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IMPROVED ESTIMATORS USING COEFFICIENT OF SKEWNESS OF AUXILIARY ATTRIBUTE

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Abstract

In this paper, some ratio estimators for the population mean using known coefficient of skewness based on auxiliary attribute have been proposed and these have been developed by combining ratio estimators for estimating population mean of variate of study using the procedure given in Kadilar and Cingi (2006). The mean square error (MSE) equations for suggested estimators have been derived and it has been concluded that the suggested estimators always perform better than the ratio estimators, given in Sections 1 and 2. The results are also supported by the three original datasets.

Key Words: Ratio Estimators, Simple Random Sampling, Auxiliary Attribute, Mean Square Error, Efficiency.

1. Introduction

Auxiliary information is usually used to enhance the precision of estimates. Many authors have proposed estimators based on the auxiliary attribute. Naik and Gupta (1996) suggested a ratio estimator when variate of study and the auxiliary attribute are positive correlated. Sing et al. (2008), Abd-Elfattah et al. (2010), Koyuncu (2012), Malik and Singh (2013), Zaman (2018) and Zaman (2019) suggested a family of estimators using information from auxiliary attribute in simple random sampling. In the present study, a set of estimators is suggested for the population mean using coefficient of skewness based on auxiliary attribute and these estimators have been developed using the style given in Kadilar and Cingi (2006).

The Naik and Gupta estimator for the population mean $\bar{Y}$ of the variate of study, which makes use of information relating the population proportion posseessing certain attribute, is defined by

$$\bar{y}_{NG} = \frac{\bar{y}}{p} P \tag{1.1}$$

Let y_i be ith characteristic of the population and ϕ_i is the case of possessing certain attributes. If ith unit has the desired characteristic, it takes the value 1, if not then the value 0. That is;

$$\phi_i = \begin{cases} 1 & , \quad if\ ith\ unit\ of\ the\ population\ possesses\ attribute \\ 0 & , \quad otherwise \end{cases}$$

where $\bar{y}$ is the sample mean of the study variable and $a = \sum_{i=1}^{n} \phi_i$ be the the total number of the units that possess certain attribute in the sample. $p = \frac{a}{n}$ shows the proportion of units and it is considered that the population proportion P of the form of attribute ϕ is known.

The MSE of the Naik and Gupta estimator (1996) is

$$MSE(\bar{y}_{NG}) \cong \frac{1-f}{n}\bar{Y}^2\left(C_y^2 - 2\rho_{pb}C_yC_p + C_p^2\right) \quad (1.2)$$

where, $f = \frac{n}{N}$; C_p is the population coefficient of variation of auxiliary attribute ϕ and C_y is the population coefficient of variation of the study variable. $\rho_{pb}$is the coefficient of point biserial correlation between auxiliary attribute and the study variable (Naik and Gupta, 1996).

2. Proposed Estimators

It is suggested the estimators considering the coefficient of skewness of variable are as the following;

$$zt_i = \frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2)\,;\, i = 1,2,3,4,5 \quad (2.1)$$

where, m_1 and m_2 are either real numbers of the function of the known parameter of auxiliary attribute such as C_p, $\beta_2(\phi)$ and $\beta_1(\phi)$.

The following table gives some of the ratio-type estimators of the population mean, which can be found by proper choice of constants m_1 and m_2;

Estimators	Values of	
	m_1	m_2
$\boldsymbol{zt_1 = \frac{\bar{y}}{p + \beta_1(\phi)}\left(P + \beta_1(\phi)\right)}$	1	$\beta_1(\phi)$
$\boldsymbol{zt_2 = \frac{\bar{y}}{\beta_2(\phi)p + \beta_1(\phi)}\left(\beta_2(\phi)P + \beta_1(\phi)\right)}$	$\beta_2(\phi)$	$\beta_1(\phi)$
$\boldsymbol{zt_3 = \frac{\bar{y}}{\beta_1(\phi)p + \beta_2(\phi)}\left(\beta_1(\phi)P + \beta_2(\phi)\right)}$	$\beta_1(\phi)$	$\beta_2(\phi)$
$\boldsymbol{zt_4 = \frac{\bar{y}}{C_p p + \beta_1(\phi)}\left(C_p P + \beta_1(\phi)\right)}$	C_p	$\beta_1(\phi)$
$\boldsymbol{zt_5 = \frac{\bar{y}}{\beta_1(\phi)p + C_p}\left(\beta_1(\phi)P + C_p\right)}$	$\beta_1(\phi)$	C_p

Table 1: Suggested estimators based on coefficient of skewness

where,C_p, $\beta_2(\phi)$ and$\beta_1(\phi)$ are coefficients of variation belonging to ratio of units possessing certain attributes, coefficient of population kurtosis and the population coefficient of skewness of auxiliary attribute, respectively.

The expressions for the MSE's for these suggested estimators using Taylor series approach as (for details, please see the Appendix A) are

$$MSE(zt_i) \cong \frac{1-f}{n}\bar{Y}^2\left[C_y^2 + C_p^2\xi_i\left(\xi_i - 2K_{pb}\right)\right]; \quad i = 1,2,3,4,5 \quad (2.2)$$

where, $\xi_i = \frac{m_1 P}{m_1 P + m_2}$ and $K_{pb} = \frac{\rho_{pb} C_y}{C_p}$

$m_1$and $m_2$are given in Table 1.

In this paper we have used the function given in equation (2.2) to calculate MSE values of our suggested estimators. In addition, we have suggested the estimators using the style the estimators given in Kadilar and Cingi (2006) by combining ratio estimators given in Table 1.The general state of the suggested estimators is as follows;

$$zt_{proi} = \omega\frac{\bar{y}}{p + \beta_1(\phi)}\left(P + \beta_1(\phi)\right) + (1-\omega)\frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2) \; ; i = 2,3,4,5 \quad (2.3)$$

whereω is an optimal value that makes the MSE of zt_{proi} is minimum.

Similarly, the expressions of MSE's of these estimators using Taylor series approach have been obtained and are as follows; (please see the Appendix B).

$$MSE\left(zt_{proi}\right) \cong \frac{1-f}{n}\bar{Y}^2\left[(\omega\xi_1 + \xi_i - \omega\xi_i)^2 C_p^2 - 2\rho_{pb}C_y C_p(\omega\xi_1 + \xi_i - \omega\xi_i) + C_y^2\right]; i = 2,3,\ldots,10 \quad (2.4)$$

For $m_1 = \beta_2(\phi)$ and $m_2 = \beta_1(\phi)$in table 1 ($zt_1$and zt_2);

$$zt_{pro1} = \omega\frac{\bar{y}}{p + \beta_1(\phi)}\left(P + \beta_1(\phi)\right) + (1-\omega)\frac{\bar{y}}{\beta_2(\phi)p + \beta_1(\phi)}\left(\beta_2(\phi)P + \beta_1(\phi)\right) \quad (2.5)$$

The expressions for the MSE of this estimator is computed as the following;

$$MSE\left(zt_{pro1}\right) \cong \frac{1-f}{n}\bar{Y}^2\left[C_p^2(\omega\xi_1 + (1-\omega)\xi_2)^2 - 2\rho_{pb}C_y C_p(\omega\xi_1 + \xi_2 - \omega\xi_2) + C_y^2\right] \quad (2.6)$$

where, $\xi_1 = \frac{P}{P+\beta_1(\phi)}$ and $\xi_2 = \frac{\beta_2(\phi)P}{\beta_2(\phi)P+\beta_1(\phi)}$.

The following estimator are also proposed using $zt_1$and zt_3estimators

$$zt_{pro2} = \omega\frac{\bar{y}}{p + \beta_1(\phi)}\left(P + \beta_1(\phi)\right) + (1-\omega)\frac{\bar{y}}{\beta_1(\phi)p + \beta_2(\phi)}\left(\beta_1(\phi)P + \beta_2(\phi)\right) \quad (2.7)$$

The expressions for the MSE of this estimator is the same as (2.6) except that ξ_2 in (2.6) is replaced with ξ_3.

Moreover, the following estimator combining ratio-type estimators in Table 1 ($zt_1$and zt_4) has also been suggested,

$$zt_{pro3} = \omega \frac{\bar{y}}{p + \beta_1(\phi)} \big(P + \beta_1(\phi)\big) + (1-\omega) \frac{\bar{y}}{C_p p + \beta_1(\phi)} \big(C_p P + \beta_1(\phi)\big) \tag{2.8}$$

Expressions for the MSE of this estimator is again the same as (2.6) except that ξ_2 in (2.6) is replaced with ξ_4.

Finally, the following estimator combining ratio estimators in Table 1 ($zt_1$and zt_5) is suggested,

$$zt_{pro4} = \omega \frac{\bar{y}}{p + \beta_1(\phi)} \big(P + \beta_1(\phi)\big) + (1-\omega) \frac{\bar{y}}{\beta_1(\phi) p + C_p} \big(\beta_1(\phi) P + C_p\big) \tag{2.9}$$

The expression for the MSE of the estimator is again the same as (2.6) except that t ξ_2 in (2.6) is replaced with ξ_5.

The optimum value of ω to minimize equation (2.6) can be computed as the following;

$$\omega_{opt} = \frac{\rho_{pb} C_y - C_p \xi_i}{C_p(\xi_1 - \xi_i)} \tag{2.10}$$

We have obtained minimum MSE of the suggested estimators considering the optimal values of ω in equation (2.10). All suggested estimators have the same minimum mean square error as the following:

$$MSE_{min}\big(zt_{proi}\big) = \frac{1-f}{n} \bar{Y}^2 \big[C_y^2\big(1-\rho_{pb}^2\big)\big] \quad i = 1,2,3,4,5 \tag{2.11}$$

3. Efficiency Comparison

We have compared the MSE of the suggested estimators in (2.11) with the MSE of the Naik-Gupta (1996) estimator, the ratio estimators suggested in Section 2 (zt_1- zt_5).

Comparing the MSE of the suggested estimators, presented in between (2.5) - (2.8), with the ratio estimator suggested by Naik-Gupta (1996), given in (1.1), the following condition is obtained;

$$MSE\big(zt_{proi}\big) < MSE(\bar{y}_{NG}) \quad i = 2,3,\dots,10$$

$$\frac{1-f}{n} \bar{Y}^2 \big[C_y^2\big(1-{\rho_{pb}}^2\big)\big] < \frac{1-f}{n} \bar{Y}^2 \big(C_y^2 - 2\rho_{pb} C_y C_p + C_p^2\big)$$

$$\big(\rho_{pb} C_y - C_p\big)^2 > 0 \tag{3.1}$$

When the condition (3.1) is satisfied, the suggested exponential estimators are more efficient than the ratio-type estimator proposed by Naik-Gupta (1996).

Comparing the MSE of the suggested estimators, presented in (2.11), with the mean square error of the ratio estimators in Section 2, presented in Table 1, the following condition is obtained;

$$MSE(zt_{proi}) < MSE(zt_i)$$

$$\left(C_y\rho_{pb} - C_p\xi_i\right)^2 > 0 \;\; ; i = 1,2,3,4,5 \quad (3.2)$$

It is concluded that all suggested estimators presented in Section 2 perform always better than estimators presented in Table 1, because the restriction presented in (3.2) is always satisfied.

4. Numerical Illustrations

In this section, the efficiencies of several estimators mentioned in this paper have been calculated for the three original datasets.

Population I (Source: see Sukhatme (1957), p. 279)

$$y = Number\ of\ villages\ in\ the\ circles$$

$$\phi_i = \begin{cases} 1 \quad , & if\ \ A\ circle\ consisting\ more\ than\ five\ vilages \\ 0 \quad , & otherwise \end{cases}$$

N:89	$\bar{Y}$: 3.3596	$\beta_1(\phi)$: 2.3267	ξ_4:0.1245
n: 20	P: 0.1236	ξ_1:0.0504	ξ_5:0.0969
$\beta_2(\phi)$: 3.4917	C_y: 0.6008	ξ_2:0.1565	
ρ_{pb}:0.766	C_p: 2.6779	ξ_3:0.0761	

Table 2: Population I data statistics

Population II (Source: see Sukhatme (1957), p. 279)

$$y = Area\ (in\ acres)\ under\ wheat\ crop\ in\ the\ circles$$

$$\phi_i = \begin{cases} 1 \quad , & if\ \ A\ circle\ consisting\ more\ than\ five\ vilages \\ 0 \quad , & otherwise \end{cases}$$

N:89	$\bar{Y}$: 1102.34	$\beta_1(\phi)$: 2.3267	ξ_4:0.1245
n: 20	P: 0.1236	ξ_1:0.0504	ξ_5:0.0969
$\beta_2(\phi)$: 3.4917	C_y: 0.6499	ξ_2:0.1565	
ρ_{pb}:0.624	C_p: 2.6779	ξ_3:0.0761	

Table 3: Population II data statistics

Population III (Source: see Zaman et al. (2014))

$$y = the\ number\ of\ teachers$$

$$\phi_i = \begin{cases} 1 & , \ if\ the\ number\ of\ teachers\ is\ more\ than\ 60 \\ 0 & , \ otherwise \end{cases}$$

N:111	$\bar{Y}$: 29.279	$\beta_1(\phi)$: 2.4142	ξ_4:0.1179
n: 30	P: 0.117	ξ_1:0.0462	ξ_5:0.0929
$\beta_2(\phi)$: 3.898	C_y: 0.872	ξ_2:0.1589	
ρ_{pb}:0.797	C_p: 2.758	ξ_3:0.0676	

Table 4: Population III data statistics

In Tables 2, 3 and 4, it is observed from the statistics about the populations for different sample sizes $n = 20, n = 30$ that the sample size has no effect on performance of the estimators.

Estimator	MSE		
	Population I	Population II	Population III
t_{NG}	2.2168	255377.7	94.532
zt_1	0.1659	20754.93	16.1946
zt_2	0.2347	28165.04	19.8603
zt_3	0.1761	21851.34	16.5799
zt_4	0.2066	25134.65	18.0606
zt_5	0.1874	23072.13	17.224
$\boldsymbol{zt_{proi}}$	**0.0652**	**12148.63**	**5.7840**

Table 5: MSE values of the ratio estimators

In Table 5, the MSE values, which are found considering equations given in Sections 1 and 2, are presented. From Table 5, it is concluded that the suggested combining estimators have the smallest MSE values among estimators, presented in Parts 1 and 2.This is an expected result, as shown in Part 3. Consequently, it is inferred that suggested combining estimators perform better than the competing estimators for these datasets.

5. Conclusion

We have improved the novel estimators combining ratio estimators mentioned ii Section 1 using information about population proportion possessing certain attribute and computed the minimum MSE expressions for the suggested estimators. Theoretically, it is shown that the suggested estimators are always more efficient than the competing estimators. These theoretical findings are supported by an application with original datasets.

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Appendices

Appendix A

In general, Taylor series approach for k variables can be as the following;

$$h(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k) = h(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_k) + \sum_{j=1}^{k} d_j(\bar{x}_j - \bar{X}_j) + R_k(\bar{X}_k, \alpha) + O_k$$

where

$$d_j = \frac{\partial h(\bar{x}_1, \bar{x}_2, \dots, \bar{x}_k)}{\partial \alpha_j}$$

and

$$R_k(\bar{X}_k, \alpha) = \sum_{j=1}^{k} \sum_{j=1}^{k} \frac{1}{2!} \frac{\partial^2 h(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_k)}{\partial \bar{X}_i \bar{X}_j} (\bar{x}_j - \bar{X}_j)(\bar{x}_i - \bar{X}_i) + O_k$$

where O_k represents the terms in the expansion of the Taylor series of more than the second degree (Wolter, 1985). When it is omitted the term $R_k(\bar{X}_k, \alpha)$, it is computed Taylor series approach for two variables as the following;

$$h(p, \bar{y}) - h(P, \bar{Y}) \cong \left.\frac{\partial h(c,d)}{\partial c}\right|_{P,\bar{Y}} (p - P) + \left.\frac{\partial h(c,d)}{\partial d}\right|_{\bar{Y},P} (\bar{y} - \bar{Y})$$

where, $h(p, \bar{y}) = zt_i$ and $h(P, \bar{Y}) = \bar{Y}$

MSE equations of the suggested estimators given in Table 1 may be computed as follows:

$$zt_i - \bar{Y} \cong \left.\frac{\partial\left(\frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2)\right)}{\partial p}\right|_{P,\bar{Y}} (p - P)$$

$$+ \left.\frac{\partial\left(\frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2)\right)}{\partial \bar{y}}\right|_{\bar{Y},P} (\bar{y} - \bar{Y})$$

$$\cong \left(\frac{-m_1 \bar{Y}}{m_1 P + m_2}\right)(p - P) + (\bar{y} - \bar{Y})$$

$$E(zt_i - \bar{Y})^2 \cong \left[\frac{(m_1 \bar{Y})^2}{(m_1 P + m_2)^2} V(p) - \frac{2(m_1 \bar{Y})}{m_1 P + m_2} Cov(p, \bar{y}) + V(\bar{y})\right]$$

$$MSE(zt_i) \cong \frac{1-f}{n} \bar{Y}^2 \left[\frac{m_1^2 P^2}{(m_1 P + m_2)^2} C_p^2 - \frac{2 m_1 P}{m_1 P + m_2} \rho_{pb} C_y C_p + C_y^2\right]$$

$$MSE(zt_i) \cong \frac{1-f}{n} \bar{Y}^2 \left[C_y^2 + C_p^2 \frac{m_1 P}{m_1 P + m_2}\left(\frac{m_1 P}{m_1 P + m_2} - \frac{2\rho_{pb} C_y}{C_p}\right)\right]$$

$$MSE(zt_i) \cong \frac{1-f}{n} \bar{Y}^2 \left[C_y^2 + C_p^2 \xi_i (\xi_i - 2K_{pb})\right]; \quad i = 1,2,3,4,5 \qquad (A.1)$$

where, $\xi_i = \frac{m_1 P}{m_1 P + m_2}$ and $K_{pb} = \frac{\rho_{pb} C_y}{C_p}$

$m_1$and $m_2$are given in Table 1.

In this paper, we have used the function given in equation (A.1) to calculate the MSE values of the suggested estimators based on coefficient of skewness.

Appendix B

The MSE equations of the combining suggested estimators presented in (2.3), (2.5) (2.6) and (2.7) are generally computed as follows:

As similar to Appendix A,

$$h(p, \bar{y}) - h(P, \bar{Y}) \cong \left.\frac{\partial h(c,d)}{\partial c}\right|_{P,\bar{Y}} (p - P) + \left.\frac{\partial h(c,d)}{\partial d}\right|_{\bar{Y},P} (\bar{y} - \bar{Y})$$

where, $h(p, \bar{y}) = zt_{proi}$ and $h(P, \bar{Y}) = \bar{Y}$

$$zt_{proi} - \bar{Y} \cong \left.\frac{\partial\left(\omega \frac{\bar{y}}{p+\beta_1(\phi)}\left(P+\beta_1(\phi)\right) + (1-\omega)\frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2)\right)}{\partial p}\right|_{P,\bar{Y}} (p-P)$$

$$+ \left.\frac{\partial\left(\omega \frac{\bar{y}}{p+\beta_1(\phi)}\left(P+\beta_1(\phi)\right) + (1-\omega)\frac{\bar{y}}{m_1 p + m_2}(m_1 P + m_2)\right)}{\partial \bar{y}}\right|_{P,\bar{Y}} (\bar{y} - \bar{Y})$$

$$\cong \left(\frac{-\omega\bar{Y}}{P+\beta_1(\phi)} + (1-\omega)\left(\frac{-m_1\bar{Y}}{m_1 P + m_2}\right)\right)(p-P) + (\bar{y}-\bar{Y})$$

$$E\left(zt_{proi} - \bar{Y}\right)^2 \cong \left[\left(\frac{\omega^2\bar{Y}^2}{\left(P+\beta_1(\phi)\right)^2} + (1-\omega)^2\left(\frac{m_1^2\bar{Y}^2}{(m_1P+m_2)^2}\right) + 2\left(\frac{-\omega\bar{Y}}{P+\beta_1(\phi)}\right)(1-\omega)\left(\frac{-m_1\bar{Y}}{m_1P+m_2}\right)\right)V(p) - 2\left(\frac{\omega\bar{Y}}{P+\beta_1(\phi)} + \frac{m_1\bar{Y}}{m_1P+m_2} - \frac{\omega m_1\bar{Y}}{m_1P+m_2}\right)Cov(p,\bar{y}) + V(\bar{y})\right]$$

$$\cong \bar{Y}^2\left[\left(\frac{\omega^2}{\left(P+\beta_1(\phi)\right)^2} + (1-\omega)^2\left(\frac{m_1^2}{(m_1P+m_2)^2}\right) + \frac{2\omega(1-\omega)m_1}{\left(P+\beta_1(\phi)\right)(m_1P+m_2)}\right)V(p) - \frac{2}{\bar{Y}}\left(\frac{\omega}{P+\beta_1(\phi)} + \frac{m_1}{m_1P+m_2} - \frac{\omega m_1}{m_1P+m_2}\right)Cov(p,\bar{y}) + \frac{V(\bar{y})}{\bar{Y}^2}\right]$$

where, $\xi_1 = \frac{P}{P+\beta_1(\phi)}$; $\xi_i = \frac{m_1 P}{m_1 P + m_2}$

$$\cong \frac{1-f}{n}\bar{Y}^2\left[\left(\frac{\omega^2\xi_1^2}{P^2} + (1-\omega)^2\xi_i^2 + 2\omega(1-\omega)\xi_1\xi_i\right)C_p^2 - 2\rho_{pb}C_yC_p(\omega\xi_1 + \xi_i - \omega\xi_i) + C_y^2\right]$$

$$MSE\left(zt_{proi}\right) \cong \frac{1-f}{n}\bar{Y}^2\left[(\omega\xi_1 + \xi_i - \omega\xi_i)^2C_p^2 - 2\rho_{pb}C_yC_p(\omega\xi_1 + \xi_i - \omega\xi_i) + C_y^2\right]; i = 2,3,\ldots,10 \quad (B.1)$$

If the equation (B.1) is optimized according to w,

$$\frac{\partial MSE\left(zt_{proi}\right)}{\partial\omega} = \frac{1-f}{n}\bar{Y}^2\left[2(\omega\xi_1 + \xi_i - \omega\xi_i)(\xi_1 - \xi_i)C_p^2 - 2\rho_{pb}C_yC_p(\xi_1 - \xi_i)\right] = 0$$

$$(\omega\xi_1 + \xi_i - \omega\xi_i)(\xi_1 - \xi_i)C_p^2 = \rho_{pb}C_yC_p(\xi_1 - \xi_i)$$

$$\omega\xi_1 + \xi_i - \omega\xi_i = \rho_{pb}\frac{C_y}{C_p}$$
$$\omega_{opt} = \frac{\rho_{pb}C_y - C_p\xi_i}{C_p(\xi_1 - \xi_i)} \qquad (B.2)$$

We have obtained minimum MSE of the suggested estimators using the optimal equations of ω_{opt} in $(B.2)$.

$$MSE_{min}(zt_{proi}) \cong \frac{1-f}{n}\bar{Y}^2\left[\left(\omega_{opt}\xi_1 + \xi_i - \omega_{opt}\xi_i\right)^2 C_p^2 - 2\rho_{pb}C_yC_p\left(\omega_{opt}\xi_1 + \xi_i - \omega_{opt}\xi_i\right) + C_y^2\right]$$
$$\omega_{opt}\xi_1 + \xi_i - \omega_{opt}\xi_i = \omega_{opt}(\xi_1 - \xi_i) + \xi_i$$
$$= \frac{\rho_{pb}C_y - C_p\xi_i}{C_p(\xi_1 - \xi_i)}(\xi_1 - \xi_i) + \xi_i = \rho_{pb}\frac{C_y}{C_p} \qquad (B.3)$$

Using $(B.2)$in $(B.3)$, the following equation is obtained

$$MSE_{min}(zt_{proi}) = \frac{1-f}{n}\bar{Y}^2\left[C_y^2\left(1-\rho_{pb}^2\right)\right] \quad i = 1,2,3,4,5 \qquad (B.4)$$

In this study, we have used the function presented in equation (B.4) to calculate MSE values of the proposed estimators.