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ON SHRINKAGE FACTORS IN EXPONENTIAL LIFE MODEL USING LINEX LOSS FUNCTION

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Abstract

The study proposes various choices for shrinkage factors (weights) for the scale parameter of an exponential distribution under LINEX loss function. Shrinkage factor based on the test statistic has been considered. Another choice of shrinkage factor is also considered. It is observed that these testimators perform better than other estimators in this class, as the risk (s) of these compared to best available estimator turns out to be smaller. Some of the values of degree of asymmetry and level of significance for the best performance have been reported.

Key Words: Exponential Distribution, Scale Parameter, Shrinkage Factors, Asymmetric Loss Function, Relative Risk.

1. Introduction

Exponential distributions play a vital role in life testing and reliability estimation. Of late, R.V. Hogg and S.A. Kingman (1984) modeled exponential distribution as a Loss Distribution to settle down the insurance claims and for some problems of Re-insurance also it fits well. So, estimation or testimation of its parameter assumes more importance. For insurance policy claims, it may be very easy to have an idea of it, from the past claims i.e. we have some knowledge about the parameter to be estimated and this may be used to get some improved estimator.

If, we do not want to utilize this prior information (a point guess) of insurance claims indiscriminately, a pre-test estimator along the lines of Thompson (1968, a) can be proposed, involving the shrinkage factor 'k '. Pandey (1983), Srivastava and Pandey (1985) and Srivastava and Pandey (1987) have proposed shrinkage testimators with different choices of shrinkage factors. It has been observed that by taking higher powers of shrinkage factor, the proposed testimators perform better, which was established by taking the Square of the shrinkage factor by Srivastava and Shah (2012, 2015) among others.

Several authors have considered many choices of shrinkage factor and it should lie between '0' and '1'. But in all the choices of 'k' or 'k^2 these limits are not attained unless $\chi_1^2 = 0$ or $\chi_2^2 = \infty$. So, we have proposed another choice of and then it exactly lies between '0' and '1'. By taking the shrinkage factor as

$$k' = \left(\frac{2n\bar{x}}{\theta_0 \chi'^2} - \frac{\chi_1^2}{\chi'^2} \right)$$ where $\chi'^2 = (\chi_2^2 - \chi_1^2)$. In this case the exact limits '0'and'1' are attained.

We have proposed two testimators for θ , viz. $\hat{\theta}_{ST_1}$ (which considers the shrinkage factor square) and $\hat{\theta}_{ST_2}$ using k′.

First utilizing the sample information we compute the sample mean, $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ which is UMVUE, when no other information is available.

Next we test the hypothesis Ho: θ = θo against two sided alternative H_1: θ ≠ θo at α % level of using the test statistic $\frac{2n\bar{x}}{\theta_0}$ which follows χ^2 - distribution with 2n degrees of freedom. If the null hypothesis is accepted, it is suggested that shrinkage estimator with shrinkage factor 'k' be used; else do not consider the guess value and use $\bar{x}$, in case the null hypothesis is rejected.

Symmetric loss function penalizes the under and over estimation equally. Hence, several authors have used and advocated the use of 'asymmetric' loss functions, particularly in situations where the over / under estimation are not of same consequences, especially while dealing with claim settlements etc. Varian (1975), Zellner (1986), Basu and Ebrahimi (1991) are prominent among others, for suggesting the convenience and supremacy of the asymmetric loss function, which includes SELF as a special case.

The loss function proposed by Basu and Ebrahimi is defined as:

$$L(\Delta) = b[e^{a\Delta} - a\Delta - 1],\ b>0,\ a \neq 0 \tag{1.1}$$

Where $\Delta = \left(\frac{\hat{\theta}}{\theta} - 1 \right)$

Where 'a' indicates the degree and direction of asymmetry and when overestimation appears to be more serious than underestimation 'a' assumes positive values, where as in the situations. When under estimation is more serious negative values of 'a' are considered. For Δ < 0 the loss function rises exponentially and it is almost linear for positive values of Δ.

In section 2, we define the shrinkage testimators. The risk(s) expressions have been given in section 3. Section 4 is devoted to obtain the Relative Risk(s). Proposed testimators are compared with the best estimator for its performance in section 5.

2. The Testimator (s)

We define the shrinkage testimator $\hat{\theta}_{ST_1}$ of θ as:

$$\hat{\theta}_{ST_1} = \begin{Bmatrix} k\bar{x} + (1-k)\theta_0, & \text{if Ho is accepted} \\ \bar{x}, & \text{Otherwise} \end{Bmatrix} \tag{2.1}$$

where k = $\frac{2n\bar{x}}{\theta_0 \chi^2}$, and $\frac{2n\bar{x}}{\theta_0}$ follows χ^2 distribution with 2n degrees of freedom. The performance of this estimator has been studied by Srivastava and Shah (2010).

Now, squaring 'k', $\hat{\theta}_{ST_1}$ can be re-written as:

$$\hat{\theta}_{ST_1} = \begin{cases} \left(\frac{2n\bar{x}}{\theta_0 \chi^2}\right)^2 \bar{x} + \left[1 - \left(\frac{2n\bar{x}}{\theta_0 \chi^2}\right)^2\right] \theta_0, & \text{if Ho is not rejected} \\ \bar{x} \quad , & \text{Otherwise} \end{cases} \tag{2.2}$$

With the other choice of shrinkage factor 'k′'we have proposed another testimator $\hat{\theta}_{ST_2}$ which is given as follows

$$\hat{\theta}_{ST_2} = \begin{cases} \left(\frac{2n\bar{x}}{\theta_0 \chi'^2} - \frac{\chi_1^2}{\chi'^2}\right)\bar{x} + \left[1 + \frac{\chi_1^2}{\chi'^2} - \frac{2n\bar{x}}{\theta_0 \chi'^2}\right] \theta_0, & \text{if Ho is not rejected} \\ \bar{x}, \quad \text{Otherwise} \end{cases} \tag{2.3}$$

where χ'^2 is defined earlier.

Now we derive the risk(s) of $\hat{\theta}_{ST1}$ and $\hat{\theta}_{ST_2}$ in section-3.

3. Derivation of Risk(s)

The risk of $\hat{\theta}_{ST_1}$ using L (Δ) can be defined as

$$R(\hat{\theta}_{ST_1}) = E[\hat{\theta}_{ST_1} \mid L(\Delta)]$$

$$= E\left[\left(\frac{2n\bar{x}}{\theta_0 \chi^2}\right)^2 \bar{x} + \left[1 - \left(\frac{2n\bar{x}}{\theta_0 \chi^2}\right)^2\right]\theta_0 \Big/ \chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right] \cdot p\left[\chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2\right]$$
$$+ E\left[\bar{x} \,\middle|\, \frac{2n\bar{x}}{\theta_0} < \chi_1^2 \cup \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \cdot p\left[\frac{2n\bar{x}}{\theta_0} < \chi_1^2 \cup \frac{2n\bar{x}}{\theta_0} > \chi_2^2\right] \tag{3.1}$$

$$= e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left[\frac{\left(\frac{2n\bar{x}}{\theta_0\chi^2}\right)^2(\bar{x}-\theta_0)+\theta_0}{\theta}\right]} f(\bar{x})\, d\bar{x} -$$

$$a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left[\frac{\left(\frac{2n\bar{x}}{\theta_0\chi^2}\right)^2(\bar{x}-\theta_0)+\theta_0}{\theta} - 1\right] f(\bar{x})\, d\bar{x}$$

$$- \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} + e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left(\bar{x}/\theta\right)} f(\bar{x})\, d\bar{x}$$

$$-a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left(\frac{\bar{x}}{\theta} - 1\right) f(\bar{x})\, d\bar{x} - \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} \tag{3.2}$$

Where $f(\bar{x}) = \frac{1}{\Gamma n}\left(\frac{n}{\theta}\right)^n (\bar{x})^{n-1} e^{\frac{-n\bar{x}}{\theta}}$

Integrating (3.2) we get

$$R(\hat{\theta}_{ST_1}) = I_1^* - I_2^* + \left\{ I\left(\frac{\chi_1^2\phi}{2}, n\right) - I\left(\frac{\chi_2^2\phi}{2}, n\right) + 1 \right\} \left\{ \frac{e^{-a}}{\left(1 - a/n\right)^n} - 1 \right\} -$$

$$a\left\{ I\left(\frac{\chi_1^2\phi}{2}, n+1\right) - I\left(\frac{\chi_2^2\phi}{2}, n+1\right) \right\} - \left\{ I\left(\frac{\chi_2^2\phi}{2}, n\right) - I\left(\frac{\chi_1^2\phi}{2}, n\right) \right\}(a+1) \tag{3.3}$$

Where

$$I_1^* = e^{a\phi - a} \int_{\frac{\chi_1^2\phi}{2}}^{\frac{\chi_2^2\phi}{2}} e^{a\left[\frac{4t^3}{n\phi^2(\chi^2)^2} - \frac{4t^2}{\phi(\chi^2)^2}\right]} \frac{1}{\Gamma n} e^{-t}\, t^{n-1}\, dt$$

and

$$I_2^* = a \int_{\frac{\chi_1^2\phi}{2}}^{\frac{\chi_2^2\phi}{2}} \left\{ \left[\left(\frac{2t}{\phi\chi^2} \right)^2 \left(\frac{t}{n} - \phi \right) \right] + \phi - 1 \right\} \frac{1}{\Gamma n} e^{-t}\, t^{n-1}\, dt$$

Again, the risk of $\hat{\theta}_{ST_2}$ using L(Δ) can be defined as

$$R(\hat{\theta}_{ST_2}) = E[\hat{\theta}_{ST_2} \mid L(\Delta)]$$

$$= E\left[\left(\frac{2n\bar{x}}{\theta_0\chi'^2} - \frac{\chi_1^2}{\chi'^2} \right)\bar{x} + \left[1 + \frac{\chi_1^2}{\chi'^2} - \frac{2n\bar{x}}{\theta_0\chi'^2} \right]\theta_0 \Big/ \chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2 \right]$$

$$\cdot p\left[\chi_1^2 < \frac{2n\bar{x}}{\theta_0} < \chi_2^2 \right] + E\left[\bar{x} \,\middle|\, \frac{2n\bar{x}}{\theta_0} < \chi_1^2 \cup \frac{2n\bar{x}}{\theta_0} > \chi_2^2 \right] \cdot p\left[\frac{2n\bar{x}}{\theta_0} < \chi_1^2 \cup \frac{2n\bar{x}}{\theta_0} > \chi_2^2 \right] \tag{3.4}$$

$$= e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left[\frac{\left(\frac{2n\bar{x}}{\theta_0\chi'^2} - \frac{\chi_1^2}{\chi'^2}\right)(\bar{x}-\theta_0)+\theta_0}{\theta}\right]} f(\bar{x})\, d\bar{x} - a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left[\frac{\left(\frac{2n\bar{x}}{\theta_0\chi'^2} - \frac{\chi_1^2}{\chi'^2} \right)(\bar{x}-\theta_0)+\theta_0}{\theta} - 1 \right] f(\bar{x})\, d\bar{x}$$

$$- \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} + e^{-a} \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} e^{a\left(\bar{x}/\theta\right)} f(\bar{x})\, d\bar{x} - a \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} \left(\frac{\bar{x}}{\theta} - 1 \right) f(\bar{x})\, d\bar{x} - \int_{\frac{\chi_1^2\theta_0}{2n}}^{\frac{\chi_2^2\theta_0}{2n}} f(\bar{x})\, d\bar{x} \tag{3.5}$$

Now integrating (3.5) using standard integration results we obtain

$$R(\hat{\theta}_{ST_2}) = I_1^* - I_2^* + \left\{ I\left(\frac{\chi_1^2\varphi}{2}, n\right) - I\left(\frac{\chi_2^2\varphi}{2}, n\right) + 1 \right\} \left\{ \frac{e^{-a}}{\left(1 - a/n\right)^n} - 1 \right\} -$$
$$a\left\{ I\left(\frac{\chi_1^2\varphi}{2}, n+1\right) - I\left(\frac{\chi_2^2\varphi}{2}, n+1\right) \right\} - \left\{ I\left(\frac{\chi_2^2\varphi}{2}, n\right) - I\left(\frac{\chi_1^2\varphi}{2}, n\right) \right\} (a+1) \tag{3.6}$$

Where

$$I_1^* = e^{a(\phi-1)} \int_{\frac{\chi_1^2\phi}{2}}^{\frac{\chi_2^2\phi}{2}} e^{a\left[\frac{2t^2}{n\phi\chi'^2} - \frac{2t}{\chi'^2} - \frac{\chi_1^2 t}{\chi'^2 n} + \frac{\chi_1^2 \phi}{\chi'^2}\right]} \frac{1}{\Gamma n} e^{-t} t^{n-1}\, dt$$

and

$$I_2^* = a(\phi-1) \int_{\frac{\chi_1^2\phi}{2}}^{\frac{\chi_2^2\phi}{2}} a\left[\frac{2t^2}{n\phi\chi'^2} - \frac{2t}{\chi'^2} - \frac{\chi_1^2 t}{\chi'^2 n} + \frac{\chi_1^2 \phi}{\chi'^2}\right] \frac{1}{\Gamma n} e^{-t} t^{n-1}\, dt$$

4. Relative Risk(s)

In order to see the performance of the proposed testimators we compare its risk with the Uniformly Most Powerful Estimator $\bar{x}$.

To facilitate this, risk of $\bar{x}$ using L(Δ) is defined by

$$R_E(\bar{x}) = E[\bar{x} \mid L(\Delta)]$$
$$= e^{-a} \int_0^\infty e^{a\left(\bar{x}/\theta\right)} f(\bar{x})\, d\bar{x}$$
$$- a \int_0^\infty \left(\frac{\bar{x}}{\theta} - 1\right) f(\bar{x})\, d\bar{x} - \int_0^\infty f(\bar{x})\, d\bar{x} \tag{4.1}$$

Again integrating (4.1) we get

$$R_E(\bar{x}) = \frac{e^{-a}}{(1-a/n)^n} - 1 \tag{4.2}$$

The relative risk of the proposed testimator as compared to that of $\bar{x}$ using L(Δ) can be defined as-

$$RR_1 = \frac{R_E(\bar{x})}{R(\hat{\theta}_{ST_1})} \tag{4.3}$$

Using equations (4.2) and (3.2) we obtain the mathematical expression for RR_1 in (4.3), it indicates that RR_1 depends onϕ, n, α, and 'a '.The performance of $\hat{\theta}_{ST_1}$ is studied for, ϕ = 0.2 (0.2)…1.6, α = 1%, 5%, 10%, n = 5, 8, 10 and a = ±1, ±2, ±3.

Next, the Relative Risk of $\hat{\theta}_{ST_2}$ with respect to $\bar{x}$ using L(Δ) is given by

$$RR_2 = \frac{R_E(\bar{x})}{R(\hat{\theta}_{ST_2})} \tag{4.4}$$

The equations (4.2) and (3.4) are used for RR_2 expression (4.4). Again RR_2 can be evaluated for ϕ, n, α and 'a'.

The performance of $\hat{\theta}_{ST_2}$ is studied for several values of above mentioned quantities considered earlier for $\hat{\theta}_{ST_1}$ with these many choices there will be several tables of the relative risk(s). Few graphs of RR_1 and RR_2 for the numerical values considered are provided in the appendix. However, our conclusions based on all the graphs are given in the next section.

5. Conclusions

Following are the conclusions for the proposed testimators.

For $\hat{\theta}_{ST_1}$

For different values of n = 5,8,10 we fix α = 1% and the degree of asymmetry is varied for 'a' = ±1, ±2, ±3, the values of relative risk of $\hat{\theta}_{ST_1}$ are higher for the different choices of n^s and a^s and the range of Ø considered here. All the positive values 'a' may be considered in particular a=3.Therefore it is recommended to use this estimator for a=3, α = 1%.

Now take α = 5% for the same data set, again $\hat{\theta}_{ST_1}$ performs better than the simple mean for complete range of 'Ø' taken here. It is observed further that magnitude of relative risk is higher for α = 1% than for α = 5%.

Next a higher value of α = 10% is taken to see the impact of higher level of significance still it is found that $\hat{\theta}_{ST_1}$ fairs better for positive values of 'a' specially for a=2. It is observed that the testimators perform better for 'lower' values level of significance whenever 'a' assumes positive values.

So, $\hat{\theta}_{ST_1}$ is recommended for different data sets considered here, as the proposed testimator performs better than usual estimator. Specially one may consider α = 1% and a=3.

For $\hat{\theta}_{ST_2}$

The another testimator $\hat{\theta}_{ST_2}$ performs better than the usual estimator as it has 'smaller' risk for the data considered here . In fact $\hat{\theta}_{ST_2}$ has lower risk in the entire range of Ø = 0.2(0.2) 1.6. Risk decreases for large values in particular when n = 10. A lower negative value of'a'with a lower value of α can be considered. Further, for smaller values of n i.e. n = 5 and n = 8 also $\hat{\theta}_{ST_2}$ has smaller risk values and in particular for n = 5 and a = 3 its performance is the best.
Considering α = 5% for different values of n = 5, 8, and 10, $\hat{\theta}_{ST_2}$ behaves nicely for positive values of 'a' and for different values in particular take a small sample i.e. n=5, however other values may be also be considered but with little lower gain in relative risk values. The performance remains true for the whole range of Ø.

Considering a high level of significance at α = 10% and same set of values for sample sizes and degree of asymmetry again $\hat{\theta}_{ST_2}$ outperforms the conventional estimator for almost all the values considered here particularly for n = 5 and a = 2, a = 3 the risk values are lowest.

1. Testimators $\hat{\theta}_{ST_1}$ and $\hat{\theta}_{ST_2}$ outperform the UMVUE for entire range of Ø = 0.2 (0.2) 1.6, different sample sizes considered for risk evaluation and in those situations where over estimation is more serious than the underestimation. Does not perform so nicely in the reverse situations.

2. A comparison of values of relative risk(s) of $\hat{\theta}_{ST_1}$ and $\hat{\theta}_{ST_2}$, reveals that the relative risk is higher for $\hat{\theta}_{ST_1}$,so a test statistic dependent 'square' of shrinkage factor (weight) is suggested.

It is observed that using the LINEX los function the effective range of Ø for which $\hat{\theta}_{ST_1}$ or $\hat{\theta}_{ST_2}$ perform better than the usual estimator increases as compared to the same when a symmetric loss function is used. In fact the ranges are Ø= 0.2(0.2) 1.6 for LINEX loss function where as it is Ø = 0.6 (0.2) 1.2 for SELF.

3. The proposed testimators perform better for small sample size, small level of significance and in the situations where overestimation is more penalized than the underestimation which is mostly the case in insurance and re-insurance problems.

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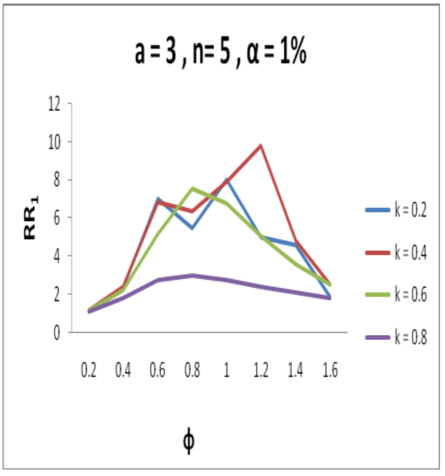
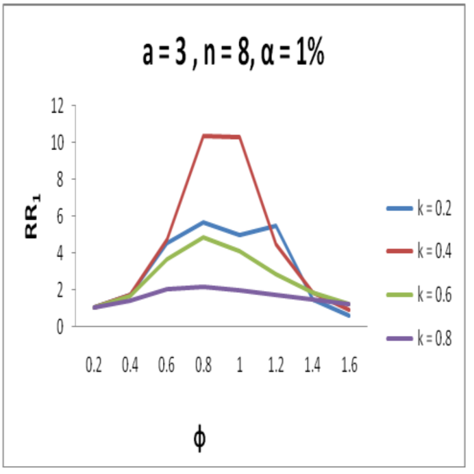
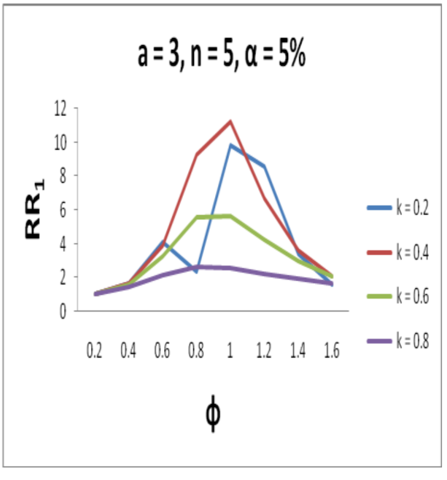
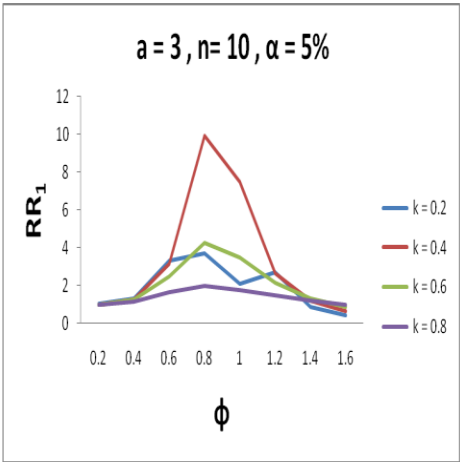
Appendix

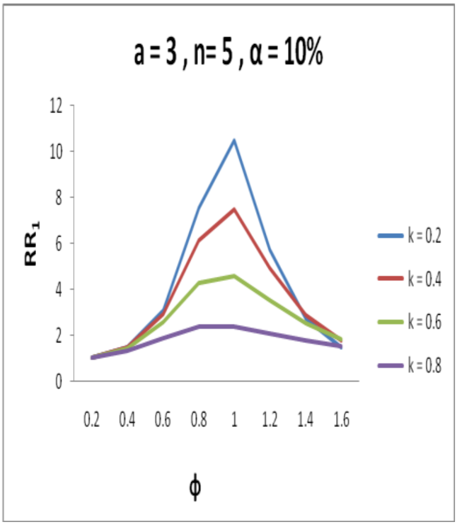
Graphs of Relative Risk(s) $\hat{\theta}_{ST_1}$

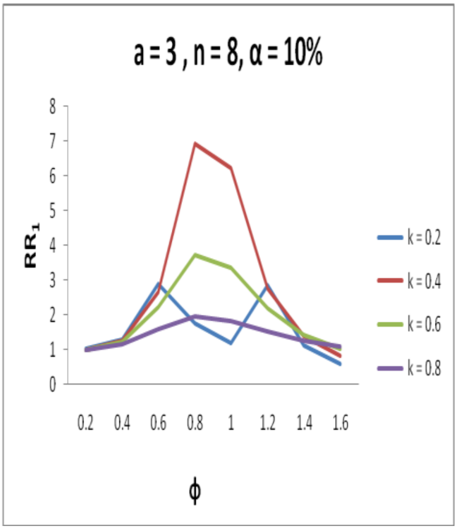









a = 3 , n= 5 , α = 10%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8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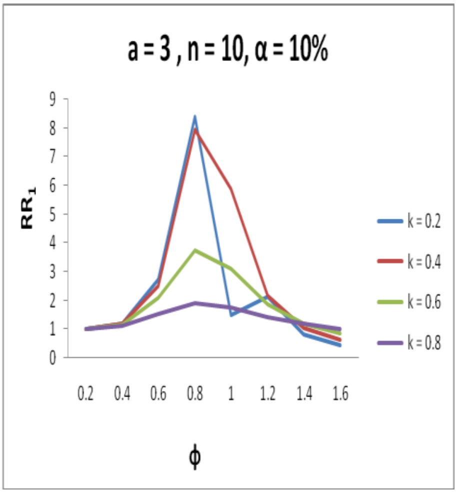
RR1
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8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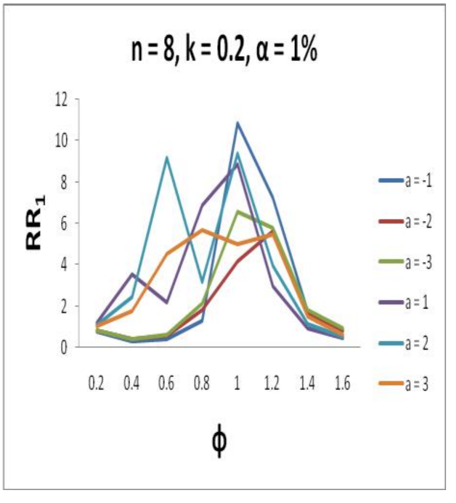
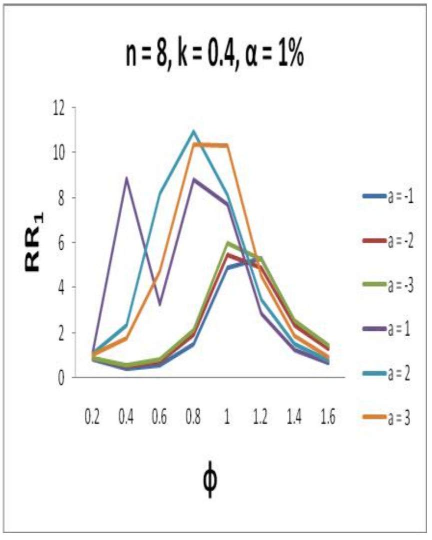
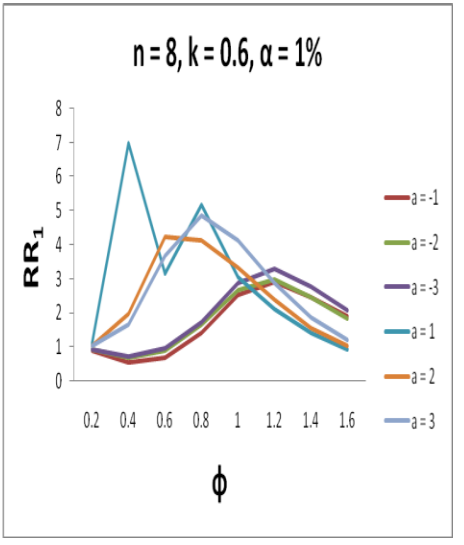
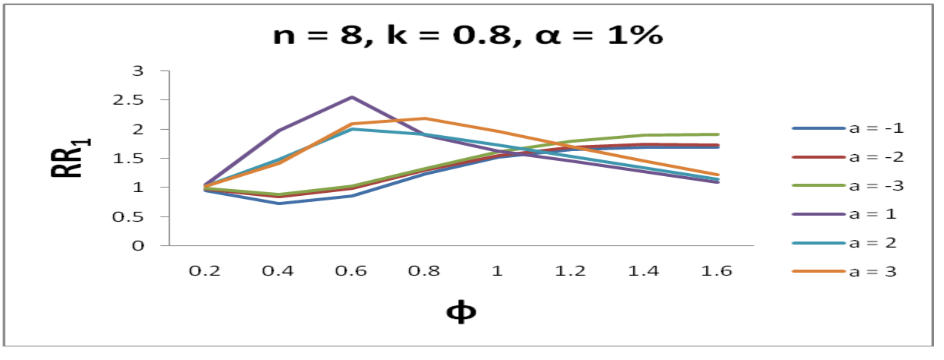
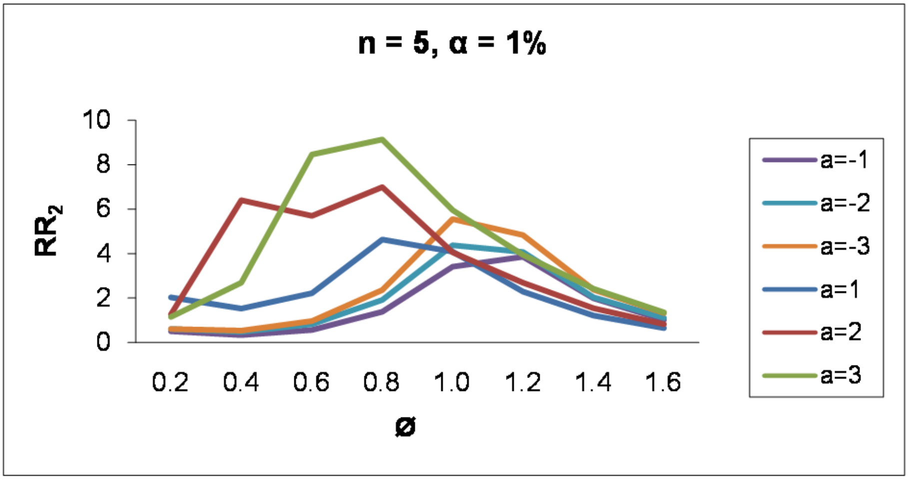
RR1
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
k = 0.2
k = 0.4
k = 0.6
k = 0.8


n = 8, k = 0.2, α = 1%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

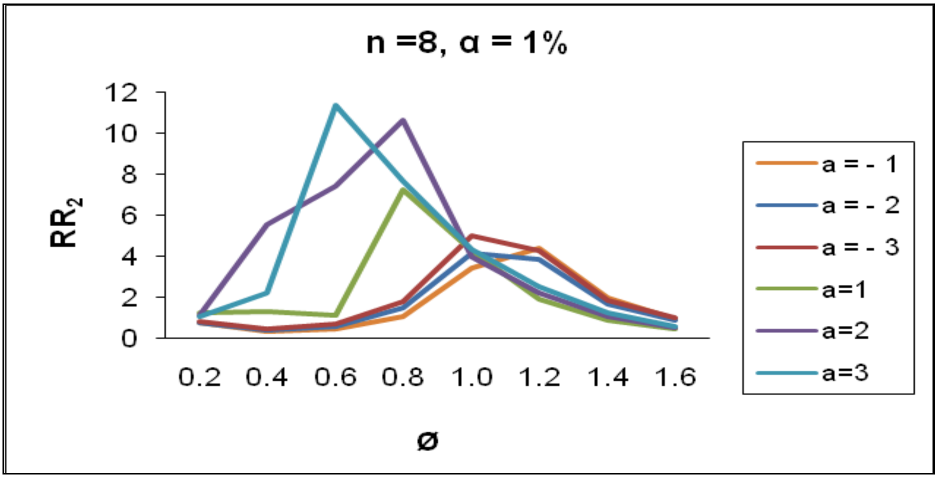

n = 8, k = 0.4, α = 1%
RR1
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

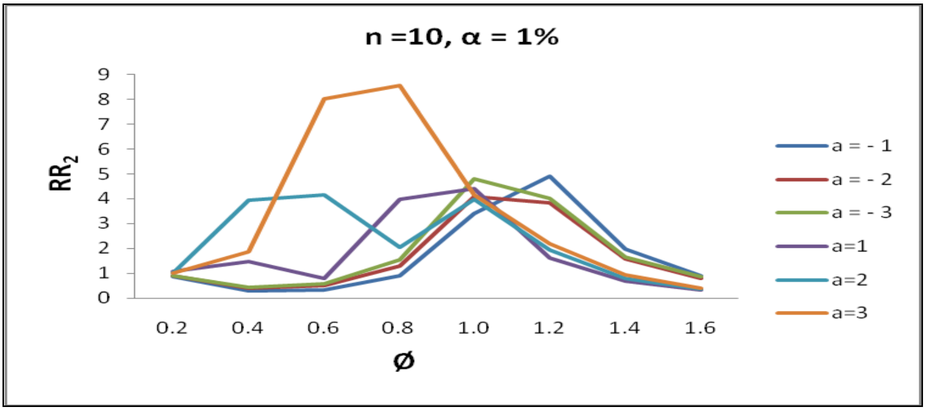

n = 8, k = 0.6, α = 1%
RR1
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

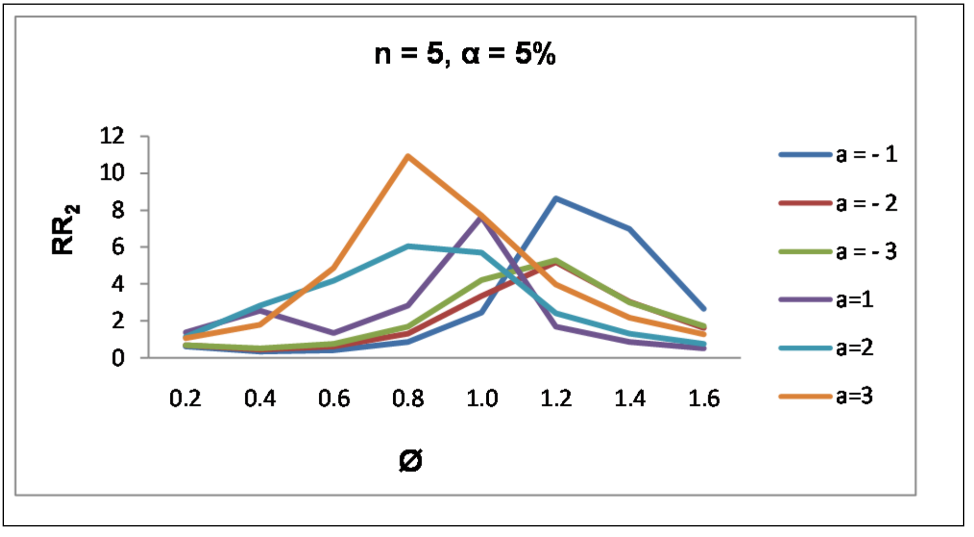

n = 8, k = 0.8, α = 1%
RR1
3
2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
ϕ
a = -1
a = -2
a = -3
a = 1
a = 2
a = 3

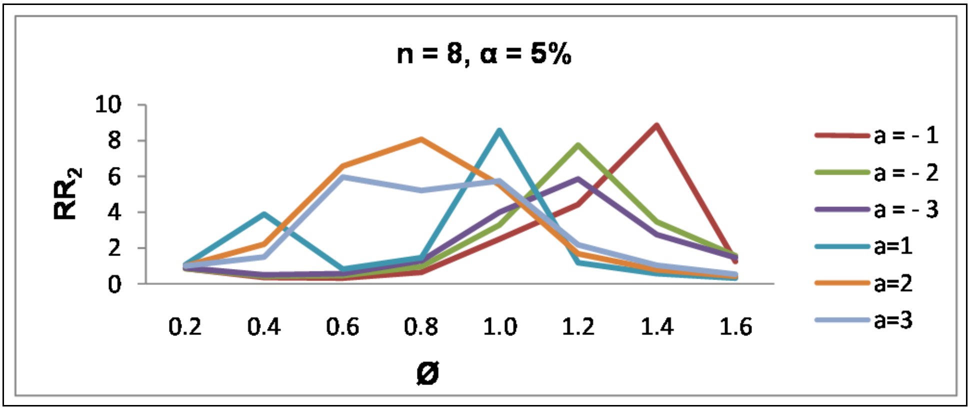


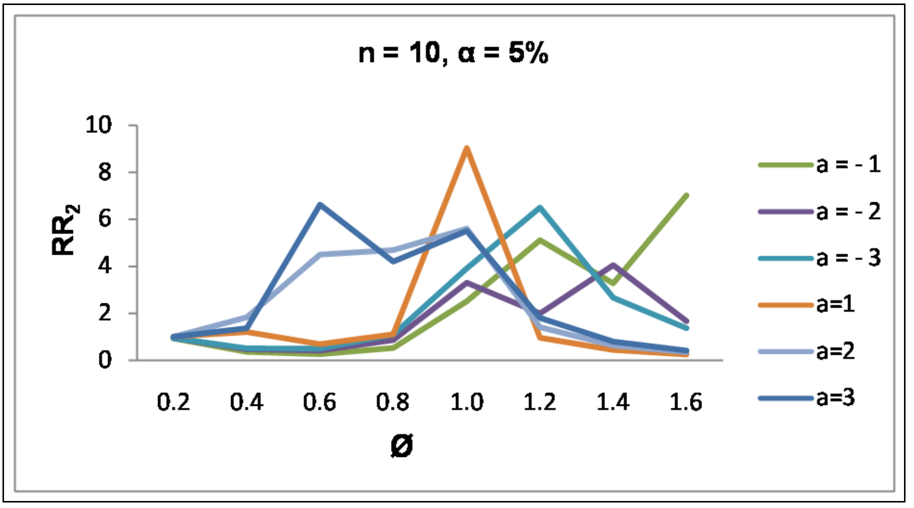
Graphs of Relative Risk for $\hat{\theta}_{ST_2}$

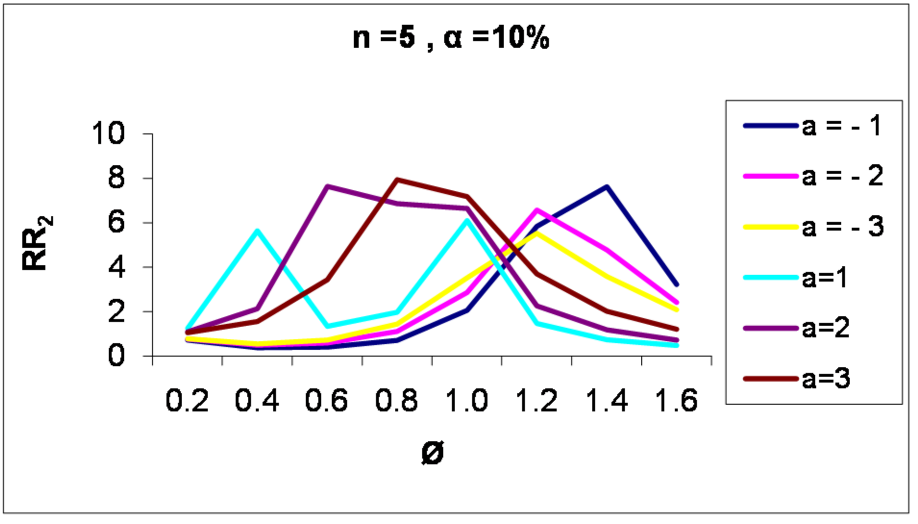


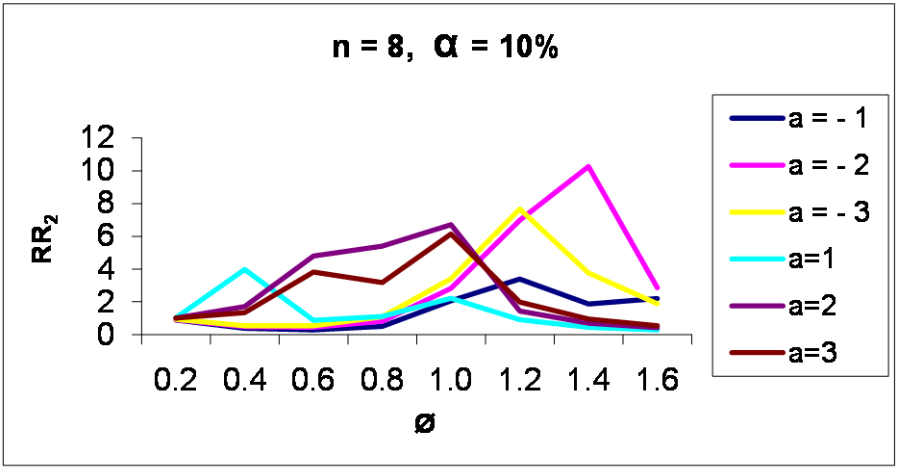

n =10, α = 1%
RR2
0
1
2
3
4
5
6
7
8
9
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3

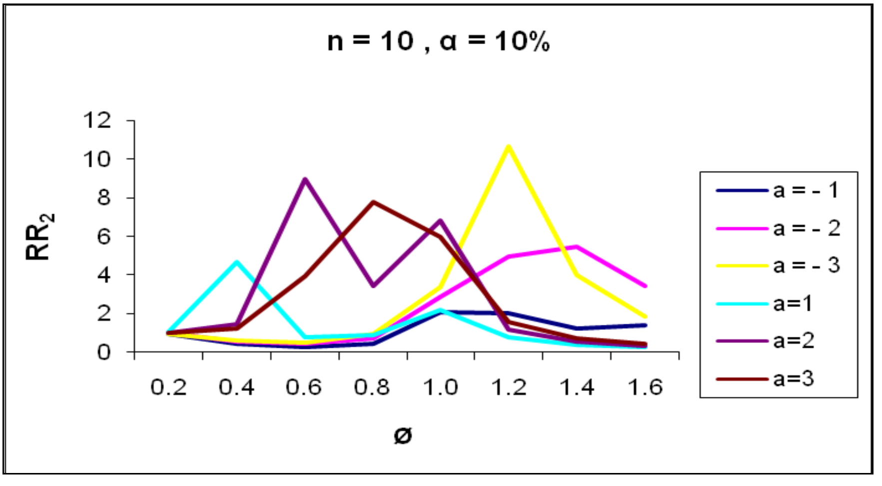

n = 5, α = 5%
RR2
0
2
4
6
8
10
12
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 8, α = 5%
RR2
0
2
4
6
8
10
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 10, α = 5%
RR2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n =5 , α =10%
RR2
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 8, α = 10%
RR2
12
10
8
6
4
2
0
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3


n = 10 , α = 10%
RR2
12
10
8
6
4
2
0
0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6
ø
a = - 1
a = - 2
a = - 3
a=1
a=2
a=3

s