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AN INTERACTIVE APPROACH TO PROBABILISTIC INTUITIONISTIC FUZZY MULTI-CRITERIA DECISION MAKING IN STOCK SELECTION PROBLEM

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Abstract

The concurrence of randomness and imprecision exists in decision making problems (DMPs). To describe unpredictability, fuzziness and statistical uncertainty in a single frame, we have developed an interactive approach to probabilistic intuitionistic fuzzy MCDM method, in which assessment of alternative over attributes are provided by probabilistic intuitionistic fuzzy elements (PIFEs). In proposed methodology a conversion method to convert fuzzy sets to intuitionistic fuzzy sets is also used. To completely describe statistical and non-statistical uncertainty, suitable probability distribution function is associated to the both belongingness values and non-belongingness values of each one entity in constructed IFS. The core intention of this paper is to propose a PIF-TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method for MCDM problem. Firstly, we develop distance measures for PIFEs. Probabilistic intuitionistic fuzzy positive and negative ideal solutions are also defined. A real life case study is in use as an example to illustrate the methodology of developed PIF-TOPSIS method and to find the ranking of organizations using real data. The decision making framework of proposed PIF-TOPSIS method is superior to other MCDM methods, because of introducing probabilistic information in IFEs, which can be useful to ensure the integrality and accuracy of intuitionistic fuzzy information.

Key Words: Probabilistic Intuitionistic Fuzzy Set, Multi-Criteria Decision Making, PIF-PIS, PIF-NIS, Distance Measure, TOPSIS.

1. Introduction

Uncertainty management in decision making problems (DMPs) has been an imperative issue for the researchers. It is sometimes impossible or very tough to obtain sufficient and accurate data due to inadequate awareness of decision makers (DMs), the behavior of objects, uncertainty with randomness of events. There may be various uncertainties in DMPs due to either fuzziness or randomness or both. These uncertainties can be categorized into stochastic/statistical and nonstochastic/non-statistical uncertainty [1]. Generally, statistical uncertainty possibly handled effectively by probabilistic modeling [2, 3]. In contrast, the fuzzy set theory (FST) [4, 5] has been witnessed to be powerful tool to handle nonstatistical uncertainties. Several theories have been developed to deal with nonstochastic uncertainty, among which FST [4] is expansively researched and effectively applied in DMPs [6- 12]. Type-2 fuzzy sets [13], interval-valued fuzzy set [4] are various extension of fuzzy sets (FSs), which are extremely used in MCDM problems to take in nonstochastic uncertainty, randomness and hesitation. Atanassov [14] used three parameters to characterize intuitionistic fuzzy set (IFS) to tackle non-determinism within system occurring due to hesitation of DMs. Various scholars [15-19] proposed IF decision making methods.

Both types of uncertainties, stochastic and non-stochastic exist simultaneously in the system. Probabilistic and fuzzy models individually process only one phase of uncertainty and do not have capable to tackle both type of uncertainties simultaneously. Comprehensive occurrence of randomness, hesitation, statistical uncertainty and imprecision in real life DMPs had fascinated researchers to integrate probability theory with FST. The integration of FST with probabilistic theory has been studied [20-23]. Probabilistic fuzzy set (PFS) was introduced [23] by describing center and width of membership function, which makes it able to tackle with both types of uncertainties in a single framework. PFS is applied for stochastic modeling [22, 23].

The occurrence probabilities of the elements in IFS and their extension are assumed to be equal, which is obviously unsuitable for the evaluation and hesitation judgments of DMs in most of the real life problems. Bifuzzy (intuitionistic) probabilistic fuzzy set was introduced [24, 25] by describing the belongingness and non-belongingness functions of an object with randomness of choosing them. For example, a DM provides probabilistic intuitionistic fuzzy element (PIFE) $\{x, < 0.4 \mid 0.4, 0.5 \mid 0.6 >\}$ in which 0.4 represents the belongingness value and 0.6 is non-belongingness value with corresponding probabilities 0.4 and 0.6 respectively. Intuitionistic probabilistic fuzzy set (IPFS) can easily overcome the gap between fuzziness and probability and has least statistical uncertainty. Various researchers [26-28] have developed TOPSIS method for DMPs.

In this paper, limitations of usual IFS theory in some particular situations are discussed which are leading to the motivations for IFS with probabilistic theory. Objective of this study is to develop a decision making methodology to TOPSIS method in PIF environment. The decision making framework of proposed PIF-TOPSIS method is more superior to other MCDM methods, because of introducing probabilistic information in IFEs, which can be useful to ensure the integrality and accuracy of IF information.

The remaining of the research paper is ordered as follows: in section 2 the basic terminology and definition of FS, IFS and PIFS are described. In this section, a conversion method is also presented. Afterwards, step wise procedure of proposed PIF TOPSIS is described in section 3. The flow of calculation and results are presented in section 4. The conclusion is given at the end of this paper.

2. Preliminaries

In this segment, we have briefly reviewed some elementary definitions related to FS, IFS and PIFS. Conversion theorem [29] for IFS is also discussed in this section.

2.1. Fuzzy set and Intuitionistic fuzzy set

In 1965, Zadeh [4] established the theory of fuzzy set to deal with non-stochastic uncertainty and vagueness. Atanassov [14] established the concept of IFS.

This idea seems to be useful in modeling the uncertainty because of hesitation in system in many real life situations [30].

Definition 1: [4] Let X be universal set then the fuzzy set A is characterized by membership degree $\mu_A : X \to [0,1]$ and represented by the following order pair:

$$A = \{(x, \mu_A(x)) : \forall x \in X\} \tag{1}$$

Definition 2: [14] An IFS A on a universal set X is defined as an object of the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\} \tag{2}$$

Where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ represents the belongingness and non- belongingness grade of an element $x \in A \subset X$ respectively.

2.2. Probabilistic intuitionistic fuzzy set

Both types of uncertainties, stochastic and nonstochastic exist simultaneously in the system. Probabilistic and fuzzy models individually process only one phase of uncertainty and do not have capability to tackle both types of uncertainties concurrently. The concurrence of randomness, hesitation, statistical uncertainty and imprecision in real life decision making problems has attracted scholars to incorporate probability theory with FST. The integration of FST with probabilistic theory has been studied [20-23]. PFS was developed [23] by describing center and width of membership function, which makes it able to tackle both types of uncertainties in a solitary framework. Recently, PFS is applied for stochastic modeling [23]. Bifuzzy (intuitionistic) probabilistic fuzzy set was introduced [24,25] by describing the belongingness and non-belongingness functions of an object with randomness of choosing them.

Definition 3: Let X be the reference set, then a PIFS I_P on X is expressed as follows:

$$I_P = \{\langle x, \mu_A(x) \mid p(x), \nu_A(x) \mid q(x)\rangle \mid x \in X\} \tag{3}$$

where $\mu_A(x)$ and $\nu_A(x)$ represent the grade of intuitionistic fuzzy membership and non-membership of an element $x \in I_P \subset X$ correspondingly. $p(x)$ and $q(x)$ denoting the corresponding probabilistic information of probabilistic intuitionistic fuzzy set I_P with $\sum_{i=1}^{\#h} p(x) + q(x) \le 1$.

2.3. A Methodology of construct IFS from fuzzy set

A technique to construct IFS from FSs proposed by Jurio *et al.* [29] is presented in this subsection. Corresponding to a given FS, IFS may be constructed by modifying the value of membership.

Let $FS_S(P)$ denotes the set of all FSs in the universal set P with $A_F \in FS_S(P)$ and let $\pi, \delta : P \to [0,1]$. Then

$$I = \{\langle p_i, \hat{f}(\mu_{A_F}(p_i), \pi(p_i), \delta(p_i))\rangle \forall p_i \in P\} \tag{4}$$

denotes an IFS, the mapping

$$\hat{f} : [0,1]^2 \times [0,1] \to L^* \quad \text{given by}$$

$$\hat{f}(x, y, \delta) = (\hat{f}_\mu(x, y, \delta), \hat{f}_V(x, y, \delta)) \quad \text{where}$$

$$\hat{f}(x, y, \delta) = x(1 - \delta y),\ \hat{f}_V(x, y, \delta) = 1 - x(1 - \delta y) - \delta y \tag{5}$$

and

$$L^* = \{(x, y) : (x, y) \in [0,1] \times [0,1]\, with\ x + y \le 1\} \tag{6}$$

3. An algorithm of proposed PIF TOPSIS

Step to step algorithm of developed PIF TOPSIS method is presented in the following section. Assumes that $A = \{A_1, A_2, ..., A_m\}$ and $C = \{C_1, C_2, ..., C_n\}$ are sets of m alternatives and n criteria correspondingly. Following are various steps in PIF TOPSIS method.

Step 1: Intuitionistic fuzzy decision matrix (IFDM) $I = (I_{ij})_{m\times n}$ of IF value $I_{ij} = (\mu_{ij}, v_{ij})$ is constructed using construction theorem. Here μ_{ij} and v_{ij} are membership and non-membership grade of alternative A_i under the criteria C_j. This step includes following sub steps.

(i) Define the universal set for each alternative for each criterion.

(ii) Construct the suitable FSs for each criterion.

(iii) IFSs are constructed using construction theorem corresponding to FSs obtained in above sub step.

The IFDM can be symbolized as follows:

$$I = \begin{bmatrix} (\mu_{A_1}(x_1), v_{A_1}(x_1)) & (\mu_{A_1}(x_2), v_{A_1}(x_2)).............. & (\mu_{A_1}(x_n), v_{A_1}(x_n)) \\ (\mu_{A_2}(x_1), v_{A_2}(x_1)) & (\mu_{A_2}(x_2), v_{A_2}(x_2))............. & (\mu_{A_2}(x_n), v_{A_2}(x_n)) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ (\mu_{A_m}(x_1), v_{A_m}(x_1)) & (\mu_{A_m}(x_2), v_{A_m}(x_2))............ & (\mu_{A_m}(x_n), v_{A_m}(x_n)) \end{bmatrix} \tag{7}$$

Step 2: In this step, suitable probability distribution function is associated to the both membership and non-membership values of each entity $I_{ij} = (\mu_{ij}, v_{ij})$ of IFDM $I = (I_{ij})_{m\times n}$ constructed in step 1 to construct probabilistic intuitionistic fuzzy decision matrix (PIFDM) $I_{Pij} = (\langle \mu_A(x) \mid p_{ij}, v_A(x) \mid q_{ij} \rangle)$. Here p_{ij} and q_{ij} denotes the corresponding probabilistic information of probabilistic intuitionistic fuzzy set I_P having condition $\sum_{i,j} p_{(ij)} + q_{(ij)} \le 1$. Suitable probability distribution function is associated to the both membership and non-membership values as follows:

$$\mu(p_{ij}) = \frac{\mu(p_{ij})}{\mu(p_{ij}) + \nu(p_{ij})}, \quad \nu(q_{ij}) = \frac{\nu(q_{ij})}{\mu(q_{ij}) + \nu(q_{ij})} \text{ for all } j \in B$$

$$\mu(p_{ij}) = 1 - \frac{\mu(p_{ij})}{\mu(p_{ij}) + \nu(p_{ij})}, \quad \nu(q_{ij}) = 1 - \frac{\nu(q_{ij})}{\mu(q_{ij}) + \nu(q_{ij})} \text{ for all } j \in C$$

Here *B* and *C* denotes benefit and cost criteria correspondingly.

The PIFDM $I = (I_{Pij})_{m \times n}$ can be represnted as follows:

$$I_P = \begin{bmatrix} (\mu_{A_1}(x_1) \mid p_{11}, \nu_{A_1}(x_1) \mid q_{11}) & (\mu_{A_1}(x_2) \mid p_{12}, \nu_{A_1}(x_2) \mid q_{12}) \ldots\ldots & (\mu_{A_1}(x_n) \mid p_{1n}, \nu_{A_1}(x_n) \mid q_{1n}) \\ (\mu_{A_2}(x_1) \mid p_{21}, \nu_{A_2}(x_1) \mid q_{21}) & (\mu_{A_2}(x_2) \mid p_{22}, \nu_{A_2}(x_2) \mid q_{22}) \ldots\ldots & (\mu_{A_2}(x_1) \mid p_{2n}, \nu_{A_2}(x_n) \mid q_{2n}) \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ (\mu_{A_m}(x_1) \mid p_{m1}, \nu_{A_m}(x_1) \mid q_{m1}) & (\mu_{A_m}(x_2) \mid p_{m2}, \nu_{A_m}(x_2) \mid q_{m2}) \ldots\ldots & (\mu_{A_m}(x_1) \mid p_{mn}, \nu_{A_m}(x_n) \mid q_{mn}) \end{bmatrix} \quad (8)$$

Step 3: PIF-PIS $(\hbar_P^{+})$ and PIF-NIS $(\hbar_P^{-})$ are defined as follows:

$$\hbar_P^{+} = (\mu_j^{+}, \nu_j^{+}) = \begin{pmatrix} MAX\mu_{ij} \mid p_{ij}, MIN\nu_{ij} \mid q_{ij}, for \in B) \\ MIN\mu_{ij} \mid p_{ij}, MAX\nu_{ij} \mid q_{ij}, for \in C) \end{pmatrix} \quad (9)$$

$$\hbar_P^{-} = (\mu_j^{-}, \nu_j^{-}) = \begin{pmatrix} MIN\mu^{l}_{\hat{\alpha}_{ij}} \mid p_{ij}, MAX\nu^{l}_{\hat{\alpha}_{ij}} \mid q_{ij}, for \in B) \\ MAX\mu^{l}_{\hat{\alpha}_{ij}} \mid p_{ij}, MIN\nu^{l}_{\hat{\alpha}_{ij}} \mid q_{ij}, for \in C) \end{pmatrix} \quad (10)$$

Here *B* and *C* are group of benefit and cost criteria respectively.

Step 4: Distance $d(A_i, \hbar_P^{+})$ among the A_i and PIF-PIS $(\hbar_P^{+})$ and the distance $d(A_i, \hbar_P^{-})$ between the alternative A_i and PIF-NIS $(\hbar_P^{-})$, is calculated using following distance measure:

$$d(A_i, \hbar_P^{+}) = \frac{1}{2}\sum_{j=1}^{n} w_j \,.\, Max\left(\left|(\mu_{ij} \mid p_{ij}) - (\mu_j^{+} \mid p_j)\right|, \left|(\nu_{ij} \mid q_{ij}) - (\nu_j^{+} \mid q_j)\right|\right) \quad (11)$$

$$d(A_i, \hbar_P^{-}) = \frac{1}{2}\sum_{j=1}^{n} w_j \,.\, Max\left(\left|(\mu_{ij} \mid p_{ij}) - (\mu_j^{-} \mid p_j)\right|, \left|(\nu_{ij} \mid q_{ij}) - (\nu_j^{-} \mid q_j)\right|\right) \quad (12)$$

Step 5: The closeness coefficient value of each alternative is calculated using subsequent expression.

$$Cc_j = \frac{d(A_i, \hbar_P^-)}{d(A_i, \hbar_P^+) + d(A_i, \hbar_P^-)}, \; i = 1, 2, ..., m \quad (13)$$

Step 6: Ranking of all the alternatives is given according to the decreasing value of closeness coefficient.

4. A real case study

In this section, developed PIF-TOPSIS method is applied to rank seven organizations, S.B.I. (A_1), InfoTech Enterprises (A_2), ITC (A_3), H.D.F.C. Bank (A_4), Tata Steel (A_5), Tata Motors (A_6), Bajaj Finance(A_7) using real data.

4.1. Implementation of proposed method

Seven organizations ($A_{i,}$ *i =1, 2, 3…6,7*) are evaluated for their performance on the basis of following seven criteria ($C_{i,}$ *i =1, 2, 3…6,7*).

1. Earnings per share(EPS) (C_1)
2. Face value (C_2)
3. Book value (C_3)
4. Deliverables (C_4)
5. Put-Call Ratio (C_5)
6. Dividend yield (C_6)
7. Price to earnings ratio) (C_7)

C_1, C_2, C_3 and C_4 are fit in to benefit criteria i.e, high value designates good quality growth and C_5, C_6 and C_7 belong to cost criteria i.e., low value designates indicate good growth.. Actual numerical values of these seven criteria of the organizations are retrieved from http://*www.moneycontrol.comfrom* date *20.7.2017 to 27.7.2017* and average of this information is tabulated in Table 1:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	13.15	1.00	196.53	45.21	19.27	260	23.48
A_2	61.18	5.00	296.12	71.41	14.98	515	15.40
A_3	8.54	1.00	37.31	75.74	30.52	475	33.55
A_4	59.07	2.00	347.59	59.77	28.50	550	30.17
A_5	22.25	2.00	237.82	66.95	5.98	25	20.17
A_6	35.47	1.12	511.31	26.35	7.95	100	16.12
A_7	36.64	2.00	174.60	39.87	45.39	245	46.99

Table1: Average of actual numerical value of criteria

Following are the stepwise computations in ranking the organizations using proposed TOPSIS method with probabilistic intuitionistic fuzzy information.

Step 1: Crisp numerical values of Table 1 are fuzzified using FSs for each criterion and following FDM (Table 2) is obtained.

$$A_1 = 0.128/13.15 + 0.158/1.0 + 0.272/196.53 + 0.297/45.21 + 0.287/19.27 + 0.269/260 + 0.311/23.48$$
$$A_2 = 0.595/61.18 + 0.788/5.0 + 0.41/296.12 + 0.469/71.41 + 0.223/14.98 + 0.534/515 + 0.204/15.40$$
$$A_3 = 0.083/8.54 + 0.158/1.0 + 0.052/37.31 + 0.497/75.74 + 0.455/30.52 + 0.492/475 + 0.444/33.55$$
$$A_4 = 0.574/59.07 + 0.315/2.0 + 0.481/347.59 + 0.392/59.77 + 0.425/28.50 + 0.57/550 + 0.4/30.17$$
$$A_5 = 0.216/22.25 + 0.315/2.0 + 0.329/237.82 + 0.4397/66.95 + 0.089/5.98 + 0.026/25 + 0.267/20.17$$
$$A_6 = 0.345/35.47 + 0.177/1.12 + 0.708/511.31 + 0.173/26.35 + 0.119/7.957 + 0.104/100 + 0.214/16.12$$
$$A_7 = 0.356/36.64 + 0.315/2.0 + 0.242/174.60 + 0.262/39.87 + 0.677/45.39 + 0.254/245 + 0.622/46.99$$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	0.128	0.158	0.272	0.297	0.287	0.269	0.311
A_2	0.595	0.788	0.41	0.469	0.223	0.534	0.204
A_3	0.083	0.158	0.052	0.497	0.455	0.492	0.444
A_4	0.574	0.315	0.481	0.392	0.425	0.57	0.4
A_5	0.216	0.315	0.329	0.439	0.089	0.026	0.267
A_6	0.345	0.177	0.708	0.173	0.119	0.104	0.214
A_7	0.356	0.315	0.242	0.262	0.677	0.254	0.622

Table 2: Fuzzy decision matrix

Construction theorem (Section 2.3) is applied to construct IFDM (Table 3) from FDM (Table 2).

$\pi = 0.083$, $\delta = 0.595$ are used to construct the IFN of A_1 against criterion C_1.

$$\mu(C_1) = 0.128\,(1 - 0.595 \times 0.083) = 0.122$$

$$\nu(C_1) = 1 - 0.128\,(1 - 0.595 \times 0.083) - 0.595 \times 0.083 = 0.829$$

Therefore, IFN corresponding to fuzzy number of A_1 against criterion C_1 is (0.122, 0.829). Similarly other IFNs are constructed to have IF decision matrix (Table 3).

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	(0.122, 0.829)	(0.138, 0.738)	(0.262, 0.701)	(0.271, 0.643)	(0.27, 0.67)	(0.265, 0.72)	(0.272, 0.602)
A_2	(0.566, 0.385)	(0.69, 0.186)	(0.395, 0.569)	(0.428, 0.486)	(0.21, 0.73)	(0.526, 0.46)	(0.178, 0.695)
A_3	(0.079, 0.872)	(0.138, 0.738)	(0.05, 0.913)	(0.454, 0.46)	(0.428, 0.512)	(0.485, 0.5)	(0.388, 0.485)
A_4	(0.546, 0.405)	(0.276, 0.6)	(0.463, 0.5)	(0.359, 0.555)	(0.399, 0.54)	(0.561, 0.424)	(0.349, 0.524)
A_5	(0.206, 0.745)	(0.276, 0.6)	(0.317, 0.646)	(0.402, 0.512)	(0.084, 0.856)	(0.026, 0.96)	(0.233, 0.64)
A_6	(0.328, 0.623)	(0.155, 0.721)	(0.681, 0.282)	(0.158, 0.756)	(0.111, 0.828)	(0.102, 0.883)	(0.186, 0.687)
A_7	(0.339, 0.612)	(0.276, 0.6)	(0.233, 0.73)	(0.239, 0.675)	(0.636, 0.304)	(0.25, 0.735)	(0.543, 0.33)

Table 3: Intuitionistic fuzzy decision matrix

Step 2: PIFDM (Table 4) of probability intuitionistic fuzzy value $I_{ij} = (\mu_{ij} \mid p_{ij}, v_{ij} \mid p_{ij})$ is constructed. Probabilities to both membership and non-membership values are associated as follows:

$$\mu(p_{11}) = \frac{0.122}{0.122 + 0.829} = 0.128 \ , \qquad v(p_{11}) = \frac{0.829}{0.122 + 0.829} = 0.872$$

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	{(0.122| 0.128), (0.829| 0.872)}	{(0.138| 0.158), (0.738| 0.842)}	{(0.262| 0.272), (0.701| 0.728)}	{(0.271| 0.297), (0.566| 0.595)}	{(0.385| 0.405), (0.69| 0.788)}	{(0.186| 0.212), (0.395| 0.41)}	{(0.569| 0.59), (0.428| 0.469)}
A_2	{(0.079| 0.083), (0.872| 0.917)}	{(0.138| 0.158), (0.738| 0.842)}	{(0.05| 0.052), (0.913| 0.948)}	{(0.454| 0.497), (0.546| 0.574)}	{(0.405| 0.426), (0.276| 0.315)}	{(0.6| 0.680), (0.463| 0.381)}	{(0.5| 0.519), (0.359| 0.392)}
A_3	{(0.206| 0.216), (0.745| 0.784)}	{(0.276| 0.315), (0.6| 0.685)}	{(0.317| 0.329), (0.646| 0.671)}	{(0.402| 0.439), (0.328| 0.345)}	{(0.623| 0.655), (0.155| 0.177)}	{(0.721| 0.823), (0.681| 0.708)}	{(0.282| 0.292), (0.158| 0.173)}

A_4	{(0.339| 0.356), (0.612| 0.644)}	{(0.276| 0.315), (0.6| 0.685)}	{(0.233| 0.242), (0.73| 0.758)}	{(0.239| 0.262), (0.643| 0.703)}	{(0.27| 0.713), (0.67| 0.287)}	{(0.265| 0.731), (0.72| 0.269)}	{(0.272| 0.689), (0.602| 0.311)}
A_5	{(0.486| 0.531), (0.21| 0.777)}	{(0.73| 0.223), (0.526| 0.466)}	{(0.46| 0.534), (0.178| 0.796)}	{(0.695| 0.204), (0.46| 0.503)}	{(0.428| 0.545), (0.512| 0.455)}	{(0.485| 0.508), (0.5| 0.492)}	{(0.388| 0.556), (0.485| 0.444)}
A_6	{(0.555| 0.608), (0.399| 0.575)}	{(0.54| 0.425), (0.561| 0.43)}	{(0.424| 0.57), (0.349| 0.6)}	{(0.524| 0.4), (0.512| 0.561)}	{(0.084| 0.911), (0.856| 0.089)}	{(0.026| 0.974), (0.96| 0.026)}	{(0.233| 0.733), (0.64| 0.267)}
A_7	{(0.756| 0.827), (0.111| 0.881)}	{(0.828| 0.119), (0.102| 0.896)}	{(0.883| 0.104), (0.186| 0.786)}	{(0.687| 0.214), (0.675| 0.738)}	{(0.636| 0.323), (0.304| 0.677)}	{(0.25| 0.746), (0.735| 0.254)}	{(0.543| 0.378), (0.33| 0.622)}

Table 4: Probabilistic intuitionistic fuzzy decision matrix

Step 3: PIF-PIS ($\hbar_P^{+}$) and PIF-NIS ($\hbar_P^{-}$) are obtained as follows:

$$\hbar_P^{+} = (\mu_j^{+}, \nu_j^{+}) = \{(0.34,0.16),(0.54,0.04),(0.48,0.08),(0.23,0.23),(0.08,0.23),(0.03,0.246),(0.14,0.22)\}$$

$$\hbar_P^{-} = (\mu_j^{-}, \nu_j^{-}) = \{(0.01,0.72),(0.02,0.62),(0.003,0.87),(0.03,0.63),(0.23,0.08),(0.25,0.025),(0.22,0.14)\}$$

Step 4: Distance $d(A_i, \hbar_P^{+})$ between A_i and PIF-PIS $(\hbar_P^{+})$ and the distance $d(A_i, \hbar_P^{-})$ between A_i and PIF-NIS $(\hbar_P^{-})$ respectively are calculated and are shown in following table (Table 5).

Alternative	$d(A_i, \hbar_P^{+})$	$d(A_i, \hbar_P^{-})$
A_1	0.582	0.355
A_2	0.32	0.582
A_3	0.784	0.394
A_4	0.457	0.607
A_5	0.457	0.432
A_6	0.555	0.784
A_7	0.472	0.329

Table 5: Distance of alternatives from PIF-PIS and PIF-NIS

Step 5: Closeness coefficients $Cc_i = \frac{d(A_i, \hbar_P^-)}{d(A_i, \hbar_P^+) + d(A_i, \hbar_P^-)}$, $i = 1, 2, \ldots\ldots, 7$ for each alternative are computed and are shown in following table (Table 6).

Cc_1	Cc_2	Cc_3	Cc_4	Cc_5	Cc_6	Cc_7
0.3791	0.6449	0.3343	0.5703	0.4862	0.5855	0.4108

Table 6: Closeness coefficients value

Step 6: Organizations are ranked as $A_2 > A_6 > A_4 > A_5 > A_7 > A_1 > A_3$.in the accordance with decreasing value of closeness coefficient. This shows that the most and the least desirable alternative confirmed by using proposed PIF TOPSIS method is A_2 and A_3 respectively.

4.2 Comparative Analysis

To demonstrate the recital of our proposed PIF TOPSIS method, we make a comparative analysis with IF TOPSIS method developed by Joshi & Kumar [18]. The ranking results based on Joshi & Kumar [18] are $A_2 > A_5 > A_6 > A_4 > A_1 > A_7 > A_3$. It is clear from above results that the ranking of alternatives derived from above proposed approach is slightly different from results derived by Joshi & Kumar's [18] method. But the most and the least desirable alternative confirmed by using proposed method and Joshi & Kumar's [18] methods are same.

Method	Ranking	Best / Worst
Proposed method	$A_2 > A_5 > A_4 > A_6 > A_7 > A_1 > A_3$	A_2/A_3
Joshi & Kumar's method	$A_2 > A_5 > A_6 > A_4 > A_1 > A_7 > A_3$	A_2/A_3

Table 7: Ranking of alternatives by different methods

This slight difference in ranking is due to reason that we have introduced probabilistic information in IFEs, which can be useful to ensure the integrality and accuracy of intuitionistic fuzzy information. The decision making framework of proposed PIF-TOPSIS method make decision results more reasonable and accurate.

5. Conclusion

In this research paper, we have developed an interactive approach to solve real life DMPs under probabilistic intuitionistic fuzzy environment. The main advantage of PIFS is that it can effectively handle comprehensive concurrence of randomness and fuzziness. Hence, PIFS can handle both statistical and non-statistical uncertainty. The core intention of this paper is to propose a PIF TOPSIS method for MCDM problem. In this proposed method a conversion theorem to convert fuzzy set to intuitionistic fuzzy set is also used. Firstly, we develop distance measures for PIFEs. Distance measures are also extended for PIFS and were used in proposed TOPSIS method to MCDM problem.

PIFPIS and PIFNIS are also defined for PIFS. A real life decision making problem is also undertaken to indicate the feasibility, reasonable and applicability of the developed multi-criteria decision making (MCDM) methodology. The decision making framework of proposed PIF-TOPSIS method make decision results more reasonable and accurate than other MCDM methods, because we have introduced suitable probabilistic information in IFEs, which can be useful to ensure the integrality and accuracy of intuitionistic fuzzy information.

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