489	3.3829	4.0177
0.9	2.1225	2.7281	3.3355	3.9437
1.0	2.1015	2.6893	3.32991	3.9207

Table 4: The values of MTSF with respect to failure rate β for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0)

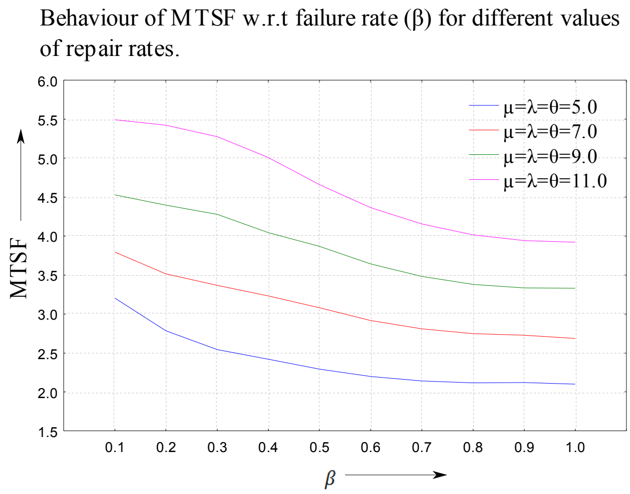

Fig. 5

β	AVAILABILITY			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	435.28	1334.65	2023.34	2770.27
0.2	202.181	427.873	732.653	1116.99
0.3	111.215	231.07	393.773	599.401
0.4	76.6134	158.625	270.27	411.591
0.5	59.8302	123.868	211.242	321.99
0.6	50.5047	104.682	178.744	272.727
0.7	44.8916	93.2158	159.386	243.438
0.8	41.3333	86.0267	147.314	225.231
0.9	38.9899	81.3769	139.578	213.628
1.0	37.3957	78.3042	134.544	206.149

Table 5: The values of Availability with respect to failure rate β for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0)

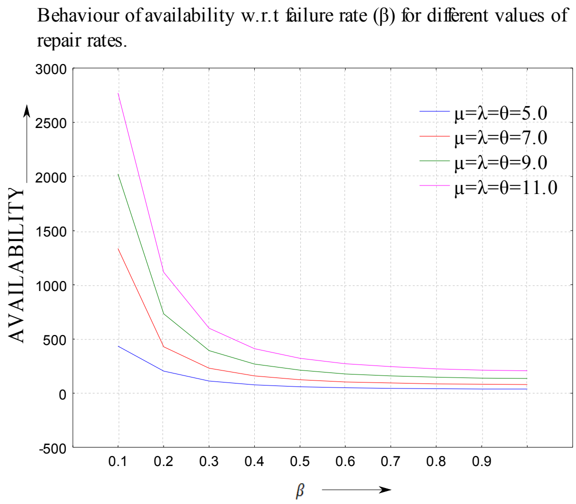

Fig. 6

β	PROFIT			
	μ=λ=θ=5.0	μ=λ=θ=7.0	μ=λ=θ=9.0	μ=λ=θ=11.0
0.1	356586	1024980	1947800	3137570
0.2	118295	290987	537674	858125
0.3	55963.5	140061	261079	419007
0.4	33083	85460.5	161556	261411
0.5	22302.2	59761.5	114735	187283
0.6	16525.4	45912.6	89450	147207
0.7	13215.5	37886.1	74734.8	123837
0.8	11254.5	33048.1	65814	109630
0.9	10079	30076.7	60294.4	100811
1.0	9377.61	28243.9	56860.7	95307.6

Table 6: The values of Profit with respect to failure rate β for different values of repair rates (μ=λ=θ=5.0, 7.0, 9.0 & 11.0)

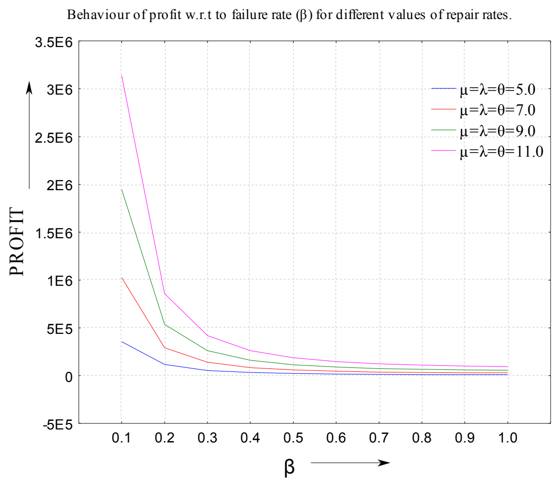

Fig. 7

From Fig. 5, Fig. 6 and Fig. 7 we observe that MTSF, Availability and Profit decrease with the increase in the failure rate for different values of repair rates. And if we increase the repair rate these characteristics also show an increase. So in order to make the system more efficient we have to maximize the repair rate and minimize the failure rate of the system.

Conclusion

After analysing the behaviour of characteristics like MTSF, Availability and Profit function graphically, we conclude that the reliability of a system can be increased by increasing the repair rate of the system and if failure rate of the system increases by any reason, it reduces the life expectancy of the system.

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