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RATIO AND RATIO TYPE SEPARATE ESTIMATOR FOR POPULATION MEAN WHEN COEFFICIENT OF VARIATION IS KNOWN

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Abstract

This paper deals with the problem of estimation of the finite population mean using auxiliary information. A ratio and ratio type separate estimator has been suggested for the finite population mean. The bias and mean squared error have been derived. The suggested estimator has been compared theoretically as well as empirically with existing estimators.

Key Words: Population Mean, Sample Mean, Stratification, Separate Type Estimator, Separate Ratio and Ratio Type Estimator.

1. Introduction

This paper is classified into four sections. First is the introduction that discuss the problem under consideration and relevant work done by other researchers. Second section is the suggested estimator in which an estimator for population mean has been suggested. Section 3 is the efficiency comparisons and in section 4 suggested estimator has been compared with other estimators empirically.

Simple random sampling has limitations of improper representation and administrative inconveniency. In case of heterogonous population also it is not used in general. In these type of situations, stratified random sampling is a good option where heterogeneous population is divided into homogenous blocks called strata and a simple random sample is drawn from each stratum. When information on parameters of auxiliary information is known for all strata, it is considered better to use separate type estimators for gain in efficiency. Suppose from the population S of size N, a sample of size n is drawn using stratified sampling. Let y be the study variate and x and z are auxiliary variates taking values y_{hi}, x_{hi}, z_{hi} (h =1,2….,L; i =1,2…, N_h).

Classical combined ratio estimator in stratified random sampling was defined and studied by Hansen et al. (1946) as

$$\hat{\bar{Y}}_{RC} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\sum_{h=1}^{L} W_h \bar{X}_h}{\sum_{h=1}^{L} W_h \bar{x}_h} \right) = \bar{y}_{st} \left(\frac{\bar{X}}{\bar{x}_{st}} \right). \tag{1.1}$$

Tailor et al. (2012) studied Singh (1967) ratio-cum-product estimator in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{st} = \bar{y}_{st} \left(\frac{\bar{X}}{\bar{x}_{st}} \right) \left(\frac{\bar{z}_{st}}{\bar{Z}} \right). \tag{1.2}$$

Parmar (2012) studied a ratio-cum-product estimator for population mean in stratified random sampling using coefficient of variation of auxiliary variate's as

$$\hat{\bar{Y}}_{RPS}^{st} = \bar{y}_{st} \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{X}_h + C_{xh} \right)}{\sum_{h=1}^{L} W_h \left(\bar{x}_h + C_{xh} \right)} \right) \left(\frac{\sum_{h=1}^{L} W_h \left(\bar{z}_h + C_{zh} \right)}{\sum_{h=1}^{L} W_h \left(\bar{Z}_h + C_{zh} \right)} \right). \tag{1.3}$$

The classical separate ratio estimator for population mean is defined as

$$\hat{\bar{Y}}_{RS} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h} \right). \tag{1.4}$$

Separate Version of Chouhan (2012) studied separate ratio type exponential estimator for the population mean is expressed

$$\hat{\bar{Y}}_{Res} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right). \tag{1.5}$$

Tailor and Lone (2014) proposed a separate ratio type estimator using coefficient of variation of auxiliary variate from each stratum as

$$\hat{\bar{Y}}_{RS}^{C} = \sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h + C_{xh}}{\bar{x}_h + C_{xh}} \right). \tag{1.6}$$

mean squared error of $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{Res}$ and $\hat{\bar{Y}}_{RS}^{C}$ are

$$MSE(\hat{\bar{Y}}_{RS}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + R_{1h}^2 S_{xh}^2 - 2R_{1h} S_{yxh} \right], \tag{1.7}$$

$$MSE(\hat{\bar{Y}}_{Res}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4} S_{xh}^2 R_{1h}^2 - S_{yh} S_{xh} \rho_{yxh} R_{1h} \right], \tag{1.8}$$

and

$$MSE(\bar{Y}_{RS}^{C}) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \theta_{1h}^2 R_{1h}^2 S_{xh}^2 - 2R_{1h} \theta_{1h} S_{yxh} \right], \tag{1.9}$$

where

$$\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right), R_{1h} = \frac{\overline{Y}_h}{\overline{X}_h} \text{ and } \theta_{1h} = \frac{\overline{X}_h}{\overline{X}_h + C_{xh}}$$

2. Suggested Estimator

Using the information on coefficient of variation of auxiliary variates x and z of each stratum i.e. C_{xh} and C_{zh}, suggested ratio and ratio type estimator for population mean as

$$\hat{\overline{Y}}^C_{RRS} = \sum_{h=1}^{L} W_h \overline{y}_h \left(\frac{\overline{X}_h + C_{xh}}{\overline{x}_h + C_{xh}}\right)\left(\frac{\overline{Z}_h + C_{zh}}{\overline{z}_h + C_{zh}}\right). \tag{2.1}$$

Let us suppose that

$\overline{y}_h = \overline{Y}_h(1+e_{0h}),\ \overline{x}_h = \overline{X}_h(1+e_{1h}),\quad \overline{z}_h = \overline{Z}_h(1+e_{2h})$

$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0$

$E(e_{0h}^2) = \gamma_h C_{yh}^2, E(e_{1h}^2) = \gamma_h C_{xh}^2, E(e_{2h}^2) = \gamma_h C_{zh}^2$

$E(e_{0h}e_{1h}) = \gamma_h \rho_{yxh} C_{xh} C_{yh}$,

$E(e_{0h}e_{2h}) = \gamma_h \rho_{yzh} C_{zh} C_{yh}$,

and

$E(e_{1h}e_{2h}) = \gamma_h \rho_{xzh} C_{xh} C_{zh}$,

In this section for a ratio and ratio type estimate has been suggested for population mean and it's bias and mean squared error are being derived.
Substituting the some values in (2.1), upto the first degree of approximation the bias and mean squared error of the suggested estimator $\hat{\overline{Y}}^C_{RRS}$ are obtained as

$$B(\hat{\overline{Y}}^C_{RRc}) = \sum_{h=1}^{L} W_h \gamma_h \left[\frac{1}{\overline{X}_h}\left(\theta_{1h}^2 S_{xh}^2 R_{1h} - \theta_{1h} S_{xh} S_{yh} \rho_{yxh}\right)\right.$$
$$\left. + \frac{1}{\overline{Z}_h}\left(\theta_{2h}^2 S_{zh}^2 R_{2h} + \theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} - \theta_{2h} S_{yh} S_{zh} \rho_{yzh}\right)\right], \tag{2.2}$$

$$MSE(\hat{\overline{y}}^C_{RRC}) = \sum_{h=1}^{L} w_h^2 \gamma_h \left[S_{yh}^2 + \theta_{1h}^2 S_{xh}^2 R_{1h}^2 + \theta_{2h}^2 S_{zh}^2 R_{2h}^2 - 2\theta_{1h} S_{yh} S_{xh} \rho_{yxh} R_{1h}\right.$$
$$\left. -2\theta_{2h} S_{yh} S_{zh} \rho_{yzh} R_{2h} + 2\theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} R_{1h} R_{2h} \right], \tag{2.3}$$

$$\theta_{1h} = \frac{\overline{X}_h}{\overline{X}_h + C_{xh}} \quad \text{and} \quad \theta_{2h} = \frac{\overline{Z}_h}{\overline{Z}_h + C_{zh}}$$

3. Efficiency Comparison

In this section suggested estimator has been compared with other considered estimator on the basis of mean squared error.

Variance of the unbiased estimator $\overline{y}_{st}$ is expressed as

$$V(\overline{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \tag{3.1}$$

Comparison of (1.7), (1.8), (1.9) and (3.1) exhibits that the estimator $\hat{\overline{Y}}_{RRc}^{C}$ would better than

(i) $\overline{Y}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big[\theta_{1h}^2 S_{xh}^2 R_{1h}^2 + \theta_{2h}^2 S_{zh}^2 R_{2h}^2 - 2\theta_{1h} S_{yh} S_{xh} \rho_{yxh} R_{1h} - 2\theta_{2h} S_{yh} S_{zh} \rho_{yzh} R_{2h} + 2\theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} R_{1h} R_{2h} \Big] < 0, \tag{3.2}$$

(ii) $\hat{\overline{Y}}_{RS}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big[R_{1h}^2 S_{xh}^2 (\theta_{1h}^2 - 1) + \theta_{2h}^2 S_{zh}^2 R_{2h}^2 - 2R_{1h} S_{yxh} (\theta_{1h} - 1) - 2\theta_{2h} S_{yh} S_{zh} \rho_{yzh} R_{2h} + 2\theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} R_{1h} R_{2h} \Big] < 0, \tag{3.3}$$

(iii) $\hat{\overline{Y}}_{Res}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \Big[S_{xh}^2 R_{1h}^2 (\theta_{1h}^2 - \frac{1}{4}) + \theta_{2h}^2 S_{zh}^2 R_{2h}^2 - R_{1h} S_{xh} S_{yh} \rho_{yxh} (2\theta_{1h} + 1) - 2\theta_{2h} S_{yh} S_{zh} \rho_{yzh} R_{2h} + 2\theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} R_{1h} R_{2h} \Big] < 0, \tag{3.4}$$

and

(iv) $\hat{\overline{Y}}_{RS}^{C}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \big[2\theta_{1h}\theta_{2h} S_{xh} S_{zh} \rho_{xzh} R_{1h} R_{2h} - 2\theta_{2h} S_{yh} S_{zh} \rho_{yzh} R_{2h} \big] < 0, \tag{3.5}$$

4. Empirical Study

In this section suggested estimators is being compared numerically with other considered estimators. For this purpose percent relative efficiencies of all estimators are calculated, **u**sing a natural data set. Details of the considered data are given below [Sources: Ministry of Agriculture] (http://agricoop.nic.in/agristatistics.html):

y:Total cereal production(Million tons),

z:Total area under cultivation for cereals (million hectares)
x:Area under total cultivation of food grain(million hectares)

$n_1 = 4$	$n_2 = 4$	$N_1 = 10$	$N_2 = 10$
$\overline{X}_1 = 107.51$	$\overline{X}_2 = 118.20$	$\overline{Z}_1 = 85.55$	$\overline{Z}_2 = 95.11$
$\overline{Y}_1 = 55$	$\overline{Y}_2 = 73.90$	$S_{x_1} = 6.29$	$S_{x_2} = 2.66$
$S_{y_1} = 7.25$	$S_{y_2} = 8.02$	$S_{z_1} = 4.31$	$S_{z_2} = 3.21$
$S_{yx_1} = 44.42$	$S_{xy_2} = 20.34$	$S_{xz_1} = 3.95$	$S_{xz_2} = 7.07$
$S_{yz_1} = 30.75$	$S_{yz_2} = 24.09$	$C_{y_1} = 0.1318$	$C_{y_2} = 0.1085$
$C_{x_1} = 0.052$	$C_{x_2} = 0.022$	$C_{z_1} = 0.050$	$C_{z_2} = 0.033$

$MSE(\hat{\hat{Y}}^C_{RRS}) - MSE(\overline{y}_{st}) < 0$	$\begin{bmatrix} R^2_{1h}S^2_{xh}\theta^2_{1h} + R^2_{2h}S^2_{zh}\theta^2_{2h} - 2\theta_{1h}R_{1h}S_{xyh} \\ -2S_{yzh}R_{2h}\theta_{2h} + 2R_{1h}R_{2h}\theta_{1h}\theta_{2h}S_{yzh} \end{bmatrix} < 0$	$-4.1 < 0$
$MSE(\hat{\hat{Y}}^C_{RRS}) - MSE(\hat{\hat{Y}}_{RS}) < 0$	$\begin{bmatrix} R^2_{1h}S^2_{xh}(\theta^2_{1h} - 1) + R^2_{2h}S^2_{zh}\theta^2_{2h} - 2(\theta_{1h} - 1)R_{1h}S_{xyh} \\ -2S_{yzh}R_{2h}\theta_{2h} + 2R_{1h}R_{2h}\theta_{1h}\theta_{2h}S_{yzh} \end{bmatrix} < 0$	$-2.0 < 0$
$MSE(\hat{\hat{Y}}^C_{RRS}) - MSE(\hat{\hat{Y}}_{Res}) < 0$	$\begin{bmatrix} R^2_{1h}S^2_{xh}(\theta^2_{1h} - \frac{1}{4}) + R^2_{2h}S^2_{zh}\theta^2_{2h} - R_{1h}S_{xyh}(2\theta_{1h} - 1) \\ -2S_{yzh}R_{2h}\theta_{2h} + 2R_{1h}R_{2h}\theta_{1h}\theta_{2h}S_{yzh} \end{bmatrix} < 0$	$-5.6 < 0$
$MSE(\hat{\hat{Y}}^C_{RRS}) - MSE(\hat{\hat{Y}}^C_{RS}) < 0$	$\left[2R_{1h}R_{2h}\theta_{1h}\theta_{2h}S_{yzh} - 2S_{yzh}R_{2h}\theta_{2h} + R^2_{2h}S^2_{zh}\theta^2_{2h} \right] < 0$	$-2.5 < 0$

Table 4.1: Empirical exhibition of the theoretical conditions obtained in section 3

Estimators	PRE'S
$\bar{y}_{st}$	100.00
$\hat{\bar{Y}}_{RS}$	197.70
$\hat{\bar{Y}}_{Res}$	137.96
$\hat{\bar{Y}}_{RS}^{C}$	197.64
$\hat{\bar{Y}}_{RRS}^{C}$	2107.33

Table 4.2: PRE'S of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{Res}$ and $\hat{\bar{Y}}_{RS}^{C}$ with respect to $\bar{y}_{st}$

5. Conclusion

From the efficiency comparisons carried out in section 3 it is observed that the suggested "ratio and ratio type estimator $\hat{\bar{Y}}_{RRS}^{C}$ would be more better than $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{Res}$, $\hat{\bar{Y}}_{RS}^{C}$ .Table 4.1 shows that conditions obtained in section 3 are satisfied empirically. Table 4.2 shows that suggested ratio and ratio type separate estimator has the maximum percent relative efficiency as compared to $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{Res}$ and $\hat{\bar{Y}}_{RS}^{C}$ and**.** Thus it can be concluded that the suggested estimator $\hat{\bar{Y}}_{RRS}^{C}$ is most efficient and recommended for estimating the population mean.

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