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GENERALIZED FAMILY OF ESTIMATORS VIA TWO AUXILIARY VARIABLES FOR POPULATION VARIANCE

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Abstract

The variance estimators have been recommended in sampling literature by many authors. It is highly popular to use auxiliary variable information to achieve more effective estimators. Moreover, the variance estimators have been studied using the information of two auxiliary variables in recent years. In this paper, we suggest three families of estimators using the information of two auxiliary variables for the estimation of the population variance in the simple random sampling method. The asymptotic expressions for the mean squared error (MSE) of the suggested family of estimators have been derived up to the first order of approximation. We show that the suggested estimators are more efficient than the classical estimators and the existing estimators in Literature under the determined conditions obtained in theory. We also support the performances of the suggested estimators with the aid of two real population data sets.

Key Words: Variance Estimator; Auxiliary Information; Mean Squared Error; Efficiency.

1. Introduction

In various sampling designs, the auxiliary information has a significant role to estimate the parameters of the population. If the auxiliary variable x has a high correlation with the study variable$y$, its use can enhance the precision of the estimators. In recent years, studies have purposed to obtain more efficiency estimators by increasing the number of the auxiliary variables. Many authors make important contributions to the estimation of the population parameters in the literature, such as Shabbir and Gupta (2017) and Muneer et al. (2017). Moreover, several authors, including Singh et al. (2011), Sidelel et al. (2014), Singh and Singh (2015a, 2015b), Shabbir and Gupta (2014), Adichwal et al. (2017) and Lone et al. (2017), have a number of estimators using two auxiliary variables for the population variance of the study variable.

To know or estimate the variation is always preferred in all areas. For example; a physician wants to know the degree of variation in the human blood pressure, an agriculturist needs the knowledge of variation in climate, a manufacture wants to know the level of variation in sales amount, etc. Hence, the variance of the study variable is a parameter that is always desired to be estimated. Many authors, including Kadilar and Cingi (2006), Gupta and Shabbir (2008), Singh and Solanki (2013), Yadav et al. (2015a, 2015b), Singh and Pal (2016, 2017) and Sanaullah et al. (2017), work on variance estimation in accordance with this purpose.

2. Estimators in Literature

Let y and (x, z) be the study variable and two auxiliary variables, respectively. They take the values y_i and x_i, z_i, respectively. Consider a finite population $P = \{P_1, P_2, \dots, P_N\}$ consisting of N units. Let the sample of size n be drawn from the population of size P using the simple random sample without replacement (SRSWOR). Assume that the population variances of the auxiliary variables S_x^2 and S_z^2 are known to estimate the population variance S_y^2 where

$$S_x^2 = \frac{1}{N}\sum_{i=1}^N (x_i - \bar{X})^2, S_z^2 = \frac{1}{N}\sum_{i=1}^N (z_i - \bar{Z})^2 \text{ and } S_y^2 = \frac{1}{N}\sum_{i=1}^N (y_i - \bar{Y})^2.$$

We use the following error terms for obtaining the first order of approximation of the proposed estimators:

$$v_0 = \frac{s_y^2}{S_y^2},\ v_1 = \frac{s_x^2}{S_x^2},\ v_2 = \frac{s_z^2}{S_z^2}, E(v_0) = E(v_1) = E(v_2) = 0,$$

$$E(v_0^2) = \theta_{400}^*/n, E(v_1^2) = \theta_{040}^*/n, E(v_2^2) = \theta_{004}^*/n,$$

$$E(v_0 v_1) = \theta_{220}^*/n, E(v_0 v_2) = \theta_{202}^*/n, E(v_1 v_2) = \theta_{022}^*/n,$$

where

$$\theta_{rst} = \mu_{rst}/\mu_{200}^{r/2}\mu_{020}^{s/2}\mu_{002}^{t/2},\ \theta_{rst}^* = \theta_{rst} - 1,$$
$$\mu_{rst} = \sum_{i=1}^N \left(y_i - \overline{Y}\right)^r (x_i - \overline{X})^s (z_i - \overline{Z})^t / N.$$

When there are two auxiliary variables, the ratio estimator of the population variance is defined by

$$s_{ratio}^2 = s_y^2 \frac{S_x^2}{s_x^2}\frac{S_z^2}{s_z^2}. \tag{2.1}$$

Up to the first order of approximation, the Mean Squared Error (MSE) of the estimator, given in (2.1), is

$$MSE(s_{ratio}^2) = \frac{S_y^4}{n}\{\theta_{400}^* + \theta_{040}^* + \theta_{004}^* - 2(\theta_{220}^* + \theta_{202}^* - \theta_{022}^*)\}. \tag{2.2}$$

The regression estimator with two auxiliary variables for the population variance of the study variable can be written as

$$s_{reg}^2 = s_y^2 + b_1(S_x^2 - s_x^2) + b_2(S_z^2 - s_z^2) \tag{2.3}$$

and the MSE of the estimator s_{reg}^2 is obtained as follows:

$$MSE(s_{reg}^2) = \frac{S_y^4}{n}\left\{\theta_{400}^* + \frac{\beta_1^2}{R_1^2}\theta_{040}^* + \frac{\beta_2^2}{R_2^2}\theta_{004}^* - 2\left(\frac{\beta_1}{R_1}\theta_{220}^* + \frac{\beta_2}{R_2}\theta_{202}^* - \frac{\beta_1\beta_2}{R_1R_2}\theta_{022}^*\right)\right\}, \tag{2.4}$$

where b_1 and b_2 indicate the estimators of the regression coefficients defined by

$\beta_1 = \frac{S_y^2\theta_{220}^*}{S_x^2\theta_{040}^*}, \beta_2 = \frac{S_y^2\theta_{202}^*}{S_z^2\theta_{004}^*}, R_1 = \frac{S_y^2}{S_x^2}$ and $R_2 = \frac{S_y^2}{S_z^2}$.

Singh and Singh (2015) developed two different modified classes of exponential estimators with the help of two auxiliary variables x and z. The estimator s_{SS1}^2 is first defined and the MSE of s_{SS1}^2 is, respectively, given by

$$s_{SS1}^2 = s_y^2 \exp\left[\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right]^{\pi_1} \exp\left[\frac{S_z^2 - s_z^2}{S_z^2 + s_z^2}\right]^{\pi_2} \tag{2.5}$$

and

$$MSE(s_{SS1}^2) = \frac{S_y^4}{n}\left\{\theta_{400}^* + \frac{1}{4}(\pi_1^2\theta_{040}^* + \pi_2^2\theta_{004}^*) - (\pi_1\theta_{220}^* + \pi_2\theta_{202}^*) + \frac{\pi_1\pi_2\theta_{022}^*}{2}\right\}, \tag{2.6}$$

where π_1 and π_2 are real constants. The optimum values of π_1 and π_2 are obtained as

$$\pi_1^{opt} = \frac{2(\theta_{202}^*\theta_{022}^* - \theta_{004}^*\theta_{220}^*)}{\theta_{022}^{*2} - \theta_{040}^*\theta_{004}^*} \text{ and } \pi_2^{opt} = \frac{2(\theta_{220}^*\theta_{022}^* - \theta_{040}^*\theta_{202}^*)}{\theta_{022}^{*2} - \theta_{040}^*\theta_{004}^*}.$$

Later, the second estimator s_{SS2}^2 is proposed and the MSE of the estimator is found as:

$$s_{SS2}^2 = s_y^2 \exp\left[\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right]^{\kappa_1} \exp\left[\frac{S_z^2 - s_z^2}{S_z^2 + s_z^2}\right]^{\kappa_2} + b_1(S_x^2 - s_x^2) + b_2(S_z^2 - s_z^2) \tag{2.7}$$

and

$$MSE(s_{SS2}^2) = \frac{1}{n}\left\{S_y^4\left(\theta_{400}^* + \frac{1}{4}(\kappa_1^2\theta_{040}^* + \kappa_2^2\theta_{004}^* + 2\kappa_1\kappa_2\theta_{022}^*)\right) - S_y^2(\kappa_1 \mathrm{M}_1 + \kappa_2 \mathrm{M}_2) + \mathrm{M}_3\right\}, \tag{2.8}$$

where κ_1 and κ_2 are suitably chosen constants and

$$\mathrm{M}_1 = S_y^2\theta_{220}^* - \beta_1 S_x^2\theta_{040}^* - \beta_2 S_z^2\theta_{022}^*,$$
$$\mathrm{M}_2 = S_y^2\theta_{202}^* - \beta_1 S_x^2\theta_{022}^* - \beta_2 S_z^2\theta_{004}^*, S_x^2 S_z^2\theta_{022}^*,$$
$$\mathrm{M}_3 = \beta_1^2 S_x^4\theta_{040}^* + \beta_2^2 S_z^4\theta_{004}^* - 2S_y^2(\beta_1 S_x^2\theta_{220}^* + \beta_2 S_z^2\theta_{202}^*) + 2\beta_1\beta_2.$$

The optimum values of κ_1 and κ_2 are obtained by

$$\kappa_1^{opt} = \frac{2(\theta_{004}^* M_1 - \theta_{022}^* M_2)}{S_y^2(\theta_{040}^*\theta_{004}^* - \theta_{022}^{*2})} \text{ and } \kappa_2^{opt} = \frac{2(\theta_{040}^* M_2 - \theta_{022}^* M_1)}{S_y^2(\theta_{040}^*\theta_{004}^* - \theta_{022}^{*2})}.$$

Singh et al. (2011) suggest an improved exponential estimator for the population variance of the study variable y as follows:

$$s_{S1}^2 = s_y^2\left[k\exp\left\{\frac{S_x^2 - s_x^2}{S_x^2 + s_x^2}\right\} + (1-k)\exp\left\{\frac{s_z^2 - S_z^2}{s_z^2 + S_z^2}\right\}\right], \tag{2.9}$$

where k is the weight that makes the MSE minimum.

The MSE expression of the estimator, given in (2.9), up to the first degree of approximation is

$$MSE(s_{S1}^2) = \frac{S_y^4}{n}\{\theta_{400}^* + \frac{1}{4}(k^2\theta_{040}^* + (1-k^2)\theta_{004}^*) - k\theta_{220}^* + (1-k)\theta_{202}^* - \frac{1}{2}k(1-k)\theta_{022}^*)\}. \quad (2.10)$$

The optimum value of k is derived as

$$k^{opt} = \frac{\theta_{004}^* + 2(\theta_{220}^* + \theta_{202}^*) + \theta_{022}^* - 6}{\theta_{040}^* + \theta_{004}^* + 2\theta_{022}^* - 4}.$$

3. Improved Estimators

Motivated by the estimators, mentioned in Section 2, we propose three classes of variance estimators in the simple random sampling as follows:

$$s_{CK1}^2 = s_y^2 \left[\frac{aS_x^2+b}{as_x^2+b}\right]^{\alpha_1} \left[\frac{cS_z^2+d}{cs_z^2+d}\right]^{\alpha_2}, \quad (3.1)$$

$$s_{CK2}^2 = s_y^2 \left\{w_1 \left[\frac{aS_x^2+b}{as_x^2+b}\right]^{\alpha_1} + w_2 \left[\frac{cS_z^2+d}{cs_z^2+d}\right]^{\alpha_2}\right\}, \quad (3.2)$$

$$s_{CK3}^2 = s_y^2 \left\{w_3 \left[\frac{aS_x^2+b}{as_x^2+b}\right]^{\alpha_5} \left[\frac{cs_z^2+d}{cS_z^2+d}\right]^{\alpha_6} + w_4 \left[\frac{as_x^{2*}+b}{aS_x^2+b}\right]^{\alpha_7} \left[\frac{cS_z^2+d}{cs_z^{2*}+d}\right]^{\alpha_8}\right\}, \quad (3.3)$$

where $a \neq 0, c \neq 0, b$ and d are the values of the known parameters of the auxiliary variables or real numbers, $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7$, and α_8 are constants and w_1, w_2, w_3, and w_4 are weights that make the MSE minimum. Here, we also use the population variance transformations as $s_x^{2*} = S_x^2 + f(S_x^2 - s_x^2), s_z^{2*} = S_z^2 + f(S_z^2 - s_z^2)$. Here, $f = \frac{n}{N-n}$.It should be noted that the proposed classes of estimators are ratio type estimators whenα_i =1 and product type estimators when α_i =-1, $i = 1,2,\ldots,8$.

We expand (3.1), up to the first order approximation, by usingϑ's and obtain

$$s_{CK1}^2 = S_y^2[1 + \vartheta_0 - \alpha_1 A\vartheta_1 - \alpha_2 B\vartheta_2 - \alpha_1 A\vartheta_0\vartheta_1 - \alpha_2 B\vartheta_0\vartheta_2 + \alpha_1^2 A^2\vartheta_1^2 + \alpha_2^2 B^2\vartheta_2^2 + \alpha_1\alpha_2 AB\vartheta_1\vartheta_2], \quad (3.4)$$

where $A = \frac{aS_x^2}{aS_x^2+b}$ and $B = \frac{cS_z^2}{cS_z^2+d}$.

After subtracting the population variance from both sides of (3.4),we square and take the expectation of the expression. Then, we obtain the MSE of the first proposed estimator s_{CK1}^2 as

$$MSE(s_{CK1}^2) = \frac{S_y^4}{n}\{\theta_{400}^* + \alpha_1^2 A^2\theta_{040}^* + \alpha_2^2 B^2\theta_{004}^* - 2(\alpha_1 A\theta_{220}^* + \alpha_2 B\theta_{202}^* - \alpha_1\alpha_2 AB\theta_{022}^*)\}. \quad (3.5)$$

The second proposed class of estimators $s^2_{CK2}$is considered in two different cases: $w_1 + w_2 = 1$ and $w_1 + w_2 \neq 1$. Therefore, we calculate the MSE expressions of s^2_{CK2} for two different conditions:

Case 1:$w_1 + w_2 = 1$ (Sum of the weights is 1).

Up to the first order approximation, using ϑ's, we expand (3.2) by

$$s^2_{CK2(1)} = S^2_y[(1 + \vartheta_0 - \alpha_2 B\vartheta_2 - \alpha_2 B\vartheta_0\vartheta_2 + \alpha_2^2 B^2\vartheta_2^2) + w_1(-\alpha_1 A\vartheta_1 + \alpha_2 B\vartheta_2 - \alpha_1 A\vartheta_0\vartheta_1 + \alpha_2 B\vartheta_0\vartheta_2 + \alpha_1^2 A^2\vartheta_1^2 + \alpha_2^2 B^2\vartheta_2^2)]. \quad (3.6)$$

The MSE of $s^2_{CK2(1)}$ is derived by subtracting S^2_y from both sides of(3.6), squaring and taking expectation as

$$MSE(s^2_{CK2(1)}) = \frac{S^4_y}{n}\{w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1\}, \quad (3.7)$$

where

$\Upsilon_1 = \theta^*_{400} + \alpha_2^2 B^2\theta^*_{004} - 2\alpha_2 B\theta^*_{202}, \Gamma_1 = \alpha_1^2 A^2\theta^*_{040} + \alpha_2^2 B^2\theta^*_{004} - 2\alpha_1\alpha_2 AB\theta^*_{022}$
and
$\Delta_1 = -\alpha_1 A\theta^*_{220} + \alpha_2 B\theta^*_{202} + \alpha_1\alpha_2 AB\theta^*_{022} - \alpha_2^2 B^2\theta^*_{004}$.

We get the optimum values of w_1 and $w_2$by

$w_1^{opt} = -\frac{\Delta_1}{\Gamma_1}$ and $w_2^{opt} = 1 - w_1^{opt}$.

When we put the optimum values of w_1 and w_2 in (3.7), we get the minimum MSE of the second proposed estimator $s^2_{CK2(1)}$ as follows:

$$MSE_{\min}(s^2_{CK2(1)}) = \frac{S^4_y}{n}\left\{\Upsilon_1 - \frac{\Delta_1^2}{\Gamma_1}\right\}. \quad (3.8)$$

Case 2:$w_1 + w_2 \neq 1$(Sum of the weights is not 1).

Expanding (3.2) with regards to ϑ's, up to the first order approximation, we have

$$s^2_{CK2(2)} = S^2_y[w_1(1 + \vartheta_0 - \alpha_1 A\vartheta_1 - \alpha_1 A\vartheta_0\vartheta_1 + \alpha_1^2 A^2\vartheta_1^2) + w_2(1 + \vartheta_0 - \alpha_2 B\vartheta_2 - \alpha_2 B\vartheta_0\vartheta_2 + \alpha_2^2 B^2\vartheta_2^2)]. \quad (3.9)$$

The MSE of $s^2_{CK2(2)}$ is derived by subtracting S^2_y from both sides of (3.9), squaring and taking expectation as

$$MSE(s^2_{CK2(2)}) = S^4_y\{w_1^2 X_1 + w_1 Y_1 + w_2^2 Z_1 + w_2 T_1 + w_1 w_2 U_1 + 1\}, \quad (3.10)$$

where

$$X_1 = 1 + (\theta^*_{400} + 2\alpha_1^2 A^2 \theta^*_{040} - 3\alpha_1 A\theta^*_{220})/n,$$
$$Y_1 = -2 + (-\alpha_1^2 A^2 \theta^*_{040} + \alpha_1 A\theta^*_{220})/n,$$
$$Z_1 = 1 + (\theta^*_{400} + 2\alpha_2^2 B^2 \theta^*_{004} - 3\alpha_2 B\theta^*_{202})/n,$$
$$T_1 = -2 + (-\alpha_2^2 B^2 \theta^*_{004} + \alpha_2 B\theta^*_{202})/n$$

and

$$U_1 = 2 + (\theta^*_{400} + \alpha_1^2 A^2 \theta^*_{040} - 2\alpha_1 A\theta^*_{220} + \alpha_2^2 B^2 \theta^*_{004} - 2\alpha_2 B\theta^*_{202} + \alpha_1\alpha_2 AB\theta^*_{022})/n.$$

We get the optimum values of w_1 and w_2 as

$w_1^{opt} = \frac{2Y_1Z_1 - U_1T_1}{U_1^2 - 4X_1Z_1}$ and $w_2^{opt} = \frac{U_1Y_1 - 2T_1X_1}{U_1^2 - 4X_1Z_1}$.

When we put the optimum values of w_1 and $w_2$in (3.10), we get the minimum MSE of the second proposed estimator as follows:

$$MSE_{\min}(s^2_{CK2(2)}) = S_y^4 \left\{ \frac{1}{(U_1^2 - 4X_1Z_1)^2} (T_1^2 U_1^2 X_1 + 12T_1^2 X_1^2 Z_1 - T_1 U_1^3 Y_1 \right. \\ -12T_1U_1X_1Y_1Z_1 + U_1^4 - 8U_1^2X_1Z_1 + 5U_1^2Y_1^2Z_1 \\ \left. +16X_1^2Z_1^2 - 4X_1Y_1^2Z_1^2) \right\}. \quad (3.11)$$

Similarly, we obtain the MSE equation of the third proposed class of estimatorss^2_{CK3} for two different conditions:

Case 1: $w_3 + w_4 = 1$ (Sum of the weights is 1).

Expanding (3.3) with respect to ϑ's, up to the first order approximation, we have

$$s^2_{CK3(1)} = S_y^2[(1 + \vartheta_0 - \alpha_7 fA\vartheta_1 + \alpha_8 fB\vartheta_2 - \alpha_7 fA\vartheta_0\vartheta_1 + \alpha_8 fB\vartheta_0\vartheta_2 \\ -\alpha_7\alpha_8 f^2 AB\vartheta_1\vartheta_2 + \frac{\alpha_7(\alpha_7 - 1)}{2} f^2 A^2 \vartheta_1^2 + \frac{\alpha_8(\alpha_8 + 1)}{2} f^2 B^2 \vartheta_2^2 \Big) \\ +w_3((\alpha_7 f - \alpha_5)A\vartheta_1 - (\alpha_8 f - \alpha_6)B\vartheta_2 + (\alpha_7\alpha_8 f^2 - \alpha_5\alpha_6)AB\vartheta_1\vartheta_2 \\ +(\alpha_7 f - \alpha_5)A\vartheta_0\vartheta_1 - \left(\frac{\alpha_7(\alpha_7 - 1)f^2}{2} - \frac{\alpha_5(\alpha_5 + 1)}{2}\right) A^2\vartheta_1^2 \\ -(\alpha_8 f - \alpha_6)B\vartheta_0\vartheta_2 - \left(\frac{\alpha_8(\alpha_8+1)f^2}{2} - \frac{\alpha_6(\alpha_6 - 1)}{2}\right) B^2\vartheta_2^2)]. \quad (3.12)$$

The MSE of $s^2_{CK3(1)}$ is derived by subtracting S_y^2 from both sides of(3.12), squaring and taking expectation as

$$MSE(s^2_{CK3(1)}) = S_y^4\{w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2\}, \quad (3.13)$$

where

$$\Upsilon_2 = 2 + [\theta^*_{400} + \alpha_7^2 f^2 A^2 \theta^*_{040} + \alpha_8^2 f^2 B^2 \theta^*_{004} - 2\alpha_7 fA\theta^*_{220} + 2\alpha_8 fB\theta^*_{202} \\ -2\alpha_7\alpha_8 f^2 AB\theta^*_{022}]/n,$$

$$\Gamma_2 = [(\alpha_7 f - \alpha_5)^2 A^2 \theta^*_{040} + (\alpha_8 f - \alpha_6)^2 B^2 \theta^*_{004} \\ -2(\alpha_7 f - \alpha_5)(\alpha_8 f - \alpha_6) AB\theta^*_{022}]/n$$

and

$$\Delta_2 = [(\alpha_7 f - \alpha_5)A\theta^*_{220} - (\alpha_8 f - \alpha_6)B\theta^*_{202} + (2\alpha_7\alpha_8 f^2 - \alpha_5\alpha_8 f - \alpha_6\alpha_7 f)AB\theta^*_{022} \\ -(\alpha_7 f - \alpha_5)\alpha_7 f A^2\theta^*_{040} - (\alpha_8 f - \alpha_6)\alpha_8 f B^2 \theta^*_{004}]/n\,.$$

We get the optimum values of w_3 and w_4 by

$w_3^{opt} = -\frac{\Delta_2}{\Gamma_2}$ and $w_4^{opt} = 1 - w_3^{opt}$.

When we put the optimum values of w_3 and $w_4$in (3.13), we get the minimum MSE of the third proposed estimator as follows:

$$MSE_{\min}(s^2_{CK3(1)}) = S_y^4\left\{\Upsilon_2 - \frac{\Delta_2^2}{\Gamma_2}\right\}. \tag{3.14}$$

Case 2: $w_3 + w_4 \neq 1$(Sum of the weights is not 1).

Expanding (3.3) according to ϑ's, up to the first order approximation, we have

$$s^2_{CK3(2)} = S_y^2[w_3(1 + \vartheta_0 - \alpha_5 A\vartheta_1 + \alpha_6 B\vartheta_2 - \alpha_5 A\vartheta_0\vartheta_1 + \alpha_6 B\vartheta_0\vartheta_2 \\ -\alpha_5\alpha_6 AB\vartheta_1\vartheta_2 + \frac{\alpha_5(\alpha_5+1)}{2}A^2\vartheta_1^2 + \frac{\alpha_6(\alpha_6-1)}{2}B^2\vartheta_2^2\Big) \\ +w_4(1 + \vartheta_0 - \alpha_7 f A\vartheta_1 + \alpha_8 f B\vartheta_2 - \alpha_7 f A\vartheta_0\vartheta_1 + \alpha_8 f B\vartheta_0\vartheta_2 \\ -\alpha_7\alpha_8 f^2 AB\vartheta_1\vartheta_2 + \frac{\alpha_7(\alpha_7-1)}{2}f^2A^2\vartheta_1^2 + \frac{\alpha_8(\alpha_8+1)}{2}f^2B^2\vartheta_2^2)]. \tag{3.15}$$

The MSE of $s^2_{CK3(2)}$ is derived by subtracting S_y^2 from both sides of (3.15), squaring and taking expectation as

$$MSE(s^2_{CK3(2)}) = S_y^4(w_3^2 X_2 - 2w_3 Y_2 + w_4^2 Z_2 - 2w_4 T_2 + 2w_3 w_4 U_2 + 1) \tag{3.16}$$

where

$$X_2 = 1 + [\theta^*_{400} + \alpha_5(2\alpha_5 + 1)A^2\theta^*_{040} + \alpha_6(2\alpha_6 - 1)B^2\theta^*_{004} \\ -4(\alpha_5 A\theta^*_{220} - \alpha_6 B\theta^*_{202} + \alpha_5\alpha_6 AB\theta^*_{022})]/n,$$

$$Y_2 = 1 + \left[-\alpha_5 A\theta^*_{220} + \alpha_6 B\theta^*_{202} - \alpha_5\alpha_6 AB\theta^*_{220} + \frac{\alpha_5(\alpha_5+1)}{2}A^2\theta^*_{040} \right. \\ \left. + \frac{\alpha_6(\alpha_6-1)}{2}B^2\theta^*_{004}\right]/n,$$

$$Z_2 = 1 + [\theta^*_{400} + \alpha_7(2\alpha_7 - 1)f^2A^2\theta^*_{040} + \alpha_8(2\alpha_8 + 1)f^2B^2\theta^*_{004} \\ -4f(\alpha_7 A\theta^*_{220} - \alpha_8 B\theta^*_{202} + \alpha_7\alpha_8 f AB\theta^*_{022})]/n,$$

$$T_2 = 1 + [\frac{\alpha_7(\alpha_7-1)}{2}f^2A^2\theta^*_{040} + \frac{\alpha_8(\alpha_8+1)}{2}f^2B^2\theta^*_{004} - \alpha_7 f A\theta^*_{220} \\ +\alpha_8 f B\theta^*_{202} - \alpha_7\alpha_8 f^2 AB\theta^*_{022}]/n$$

and

$$U_2 = 1 + [\theta^*_{400} - 2(\alpha_5 + \alpha_7 f)A\theta^*_{220} + 2(\alpha_6 + \alpha_8 f)B\theta^*_{202}$$
$$+(\alpha_5 + \alpha_7 f)(\alpha_6 + \alpha_8 f)AB\theta^*_{022} + \left((\alpha_5 + \alpha_7 f)^2 + (\alpha_5 - \alpha_7 f^2)\right)\frac{A^2\theta^*_{040}}{2}$$
$$+\left((\alpha_6 + \alpha_8 f)^2 + (\alpha_8 f^2 - \alpha_6)\right)\frac{B^2\theta^*_{004}}{2}]/n.$$

We reach the optimum values of w_3 and w_4 as

$$w_3^{opt} = \frac{Y_2Z_2 - T_2U_2}{U_2^2 - X_2Z_2} \text{ and } w_4^{opt} = \frac{U_2Y_2 - T_2X_2}{U_2^2 - X_2Z_2}.$$

When we put the optimum values of w_3 and w_4 in (3.16), we get the minimum MSE of the third proposed estimator as follows:

$$MSE_{\min}(s^2_{CK3(2)}) = S_y^4\left\{1 - \frac{T_2^2X_2 - 2T_2U_2Y_2 + Z_2Y_2^2}{U_2^2 - X_2Z_2}\right\}. \quad (3.17)$$

4. Efficiency Comparisons

Comparing the MSE equations of the proposed estimators with the mentioned estimators in Section 2, we get the efficiency conditions of the proposed classes of estimators in Subsections 4.1 – 4.3.

4.1 Efficiency Conditions of the First Proposed Estimator s^2_{CK1}

i) $$MSE(s^2_{ratio}) - MSE(s^2_{CK1}) = \{(1 - \alpha_1^2A^2)\theta^*_{040} + (1 - \alpha_2^2B^2)\theta^*_{004}$$
$$-2\left((1 - \alpha_1 A)\theta^*_{220} + (1 - \alpha_2 B)\theta^*_{202} - (1 - \alpha_1\alpha_2 AB)\theta^*_{022}\right)\} > 0, \quad (4.1)$$

ii) $$MSE(s^2_{reg}) - MSE(s^2_{CK1}) = \left\{\left(\frac{\beta_1^2}{R_1^2} - \alpha_1^2A^2\right)\theta^*_{040} + \left(\frac{\beta_2^2}{R_2^2} - \alpha_2^2B^2\right)\theta^*_{004}\right.$$
$$\left.-2\left(\left(\frac{\beta_1}{R_1} - \alpha_1 A\right)\theta^*_{220} + \left(\frac{\beta_2}{R_2} - \alpha_2 B\right)\theta^*_{202} - \left(\frac{\beta_1\beta_2}{R_1R_2} + \alpha_1\alpha_2 AB\right)\theta^*_{022}\right)\right\} > 0, \quad (4.2)$$

iii) $$MSE(s^2_{SS1}) - MSE(s^2_{CK1}) = \left\{\left(\frac{1}{4}\pi_1^2 - \alpha_1^2A^2\right)\theta^*_{040} + \left(\frac{1}{4}\pi_2^2 - \alpha_2^2B^2\right)\theta^*_{004}\right.$$
$$\left.-\left((\pi_1 - 2\alpha_1 A)\theta^*_{220} + (\pi_2 - 2\alpha_2 B)\theta^*_{202} + \left(\frac{\pi_1\pi_2}{2} - 2\alpha_1\alpha_2 AB\right)\theta^*_{022}\right)\right\} > 0, \quad (4.3)$$

iv) $$MSE(s^2_{SS2}) - MSE(s^2_{CK1}) = \left\{\left(\frac{1}{4}k_1^2 - \alpha_1^2A^2\right)\theta^*_{040} + \left(\frac{1}{4}k_2^2 - \alpha_2^2B^2\right)\theta^*_{004}\right.$$
$$+\left(\frac{k_1k_2}{2} - 2\alpha_1\alpha_2 AB\right)\theta^*_{022} + 2(\alpha_1 A\theta^*_{220} + \alpha_2 B\theta^*_{202}) + \frac{M_3}{S_y^4}$$
$$\left.-\frac{(k_1M_1 + k_2M_2)}{S_y^2}\right\} > 0, \quad (4.4)$$

v) $$MSE(s^2_{S1}) - MSE(s^2_{CK1}) = \left\{\left(\frac{k^2}{4} - \alpha_1^2A^2\right)\theta^*_{040} + \left(\frac{1-k^2}{4} - \alpha_2^2B^2\right)\theta^*_{004}\right.$$
$$\left.-(\mathrm{k} - 2\alpha_1 A)\theta^*_{220} + (1 - \mathrm{k} + 2\alpha_2 B)\theta^*_{202} - \left(\frac{\mathrm{k}(1-k)}{2} + 2\alpha_1\alpha_2 AB\right)\theta^*_{022}\right\} > 0,$$
(4.5)

When the proposed class of estimators of $s^2_{CK1}$is evaluated based on the conditions in(4.1)-(4.5), it is concluded that the proposed estimators are more efficient than the related estimators.

4.2 Efficiency Conditions of the Second Proposed Estimator s^2_{CK2}

i) $$MSE(s^2_{ratio}) - MSE(s^2_{CK2(1)}) = \{\theta^*_{400} + \theta^*_{040} + \theta^*_{004} -2(\theta^*_{220} + \theta^*_{202} - \theta^*_{022}) - (w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1)\} > 0, \quad (4.6)$$

ii) $$MSE(s^2_{ratio}) - MSE(s^2_{CK2(2)}) = \{\theta^*_{400} - 2(\theta^*_{220} + \theta^*_{202} - \theta^*_{022}) +\theta^*_{040} + \theta^*_{004} - \mathrm{n}(w_1^2X_1 + w_1Y_1 + w_2^2Z_1 + w_2T_1 + w_1w_2U_1 + 1)\} > 0, \quad (4.7)$$

iii) $$MSE(s^2_{reg}) - MSE(s^2_{CK2(1)}) = \left\{\theta^*_{400} - 2\left(\frac{\beta_1}{R_1}\theta^*_{220} + \frac{\beta_2}{R_2}\theta^*_{202} - \frac{\beta_1\beta_2}{R_1R_2}\theta^*_{022}\right) + \frac{\beta_1^2}{R_1^2}\theta^*_{040} + \frac{\beta_2^2}{R_2^2}\theta^*_{004} - (w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1)\right\} > 0, \quad (4.8)$$

iv) $$MSE(s^2_{reg}) - MSE(s^2_{CK2(2)}) = \{\theta^*_{400} + \frac{\beta_1^2}{R_1^2}\theta^*_{040} + \frac{\beta_2^2}{R_2^2}\theta^*_{004} - 2\left(\frac{\beta_1}{R_1}\theta^*_{220} + \frac{\beta_2}{R_2}\theta^*_{202} - \frac{\beta_1\beta_2}{R_1R_2}\theta^*_{022}\right) -\mathrm{n}(w_1^2X_1 + w_1Y_1 + w_2^2Z_1 + w_2T_1 + w_1w_2U_1 + 1)\} > 0, \quad (4.9)$$

v) $$MSE(s^2_{SS1}) - MSE(s^2_{CK2(1)}) = \left\{\theta^*_{400} + \frac{1}{4}(\pi_1^2\theta^*_{040} + \pi_2^2\theta^*_{004}) -(\pi_1\theta^*_{220} + \pi_2\theta^*_{202}) + \frac{\pi_1\pi_2\theta^*_{022}}{2} - (w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1) > 0, \right. \quad (4.10)$$

vi) $$MSE(s^2_{SS1}) - MSE(s^2_{CK2(2)}) = \left\{\theta^*_{400} + \frac{1}{4}(\pi_1^2\theta^*_{040} + \pi_2^2\theta^*_{004}) -(\pi_1\theta^*_{220} + \pi_2\theta^*_{202}) + \frac{\pi_1\pi_2\theta^*_{022}}{2} -n(w_1^2X_1 + w_1Y_1 + w_2^2Z_1 + w_2T_1 + w_1w_2U_1 + 1) > 0, \right. \quad (4.11)$$

vii) $$MSE(s^2_{SS2}) - MSE(s^2_{CK2(1)}) = \left\{\left(\theta^*_{400} - (w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1)\right) + \frac{1}{4}(\kappa_1^2\theta^*_{040} + \kappa_2^2\theta^*_{004} + 2\kappa_1\kappa_2\theta^*_{022}) - \frac{(\kappa_1\mathrm{M}_1 + \kappa_2\mathrm{M}_2)}{S_y^2} + \frac{\mathrm{M}_3}{S_y^4}\right\} > 0, \quad (4.12)$$

viii) $$MSE(s^2_{SS2}) - MSE(s^2_{CK2(2)}) = \left\{\frac{1}{4}(\kappa_1^2\theta^*_{040} + \kappa_2^2\theta^*_{004} + 2\kappa_1\kappa_2\theta^*_{022}) -\left(n(w_1^2X_1 + w_1Y_1 + w_2^2Z_1 + w_2T_1 + w_1w_2U_1 + 1)\right) \theta^*_{400} - \frac{(\kappa_1\mathrm{M}_1 + \kappa_2\mathrm{M}_2)}{S_y^2} + \frac{\mathrm{M}_3}{S_y^4} > 0, \right. \quad (4.13)$$

ix) $$MSE(s^2_{S1}) - MSE(s^2_{CK2(1)}) = \{\theta^*_{400} - (w_1^2\Gamma_1 + 2w_1\Delta_1 + \Upsilon_1) + \frac{1}{4}(k^2\theta^*_{040} + (1-k^2)\theta^*_{004}) - k\theta^*_{220} + (1-k)\theta^*_{202} - k(1-k)\theta^*_{022} > 0, \quad (4.14)$$

x) $$MSE(s^2_{S1}) - MSE(s^2_{CK2(2)}) = \{\theta^*_{400} + \frac{1}{4}(k^2\theta^*_{040} + (1-k^2)\theta^*_{004})$$

$$-n(w_1^2X_1 + w_1Y_1 + w_2^2Z_1 + w_2T_1 + w_1w_2U_1 + 1) - k\theta_{220}^*$$
$$+(1-k)\theta_{202}^* - k(1-k)\theta_{022}^* > 0. \qquad (4.15)$$

It is inferred that when the conditions from (4.6) to (4.15) are satisfied, the proposed class of estimators of s_{CK2}^2has smaller MSE values than the mentioned estimators for both of two cases discussed in Section 3.

4.3 Efficiency Conditions of the Third Proposed Estimator s_{CK3}^2

i) $MSE(s_{ratio}^2) - MSE(s_{CK3(1)}^2) = \{\theta_{400}^* + \theta_{040}^* + \theta_{004}^*$
$-2(\theta_{220}^* + \theta_{202}^* - \theta_{022}^*) - n(w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2)\} > 0,$ (4.16)

ii) $MSE(s_{ratio}^2) - MSE(s_{CK3(2)}^2) = \{\theta_{400}^* + \theta_{040}^* - 2(\theta_{220}^* + \theta_{202}^* - \theta_{022}^*)$
$+\theta_{004}^* - n(w_3^2X_2 - 2w_3Y_2 + w_4^2Z_2 - 2w_4T_2 + 2w_1w_2U_2 + 1)\} > 0,$ (4.17)

iii) $MSE(s_{reg}^2) - MSE(s_{CK3(1)}^2) = \left\{\theta_{400}^* - 2\left(\frac{\beta_1}{R_1}\theta_{220}^* + \frac{\beta_2}{R_2}\theta_{202}^* - \frac{\beta_1\beta_2}{R_1R_2}\theta_{022}^*\right)\right.$
$\left.+\frac{\beta_1^2}{R_1^2}\theta_{040}^* + \frac{\beta_2^2}{R_2^2}\theta_{004}^* - n(w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2)\right\} > 0,$ (4.18)

iv) $MSE(s_{reg}^2) - MSE(s_{CK3(2)}^2) = \{\theta_{400}^*$
$$+\frac{\beta_1^2}{R_1^2}\theta_{040}^* + \frac{\beta_2^2}{R_2^2}\theta_{004}^* - 2\left(\frac{\beta_1}{R_1}\theta_{220}^* + \frac{\beta_2}{R_2}\theta_{202}^* - \frac{\beta_1\beta_2}{R_1R_2}\theta_{022}^*\right)$$
$$-n(w_3^2X_2 - 2w_3Y_2 + w_4^2Z_2 - 2w_4T_2 + 2w_1w_2U_2 + 1)\} > 0, \qquad (4.19)$$

v) $MSE(s_{SS1}^2) - MSE(s_{CK3(1)}^2) = \left\{\theta_{400}^* + \frac{1}{4}(\pi_1^2\theta_{040}^* + \pi_2^2\theta_{004}^*)\right.$
$-(\pi_1\theta_{220}^* + \pi_2\theta_{202}^*) + \frac{\pi_1\pi_2\theta_{022}^*}{2} - n(w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2) > 0,$ (4.20)

vi) $MSE(s_{SS1}^2) - MSE(s_{CK3(2)}^2) = \left\{\frac{1}{4}(\pi_1^2\theta_{040}^* + \pi_2^2\theta_{004}^*) - (\pi_1\theta_{220}^* + \pi_2\theta_{202}^*)\right.$
$+\theta_{400}^* + \frac{\pi_1\pi_2\theta_{022}^*}{2} - n(w_3^2X_2 - 2w_3Y_2 + w_4^2Z_2 - 2w_4T_2 + 2w_1w_2U_2 + 1) > 0,$
(4.21)

vii) $MSE(s_{SS2}^2) - MSE(s_{CK3(1)}^2) = \left\{S_y^4(\theta_{400}^* - n(w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2))\right.$
$\left.+\frac{S_y^4}{4}(\kappa_1^2\theta_{040}^* + \kappa_2^2\theta_{004}^* + 2\kappa_1\kappa_2\theta_{022}^*) - S_y^2(\kappa_1\mathrm{M}_1 + \kappa_2\mathrm{M}_2) + \mathrm{M}_3\right\} > 0,$ (4.22)

viii) $MSE(s_{SS2}^2) - MSE(s_{CK3(2)}^2) = \left\{-\frac{(\kappa_1\mathrm{M}_1+\kappa_2\mathrm{M}_2)}{S_y^2} + \frac{(\kappa_1^2\theta_{040}^*+\kappa_2^2\theta_{004}^*+2\kappa_1\kappa_2\theta_{022}^*)}{4}\right.$
$\left.+\theta_{400}^* + \frac{\mathrm{M}_3}{S_y^4} - n(w_3^2X_2 + w_3Y_2 + w_4^2Z_2 + w_4T_2 + w_3w_4U_2 + 1)\right\} > 0,$ (4.23)

ix) $MSE(s_{S1}^2) - MSE(s_{CK3(1)}^2) = \{\theta_{400}^* - n(w_3^2\Gamma_2 + 2w_3\Delta_2 + \Upsilon_2)$
$+\frac{1}{4}(k^2\theta_{040}^* + (1-k^2)\theta_{004}^*) - k\theta_{220}^* + (1-k)\theta_{202}^* - k(1-k)\theta_{022}^* > 0,$ (4.24)

x) $$MSE(s_{S1}^2) - MSE(s_{CK3(2)}^2) = \Big\{\theta_{400}^* + \frac{1}{4}(k^2\theta_{040}^* + (1-k^2)\theta_{004}^*) - k\theta_{220}^* + (1-k)\theta_{202}^* - k(1-k)\theta_{022}^* - n(w_3^2X_2 + w_3Y_2 + w_3^2Z_2 + w_4T_2 + w_3w_4U_2 + 1)\Big\} > 0. \quad (4.25)$$

It is inferred that when the conditions (4.16) -(4.25) are satisfied, the proposed class of estimators s_{CK3}^2has smaller MSE values than the mentioned estimators for both of two cases.

5. Numerical Illustrations

We aim to support that the findings in theory are also valid in applications with the help of the following two data sets.

The values of the percent relative efficiency (PRE) are computed in order to compare the efficiency of estimators. Using the data of Populations I and II, we calculate the PRE of the proposed estimators according to the mentioned estimators in Section 2 as follows:

$$PRE = \frac{MSE(S_y^2)}{MSE(S_t^2)} \times 100, t = ratio, reg, SSi, S1, CKi, i = 1,2,3.$$

5.1 Numerical Study for Population I

We use the data set previously considered in Singh and Mangat (1996) for the Population I defined by

y: Irrigated area, x: Number of tubewells and z: Number of tractors.

$$N = 69, n = 15, \theta_{400}^* = 9.477, \theta_{040}^* = 7.945, \theta_{004}^* = 6.319, \theta_{220}^* = 8.091,$$
$$\theta_{202}^* = 7.104, \theta_{022}^* = 6.707, S_y^2 = 85938.89, S_x^2 = 12975.90, S_z^2 = 286.27,$$
$$C_y = 0.8477, C_x = 0.8422, C_z = 0.7969, \rho_{yx} = 0.922, \rho_{yz} = 0.901.$$

The correlation coefficients between the study variable and the auxiliary variables are positive. For this reason, it is taken as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_8 =$ 1and $\alpha_6 = \alpha_7 = -1$. In other words, only ratio estimators are used for this data set. Moreover, the PRE values of the proposed estimators are derived for some values of a, b, c and d given in Table 1. It is sure that one can have various new estimators by taking different values for $a, b, c$and d.

In Table 1, the PRE values of the estimators are demonstrated for the Population I. The PRE values of both the second and the third proposed classes of estimators for the second case are bigger than the PRE values of the estimators in Literature. Therefore, it is inferred that the proposed classes of estimators having the weights whose sum is not 1 are the most efficient estimators.

The proposed estimators	a	b	c	d	PRE	The mentioned estimators	PRE
s^2_{CK1}	1	$\bar{X}$	1	$\bar{Z}$	377.55	s^2_y	100.00
	C_x	θ^*_{040}	C_z	θ^*_{004}	142.31	s^2_{ratio}	109.60
	C_x	$\bar{X}$	C_z	$\bar{Z}$	497.77	s^2_{reg}	138.73
	θ^*_{040}	$\bar{X}$	θ^*_{004}	$\bar{Z}$	263.01	s^2_{SS1}	843.70
	ρ_{yx}	$\bar{X}$	ρ_{yz}	$\bar{Z}$	400.87	s^2_{SS2}	505.48
$s^2_{CK2(1)}$	1	0	1	0	812.86	s^2_{S1}	791.22
	$\bar{X}$	θ^*_{040}	$\bar{Z}$	θ^*_{004}	812.85		
	$\bar{X}$	C_x	$\bar{Z}$	C_z	812.86		
	$\bar{X}$	ρ_{yx}	$\bar{Z}$	ρ_{yz}	812.86		
	ρ_{yx}	C_x	ρ_{yz}	C_z	812.25		
$s^2_{CK2(2)}$	C_x	θ^*_{040}	C_z	θ^*_{004}	1442.14		
	$\bar{X}$	C_x	$\bar{Z}$	C_z	1412.88		
	C_x	ρ_{yx}	C_z	ρ_{yz}	1427.76		
	ρ_{yx}	C_x	ρ_{yz}	C_z	1422.65		
	ρ_{yx}	θ^*_{040}	ρ_{yz}	θ^*_{004}	1436.75		
$s^2_{CK3(1)}$	1	$\bar{X}$	1	$\bar{Z}$	504.17		
	C_x	$\bar{X}$	C_z	$\bar{Z}$	504.42		
	C_x	θ^*_{040}	C_z	θ^*_{004}	503.62		
	S_x	$\bar{X}$	S_z	$\bar{Z}$	503.05		
	ρ_{yx}	$\bar{X}$	ρ_{yz}	$\bar{Z}$	504.28		
$s^2_{CK3(2)}$	C_x	0	C_z	0	2529.67		
	S_x	$\bar{X}$	S_z	$\bar{Z}$	2521.11		
	1	ρ_{yx}	1	ρ_{yz}	2371.99		
	ρ_{yx}	C_x	ρ_{yz}	C_z	2519.99		
	θ^*_{040}	ρ_{yx}	θ^*_{004}	ρ_{yz}	2124.61		

Table 1: The PRE values of estimators for Population I

5.2 Numerical Study for Population II

We use the data set previously considered in Ahmed (1995) for the Population II defined by

y: Number of literate persons, x: Number of households and z: Total population in the village.

$N = 340, n = 50, \theta^*_{400} = 10.903, \theta^*_{040} = 8.054, \theta^*_{004} = 9.255, \theta^*_{220} = 7.313,$
$\theta^*_{202} = 9.129, \theta^*_{022} = 7.136, S^2_y = 71593.30, S^2_x = 19427.26, \rho_{yz} = 0.909,$
$S^2_z = 693780.90, C_y = 0.8450, C_x = 0.7805, C_z = 0.7746, \rho_{yx} = 0.892.$

The proposed estimators	a	b	c	d	PRE	The mentioned estimators	PRE
s^2_{CK1}	1	$\bar{X}$	1	$\bar{Z}$	115.56	s^2_y	100.00
	C_x	θ^*_{040}	C_z	θ^*_{004}	113.68	s^2_{ratio}	113.58
	C_x	$\bar{X}$	C_z	$\bar{Z}$	161.13	s^2_{reg}	135.59
	θ^*_{040}	$\bar{X}$	θ^*_{004}	$\bar{Z}$	113.82	s^2_{SS1}	583.33
	ρ_{yx}	$\bar{X}$	ρ_{yz}	$\bar{Z}$	115.80	s^2_{SS2}	796.91
$s^2_{CK2(1)}$	1	0	1	0	583.03	s^2_{S1}	333.88
	$\bar{X}$	θ^*_{040}	$\bar{Z}$	θ^*_{004}	583.04		
	$\bar{X}$	C_x	$\bar{Z}$	C_z	583.04		
	$\bar{X}$	ρ_{yx}	$\bar{Z}$	ρ_{yz}	583.04		
	ρ_{yx}	C_x	ρ_{yz}	C_z	583.03		
$s^2_{CK2(2)}$	C_x	θ^*_{040}	C_z	θ^*_{004}	820.29		
	$\bar{X}$	C_x	$\bar{Z}$	C_z	821.61		
	C_x	ρ_{yx}	C_z	ρ_{yz}	821.47		
	ρ_{yx}	C_x	ρ_{yz}	C_z	821.50		
	ρ_{yx}	θ^*_{040}	ρ_{yz}	θ^*_{004}	820.45		
$s^2_{CK3(1)}$	1	$\bar{X}$	1	$\bar{Z}$	12675.31		
	C_x	$\bar{X}$	C_z	$\bar{Z}$	12675.32		
	C_x	θ^*_{040}	C_z	θ^*_{004}	12675.33		
	θ^*_{040}	S_x	θ^*_{004}	S_z	12675.32		
	ρ_{yx}	S_x	ρ_{yz}	S_z	12675.30		
$s^2_{CK3(2)}$	C_x	θ^*_{040}	C_z	θ^*_{004}	966.27		
	S_x	$\bar{X}$	S_z	$\bar{Z}$	966.27		
	ρ_{yx}	$\bar{X}$	ρ_{yz}	$\bar{Z}$	966.04		
	ρ_{yx}	S_x	ρ_{yz}	S_z	965.76		
	θ^*_{040}	ρ_{yx}	θ^*_{004}	ρ_{yz}	966.25		

Table 2: The PRE values of estimators for Population II

In Table 2, the PRE values of the estimators are demonstrated for the Population II. Similarly with the Population I, the correlation coefficients between the study variable and the auxiliary variables are also positive in Population II. For this reason, it is taken as $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = \alpha_8 = 1$ and $\alpha_6 = \alpha_7 = -1$,i.e., only ratio estimators are used for this data set.

From Table 2, it is clearly observed that the third proposed class of estimators for both two cases and the second proposed class of estimators for the second case are more efficient than the estimators in Literature.

6. Conclusion

In this article, we have proposed three new classes of estimators using two auxiliary variables for estimating the population variance. For the second and the third proposed classes of estimators, we derive minimum MSE equations for each case. Theoretical efficiency comparisons are obtained for the proposed estimators with the estimators in Literature and then these comparisons are supported by numerical illustrations. Using two data sets, we see that the second proposed class of estimators $s^2_{CK3(2)}$ is the best estimators for Population I and the third proposed class of estimators $s^2_{CK3(1)}$ is the best estimators for Population II. We would like to point out that many more estimators could also be developed by the second and the third proposed classes of estimators using various parameters fora, b, c and d.

We utilize the weights as w_1, w_2, w_3 and $w_4$for the second and the third proposed classes of estimators. We obtain the MSE equations for different states of these weights .It can be seen from the application section that the use of weight gives effective results on the proposed estimators. The new estimators should be developed using the multi auxiliary variables with the help of weights for the different sample techniques in the forthcoming study.

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