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A COMPARISON OF PROPER AND IMPROPER PRIOR OF BAYESIAN ANALYSIS: AN APPLICATION OF MARKOV MODEL

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Abstract

This paper focuses on the application of transitional model using Bayesian approach for analyzing longitudinal binary data. Multivariate and uniform priors have been used in Bayesian analysis to estimate the parameters of Markov model. Multivariate prior is found to give better results than uniform prior.

Key Words: Bayesian Approach for Multivariate Prior (BM), Bayesian Approach for Uniform Prior (BU), Bayesian Approach under Squared Error Loss (BSE).

1. Introduction

Cause and effect relationship is the relationship in which one event (the cause) makes another event happen (the effect). A central goal of most of the researches is the identification of causal relationships that set of independent variables (the cause) has an effect on dependent variable of interest (the effect). If the outcomes of the response variable of the study are binary, then logistic regression is applied. Now-a-days, repeated measures data are widely used in many research areas. For repeated measures data, Markov based logistic regression is applied because Markov property deals with the cases where present condition depends on immediate past condition. These repeated observations of the outcome and the associated risk factors characterize the longitudinal data for the subjects of a certain population. Markov chain is a suitable probability model for longitudinal data in which at a given time, the outcome is a categorical variable. The choice of Markov chains arises because they are often a good approximation to the structure of serially dependent data. The dependence relationship is commonly assumed to be of first order. Korn and Whittemore (1979) proposed a model to incorporate role of previous state as a covariate to analyze the probability of occupying the current state. Regier (1968) introduced a two state transition matrix for estimating odds ratio. Azzalini (1994) examined the influence of time dependent covariate on the marginal distribution of the binary outcome variables in serially correlated data. Muenz-Rubinstein (1985) employed a logistic regression model to analyze the transitional probabilities from one state to another. Among the recent works on Muenz-Rubinstein model, Islam and Chowdhury (2004, 2006), Islam et al. (2008, 2012) and Chowdhury et al. (2005) are noteworthy. All of them have applied the method of maximum likelihood approach for decision-making.

In some cases, parameter behaves as a random variable. In that situation classical approaches cannot be applied. Bayesian approach helps us to deal such a situation. Bayesian estimation is extending rapidly in many areas. Noorian and Ganjali (2012) applied Bayesian analysis of transitional model for longitudinal ordinal data, but in their study, they applied Markov chain Monte Carlo (MCMC) method. Acquah (2013) has also used MCMC applied Bayesian logistic regression for economic data. Mahanta and Biswas (2016) have employed Bayesian approach in Azzalini model. Although MCMC method is well known and popular but this method is to be solved by programming. There is no theoretical idea about the procedure of estimating parameter. To get proper idea of the estimating procedure, in this paper, Muenz-Rubinstein model has been estimated theoretically and the numerical findings are obtained using R programming. Mahanta et al. (2015) have applied both Bayesian approach using uniform prior and method of maximum likelihood approach for estimating Muenz-Rubinstein model. In Bayesian approach, prior distribution is the most important ingredients, using proper prior that means the known distribution of the parameter, Bayesian approach may give more accurate results. That's why multivariate normal prior has been used in this paper.

2. Model

The transition matrix of a two states discrete time binary sequence Markov chain is as

$$P = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix}$$

where, P is the transition probability matrix, p_{00} denotes the transition probability from state 0 to 0 and p_{10} is the transition probability from state 1 to 0. At each time point, a vector of length two contains the probability of outcome of interest and its complement.

Muenz and Rubinstein used the model where the transition probabilities p_{00} and p_{10} are replaced by by logistic regressions i.e.

$$p_{00} = \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \text{ and } p_{10} = \frac{\exp(\alpha' X)}{1+\exp(\alpha' X)} \tag{1}$$

The vector X contains covariates and for the q^{th} person in the study is $X_q = (1, X_{q1},...X_{qp})$.

There are two logistic regressions, one having parameter vector $\beta = (\beta_0,...,\beta_p)'$ and the other having parameter vector $\alpha = (\alpha_0,...,\alpha_p)$. Large positive (negative) values of $\beta' X$ and $\alpha' X$ yield large (small) transition probabilities. The above transition probabilities follow multinomial distribution, for 0 to 0 transitions the joint distribution of the above Markov model is

$$\begin{aligned} f(x/\beta) &= \prod_{i=0}^{n} \left[\{p_{00}\}^{n_{00i}} \{1 - p_{00}\}^{n_{01i}} \right] \\ &= \prod_{i=0}^{n} \left[\left\{ \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \right\}^{n_{00i}} \left\{ \frac{1}{1+\exp(\beta' X)} \right\}^{n_{01i}} \right]; \ -\infty \le \beta \le +\infty \end{aligned} \tag{2}$$

where, n_{00i} and n_{01i} are the number of transitions.

3. Prior and Posterior distribution

Selection of a prior distribution is an important part in Bayesian approach. When proper information is available and parametric value lies between $-\infty$ *to* $+\infty$ then informative prior distribution is multivariate normal. The p.d.f of Multivariate (Acquah, 2013) normal distribution is

$$g(\beta) = \frac{1}{(2\pi)^{\frac{p}{2}} |\Sigma|^{\frac{1}{2}}} e^{-\frac{1}{2}(\beta-\mu)' \Sigma^{-1} (\beta-\mu)};$$
$$-\infty \le \beta \le +\infty, \quad -\infty \le \mu \le +\infty,$$
$$\Sigma > 0. \tag{3}$$

where, μ is the mean vector and Σ is the variance-covariance matrix.
Then the posterior density of β for the given sample is

$$f(\beta/X) = \frac{\prod_{i=0}^{n} \left\{ \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \right\}^{n_{00i}} \left\{ \frac{1}{1+\exp(\beta' X)} \right\}^{n_{01i}} e^{-\frac{1}{2}(\beta-\mu)' \Sigma^{-1} (\beta-\mu)}}{\int_{-\infty}^{\infty} \prod_{i=0}^{n} \left\{ \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \right\}^{n_{00i}} \left\{ \frac{1}{1+\exp(\beta' X)} \right\}^{n_{01i}} e^{-\frac{1}{2}(\beta-\mu)' \Sigma^{-1} (\beta-\mu)} d\beta} \tag{4}$$

4. Loss function

Loss function is the important ingredients for Bayesian approach. The squared error loss function is defined as

$$L\left(\hat{\beta}; \beta\right) = \left(\hat{\beta} - \beta\right)^2 \tag{5}$$

For squared error loss function, Bayes estimators (Mahanta et al., 2015) are the mean of the posterior density

$$\hat{\beta}_{BM} = \frac{\int_{-\infty}^{\infty} \beta \prod_{i=0}^{n} \left[\left\{ \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \right\}^{n_{00i}} \left\{ \frac{1}{1+\exp(\beta' X)} \right\}^{n_{00i}} \right] e^{-\frac{1}{2}(\beta-\mu)' \Sigma^{-1} (\beta-\mu)} d\beta}{\int_{-\infty}^{\infty} \prod_{i=0}^{n} \left[\left\{ \frac{\exp(\beta' X)}{1+\exp(\beta' X)} \right\}^{n_{01i}} \left\{ \frac{1}{1+\exp(\beta' X)} \right\}^{n_{01i}} \right] e^{-\frac{1}{2}(\beta-\mu)' \Sigma^{-1} (\beta-\mu)} d\beta} \tag{6}$$

The two integrals appear in the ratio cannot be solved to have a closed form. For evaluating them, we use the Lindley (1980) approximation.
Lindley (1980) suggest that, if the form of the integrals is

$$I(X) = E(u(\beta)/X) = \frac{\int u(\beta) e^{L_0(\beta)+p(\beta)} d\beta}{\int e^{L_0(\beta)+p(\beta)} d\beta} \tag{7}$$

where, $I(X)$ represent the form of the integral, L_0 is the log likelihood and $p(\beta)$ is the log of prior.

Then according to Lindley (1980), the integral can be approximately be evaluated as

$$I(X) = u(\hat{\beta}) + \frac{1}{2}\left[u''(\hat{\beta}) + 2u'(\hat{\beta})p'(\hat{\beta}) + L_0'''(\hat{\beta})u(\hat{\beta})\hat{\sigma}^2\right]\hat{\sigma}^2 \quad (8)$$

where, $u(\beta)$ is the functional form of the parameter β, that is used in expectation of posterior density and $\hat{\beta}$ is the maximum likelihood estimators of β.

From equation (3), we have

$$p(\beta) = \log g(\beta) = -\frac{p}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma| - \frac{1}{2}(\beta-\mu)'\Sigma^{-1}(\beta-\mu) \quad (9)$$

Differentiating both sides with respect to β

$$\frac{\delta p(\beta)}{\delta \beta} = -(\beta-\mu)'\Sigma^{-1} \quad (10)$$

Using equation (2) the log likelihood function is

$$L_0 = \sum_{i=1}^{n}\left[n_{00i}\log\frac{\exp(\beta'X)}{1+\exp(\beta'X)} + n_{01i}\log\frac{1}{1+\exp(\beta'X)}\right] \quad (11)$$

Differentiating successively both sides with respect to β, we have

$$\frac{\delta L_0}{\delta \beta} = \sum_{i=1}^{n}\left[n_{00i}X_i - (n_{00i}+n_{01i})\frac{\exp(\beta'X)X_i}{1+\exp(\beta'X)}\right]$$

$$\frac{\delta^2 L_0}{\delta \beta^2} = -\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\,\hat{p}_{00}\,\hat{p}_{01}$$

$$\frac{\delta^3 L_0}{\delta \beta^3} = \sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^3\,\hat{p}_{00}\,\hat{p}_{01}\left(\hat{p}_{00}-\hat{p}_{01}\right)$$

$$\therefore \hat{\sigma}^2 = -\frac{1}{L_0''} = \frac{1}{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\,\hat{p}_{00}\,\hat{p}_{01}}$$

From equation (8), Bayes estimator under squared error loss function using Multivariate normal prior is

$$\hat{\beta}_{BM} = \hat{\beta} + \hat{\beta}\left[\frac{1}{2}\frac{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^3\,\hat{p}_{00}\,\hat{p}_{01}\left(\hat{p}_{00}-\hat{p}_{01}\right)}{\left\{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\,\hat{p}_{00}\,\hat{p}_{01}\right\}^2}\right] - \frac{(\hat{\beta}-\mu)'\Sigma^{-1}}{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\,\hat{p}_{00}\,\hat{p}_{01}}. \quad (12)$$

Where, $\hat{\beta}$ is the maximum likelihood estimate of β (Mahanta et al., 2015) and $\hat{\beta}_{BM}$ is the Bayes estimator under squared error loss function for Multivariate normal prior distribution of β.

5. Bayes estimator under squared error loss function using uniform prior distribution

Mahanta et al. (2015) used uniform prior to estimate the parameter of Muenz-Rubinstein model which is given below

$$\hat{\beta}_{BU} = \hat{\beta} + \hat{\beta}\left[\frac{1}{2}\frac{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^3\,\hat{p}_{00}\,\hat{p}_{01}\left(\hat{p}_{00}-\hat{p}_{01}\right)}{\left\{\sum_{i=1}^{n}(n_{00i}+n_{01i})X_i^2\,\hat{p}_{00}\,\hat{p}_{01}\right\}^2}\right] \tag{13}$$

where, $\hat{\beta}_{BU}$ is the Bayes estimator of β for uniform prior.

6. Posterior risk function under squared error loss function

Posterior risk function is the expected value of loss function with respect to posterior density

$$\begin{aligned} R_p(\beta) &= \int\left(\hat{\beta}-\beta\right)^2 f(\beta/X)d\beta \\ &= \int\left(\hat{\beta}^2 - 2\hat{\beta}\beta + \beta^2\right)f(\beta/X)d\beta \\ &= -\hat{\beta}_{BSE}^2 + \int \beta^2 f(\beta/X)d\beta \end{aligned} \tag{14}$$

where, $\hat{\beta}_{BSE}$ is the Bayes estimator under squared error loss function.

7. Results and Discussion

In this paper, we have used pregnancy complication data obtained from Bangladesh Institute of Research for Promotion of Essential & Reproductive Health and Technologies (BIRPERHT) for the period November 1992 to December 1993. The data were collected using both cross-sectional and prospective study designs. A total of 1059 pregnant women were interviewed in the follow-up component of the study. We have estimated the parameters of Muenz-Rubinstein model using pregnancy complication data. Three covariates have been utilized in this study because of complexity to fit the model. Three highly significant covariates viz. any miscarriage, socio economic status and age at marriage are used. Bayesian approach has been applied for estimating the parameters of the model.

Table 1 reveals that in 0 to 0 transitions, any miscarriage and age at marriage are positively and economic status is negatively associated with pregnancy complication. Table 2 shows that in 1 to 0 transitions any miscarriage and economic status are positively and age at marriage is negatively associated with pregnancy complication for all cases. Also, intercept terms have been shown positive and negative index in 0 to 0 and 1 to 0 transitions respectively. Flat (uniform) prior and multivariate normal prior have been used. For multivariate normal prior, different values of mean vector and variance-covariance matrix have been taken to get precise results.

Covariates	Estimate	Posterior Risk	Prior
Constant	1.7756	0.0009	Flat or Uniform
Any miscarriage	0.1604	0.0002	
Economic Status	-0.3065	0.0002	
Age at Marriage	0.0244	0.0002	
Type-1			$\beta \sim MND(\mu, \Sigma)$
Constant	1.7752	0.0005	$\mu = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{bmatrix} \text{ \& } \Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.1603	0.0002	
Economic Status	-0.3064	0.0002	
Age at Marriage	0.0244	0.0002	
Type-2			$\beta \sim MND(\mu, \Sigma)$
Constant	1.7752	0.0006	$\mu = \begin{bmatrix} -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \end{bmatrix} \text{ \& } \Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.1603	0.0002	
Economic Status	-0.3065	0.0002	
Age at Marriage	0.0243	0.0002	
Type-3			$\beta \sim MND(\mu, \Sigma)$
Constant	1.7752	0.0005	$\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{ \& } \Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.1603	0.0002	
Economic Status	-0.3065	0.0002	
Age at Marriage	0.0243	0.0002	
Type-4			$\beta \sim MND(\mu, \Sigma)$
Constant	1.7755	0.0008	$\mu = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \text{ \& } \Sigma = \begin{bmatrix} 0.1 & -0.002 & -0.002 & -0.002 \\ -0.002 & 0.1 & -0.002 & -0.002 \\ -0.002 & -0.002 & 0.1 & -0.002 \\ -0.002 & -0.002 & -0.002 & 0.1 \end{bmatrix}$
Any miscarriage	0.1604	0.0002	
Economic Status	-0.3065	0.0002	
Age at Marriage	0.0244	0.0002	
Type-5			$\beta \sim MND(\mu, \Sigma)$
Constant	1.7755	0.0007	$\mu = \begin{bmatrix} -0.2 \\ -0.2 \\ -0.2 \\ -0.2 \end{bmatrix} \text{ \& } \Sigma = \begin{bmatrix} 0.1 & -0.002 & -0.002 & -0.002 \\ -0.002 & 0.1 & -0.002 & -0.002 \\ -0.002 & -0.002 & 0.1 & -0.002 \\ -0.002 & -0.002 & -0.002 & 0.1 \end{bmatrix}$
Any miscarriage	0.1603	0.0002	
Economic Status	-0.3065	0.0002	
Age at Marriage	0.0243	0.0002	

Table 1: Estimate the parameters and its posterior risk for different prior distributions (0 to 0 Transitions)

Covariates	Estimate	Posterior Risk	Prior
Constant	-0.7024	0.0012	Flat or Uniform
Any miscarriage	0.2856	0.0010	
Economic Status	0.2356	0.0010	
Age at Marriage	-0.1636	0.0009	
Type-1			$\beta \sim MND(\mu, \Sigma)$
Constant	-0.7017	0.0003	$\mu = \begin{bmatrix} 0.01 \\ 0.01 \\ 0.01 \\ 0.01 \end{bmatrix}$ & $\Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.2853	0.0008	
Economic Status	0.2353	0.0009	
Age at Marriage	-0.1634	0.0009	
Type-2			$\beta \sim MND(\mu, \Sigma)$
Constant	-0.7017	0.0003	$\mu = \begin{bmatrix} -0.01 \\ -0.01 \\ -0.01 \\ -0.01 \end{bmatrix}$ & $\Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.2853	0.0008	
Economic Status	0.2353	0.0009	
Age at Marriage	-0.1634	0.0009	
Type-3			$\beta \sim MND(\mu, \Sigma)$
Constant	-0.7017	0.0003	$\mu = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ & $\Sigma = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
Any miscarriage	0.2853	0.0008	
Economic Status	0.2353	0.0009	
Age at Marriage	-0.1634	0.0009	
Type-4			$\beta \sim MND(\mu, \Sigma)$
Constant	-0.7023	0.0011	$\mu = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \\ 0.2 \end{bmatrix}$ & $\Sigma = \begin{bmatrix} 0.1 & -0.002 & -0.002 & -0.002 \\ -0.002 & 0.1 & -0.002 & -0.002 \\ -0.002 & -0.002 & 0.1 & -0.002 \\ -0.002 & -0.002 & -0.002 & 0.1 \end{bmatrix}$
Any miscarriage	0.2856	0.0010	
Economic Status	0.2355	0.0010	
Age at Marriage	-0.1636	0.0009	
Type-5			$\beta \sim MND(\mu, \Sigma)$
Constant	-0.7023	0.0012	$\mu = \begin{bmatrix} -0.2 \\ -0.2 \\ -0.2 \\ -0.2 \end{bmatrix}$ & $\Sigma = \begin{bmatrix} 0.1 & -0.002 & -0.002 & -0.002 \\ -0.002 & 0.1 & -0.002 & -0.002 \\ -0.002 & -0.002 & 0.1 & -0.002 \\ -0.002 & -0.002 & -0.002 & 0.1 \end{bmatrix}$
Any miscarriage	0.2855	0.0010	
Economic Status	0.2355	0.0009	
Age at Marriage	-0.1636	0.0009	

Table 2: Estimate the parameters and its posterior risk for different prior distributions (1 to 0 Transitions)

8. Bayesian Credible Interval

If $f(\beta / X)$ is the posterior distribution given the sample, we may be interested in finding an interval such that

$$P(\beta \in (\beta_1, \beta_2) / X) = \int_{\beta_1}^{\beta_2} f(\beta / X) d\beta = 1 - \alpha \qquad (15)$$

Mahanta and Biswas (2016) used ($1-\alpha$) 100% Bayesian credible interval of β.

In Bayesian analysis, credible interval becomes the counterpart of the classical confidence interval, also credible interval may be unique for all models. The Bayesian credible interval, on the other hand, has a direct probability interpretation $P(\beta \in (\beta_1, \beta_2)/x) \geq 1-\alpha$ and is completely determined from the current observed data x and the prior distribution.

Covariates	Length of credible interval =Upper Limit-Lower Limit						Minimum Length
	Prior						
	Flat	Type-1	Type-2	Type-3	Type-4	Type-5	
0 to 0 Transitions							
Constant	0.1180	0.0888	0.0962	0.0895	0.1094	0.1072	Type-1
Any miscarriage	0.0597	0.0583	0.0572	0.0582	0.0598	0.0594	Type-2
Economic Status	0.0616	0.0558	0.0579	0.0560	0.0607	0.0613	Type-1
Age at Marriage	0.0590	0.0590	0.0588	0.0590	0.0590	0.0590	Type-2
1 to 0 Transitions							
Constant	0.1376	0.0684	0.0713	0.0699	0.1309	0.1337	Type-1
Any miscarriage	0.1226	0.1131	0.1124	0.1127	0.1222	0.1210	Type-2
Economic Status	0.1216	0.1152	0.1146	0.1149	0.1214	0.1204	Type-2
Age at Marriage	0.1205	0.1171	0.1175	0.1173	0.1198	0.1205	Type-1

Table 3: Length of Bayesian Credible interval for different prior distributions

Lengths of all Bayesian credible intervals are smaller for case of multivariate prior than flat prior. For transitions 0 to 0, minimum lengths of credible interval for any miscarriage and age at marriage are corresponding to type-2 condition of MND and for economic status corresponding to type-1 condition of MND. On the other hand in transitions 1 to 0, minimum length of credible interval for any miscarriage and economic status are corresponding to type-2 and for age at marriage corresponding to type-1 condition of MND, as revealed from Table 3. Under both the transitions for intercept term, minimum length of the credible interval has been observed corresponding to type 1 condition of MND. Here, type-1 and type-2 forms of multivariate normal distribution (MND) have mean vectors $\mu' = [0.01 \quad 0.01 \quad 0.01 \quad 0.01]$, $\mu' = [-0.01 \quad -0.01 \quad -0.01 \quad -0.01]$ with the same variance-covariance matrix $\Sigma = I_4$ respectively. Type-3, type-4 and type-5 have been shown in Table 1. All the calculations were performed by using R-Software (Version-2.10.0).

9. Conclusions

Markov chain models are widely used in medical science, engeenering, social research etc. for better prediction. Bayesian approach is the non-classical approach to estimate the parameter of any model or distribution. This method is applied when the parameters are random, that is, there is a distribution of the parameter. This paper uses Bayesian approach to estimate the parameters of Muenz-Rubinstein (1985) model. Multivariate prior and uniform (flat) priors are employed for Bayesian estimation. Form the above analysis, it has been observed that Bayesian approach for multivariate prior is better than flat prior. Thus Bayesian approach for multivariate prior can be suggested for the estimation of the parameters of Muenz-Rubinstein model.

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References

1. Acquah, H. D. (2013). Bayesian Logistic Regression Modeling via Markov Chain Monte Carlo Algorithm, Journal of Social and Development Sciences, 4 (4), p. 193-197.
2. Azzalini, A. (1994). Logistic Regression for Autocorrelated data with Application to Repeated Measure, Biometrika, 81(4), p. 767-775.
3. Chowdhury, R. I., Islam, M. A., Shah, M. A. and Enezi, N. A. (2005). A computer program to estimate the parameters of covariate dependent higher order Markov model, Computer Methods and Programs in Biomedicine, 77, p. 175-181.
4. Islam, M. A. and Chowdhury, R. I. (2004). A Three State Markov Model for Analyzing Covariate Dependence, International Journal of Statistical Sciences, 3, p. 241-249.
5. Islam, M. A. and Chowdhury, R. I. (2006). A higher order Markov model for analyzing covariate dependence, Applied Mathematical Modelling, 30, p. 477–488.
6. Islam, M. A., Chowdhury, R. I. and Singh, K. P. (2008). Covariate Dependent Markov Models for Analysis of Repeated Binary Outcomes, Journal of Modern Applied Statistical Methods, 6(2), p. 561-572.
7. Islam, M. A., Chowdhury, R. I. and Singh, K. P. (2012). A Markov Model for Analyzing Polytomous Outcome Data, Pakistan Journal of Statistics and Operation Research, 8 (3), p. 593-603.
8. Kalbfleisch, J. D. and Lawless, J. F. (1985). The analysis of panel data under a Markov assumption, Journal of American Statistical Association, 88, p. 863–871.
9. Kass and Raftery, A. E. (1995). Bayes Factor. Journal of American Statistical Association , 90, p. 773-795.
10. Korn, E.L. and Whittemore, A.S. (1979). Methods of Analyzing Panel Studies of Acute Health Effects of Air Pollution, Biometrics, 35, p. 795-802.
11. Lindley, D. V. (1980). Approximate Bayes methods, Bayesian Statistics, Valencia, Spain: University Press.
12. Mahanta, J., Biswas, S. C., Roy, M. K. and Islam, M. A. (2015). A

Comparison of Bayesian and Classical Approach for Estimating Markov Based Logistic Model, American Journal of Mathematics and Statistics, 5(4), p. 178-183.

13. Mahanta, J. and Biswas, S. C. (2015). Comparison of Bayesian approach and classical approach for estimating the parameter of Markov model, Journal of reliability and statistical studies, 9(2), p. 81-90.
14. Muenz, K.R. and Rubinstein, L.V. (1985). Markov chain for covariance dependence of binary sequences, Biometrics, 41, p. 91-101.
15. Noorian, S. and Ganjali, M. (2012). Bayesian Analysis of Transition Model for Longitudinal Ordinal Response Data: Application to Insomnia Data, International Journal of Statistics in Medical Research, 1, p. 148-161
16. Regier, M. H. (1968). A Two State Markov Model for Behavior Change, Journal of American Statistical Association, 63, p. 993-999.

APPENDIX

```
library(foreign,MASS)
data<-
as.matrix(read.table("D:/three.txt"))
initial<-c(.1,.1,.1,.1)
a=0.1
b=-0.002
c=-0.2
M<- matrix(c(c,c,c,c),nrow=4)
S=
matrix(c(a,b,b,b,b,a,b,b,b,b,a,b,b,b,b,a
),ncol=4,byrow=T)
Rub1<-function(data,initial)
{

        id<-data[,1]
        fup<-data[,2]
        A<-data[,3]
        B<-data[,4]
        C<-data[,5]
        count<-0
        k<-0
repeat {

        b0<-initial[1]
        b1<-initial[2]
        b2<-initial[3]
        b3<-initial[4]

        b<-as.vector(c(b0,b1,b2,b3))

        infb=matrix(c(0,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0),ncol=4,byrow=T)
        k<-k+1

x=cbind(1,A,B,C)
g00=exp(x%*%b)/(1+exp(x%*%b))
g01=1/(1+exp(x%*%b))

sb=colSums(x*as.vector(data[,6]-
(data[,6]+data[,7])*g00))

for(i in 1:ncol(x)){
for(j in 1:ncol(x)){
for(l in 1:ncol(x)){
infb[i,j]=(sum(x[,i]*x[,j]*(as.matrix(data[
,6]+data[,7]*(g01)*(g01)))))
}
}
}
Fisinvb=solve(infb)
count<-count+1
```

```
se<-matrix(cbind(sqrt(c(Fisinvb [1,1],
Fisinvb [2,2], Fisinvb [3,3], Fisinvb
[4,4]))),nrow=4)
h00=( exp(x%*%(lik
)))/(1+exp(x%*%(lik)))
h01=1/(1+exp(x%*%(lik)))
denb<-
(sum(x[,i]*x[,j]*(as.matrix(data[,6]+data
[,7]*(h00)*(h01)))))
neob<-
(sum(x[,i]*x[,j]*x[,l]*(as.matrix(data[,6]+
data[,7]*(h00)*(h01)*(h00-h01)))))
bse<-lik+lik*(neob/(2*(denb)^2))
cat("The Bayes estimate for squared
error is\n")
print(bse)
#calculation of posterior risk
bse2<-
bse*bse+bse*bse*(neob/(2*(denb)^2))+(
1/denb)
rbse<-(-bse*bse)+bse2
cat("The posterior risk of BSE is\n")
print(rbse)
#calculation of Bayesian estimate for
Mutivariate prior
bse1<-bse-(S%*%(lik-M))/denb
cat("The Bayes estimate for squared
error for Multivariate prior is\n")
print(bse1)
#Calculation its posterior risk
bse21<-
bse1*bse1+bse1*bse1*(neob/(2*(denb)
^2))+(1/denb)-2*bse1*(S%*%(bse1-
M))/denb
rbse1<-(-bse1*bse1)+bse21
cat("The posterior risk of BSE  for
Multivariate prior is\n")
print(rbse1)
#Calculation of Bayes factor
bb00=( exp(x%*%(
bse1)))/(1+exp(x%*%( bse1)))
bb01=1/(1+exp(x%*%( bse1)))
sbc=colSums(x*as.vector(data[,6]))%*%
bse1
sbd=colSums(as.vector(data[,6]+data[,
7])*log(1/bb01))
#Likelihood of multivariate prior
sbf=sbc-sbd
bm00=(
exp(x%*%(bse)))/(1+exp(x%*%(bse)))
bm01=1/(1+exp(x%*%(bse)))
```

```
lik<-b+Fisinvb%*%sb
conv<-abs(initial-lik)
if(conv[1]<=0.001 && conv[2]<=0.001
&& conv[3]<=0.001 &&
conv[4]<=0.001)
       break
initial<-lik
}
sbmc=colSums(x*as.vector(data[,6]))%*
%bse
sbmd=colSums(as.vector(data[,6]+data
[,7])*log(1/bm01))
#Likelihood of uniform prior
sbmf=sbmc-sbmd
bfactor=sbf/sbmf
cat("Bayes factor of Multivariate prior
with respect to uniform prior\n")

print(bfactor)

}
Rub1(data, initial)
```