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GENERALIZED FAMILY OF ESTIMATORS IN STRATIFIED RANDOM SAMPLING USING SUB-SAMPLING OF NON-RESPONDENTS

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Abstract

Generalized family of estimators have been proposed for the estimation of population mean using two auxiliary variables in stratified random sampling under non-response. The bias and the mean square error expressions of the adapted and suggested family estimators have been derived up to first order of approximation. The mathematical conditions have been obtained for which the suggested family of estimators are more efficient than the stratified mean estimator and ratio estimator. Empirical study is also provided in support of theoretical findings. In addition, the expressions for optimum sample size of the each stratum in respect to cost of the survey have also obtained.

Key Words: Ratio Estimator, Product Estimator, Bias, Mean Square Error, Stratified Random Sampling.

1. Introduction

In sample survey, non-response creates problem for the estimation. This problem occurs due to lack of failure to obtain the desired information. Non-response cannot be removed by only increasing the sample size. It has been observed that under non–response the bias in estimates increases that eventually reduces the efficiency of the estimate. The method of sub-sampling to obtain responses has been a popular method in case of non-response, was first introduced by Hansen and Hurwitz (1946). Further, Madow et al. (1983) presented weighting adjustments and imputation adjustments are used to deal with the problem of non-response. Cochran (1977), followed Hansen and Hurwitz (1946) methodology and presented ratio estimator for simple random sampling under non-response. For this situation, many statisticians such as Rao (1986), Okafor and Lee(2000), Kadilar and Cingi (2005),Khan et al. (2008). Ismail et al.(2011),Olufadi (2013) and some more statisticians developed various estimators in simple random sampling.

Kadilar and Cingi(2003) modified the estimators introduced by Upadhyaya and Singh to stratified random sampling. Further, some authors presented their work under the situation of non-response for stratifies random sampling such as Chauhdary et al.

(2009), Chauhdary etal. (2012), Sanaullah et al. (2012), Sharma and Singh (2015), Singh and Pal (2015), Sanaullah et al.(2015) and Pal and Singh (2016,2018).

In this study, we have proposed generalized family of estimators using two auxiliary variables under stratified sampling in the presence of non-response, when non-response is observed on all variables.

2. Sampling Methodology

Consider afinite population of$N$and is divided into L homogenous strata, such that $\sum_{h=1}^{L} N_h = N$, where N_h is the size of h^{th}stratum ($h=1,2,3,..., L$). Let a sample of size $n_h$drawn from each stratum by simple random sample such that $\sum_{h=1}^{L} n_h = n$, where $n_h$is the stratum sample size. Let (y_{hi}, x_{hi}, z_{hi}) be the observations of study variable (y) and auxiliary variables (x and z) on the i^{th} unit of h^{th}stratum, respectively. Moreover, $\bar{y}_h, \bar{x}_h$ and $\bar{z}_h$ be the sample means of h^{th}stratum corresponding to the population means $\bar{Y}_h, \bar{X}_h$ and $\bar{Z}_h$ of y, x and z respectively.It is assumed that $n_{h(1)}$ units respond and $n_{h(2)}$ unit will not such that $n_{h(1)} + n_{h(2)} = n_h$. A sub-sample of size r_h ($r_h = n_{h(2)} / f_h$) drawn at random out of $n_{h(2)}$ non-respondent units by following the method of Hansen and Hurwitz (1946), where $1/ f_h (f_h>1)$ denotes the sampling fraction among the non-respondent group in the h^{th} stratum. And it is assume that allthe selected $r_h (\subset n_{h(2)})$ units will respond on the second call. Assume a dummy variable as U_h which obtains the values of u_{hi} on the i^{th} population unit of stratum h and has mean $\bar{U}_h$. After this, $\bar{U}_h$ may stand for Y_h; X_h or for a second auxiliary variable Z_h. Let;

$$\bar{u}_{n_{h(1)}} = \frac{\sum_{i=1}^{n_{h(1)}} u_{i(1)}}{n_{h(1)}}, \quad \bar{u}_{r_h} = \frac{\sum_{i=1}^{r_h} u_{i(2)}}{r_h},$$

$$\text{So } \bar{u}_h^* = \frac{n_{h(1)}}{n_h} \bar{u}_{(1)n_{h1}} + \frac{r_h}{n_h} \bar{u}_{(2)r_h}. \tag{1}$$

where $\bar{u}_{(1)n_{h(1)}}$ and $\bar{u}_{(2)r_h}$ is mean of $n_{h(1)}$ respondents on first call and is mean of r_h units respond on the second call respectively, and $\bar{u}_h^*$ denotes the unbiased Hansen-Hurwitz (1946) of $\bar{U}_h$ for stratum h.

- A modified Hansen and Hurwitz (1946) unbiased estimator for stratified sampling may be given as,

$$t_{HH} = \sum_{h=1}^{L} P_h \bar{u}_h^*, \tag{2}$$

The variance of t_{HH} may be formed as

$$Var(t_{HH}) = \sum_{h=1}^{L} \lambda_h P_h^2 S_{u_h}^2 + \sum_{h=1}^{L} \lambda_h^* P_h^2 S_{u_{h(2)}}^2. \tag{3}$$

where $S_{uh}^2 = \sum_{i=1}^{N_h}\left(u_{hi} - \bar{U}_h\right)^2 / \left(N_h - 1\right)$, and $S_{uh(2)}^2 = \sum_{i=1}^{N_{h(2)}}\left(u_i - \bar{U}_{h(2)}\right)^2 / \left(N_{h(2)} - 1\right)$, be the mean square error of the entire group and the non-response group of the study variable with

$$P_h = N_h / N,\quad \lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right),\ \lambda_h^* = \left(\frac{f_h - 1}{n_h}\right)W_{h(2)}, W_{h(2)} = N_{h(2)} / N_h, \text{ and } f_h = n_{h(2)} / r_h.$$

- The modified ratio and product estimator presented by Cochran (1977)for stratified random sampling under non-response may be written as:

$$t_R = \bar{y}_{st}^*\left[\frac{\bar{X}}{\bar{x}_{st}^*}\right]\left[\frac{\bar{Z}}{\bar{z}_{st}^*}\right], \tag{4}$$

$$t_P = \bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]\left[\frac{\bar{z}_{st}^*}{\bar{Z}}\right], \tag{5}$$

where $\bar{y}_{st}^* = \sum_{h=1}^{L} P_h \bar{y}_h^*$ and $\bar{Y} = \sum_{h=1}^{L} P_h \bar{Y}_h$ are the sample and populations means of Y respectively. Similarly, we can define notations for $\bar{x}_{st}^*$ and $\bar{z}_{st}^*$.

The mean square error of the estimators t_R and t_P are given respectively as,

$$MSE\left(t_R\right) \approx \bar{Y}^2 \sum_{h=1}^{L}\left[\lambda_h\left(\frac{S_{hy}^2}{\bar{Y}^2} + \frac{S_{hx}^2}{\bar{X}^2} + \frac{S_{hz}^2}{\bar{Z}^2} - 2\frac{S_{hxy}}{\bar{Y}\bar{X}} - 2\frac{S_{hyz}}{\bar{Y}\bar{Z}} + 2\frac{S_{hyz}}{\bar{X}\bar{Z}}\right) + \lambda_h^*\left(\frac{S_{hy(2)}^2}{\bar{Y}^2} + \frac{S_{hx(2)}^2}{\bar{X}^2} + \frac{S_{hz(2)}^2}{\bar{Z}^2} - 2\frac{S_{hxy(2)}}{\bar{Y}\bar{X}} - 2\frac{S_{hyz(2)}}{\bar{Y}\bar{Z}} + 2\frac{S_{hxz(2)}}{\bar{X}\bar{Z}}\right)\right], \tag{6}$$

and

$$MSE\left(t_P\right) \approx \bar{Y}^2 \sum_{h=1}^{L}\left[\lambda_h\left(\frac{S_{hy}^2}{\bar{Y}^2} + \frac{S_{hx}^2}{\bar{X}^2} + \frac{S_{hz}^2}{\bar{Z}^2} + 2\frac{S_{hxy}}{\bar{Y}\bar{X}} + 2\frac{S_{hyz}}{\bar{Y}\bar{Z}} + 2\frac{S_{hyz}}{\bar{X}\bar{Z}}\right) + \lambda_h^*\left(\frac{S_{hy(2)}^2}{\bar{Y}^2} + \frac{S_{hx(2)}^2}{\bar{X}^2} + \frac{S_{hz(2)}^2}{\bar{Z}^2} + 2\frac{S_{hxy(2)}}{\bar{Y}\bar{X}} + 2\frac{S_{hyz(2)}}{\bar{Y}\bar{Z}} + 2\frac{S_{hxz(2)}}{\bar{X}\bar{Z}}\right)\right]. \tag{7}$$

- A ratio estimator for stratified sampling when non-response is present only on study variable was presented by Chaudhary et al. (2009) as,

$$t_C = \bar{y}_{st}^*\left[\frac{a\bar{X} + b}{\alpha\left(a\bar{x}_{st} + b\right) + \left(1 - \alpha\right)\left(a\bar{X} + b\right)}\right]^g, \tag{8}$$

The mean square error of $t_C$is

$$MSE\left(t_C\right) \approx \sum_{h=1}^{L} P_h^2 \lambda_h\left[S_{yh}^2 + \alpha^2 v^2 g^2 R^2 S_{xh}^2 - 2\alpha v g R \rho_{xyh} S_{xh} S_{yh}\right] + \sum_{h=1}^{L} P_h^2 \lambda_h^* S_{yh(2)}^2. \tag{9}$$

where $v = \frac{a\bar{X}}{a\bar{X} + b}$ and $R = \frac{\bar{Y}}{\bar{X}}$.

The value of MSE(t_C) is minimum for $\alpha = \frac{\sum_{h=1}^{L} P_h^2 \lambda_h \rho_{xyh} S_{yh} S_{xh}}{vgR \sum_{h=1}^{L} P_h^2 \lambda_h S_{xh}}$.

In this study we have provided a generalized class of estimators for estimating population mean using two auxiliary variables and also considering a situation of non-response in stratified random sampling. Hansen and Hurwitz's (1946) sub-sampling has been used to deal with the survey non-response. Also it has been shown that the proposed class of estimators is more efficient as they attain minimum mean square errors than the some existing estimator. Further the estimators discussed in the study are simpler and can be implemented into computer software and also are cost effective estimators as more efficient results can be obtained at minimum cost and with minimum mean square error. What sample size is desired to achieve these objectives has also been discussed in the study. In the following, the study is presented in seven sections. Section-II presents basic notation used in the text, and estimation results in stratified random sampling under the situation of non-response. Some existing estimators are also presented along their mean square errors. In Section-III an adapted family of estimators is presented with some properties of the estimators. Similarly, we propose a generalized family of estimators in Section-IV and some properties e.g. bias, mean square of errors, optimal conditions to get minimum mean square errors. Section-V is given for mathematical comparisons and to derive some conditions under which the proposed estimator perform better than some existing estimators. Further issues such as optimal sample size, and minimum survey cost to attain maximum efficiency of the estimates are discussed in Section-VI, and in Section-VII a numerical study is given to show the performance of the proposed estimators. Finally in Section-VIII conclusion based on numerical illustration is drawn.

3. Adapted Family of Estimators

Ismail et al. (2011) introduced an estimator for population mean for non-response in two-phase simple random sampling given by

$$t_I = \bar{y}^* \left[\frac{\bar{x}^* - w\left(\bar{x}^* - \bar{x}_1\right)}{\bar{x}_1 + w\left(\bar{x}^* - \bar{x}_1\right)} \right], \tag{10}$$

we adapt this estimator to a generalized family of estimatorsusing two auxiliary variablessay x and z under stratified sampling in the presence of non-response as,

$$k_a = \bar{y}_{st}^* \left[\frac{\bar{x}_{st}^* - w_X\left(\bar{x}_{st}^* - \bar{X}\right)}{\bar{X} + w_X\left(\bar{x}_{st}^* - \bar{X}\right)} \right]^{g_X} \left[\frac{\bar{z}_{st}^* - w_Z\left(\bar{z}_{st}^* - \bar{Z}\right)}{\bar{Z} + w_Z\left(\bar{z}_{st}^* - \bar{Z}\right)} \right]^{g_Z}, \tag{11}$$

Where g_X and g_Z are known constants, w_X and w_Z are assumed to be unknown, whose values are to estimate such that the MSE of*k*is minimum. Further, it is observed that for various values of w_X and w_Z, some ratio and product estimators are obtained. The class of estimators for adapted family has been given in Table 1.

In order to obtain the bias and the MSE, if we assume that

$\bar{y}_{st}^* = \bar{Y}\left(1+e_o^*\right), \bar{x}_{st}^* = \bar{X}\left(1+e_1^*\right), \bar{z}_{st}^* = \bar{Z}\left(1+e_2^*\right)$ and $E(e_i^*) = 0$ where $i = 0,1,2$,

then in stratified random sampling (without replacement)

Ratio Estimator	Product Estimator	g_X	g_Z	w_X	w_Z
	$k_a^2=\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]\left[\frac{\bar{z}_{st}^*}{\bar{Z}}\right]$	1	1	0	0
$k^1=\bar{y}_{st}^*\left[\frac{\bar{X}}{\bar{x}_{st}^*}\right]\left[\frac{\bar{Z}}{\bar{z}_{st}^*}\right]$		1	1	1	1
$k_a^3=\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*-2\left(\bar{x}_{st}^*-\bar{X}\right)}{\bar{X}+2\left(\bar{x}_{st}^*-\bar{X}\right)}\right]\left[\frac{\bar{Z}}{\bar{z}_{st}^*}\right]$		1	1	2	1
	$k_a^4=\bar{y}_{st}^*\left[\frac{2\bar{x}_{st}^*-\bar{X}}{2\bar{X}-\bar{x}_{st}^*}\right]\left[\frac{\bar{z}_{st}^*}{\bar{Z}}\right]$	1	1	-1	0
	$k_a^6=\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]\left[\frac{2\bar{z}_{st}^*-\bar{Z}}{2\bar{Z}-\bar{z}_{st}^*}\right]$	1	1	0	-1
$k_a^5=\bar{y}_{st}^*\left[\frac{\bar{X}}{\bar{x}_{st}^*}\right]\left[\frac{\bar{z}_{st}^*-2\left(\bar{z}_{st}^*-\bar{Z}\right)}{\bar{Z}+2\left(\bar{z}_{st}^*-\bar{Z}\right)}\right]$		1	1	1	2
$k_a^7=\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*-2\left(\bar{x}_{st}^*-\bar{X}\right)}{\bar{X}+2\left(\bar{x}_{st}^*-\bar{X}\right)}\right]\left[\frac{\bar{z}_{st}^*-2\left(\bar{z}_{st}^*-\bar{Z}\right)}{\bar{Z}+2\left(\bar{z}_{st}^*-\bar{Z}\right)}\right]$		1	1	2	2

Table 1: Class of Estimator for Adapted Family of Estimators

$$
\begin{aligned}
&E\left(e_0^*\right)^2=\frac{1}{\bar{Y}^2}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{yh}^2+\lambda_h^* S_{yh2}^2\right)=V_{020}^*, E\left(e_1^*\right)^2=\frac{1}{\bar{X}^2}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{xh}^2+\lambda_h^* S_{xh2}^2\right)=V_{200}^*,\\
&E\left(e_2^*\right)^2=\frac{1}{\bar{Z}^2}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{zh}^2+\lambda_h^* S_{zh2}^2\right)=V_{002}^*, E\left(e_o^* e_1^*\right)=\frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{yxh}+\lambda_h^* S_{yxh2}\right)=V_{110}^*,\\
&E\left(e_o^* e_2^*\right)=\frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{yzh}+\lambda_h^* S_{yzh2}\right)=V_{011}^*, E\left(e_1^* e_2^*\right)=\frac{1}{\bar{X}\bar{Z}}\sum_{h=1}^{L}P_h^2\left(\lambda_h S_{xzh}+\lambda_h^* S_{xzh2}\right)=V_{101}^*.
\end{aligned}
\tag{12}
$$

where $V_{r,s,t}^*=\sum_{h=1}^{L}P_h^{r+s+t}\frac{E\left(\left(\bar{x}_h^*-\bar{X}_h\right)^r\left(\bar{y}_h^*-\bar{Y}_h\right)^s\left(\bar{z}_h^*-\bar{Z}_h\right)^t\right)}{\bar{X}^r\bar{Y}^s\bar{Z}^t}$

On rewriting (11) as,

$$
k_a=\bar{Y}\left(1+e_0^*\right)\left[\left(1+e_1^*-w_X e_1^*\right)\left(1+w_X e_1^*\right)^{-1}\right]^{gX}\left[\left(1+e_2^*-w_Z e_1^*\right)\left(1+w_Z e_2^*\right)^{-1}\right]^{gZ},
\tag{13}
$$

Expanding the right hand side of (13) up to first order approximation, we obtain,

$$k_a = \bar{Y}\left[1 + e_0^* - e_1^*(2w_X - 1)g_X - e_2^*(2w_Z - 1)g_Z + e_1^* e_2^*(2w_X - 1)(2w_Z - 1)g_X g_Z - e_0^* e_1^*(2w_X - 1)g_X - e_0^* e_2^*(2w_Z - 1)g_Z\right] \tag{14}$$

Or

$$k_a - \bar{Y} = \bar{Y}\left[e_0^* - \omega_X e_1^* - \omega_Z e_2^* + \omega_X \omega_Z e_1^* e_2^* - \omega_X e_0^* e_1^* - \omega_Z e_0^* e_2^*\right], \tag{15}$$

where $(2w_X - 1)g_X = \omega_X$ and $(2w_Z - 1)g_Z = \omega_Z$

Taking expectations of both sides in(15),we get the expressionof the bias as,

$$Bias(k_a) = \bar{Y}\left[\omega_X \omega_Z V_{101}^* - \omega_X V_{110}^* - \omega_Z V_{011}^*\right], \tag{16}$$

Squaring both sides of(15) and retain terms up to the order n^{-1} then we take expectation to get the MSE of the estimator k_a as,

$$MSE(k_a) = \bar{Y}^2\left[V_{020}^* + \omega_X^2 V_{200}^* + \omega_Z^2 V_{002}^* - 2\omega_X V_{110}^* - 2\omega_Z V_{011}^* + 2\omega_X \omega_Z V_{101}^*\right], \tag{17}$$

The MSE of$k_a$is minimizedfor the optimal values of ω_X and ω_Z as,

$$\omega_X = \frac{\left(V_{011}^* V_{101}^* - V_{110}^* V_{002}^*\right)}{\left(V_{101}^{*2} - V_{002}^* V_{200}^*\right)} \quad \text{and} \quad \omega_Z = \frac{\left(V_{011}^* V_{101}^* - V_{011}^* V_{200}^*\right)}{\left(V_{101}^{*2} - V_{002}^* V_{200}^*\right)}, \tag{18}$$

Substitute the optimal values of (18) in (17), results the minimum value of $MSE(k_a)$as,

$$\text{MSE}(k_a) = \bar{Y}^2\left[V_{020}^* - \left(\frac{V_{002}^* V_{110}^{*2} + V_{200}^* V_{011}^{*2} - 2V_{101}^* V_{011}^* V_{110}^*}{V_{200}^* V_{002}^* - V_{101}^{*2}}\right)\right]. \tag{19}$$

For ratio estimators presented in Table 1, we can express the *MSE* expression in (19) as,

$$MSE(k_a^i) = \begin{cases} \bar{Y}^2\left[V_{020}^* + V_{200}^* + V_{002}^* - 2V_{110}^* - 2V_{011}^* + 2V_{101}^*\right] & i = 1 \\ \bar{Y}^2\begin{bmatrix} V_{020}^* + v_{x(\frac{i-1}{2})}^2 V_{200}^* + v_{z(\frac{i-1}{2})}^2 V_{002}^* \\ -2v_{x(\frac{i-1}{2})} V_{110}^* - 2v_{z(\frac{i-1}{2})} V_{011}^* \\ +2v_{x(\frac{i-1}{2})} v_{z(\frac{i-1}{2})} V_{101}^* \end{bmatrix} & i = 3,5,7 \end{cases} \tag{20}$$

and for product estimators, the MSE expression can be given as,

$$MSE(k_a^j) = \begin{cases} \bar{Y}^2\left[V_{020}^* + V_{200}^* + V_{002}^* + 2V_{110}^* + 2V_{011}^* + 2V_{101}^*\right] & j = 2 \\ \bar{Y}^2\begin{bmatrix} V_{020}^* + v_{x(\frac{j}{2}-1)}^2 V_{200}^* + v_{z(\frac{j}{2}-1)}^2 V_{002}^* + 2v_{x(\frac{j}{2}-1)} V_{110}^* \\ +2v_{z(\frac{j}{2}-1)} V_{011}^* + 2v_{x(\frac{j}{2}-1)} v_{z(\frac{j}{2}-1)} V_{101}^* \end{bmatrix} & j = 4,6,8 \end{cases}, \tag{21}$$

where $v_{X1} = \dfrac{\bar{X}}{\bar{X} + \rho_{xy}}$ and $v_{Z1} = \dfrac{\bar{Z}}{\bar{Z} + \rho_{yz}}$,

$v_{X2} = \dfrac{\sigma_X \bar{X}}{\sigma_X \bar{X} + 1}$ and $v_{Z2} = \dfrac{\sigma_Z \bar{Z}}{\sigma_Z \bar{Z} + 1}$,

$$v_{X3} = \frac{\rho_{xy}\bar{X}}{\rho_{xy}\bar{X}+1} \text{ and } v_{Z3} = \frac{\rho_{yz}\bar{Z}}{\rho_{yz}\bar{Z}+1}.$$

4. Proposed Family of Estimators

In this section, we have proposed a generalized family of estimatorsgiven by,

$$k_p = \eta\bar{y}^*_{st}\left[\frac{\bar{x}^*_{st} - w_X\left(\bar{x}^*_{st} - \bar{X}\right)}{\bar{X} + w_X\left(\bar{x}^*_{st} - \bar{X}\right)}\right]^{gX}\left[\frac{\bar{z}^*_{st} - w_Z\left(\bar{z}^*_{st} - \bar{Z}\right)}{\bar{Z} + w_Z\left(\bar{z}^*_{st} - \bar{Z}\right)}\right]^{gZ}, \tag{22}$$

where η (≠0) is a constant.We develop some ratio-cum-ratio, and product-cum-product estimators by assuming different values of parameter in (22)as shown in Table 2.
On rewriting we may get (22) as

$$k_p = \eta\bar{Y}\left(1+e^*_0\right)\left[\left(1+e^*_1 - w_X e^*_1\right)\left(1+w_X e^*_1\right)^{-1}\right]^{gX}\left[\left(1+e^*_2 - w_Z e^*_2\right)\left(1+w_Z e^*_2\right)^{-1}\right]^{gZ}, \tag{23}$$

Solving (23), neglecting terms of e's having power higher than two, we have

$$k_p - \bar{Y} = \eta\bar{Y}\left[1 + e^*_0 - e^*_1\left(2w_X - 1\right)g_X - e^*_2\left(2w_Z - 1\right)g_Z - e^*_0 e^*_1\left(2w_X - 1\right)g_X \right.$$
$$\left. - e^*_0 e^*_2\left(2w_Z - 1\right)g_Z + e^*_1 e^*_2\left(2w_X - 1\right)\left(2w_Z - 1\right)g_X g_Z\right] - \bar{Y}, \tag{24}$$

Applying similar procedure to (24) as described in section 2, the expression of the bias and MSE of k_p can be easily obtained as,

$$Bias\left(k_p\right) = \bar{Y}\left[\eta\left\{1 - \omega_X V^*_{110} - \omega_Z V^*_{011} + \omega_X\omega_Z V^*_{101}\right\} - 1\right], \tag{25}$$

and

$$MSE\left(k_p\right) = \bar{Y}^2\left[\eta^2\left\{V^*_{020} + \omega_X^2 V^*_{200} + \omega_Z^2 V^*_{002}\right\} - 2\left(2\eta^2 - \eta\right)\left\{\omega_X\left(V^*_{110} - \omega_Z V^*_{101}\right) + \omega_Z V^*_{011}\right\} + \left(\eta - 1\right)^2\right] \tag{26}$$

Minimization of (26) with respect to η, ω_X and ω_Z yields the optimum values as,

$$\omega_X = \frac{\left(V^*_{011}V^*_{101} - V^*_{110}V^*_{002}\right)}{\left(V^{*2}_{101} - V^*_{002}V^*_{200}\right)}, \omega_Z = \frac{\left(V^*_{011}V^*_{101} - V^*_{011}V^*_{200}\right)}{\left(V^{*2}_{101} - V^*_{002}V^*_{200}\right)}, \tag{27}$$

and

$$\eta = \frac{1}{1 + V^*_{020} - \left(\dfrac{V^*_{002}V^{*2}_{110} + V^*_{200}V^{*2}_{011} - 2V^*_{101}V^*_{011}V^*_{110}}{V^*_{200}V^*_{002} - V^{*2}_{101}}\right)}. \tag{28}$$

Thus, the minimum *MSE* (k_p) is obtained as

$$\min MSE\left(k_p\right) = \bar{Y}^2\left[1 - \frac{1}{1 + V^*_{020} - \left(\dfrac{V^*_{002}V^{*2}_{110} + V^*_{200}V^{*2}_{011} - 2V^*_{101}V^*_{011}V^*_{110}}{V^*_{200}V^*_{002} - V^{*2}_{101}}\right)}\right]. \tag{29}$$

For ratio-cum-ratio estimators presented in Table 2, we can express the *MSE* expression in (27) as,

$$MSE\left(k_p^1\right)=\bar{Y}^2\left[\hat{\eta}^2\left\{V_{020}^*+V_{200}^*+V_{002}^*\right\}-2\left(2\hat{\eta}^2-\hat{\eta}\right)\left(V_{110}^*-V_{101}^*+V_{011}^*\right)+\left(\hat{\eta}-1\right)^2\right] \quad \textit{and}$$

$$MSE\left(k_p^i\right)=\bar{Y}^2\left[\eta^{+2}\left\{V_{020}^*+\nu_{x\left(\frac{i-1}{2}\right)}^2V_{200}^*+\nu_{z\left(\frac{i-1}{2}\right)}^2V_{002}^*\right\}\right.$$
$$\left.-2\left(2\eta^{+2}-\eta\right)\left(\nu_{x\left(\frac{i-1}{2}\right)}V_{110}^*-\nu_{x\left(\frac{i-1}{2}\right)}\nu_{z\left(\frac{i-1}{2}\right)}V_{101}^*+\nu_{z\left(\frac{i-1}{2}\right)}V_{011}^*\right)+\left(\eta^+-1\right)^2\right] \tag{30}$$

where $i=3,5,7$

Ratio Estimator	Product Estimator	w_X	w_Z	η
	$k_p^2=\eta\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]\left[\frac{\bar{z}_{st}^*}{\bar{Z}}\right]$	0	0	η
$k_p^1=\eta\bar{y}_{st}^*\left[\frac{\bar{X}}{\bar{x}_{st}^*}\right]\left[\frac{\bar{Z}}{\bar{z}_{st}^*}\right]$		1	1	η
$k_p^3=\eta\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*-2\left(\bar{x}_{st}^*-\bar{X}\right)}{\bar{X}+2\left(\bar{x}_{st}^*-\bar{X}\right)}\right]\left[\frac{\bar{Z}}{\bar{z}_{st}^*}\right]$		2	1	η
	$k_p^4=\eta\bar{y}_{st}^*\left[\frac{2\bar{x}_{st}^*-\bar{X}}{2\bar{X}-\bar{x}_{st}^*}\right]\left[\frac{\bar{z}_{st}^*}{\bar{Z}}\right]$	-1	0	η
	$k_p^6=\eta\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*}{\bar{X}}\right]\left[\frac{2\bar{z}_{st}^*-\bar{Z}}{2\bar{Z}-\bar{z}_{st}^*}\right]$	0	-1	η
$k_p^5=\eta\bar{y}_{st}^*\left[\frac{\bar{X}}{\bar{x}_{st}^*}\right]\left[\frac{\bar{z}_{st}^*-2\left(\bar{z}_{st}^*-\bar{Z}\right)}{\bar{Z}+2\left(\bar{z}_{st}^*-\bar{Z}\right)}\right]$		1	2	η
$k_p^7=\eta\bar{y}_{st}^*\left[\frac{\bar{x}_{st}^*-2\left(\bar{x}_{st}^*-\bar{X}\right)}{\bar{X}+2\left(\bar{x}_{st}^*-\bar{X}\right)}\right]\left[\frac{\bar{z}_{st}^*-2\left(\bar{z}_{st}^*-\bar{Z}\right)}{\bar{Z}+2\left(\bar{z}_{st}^*-\bar{Z}\right)}\right]$		2	2	η

Table 2: Class of Estimator for Suggested Family of Estimator

and for product-cum-product estimators, the MSE expression can be given as,

$$MSE\left(k_p^2\right)=\bar{Y}^2\left[\dot{\eta}^2\left\{V_{020}^*+V_{200}^*+V_{002}^*\right\}+\left(\dot{\eta}-1\right)^2+2\left(2\dot{\eta}^2-\dot{\eta}\right)\left(V_{110}^*-V_{101}^*+V_{011}^*\right)\right] \quad \textit{and}$$

$$MSE\left(k_p^j\right)=\bar{Y}^2\left[\eta^{\circ 2}\left\{V_{020}^*+\nu_{x\left(\frac{i-1}{2}\right)}^2V_{200}^*+\nu_{z\left(\frac{i-1}{2}\right)}^2V_{002}^*\right\}\right.$$
$$\left.+2\left(2\eta^{\circ 2}-\eta^{\circ}\right)\left(\nu_{x\left(\frac{j}{2}-1\right)}V_{110}^*-\nu_{x\left(\frac{j}{2}-1\right)}\nu_{z\left(\frac{j}{2}-1\right)}V_{101}^*+\nu_{z\left(\frac{j}{2}-1\right)}V_{011}^*\right)+\left(\eta^{\circ}-1\right)^2\right] \textit{ where } j=4,6 \tag{31}$$

The optimal values of η's given as, $\hat{\eta}=\frac{\hat{A}}{\hat{B}}$, $\eta^{+}=\frac{A^{+}}{B^{+}}$, $\dot{\eta}=\frac{\dot{A}}{\dot{B}}$ and $\eta^{\circ}=\frac{A^{\circ}}{B^{\circ}}$

where

$$\hat{A}=1-V^{*}_{110}-V^{*}_{011}+V^{*}_{101}\ \hat{A}=1-V^{*}_{110}-V^{*}_{011}+V^{*}_{101},\hat{B}=1+V^{*}_{020}+V^{*}_{200}+V^{*}_{002}-4\left(V^{*}_{110}-V^{*}_{101}+V^{*}_{011}\right),$$

$$A^{+}=1-\nu_{x\left(\frac{i-1}{2}\right)}V^{*}_{110}-\nu_{z\left(\frac{i-1}{2}\right)}V^{*}_{011}+\nu_{x\left(\frac{i-1}{2}\right)}\nu_{z\left(\frac{i-1}{2}\right)}V^{*}_{101}$$

$$B^{+}=1+V^{*}_{020}+\nu^{2}_{x\left(\frac{i-1}{2}\right)}V^{*}_{200}+\nu^{2}_{z\left(\frac{i-1}{2}\right)}V^{*}_{002}-4\left(\nu_{x\left(\frac{i-1}{2}\right)}V^{*}_{110}+\nu_{z\left(\frac{i-1}{2}\right)}V^{*}_{011}-\nu_{x\left(\frac{i-1}{2}\right)}\nu_{z\left(\frac{i-1}{2}\right)}V^{*}_{101}\right)$$

$$\dot{A}=1+V^{*}_{110}+V^{*}_{011}+V^{*}_{101},\dot{B}=1+V^{*}_{020}+V^{*}_{200}+V^{*}_{002}+4\left(V^{*}_{110}+V^{*}_{101}+V^{*}_{011}\right)$$

$$A^{\circ}=1+\nu_{x\left(\frac{j-1}{2}\right)}V^{*}_{110}-\nu_{z\left(\frac{j-1}{2}\right)}V^{*}_{011}+\nu_{x\left(\frac{j-1}{2}\right)}\nu_{z\left(\frac{j-1}{2}\right)}V^{*}_{101}$$

$$B^{\circ}=1+V^{*}_{020}+\nu^{2}_{x\left(\frac{j-1}{2}\right)}V^{*}_{200}+\nu^{2}_{z\left(\frac{j-1}{2}\right)}V^{*}_{002}$$

$$+4\left(\nu_{x\left(\frac{j-1}{2}\right)}V^{*}_{110}+\nu_{z\left(\frac{j-1}{2}\right)}V^{*}_{011}-\nu_{x\left(\frac{j-1}{2}\right)}\nu_{z\left(\frac{j-1}{2}\right)}V^{*}_{101}\right)$$

By substitution the above values of η's, the $MSE(k^{i}_{p})$ and $MSE(k^{j}_{p})$ are minimized as,

$$MSE\left(k^{i}_{p}\right)=\begin{cases}\bar{Y}^{2}\left\{1-\frac{\hat{A}^{2}}{\hat{B}}\right\} & i=1\\ \bar{Y}^{2}\left\{1-\frac{A^{+2}}{B^{+}}\right\} & i=3,5,7\end{cases}\tag{32}$$

and

$$MSE\left(k^{j}_{p}\right)=\begin{cases}\bar{Y}^{2}\left\{1-\frac{\dot{A}^{2}}{\dot{B}}\right\} & j=2\\ \bar{Y}^{2}\left\{1-\frac{A^{\circ 2}}{B^{\circ}}\right\} & j=4,6\end{cases}\tag{33}$$

5. Efficiency Comparison

Now we compare the generalize family of estimator with stratified mean estimator, ratio estimator and the class of estimators. The following notations will be considered for comparison

$$\pi_{1}=\left(\frac{V^{*}_{002}V^{*2}_{110}+V^{*}_{200}V^{*2}_{011}-2V^{*}_{101}V^{*}_{011}V^{*}_{110}}{V^{*}_{200}V^{*}_{002}-V^{*2}_{101}}\right)$$

i) $Var\left(t_{HH}\right)>MSE\left(k_{p}\right)$

If $\eta > 1 - V^*_{020}$ (34)

ii) $MSE(k_a) < Var(t_{HH})$

If $\pi_1 > 0$ (35)

iii) $MSE(k_p) < MSE(t_R)$,

If $\eta > 1 - \left(V^*_{020} + V^*_{200} + V^*_{002} - 2V^*_{011} - 2V^*_{110} + 2V^*_{101}\right)$ (36)

iv) $MSE(k_a) < MSE(t_R)$,

If $\pi_1 > 2V^*_{011} + 2V^*_{110} - 2V^*_{101} - V^*_{200} - V^*_{002}$ (37)

v) $MSE(t_p) < MSE(k_a)$,

If $\eta > 1 - V^*_{020} - \pi_1$ (38)

6. Cost Function and Sample Size Estimation

In this section we are discussing a general procedure for how survey cost can be minimized and also what would be the optimal sample size to attain the minimum variance as given in (19) or (32).

The cost function is considered tobe

$$C' = \sum_{h=1}^{L} c_{ho} n_h + \sum_{h=1}^{L} c_{h1} n_h + \sum_{h=1}^{L} c_{h2} \frac{n_{h(2)}}{f_h} \tag{39}$$

where

c_{ho}= The per unit cost of making first attempt

c_{h1}= The per unit cost for processing the result of all characteristics in first attempt

c_{h2}= The per unit cost for processing the result of all characteristics in second attempt inthe h^{th}stratum.

The total expected cost of the survey could be given as

$$C = E(C') = \sum_{h=1}^{L} \left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{f_h} \right) n_h \tag{40}$$

Let the variance of an estimator t in the presence of non-response be represented by

$$Var(t) = \frac{V_0}{n_h} + \frac{f_h}{n_h} V_1 + \text{(terms independent of } n_h \text{ and } r_h) \tag{41}$$

whereV_0 and V_1 are the coefficients of the terms of $\frac{1}{n_h}$ and $\frac{f_h}{n_h}$ in the expressions of variance of $t=k_a$ or k_p.

For the fixed cost $C \le C'$, let us define a function by

$$\varphi = Var(t) + \delta \left[\sum_{h=1}^{L} \left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{f_h} \right) n_h - C' \right] \tag{42}$$

where δ is the Langrange's multiplier

Now differentiate Φ with respect to n_h and f_h, and equating to zero, we may get

$$n_{h_{opt}} = \sqrt{\frac{V_0 + f_h V_1}{\delta \left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{k_h} \right)}} \tag{43}$$

and

$$f_{h_{opt}} = \sqrt{\frac{V_0 c_{h2} W_{h2}}{(c_{h0} + c_{h1} W_{h1}) V_1}} \tag{44}$$

Substituting the values of n_{hopt} and f_{hopt} in (42),we have

$$\sqrt{\delta} = \frac{1}{C} \sum_{h=1}^{L} \sqrt{\left[\left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{f_{h_{opt}}} \right) \left(V_0 + V_1 f_{h_{opt}} \right) \right]} \tag{45}$$

Thus the minimum value of $Var(t)$ is given as

$$Var(t)_{\min} = \frac{1}{C} \sum_{h=1}^{L} \left[\sqrt{\left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{f_{h_{opt}}} \right) \left(V_0 + V_1 f_{h_{opt}} \right)} \right]^2 + \left(terms\ independent\ of\ n_h and\ f_h \right) \tag{46}$$

By ignoring the terms independent of n_h *and* f_h , we have,

$$Var(t)_{\min} = \frac{1}{C} \sum_{h=1}^{L} \left[\left(c_{ho} + c_{h1} W_{h1} + \frac{c_{h2} W_{h2}}{f_{h_{opt}}} \right) \left(V_0 + V_1 f_{h_{opt}} \right) \right] \tag{47}$$

Sample size given in (43) will minimize the survey cost and further the estimates can be made with minimum variance expressed in (47).

7. Empirical Study

For empirical study, we have considered two different populations. The description of the population is given below:

Population-I: (Source: Koyuncu and Kadilar (2009))

We consider No. of teachers as study variable *(Y)*, No. of students as auxiliary variable *(X)*, and No. of classes in primary and secondary schools as another auxiliary variable *(Z)* for 923 districts at six 6 regions (1: Marmara, 2: Agean, 3: Mediterranean, 4: Central Anatolia, 5: Black Sea, and 6: East and Southeast Anatolia) in Turkey in 2007.

Population-II: (Source: detailed livelihood assessment of flood affected districts of Pakistan September 2011, Food Security Cluster, Pakistan)

We consider food expenditure as study variable *(Y)*, household earn as auxiliary variable *(X)*, and total expenditure in May (2011) as another auxiliary variable *(Z)* for (6940) male and (1678) female households in flood affected districts of Pakistan. Further descriptive statistics for the two populations are given in Table 3.

We have used Neyman allocation to allocate the samples to different strata. The Table 4-5 shows the PRE of stratified ratio and product estimators, k_a , k_p and their class of estimator. The summary statistics of data are presented in Table 3.

Stratified Mean, S.D.'s and Correlation Coefficients						
h	Population-I					
	1	2	3	4	5	6
N_h	127	117	103	170	205	201
n_h	31	21	29	38	22	39
n'_h	70	50	75	95	70	90
S_{yh}	883.84	644.92	1033.40	810.58	403.65	711.72
S_{xh}	30486.7	15180.77	27549.69	18218.93	8497.77	23094.14
S_{zh}	555.58	365.46	612.95	458.03	260.85	397.05
$\bar{Y}_h$	703.74	413	573.17	424.66	267.03	393.84
$\bar{X}_h$	20804.59	9211.79	14309.30	9478.85	5569.95	12997.59
$\bar{Z}_h$	498.28	318.33	431.36	311.32	227.20	313.71
ρ_{xyh}	0.9360	0.9960	0.9940	0.9830	0.9890	0.9650
ρ_{xzh}	0.9396	0.9696	0.9770	0.9640	0.9670	0.9960
ρ_{yzh}	0.9790	0.9760	0.9840	0.9830	0.9640	0.9830
W_h=10% Non-response						
S_{yh2}	510.57	386.77	1872.88	1603.30	264.19	497.84
S_{xh2}	9446.93	9198.29	52429.99	34794.9	4972.56	12485.10
S_{zh2}	303.92	278.51	960.71	821.29	190.85	287.99
ρ_{xy2}	0.9961	0.9975	0.9998	0.9741	0.9950	0.9284
ρ_{xz2}	0.9901	0.9895	0.9964	0.9609	0.9865	0.9752
ρ_{yz2}	0.9931	0.9871	0.9972	0.9942	0.9850	0.9647
W_h=20% Non-response						
S_{yh2}	396.77	406.15	1654.40	1333.35	335.83	903.91
S_{xh2}	7439.16	8880.46	45784.78	29219.30	6540.43	28411.44
S_{zh2}	244.56	274.42	965.42	680.28	214.49	469.86
ρ_{xy2}	0.9954	0.9931	0.9960	0.9761	0.9966	0.9869
ρ_{xz2}	0.9897	0.9884	0.9789	0.9629	0.9820	0.9825
ρ_{yz2}	0.9898	0.9798	0.9846	0.9940	0.9818	0.9874
W_h=30% Non-response						
S_{yh2}	500.26	356.95	1383.70	1193.47	289.41	825.24
S_{xh2}	14017.99	7812.00	38379.77	26090.60	5611.32	24571.95
S_{zh2}	284.44	247.63	811.21	631.28	188.30	437.90
ρ_{xy2}	0.9639	0.9919	0.9955	0.9801	0.9961	0.9746
ρ_{xz2}	0.9107	0.9848	0.9771	0.9650	0.9794	0.9642
ρ_{yz2}	0.9739	0.9793	0.9839	0.9904	0.9799	0.9829

Stratified Mean, S.D.'s and Correlation Coefficients			
h	Population II		
	1	2	3
N_h	21	34	26
n_h	06	04	02
n'_h	15	17	08
S_{yh}	12.14	8.34	5.47
S_{xh}	76.71	31.94	49.55
S_{zh}	19.48	07.10	13.21
$\bar{Y}_h$	37.55	37.25	26.39
$\bar{X}_h$	116.57	093.00	26.39
$\bar{Z}_h$	114.14	106.50	118.9
ρ_{xyh}	0.7914	0.8339	0.770
ρ_{xzh}	0.9894	0.8820	0.967
ρ_{yzh}	0.7781	0.6651	0.594
W_h=10% Non-response			
S_{yh2}	08.66	10.05	03.95
S_{xh2}	42.14	13.28	74.22
S_{zh2}	6.25	5.20	20.53
ρ_{xy2}	0.9997	0.9995	0.984
ρ_{xz2}	0.9707	1.0000	0.999
ρ_{yz2}	0.9649	0.9996	0.982
W_h=20% Non-response			
S_{yh2}	7.96	8.47	4.06
S_{xh2}	36.50	25.82	59.32
S_{zh2}	5.20	8.18	16.54
ρ_{xy2}	0.9905	0.8026	0.860
ρ_{xz2}	0.9623	0.9858	0.996
ρ_{yz2}	0.9297	0.8062	0.811
W_h=30% Non-response			
S_{yh2}	12.70	09.86	4.50
S_{xh2}	37.69	24.02	52.26
S_{zh2}	9.42	6.83	14.54
ρ_{xy2}	0.9288	0.8335	0.828
ρ_{xz2}	0.9062	0.8859	0.991
ρ_{yz2}	0.9696	0.5877	0.754

Table 3: Data Statistics for the two populations

W_h	K	t_{HH}	t_R	$k_p^{\min}$	k_p^1	k_p^3	k_p^5	k_p^7
10%	2	100	148.0106	2589.6629	148.0106	12.09883	19.0051	5.3156
	2.5	100	154.8764	2671.5116	154.8764	12.46886	19.5063	5.4495
	3	100	160.8565	2750.3186	160.8565	12.78217	19.9274	5.5618
20%	2	100	149.4549	2904.3771	149.4549	12.00532	19.0984	5.2994
	2.5	100	155.2229	3095.1074	155.2229	12.26011	19.5129	5.3974
	3	100	159.6785	3260.7570	159.6785	12.45115	19.8241	5.4705
30%	2	100	150.7534	2825.1928	150.7534	12.15578	19.0983	5.3347
	2.5	100	156.4283	2967.2440	156.4283	12.43145	19.4761	5.4340
	3	100	160.6685	3083.5557	160.6685	13.34521	19.7500	5.5058

Table 4: PRE Values for Adapted class of Estimators

W_h	K	t_{HH}	t_R	$k_p^{\min}$	k_p^1	k_p^3	k_p^5	k_p^7
10%	2	100	148.0106	2590.0436	148.3917	12.4799	19.3862	5.6968
	2.5	100	154.8764	2671.9289	155.3048	12.8972	19.9346	5.8779
	3	100	160.8565	2750.7938	161.3321	13.2577	20.4030	6.0374
20%	2	100	149.4549	2904.5667	149.9196	12.4699	19.5630	5.7641
	2.5	100	155.2229	3095.7143	155.7766	12.8137	20.0666	5.9511
	3	100	159.6785	3261.3999	160.3212	13.0937	20.4667	6.1132
30%	2	100	150.7534	2825.6926	151.2532	12.6555	19.5980	5.8346
	2.5	100	156.4283	2967.8502	157.0346	13.0378	20.0824	6.0404
	3	100	160.6685	3084.2694	161.3815	13.3452	20.4629	6.2187

Table 5: PRE Values for the Proposed Estimator

8. Conclusion

In this paper, we have suggested generalized family of estimators for single phase stratified sampling under the situation of incomplete response on all variables. It is clearly noticed from Table 4 and 5 that the proposed family of estimators k_a and k_p are efficient as compare to mean estimator, stratified ratio estimator. So we may conclude that the suggested generalized family of estimators has shown better performance than the class of estimators and the available estimators.

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