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BAYES ESTIMATORS OF SHAPE PARAMETER OF PARETO DISTRIBUTION UNDER TWO DIFFERENT LOSS FUNCTIONS

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Abstract

In this paper, Bayes estimators of the shape parameter θ of Pareto distribution have been attained for different priors. The paper also discusses the comparison of Bayes estimators of θ and other estimators like, uniformly minimum variance unbiased estimator (UMVUE) and Maximum likelihood estimator (MLE) of θ under Two loss functions namely, Asymmetric Precautionary Loss Function (APLF) and Squared Error Loss Function (SELF). The results have been illustrated using a simulation study with varying sample sizes through R software.

Key Words: Maximum Likelihood Estimator, Prior, Bayes Estimator, Shape Parameter, Squared Error Loss Function And Asymmetric Precautionary Loss Function.

1. Introduction

The Pareto distribution was discovered by a very famous economist Wilfredo Pareto (1848-1923). It is also known as power law probability distribution or a model for the distribution of income. This distribution is not only useful in economics but also useful in medical, demographic, biological and sociological fields. It also plays an important role in Reliability Theory and Queuing Models in Engineering and Operations Research, respectively.

A random variable T is said to have a Pareto distribution with two parameters, if its probability density function is given by:

$$f(t,\alpha,\theta)=\frac{\theta\alpha^{\theta}}{t^{\theta+1}} \qquad t \geq \alpha,\ \alpha > 0,\ \theta > 0 \tag{1}$$

$$= 0, \qquad \text{otherwise}$$

Where, t is a random variable, θ is a shape parameter and α is a scale parameter.

The cumulative distribution function of (1) is given by-

$$F(t) = 1 - \left(\frac{\alpha}{t}\right)^{\theta} \quad \alpha > 0,\ \theta > 0,\ t \geq \alpha, \tag{2}$$

The reliability function of Pareto distribution is

$$R(t) = P(T \geq t) = \int_t^{\infty} f(t, \alpha, \theta)\, dt$$

$$R(t) = \left(\frac{\alpha}{t}\right)^{\theta} \qquad \theta > 0,\ \alpha > 0, t \geq \alpha, \tag{3}$$

The hazard function of Pareto distribution is

$$h(t) = \frac{f(t, \alpha, \theta)}{R(t)} = \frac{\theta}{t} \qquad \theta > 0,\ \alpha > 0,\ t \geq \alpha, \tag{4}$$

Moments

r^th^ moment about origin

$$\mu_r' = E(T^r) = \frac{\theta\alpha^r}{\theta - r} \qquad [\theta > r]$$

$$\mu_1' = E(T) = \frac{\theta\alpha}{\theta - 1} \qquad [\theta > 1] \tag{5}$$

$$\mu_2 = V(T) = \frac{\theta\alpha^2}{(\theta-2)(\theta-1)^2} \qquad [\theta > 2] \tag{6}$$

$$\mu_3 = \frac{2\alpha^3\theta(\theta+1)}{(\theta-3)(\theta-2)(\theta-1)^3} \qquad [\theta > 3] \tag{7}$$

$$\mu_4 = \frac{3\alpha^4\theta(3\theta^2+\theta+2)}{(\theta-4)(\theta-3)(\theta-2)(\theta-1)^4} \qquad [\theta > 4] \tag{8}$$

Coefficients of Skewness and Kurtosis

$$\gamma_1 = \frac{2(\theta+1)}{(\theta-3)}\sqrt{\frac{\theta-2}{\theta}} \qquad [\theta > 3] \tag{9}$$

$$\gamma_2 = \frac{6(\theta^3+\theta^2-6\theta-2)}{\theta(\theta-3)(\theta-4)} \qquad [\theta > 4] \tag{10}$$

Since γ_1 and γ_2 are positive, hence the distribution is positively skewed and leptokurtic.

2. Classical Estimation

A lot of work has been done about the inferences of Pareto distribution. Quandt (1966) used some old and new techniques of estimation to obtain different estimators for various parameters of Pareto distribution. Hosking and Wallis (1987) have shown that uniform, exponential and Pareto distributions are some special cases of generalized Pareto distribution. Rytgaard (1990) has obtained MLE and moment estimators of scale and shape parameters of Pareto distribution. A comparison has also been made by him between these two methods by using simulation and it is found that the maximum likelihood method is superior to method of moments.

In this part of the paper, the Maximum Likelihood Estimator and Uniformly Minimum Variance Unbiased Estimator for the shape parameter of Pareto distribution are discussed.

2.1 Maximum likelihood estimators

Let $t_1, t_2, t_3, \ldots.., t_n$ be a random sample of size n from the proposed life testing model, whose p.d.f. is shown in (1).
Then Likelihood function (L) is given by

$$L = L(t_i, \alpha, \theta) = \prod_{i=1}^{n} \frac{\theta \alpha^{\theta}}{t_i^{\theta+1}} = \frac{\theta^n \alpha^{n\theta}}{\prod_{i=1}^{n} t_i^{\theta+1}} = \theta^n \alpha^{n\theta} e^{-(\theta+1)\sum_{i=1}^{n} \log_e t_i} \tag{11}$$

$$L = \theta^n e^{n\theta \log_e \alpha} e^{-(\theta+1)\sum_{i=1}^{n} \log_e t_i} \tag{12}$$

To obtain MLE, we solve the following equation

$$\frac{\partial}{\partial \theta} \log_e L = \frac{n}{\theta} + n \log_e \alpha - \sum_{i=1}^{n} \log_e t_i = 0, \tag{13}$$

which gives $$\hat{\theta}_{MLE} = \frac{n}{\sum_{i=1}^{n} \log_e \left(\frac{t_i}{\alpha} \right)} \tag{14}$$

and $$MSE(\hat{\theta}_{MLE}) = \frac{\theta^2 (n+2)}{(n-1)(n-2)} \tag{15}$$

2.2 Exponential family and uniformly minimum variance unbiased estimator

An exponential family with one parameter of density f (t, θ) can be expressed as
f (t, θ) = a(θ) b(t) exp[c(θ) d(t)]

The Pareto distribution belongs to the exponential family because its density function (1) can be written as

$$f(t, \theta, \alpha) = \theta e^{\theta \log_e \alpha - (\theta+1) \log_e t} = \theta e^{\theta \log_e \alpha - \theta \log_e t - \log_e t} = \theta e^{-\log_e t} e^{-\theta \log_e \left(\frac{t}{\alpha} \right)}$$

Where $a(\theta) = \theta, \quad b(t) = e^{-\log_e t}, \quad c(\theta) = -\theta, \quad d(t) = \log_e \left(\frac{t}{\alpha} \right),$ (16)

Theorem: Let $T_1, T_2, \ldots\ldots, T_n$ be a random sample from f $(\cdot,\theta), \theta \in \Theta$, where Θ is an interval (possibly infinite). If f(t, θ) = a(θ) b(t) exp[c(θ) d(t)], i.e. f $(\cdot,\theta)$ is a member of one parameter exponential family, Then $\left(\sum_{i=1}^{n} d(T_i)\right)$ is a complete sufficient statistic.

Therefore, statistic $\left(\sum_{i=1}^{n} \log_e \left(\frac{t_i}{\alpha}\right)\right)$ is a complete sufficient statistic for θ.

$$\textit{If } T \sim \textit{Pareto } (\alpha, \theta), \textit{ Then } \ln\left(\frac{t}{\alpha}\right) \sim \exp(\theta)$$

$$\Rightarrow P = \left(\sum_{i=1}^{n} \log_e \left(\frac{t_i}{\alpha}\right)\right) \sim Gamma\ (n, \theta) \tag{17}$$

Now the probability density function of P is given by

$$g(p, n, \theta) = \frac{\theta^n}{\Gamma n} p^{n-1} e^{-\theta p}; \qquad p \geq 0, \theta > 0 \tag{18}$$

$$\Rightarrow E\left(\frac{P}{n}\right) = \frac{1}{\theta} \qquad \text{and} \qquad E\left(\frac{1}{P}\right) = \int_0^\infty \frac{1}{p} \frac{\theta^n}{\Gamma n} p^{n-1} e^{-\theta p}\, dp = \frac{\theta}{n-1}$$

$$\Rightarrow E\left(\frac{n-1}{P}\right) = \theta \tag{19}$$

Here, $\frac{n-1}{P}$ is unbiased estimator for θ and P represents a complete sufficient statistics for θ. Thus, by Lehmann-Scheffe Theorem, the UMVUE of θ denoted by $\widehat{\theta}_{UMBUE}$ is given by

$$\widehat{\theta}_{UMBUE} = \frac{n-1}{P} \tag{20}$$

and
$$MSE(\widehat{\theta}_{UMVUE}) = \frac{\theta^2}{(n-2)} \tag{21}$$

3. Bayesian Estimation

Bayesian analysis is an important and popular approach to the statisticians. In this approach, parameters are considered as random variables and data are considered fixed. In lifetime distributions, this analysis plays a very important role but its implementation is so tough because if someone is interested to implement Bayesian analysis using lifetime models, the likelihood function and the prior provide quite difficult posterior forms which are mostly impossible to analyze analytically. It is also very challenging for the usual numerical perspective. A lot of work in Bayesian estimation of Pareto distribution has been done by many authors. Giorgi and Crescenzi (2001) have proposed Bayes estimators of Bonferroni index (*B*) from a Pareto type-I population under squared error loss function by using truncated Erlang prior and the translated exponential prior. Ertefaie and Parsian (2005) have estimated Bayes estimators for the parameters of Pareto distribution under LINEX loss function with unknown scale and shape parameters. Kifayat et al. (2012) have analyzed power distribution by using gamma and Rayleigh as informative priors and Jeffreys and uniform as non-informative priors and draw some conclusions regarding Bayesian estimation. Setiya and Kumar (2013) have analyzed Pareto distribution and drew some Bayes estimators of the related parameters for different priors with SELF and APLF through Lindley's approach. They also conquered Bayes estimation of reliability and hazard rate functions. Rasheed and Al-Gazi (2014) have obtained Bayes estimators of the shape parameter of the Pareto distribution under two different loss functions. Setiya and Kumar (2016) have used two different methods to obtain the Bayes estimators of the parameters of a Pareto distribution.

In this part, we attain Bayes estimators of the θ of (1) considering α as fixed, under different priors viz. Jeffrey's, exponential and gamma.

3.1 Prior distribution

In Bayesian inference, the prior distribution represents the information about an uncertain parameter θ. The posterior distribution, which is useful for future inferences and making decision, is the product of prior distribution and the probability distribution of new data set. The derivation of the prior distribution based on evidence other than the current data is impossible or rather problematic because the likelihood function and the prior provide quite difficult posterior forms which are impossible to analyze analytically and are even very challenging from the usual numerical perspective. Hence, it is necessary to employ as minor subjective input as possible, so that the result may look merely based on sampling model and present data set. Here we are using Jeffrey's, exponential and gamma as a prior.

3.1.1 Jeffrey's prior

Jeffrey's suggested a proper rule for obtaining a non-informative prior. It is proportional to the square root of the determinant of the Fisher information:

$$g(\theta) \propto \sqrt{|I(\theta)|}$$

Where θ = k-vector valued parameter and $I(\theta)_{k\mathrm{x}k}$ = Fisher's information matrix.

If we consider θ as a scalar parameter, Jeffrey's non-informative prior for θ is $g(\theta) \propto \sqrt{I(\theta)}$. Thus, we consider

$$g(\theta) \propto \frac{1}{\theta} \qquad \Rightarrow \qquad g(\theta) = c\,\frac{1}{\theta} \tag{22}$$

Where c is a constant.

3.1.2 Exponential prior

$$g(\theta) = \frac{1}{\beta} e^{-\theta/\beta} \qquad ; \quad \theta > 0,\ \beta > 0 \tag{23}$$

3.1.3 Gamma prior

$$g(\theta)=\frac{a^{\lambda}}{\Gamma\lambda}e^{-a\theta}\theta^{\lambda-1} \quad ;\ \theta>0,\ \lambda>0, a>0 \tag{24}$$

3.2 Posterior distribution

The posterior distribution of θ given the random sample for fixed α is given by

$$\pi(\theta/t_1,t_2,....,t_n) = \frac{L(t_1,t_2,.......,t_n|\theta)g(\theta)}{\int_{\Theta} L(t_1,t_2,\_\_\_\_,t_n|\theta)g(\theta)d\theta}$$

Under Jeffrey's Prior, it becomes

$$\pi(\theta/t_1,t_2,....,t_n) = \frac{P^n}{\Gamma n}e^{-P\theta}\theta^{n-1} \quad ;\ P>0, n>0, \theta>0 \tag{25}$$

Under Exponential Prior, we have

$$\pi(\theta/t_1,t_2,....,t_n) = \frac{\left(P+\frac{1}{\beta}\right)^{n+1}}{\Gamma n+1}e^{-(P+1/\beta)\theta}\theta^{n} \quad ;\ P>0, n>0, \beta>0, \theta>0 \tag{26}$$

and under Gamma Prior, we have

$$\pi(\theta/t_1,t_2,....t_n) = \frac{(P+a)^{n+\lambda}}{\Gamma n+\lambda}e^{-(P+a)\theta}\theta^{n+\lambda-1} \quad ;\ P>0, n>0, \lambda>0, \theta>0, a>0 \tag{27}$$

3.3 Loss function

The loss function plays an important role in Bayesian inference. A loss function is a background mathematical convention which requires much greater attention than the traditional manner. The estimator which has least expected loss is preferred to use as compared to the others. Most authors prefer posterior mean as the Bayesian estimate by using simple quadratic (symmetric) loss function. However, in practice, the real loss function is often not symmetric. In this part, we have used two loss functions (i) Squared Error Loss Function (SELF) and (ii) Asymmetric Precautionary Loss Function (APLF).

3.3.1 Squared Error Loss Function (SELF)

A squared error loss function (SELF) is very commonly used loss function and is given as

$$L(\hat{\theta},\theta)=(\hat{\theta}-\theta)^2 \tag{28}$$

It is also known as symmetric loss function because it allocates equal losses to over estimation and under estimation.

3.3.2 Asymmetric Precautionary Loss Function (APLF)

An asymmetric precautionary loss function (APLF) is given as

$$L(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \tag{29}$$

4. Estimation Under Square Error Loss

In this part, we obtain posterior expected losses of Bayes estimator of θ for different priors under SELF. Posterior expected loss (ρ) of Bayes estimator is given by

$$\rho=\int_0^\infty (\hat{\theta}-\theta)^2\pi(\theta/t_1,t_2,......,t_n)\,d\theta$$

Under Jeffrey's Prior

$$\rho=\int_0^\infty (\hat{\theta}-\theta)^2\pi(\theta/t_1,t_2,......,t_n)\,d\theta$$

$$\rho=\hat{\theta}_J^{\,2}+\frac{n(n+1)}{P^2}-\frac{2\hat{\theta}_J n}{P} \tag{30}$$

$$Solving\ \frac{\partial\rho}{\partial\hat{\theta}_J}=0,$$

We obtain Bayes Estimator as

$$\hat{\theta}_J=\frac{n}{P} \tag{31}$$

Under Exponential Prior

$$\rho=\hat{\theta}_E^{\,2}+\frac{(n+2)(n+1)}{\left(P+\frac{1}{\beta}\right)^2}-\frac{2\hat{\theta}_E(n+1)}{\left(P+\frac{1}{\beta}\right)} \tag{32}$$

$$Solving\ \frac{\partial\rho}{\partial\hat{\theta}_E}=0$$

We obtain Bayes Estimator as

$$\hat{\theta}_E=\frac{n+1}{\left(P+\frac{1}{\beta}\right)} \tag{33}$$

Under Gamma Prior

$$\rho=\hat{\theta}_G^{\,2}+\frac{(n+\lambda)(n+\lambda+1)}{(P+a)^2}-\frac{2\hat{\theta}_G(n+\lambda)}{(P+a)} \tag{34}$$

$$Solving\ \frac{\partial \rho}{\partial \hat{\theta}_G} = 0$$

We obtain Bayes Estimator as

$$\hat{\theta}_G = \frac{n+\lambda}{(P+a)} \tag{35}$$

5. Estimation under Asymmetric Precautionary Loss

In this part, we obtain posterior expected losses of Bayes estimator of θ for different priors under APLF. Posterior expected loss of Bayes estimator is given as

$$\rho_A = \int_0^\infty \left(\frac{(\hat{\theta}-\theta)^2}{\hat{\theta}} \right) \pi(\theta / x_1, x_2, \ldots\ldots, x_n)\, d\theta$$

Under Jeffrey's Prior

$$\rho_A = \hat{\theta}_J + \frac{n(n+1)}{\hat{\theta}_J P^2} - \frac{2n}{P} \tag{36}$$

$$Solving\ \frac{\partial \rho_A}{\partial \hat{\theta}_J} = 0,$$

We obtain Bayes Estimator as

$$\hat{\theta}_J = \frac{\sqrt{n(n+1)}}{P} \tag{37}$$

Under Exponential Prior

$$\rho_A = \hat{\theta}_E + \frac{(n+2)(n+1)}{\hat{\theta}_E \left(P + \frac{1}{\beta} \right)^2} - \frac{2(n+1)}{\left(P + \frac{1}{\beta} \right)} \quad (3$$

$$Solving\ \frac{\partial \rho_A}{\partial \hat{\theta}_E} = 0,$$ we obtain Bayes estimator as

$$\hat{\theta}_E = \frac{\sqrt{(n+1)(n+2}}{\left(P + \frac{1}{\beta} \right)} \tag{39}$$

Under Gamma Prior

$$\rho = \hat{\theta}_G + \frac{(n+\lambda)(n+\lambda+1)}{\hat{\theta}_G (P+a)^2} - \frac{2(n+\lambda)}{(P+a)} \tag{40}$$

$$Solving\ \frac{\partial \rho_A}{\partial \hat{\theta}_G} = 0,$$

We obtain Bayes Estimator as

$$\widehat{\theta}_G = \frac{\sqrt{(n+\lambda)(n+\lambda+1)}}{(P+a)} \tag{41}$$

6. Simulation Study

For simulation study, we have used R = 1000 replications, for samples of sizes n = 20, 50, and 100 respectively from Pareto distribution for shape parameter θ =2 and fixed α = 4 and 5 respectively. We have chosen β=1, 2 for the Exponential prior and (λ,a)= (1,1), (1,2) and (2,1), (2,2) respectively for gamma prior. After estimating the parameters, mean square error may be calculated by

$$MSE = \frac{\sum_{i=1}^{R} (\hat{\theta}_i - \theta)^2}{R}$$

The simulation study results for estimating the shape parameter (θ) of Pareto distribution when the scale parameter (α) is known, are précised and tabulated in Tables (1), (2) and (3) which comprise the MLE, UMVUE, Bayes Estimator values and MSE's for estimating the shape parameter (θ).

N	**α**	**θ**	$\widehat{\omega}_{MLE}$	**MSE**	$\widehat{\omega}_{UMVUE}$	**MSE**
20	4	2	2.09661	0.29902	1.99178	0.233065
	5	2	2.09661	0.29902	1.99178	0.233065
50	4	2	2.039578	0.093875	1.998786	0.084956
	5	2	2.039833	0.093897	1.999036	0.084976
100	4	2	2.02073	0.043398	2.000523	0.041283
	5	2	2.01381	0.043083	1.993672	0.040984

N	**α**	**θ**	$\widehat{\omega}^{J}_{SELF}$	**MSE**	$\widehat{\omega}^{J}_{APLF}$	**MSE**
20	4	2	2.09661	0.253066	2.173701	0.280438
	5	2	2.09661	0.262054	2.148386	0.287265
50	4	2	2.039578	0.087722	2.059873	0.091464
	5	2	2.039833	0.087716	2.06013	0.091467
100	4	2	2.02073	0.044962	2.030809	0.045927
	5	2	2.01381	0.042759	2.023854	0.043563

Table 1: Bayes Estimators of θ and their corresponding MSE's under Jeffrey's Prior

N	α	θ	β	$\widehat{\theta}_{MLE}$	MSE	$\widehat{\theta}_{UMVUE}$	MSE
20	4	2	1	2.1085	0.3021	2.0031	0.2355
	5	2	1	2.100975	0.298054	1.995927	0.232313
	4	2	2	2.07663	0.29189	1.972799	0.227508
	5	2	2	2.096164	0.298432	1.991356	0.232607
50	4	2	1	2.041914	0.094077	2.001076	0.085139
	5	2	1	2.051249	0.094859	2.010224	0.085846
	4	2	2	2.027894	0.092915	1.987336	0.084087
	5	2	2	2.047207	0.094663	2.006263	0.085669
100	4	2	1	2.009327	0.042868	1.989233	0.040779
	5	2	1	2.011255	0.0429	1.991142	0.040809
	4	2	2	2.01662	0.043165	1.996454	0.041061
	5	2	2	2.027961	0.043707	2.007681	0.041577

N	α	θ	β	$\widehat{\theta}^{E}_{SELF}$	MSE	$\widehat{\theta}^{E}_{APLF}$	MSE
20	4	2	1	1.993232	0.1774	2.040138	0.187451
	5	2	1	1.987943	0.156399	2.034724	0.164900
	4	2	2	2.067829	0.202798	2.11649	0.221205
	5	2	2	2.085902	0.223879	2.134988	0.245032
50	4	2	1	1.999487	0.075522	2.018995	0.077363
	5	2	1	2.008328	0.07293	2.027922	0.075069
	4	2	2	2.026475	0.086924	2.046246	0.090052
	5	2	2	2.045391	0.088773	2.065347	0.092683
100	4	2	1	1.989064	0.037862	1.998886	0.038117
	5	2	1	1.990981	0.033366	2.000813	0.033615
	4	2	2	2.016263	0.038429	2.02622	0.039229
	5	2	2	2.027461	0.044446	2.037473	0.045529

Table 2: Bayes Estimators of θ and their corresponding MSE's under exponential prior

N	α	(θ)	λ	A	$\hat{\theta}_{MLE}$	MSE	$\hat{\theta}_{UMVUE}$	MSE
20	4	2	1	1	2.105278	0.300151	2.000014	0.233947
	4	2	1	2	2.087662	0.295894	1.983279	0.230629
	4	2	2	1	2.096918	0.2999	1.992072	0.233752
	4	2	2	2	2.096866	0.298296	1.992022	0.232502
	5	2	1	1	2.093407	0.298093	1.988736	0.232343
	5	2	1	2	2.083026	0.294643	1.978875	0.229654
	5	2	2	1	2.095028	0.29998	1.990277	0.233814
	5	2	2	2	2.087704	0.295921	1.983319	0.23065
50	4	2	1	1	2.045345	0.094408	2.004438	0.085438
	4	2	1	2	2.049513	0.094782	2.008522	0.085777
	4	2	2	1	2.045357	0.094415	2.00445	0.085445
	4	2	2	2	2.040399	0.09408	1.999591	0.085142
	5	2	1	1	2.048544	0.094651	2.007573	0.085658
	5	2	1	2	2.050968	0.094766	2.009948	0.085763
	5	2	2	1	2.047524	0.094431	2.007434	0.085472
	5	2	2	2	2.045761	0.094349	2.004846	0.085385
100	4	2	1	1	2.026022	0.043585	2.005761	0.041461
	4	2	1	2	2.021627	0.043363	2.00141	0.04125
	4	2	2	1	2.02069	0.043348	2.000483	0.041236
	4	2	2	2	2.020108	0.043321	1.999907	0.04121
	5	2	1	1	2.024666	0.043533	2.00442	0.041412
	5	2	1	2	2.021688	0.043416	2.001471	0.041301
	5	2	2	1	2.024325	0.042353	2.00321	0.041251
	5	2	2	2	2.020456	0.043354	2.000251	0.041242

N	α	θ	λ	A	$\hat{\theta}^{G}_{SELF}$	MSE	$\hat{\theta}^{G}_{APLF}$	MSE
20	4	2	1	1	1.991094	0.166548	2.03795	0.175836
	4	2	1	2	2.069711	0.193883	1.927635	0.13971
	4	2	2	1	2.077223	0.209428	2.123908	0.228066
	4	2	2	2	2.078078	0.19344	1.934853	0.137044
	5	2	1	1	1.980166	0.180782	2.026765	0.189695
	5	2	1	2	1.971624	0.173619	1.838463	0.149495
	5	2	2	1	2.075058	0.219762	2.121695	0.238671
	5	2	2	2	2.06974	0.194075	1.927653	0.139841

50	4	2	1	1	2.002699	0.076515	2.022238	0.078502
	4	2	1	2	2.045972	0.081346	1.985991	0.069044
	4	2	2	1	2.041977	0.081332	2.061518	0.084885
	4	2	2	2	1.997947	0.081118	2.017439	0.083008
	5	2	1	1	2.045077	0.079567	2.064648	0.083205
	5	2	1	2	2.008116	0.070424	1.949654	0.063695
	5	2	2	1	2.043717	0.078657	2.063128	0.083356
	5	2	2	2	2.04245	0.077108	1.982767	0.065682
100	4	2	1	1	2.005258	0.038516	2.015161	0.039099
	4	2	1	2	2.020837	0.036509	1.990933	0.033721
	4	2	2	1	2.019897	0.038692	2.029774	0.039558
	4	2	2	2	1.999531	0.037365	2.009406	0.037824
	5	2	1	1	2.023778	0.040382	2.033675	0.041341
	5	2	1	2	2.001039	0.039888	1.97148	0.038004
	5	2	2	1	2.022876	0.039842	2.028769	0.040125
	5	2	2	2	2.019652	0.040193	1.989753	0.03722

Table 3: Bayes Estimators of θ with their corresponding MSE under gamma prior

7. Conclusion

By comparing the results of our study, it is found that in most of the cases Bayes estimators with squared error loss function (SELF) under exponential priors have the least MSE whereas UMVUE has least MSE than MLE, Bayes estimators with SELF and APLF under Jeffrey prior. Bayes estimator with APLF has least MSE than MLE, UMVUE and Bayes estimator with SELF under gamma prior when α =5 and Bayes estimator with SELF has least MSE than MLE, UMVUE and Bayes estimator with APLF under gamma priors when α =4. Thus, Bayes estimator of shape parameter θ under SELF using exponential prior is more efficient than MLE, Bayes estimator under APLF and UMVUE.

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