Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online): 2229-5666
Vol. 10, Issue 2 (2017): 43-63

A NEW FAMILY OF ESTIMATORS FOR MEAN ESTIMATION ALONG SIDE THE SENSITIVITY ISSUE

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Received May 29, 2017
Modified September 28, 2017
Accepted October 12, 2017

Abstract
In this article, we have envisaged a new family of estimators for finite population mean of the study variable Y under simple random sampling (SRS) utilizing one auxiliary variable. The work is also extended for the case when study variable has sensitive nature. Optimum properties such as bias and mean square error (MSE) of the proposed family of estimators have been determined for both cases. It has been shown that the proposed family of estimators is more efficient than existing estimators. In the support of the theoretical proposed work, we have given numerical illustration.

Key Words: Mean Square Error, Scrambled Response, Simple Random Sampling.

1. Introduction
In survey sampling, precision of the estimates of the finite population mean of study variable y can be improved considerably by the use of known supplementary/auxiliary information. Ratio, product and regression are the most commonly used techniques for utilization of auxiliary information. Cochran (1940) Introduced ratio estimator by utilizing auxiliary information under simple random sampling. Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Singh and Tailor (2003) introduced some ratio-type estimators by adapting Cochran (1940) and using available parameters of auxiliary variable. Kadilar and Cingi (2004) extend the work in this field by combining the idea of ratio and regression estimators. Several authors namely c.f. (Jeelani et al. (2013), Subramani and Kumarapandiyan (2012a,2012b,2012c), Yan and Tian (2010), Subramani and Prabavathy (2014), Abid et al. (2016) etc) followed the strategy of Kadilar and Cingi (2004). Taking motivation from Bahl and Tuteja (1991), Grover and Kaur (2011) proposed an improved family of estimators. Further, Grover and Kaur (2014) introduced the generalized version of Grover and Kaur (2011) and Shabbir and Gupta (2011) estimators. Singh et al. (2016) and Tarray (2016) studied the class of estimators of different population parameters. Singhand Tarray (2014) studied the estimation of population proportion possessing the sensitive attribute.

Let $\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$ and $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ be the sample means of the study and auxiliary variables having size n respectively. To obtain the bias and MSE let us define $e_o = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$ and $e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$ such that

$$E(e_o) = E(e_1) = 0,$$
$$E(e_0^2) = \left(\frac{1-f}{n}\right) C_y^2 = b_0,$$
$$E(e_1^2) = \left(\frac{1-f}{n}\right) C_x^2 = b_1,$$
$$E(e_o e_1) = \left(\frac{1-f}{n}\right) \rho C_y C_x = b_{01}.$$

where we used b_0, b_1 and b_{01} notations for abbreviation. In this section firstly we reviewed some important estimators of population mean and their MSEs.

The variance of unbiased estimator $\bar{y}$ is given by

$$\hat{D}_0 = V(\bar{y}) = \bar{Y}^2 b_0 \tag{1}$$

Cochran (1940) developed the following mean per unit estimator as follows

$$\hat{D}_r = \bar{y}\left[\frac{\bar{X}}{\bar{x}}\right] \tag{2}$$

The MSE of $\hat{D}_r$ is

$$MSE(\hat{D}_r) = \bar{Y}^2 \left[b_0 + b_1 - 2b_{01}\right] \tag{3}$$

The conventional regression estimator is given by

$$\hat{D}_{reg} = \bar{y} + b(\bar{X} - \bar{x}) \tag{4}$$

The MSE of $\hat{D}_{reg}$ is

$$MSE(\hat{D}_{reg}) = \bar{Y}^2 \left[b_0 - \frac{b_{01}^2}{b_1}\right] \tag{5}$$

Bahl and Tuteja (1991) developed the following exponential estimator

$$\hat{D}_{bt} = \bar{y} \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{6}$$

The MSE of $\hat{D}_{bt}$ is

$$MSE(\hat{D}_{bt}) = \bar{Y}^2 \left[b_0 - \frac{b_1}{4} - b_{01}\right] \tag{7}$$

We can describe some of the estimators which are suggested by different authors as given below:

$$\hat{D}_i = \psi\lambda_i, \qquad for\ i = 1,2,.....,26 \tag{8}$$

where $\psi = \left[\bar{y} + b(\bar{X} - \bar{x})\right]$, $\gamma_i = \dfrac{c\bar{Y}}{c\bar{X} + d}$ $\lambda_i = \dfrac{c\bar{X} + d}{c\bar{x} + d}$.

We can generate some important studies from (8) as given in Table 1. Kadilar and Cingi (2004) (for i=1,2,.....,5) developed a class of ratio estimators by using the conventional descriptives of population. After that a number of researchers adopted their strategy and provided several modified ratio estimators (c.f., Kadilar and Cingi (2006); (for i=6,7,.....,10), Yan and Tian (2010); (for i=11,12), (Subramani and Kumarapandiyan (2012a, 2012b, 2012c); (for i=13,14,..,16), Jeelani et al. (2013); (for i=17) and Abid et al. (2016); (for i=18,19,....,26) are given by

Est	c	d	Est	c	d
$\hat{D}_1 = \psi\lambda_1$	1	0	$\hat{D}_{14} = \psi\lambda_{14}$	C_x	M_d
$\hat{D}_2 = \psi\lambda_2$	1	C_x	$\hat{D}_{15} = \psi\lambda_{15}$	$\beta_1(x)$	M_d
$\hat{D}_3 = \psi\lambda_3$	1	$\beta_2(x)$	$\hat{D}_{16} = \psi\lambda_{16}$	$\beta_2(x)$	M_d
$\hat{D}_4 = \psi\lambda_4$	$\beta_2(x)$	C_x	$\hat{D}_{17} = \psi\lambda_{17}$	$\beta_1(x)$	$Q.D$
$\hat{D}_5 = \psi\lambda_5$	C_x	$\beta_2(x)$	$\hat{D}_{18} = \psi\lambda_{18}$	1	TM
$\hat{D}_6 = \psi\lambda_6$	1	ρ	$\hat{D}_{19} = \psi\lambda_{19}$	C_x	TM
$\hat{D}_7 = \psi\lambda_7$	C_x	ρ	$\hat{D}_{20} = \psi\lambda_{20}$	ρ	TM
$\hat{D}_8 = \psi\lambda_8$	ρ	C_x	$\hat{D}_{21} = \psi\lambda_{21}$	1	MR
$\hat{D}_9 = \psi\lambda_9$	$\beta_2(x)$	ρ	$\hat{D}_{22} = \psi\lambda_{22}$	C_x	MR
$\hat{D}_{10} = \psi\lambda_{10}$	ρ	$\beta_2(x)$	$\hat{D}_{23} = \psi\lambda_{23}$	ρ	MR
$\hat{D}_{11} = \psi\lambda_{11}$	1	$\beta_1(x)$	$\hat{D}_{24} = \psi\lambda_{24}$	1	HL
$\hat{D}_{12} = \psi\lambda_{12}$	$\beta_1(x)$	$\beta_2(x)$	$\hat{D}_{25} = \psi\lambda_{25}$	C_x	HL
$\hat{D}_{13} = \psi\lambda_{13}$	1	1	$\hat{D}_{26} = \psi\lambda_{26}$	ρ	HL

Table 1: Estimators of Kadilar and Cingi (2004) and those based on their adaptation

For the details about the terms mentioned in Table 1 such that $(M_d, Q.D, TM, MR, HL)$ see Abid et al. (2016). The MSE of $\hat{D}_i$ $(for\ i = 1,2,.....,26)$, given by

$$MSE(\hat{D}_i) = f'\left[\gamma_i^2 S_x^2 + S_y^2(1-\rho^2)\right] \tag{9}$$

Kadilar and Cingi (2006b) proposed four new estimators by merging the estimators of Kadilar and Cingi (2004) are given by

$$\hat{D}_{KC1} = \omega_1^{KC1} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X} + \omega_2^{KC1} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x} + C_x} (\bar{X} + C_x) \tag{10}$$

$$\hat{D}_{KC2} = \omega_1^{KC2} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X} + \omega_2^{KC2} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x} + \beta_2(x)} (\bar{X} + \beta_2(x)) \tag{11}$$

$$\hat{D}_{KC3} = \omega_1^{KC3} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X} + \omega_2^{KC3} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}\beta_2(x) + C_x} (\bar{X}\beta_2(x) + C_x) \tag{12}$$

$$\hat{D}_{KC4} = \omega_1^{KC4} \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X} + \omega_2^{KC4} \frac{\bar{y} + b(\bar{X} - \bar{x})}{C_x\bar{x} + \beta_2(x)} (C_x\bar{X} + \beta_2(x)) \tag{13}$$

where the optimum values of ω_1^{KCi} and ω_2^{KCi} for (i=1,2,3,4) are

$$\omega_1^{KC1(opt)} = \frac{\gamma_{10}}{\gamma_{10} - \gamma_9}, \omega_2^{KC1(opt)} = \frac{\gamma_9}{\gamma_9 - \gamma_{10}}, \omega_1^{KC2(opt)} = \frac{\gamma_{11}}{\gamma_{11} - \gamma_9}, \omega_2^{KC2(opt)} = \frac{\gamma_9}{\gamma_9 - \gamma_{11}}$$

$$\omega_1^{KC3(opt)} = \frac{\gamma_{12}}{\gamma_{12} - \gamma_9}, \omega_2^{KC3(opt)} = \frac{\gamma_9}{\gamma_9 - \gamma_{12}}, \omega_1^{KC4(opt)} = \frac{\gamma_{13}}{\gamma_{13} - \gamma_9}, \omega_2^{KC4(opt)} = \frac{\gamma_9}{\gamma_9 - \gamma_{13}}$$

The MSE minimum of $\hat{D}_{KCi}$ $(i = 1,2,3,4)$ is

$$MSE(\hat{D}_{KCi}) = MSE(\hat{D}_{\text{Re}g}) \tag{14}$$

Similarly, by taking motivation from Kadilar and Cingi (2006b) one can merge any two estimators $\hat{D}_i$ $(for\ i = 1,2,.....,26)$ and get results equal to regression estimator.

Grover and Kaur (2011) introduced thefollowing exponential estimation of population mean as follows

$$\hat{D}_{lp} = [q_{lp1}\bar{y} + q_{lp2}(\bar{X} - \bar{x})]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right], \tag{15}$$

The MSE of $\hat{D}_{lp}$ is

$$MSE(\hat{D}_{lp}) = \bar{Y}^2 + q_{lp1}^2\upsilon_{Alp} + q_{lp2}^2\upsilon_{Blp} + 2q_{lp1}q_{lp2}\upsilon_{Clp} - 2q_{lp1}\upsilon_{Dlp} - 2q_{lp2}\upsilon_{Elp}$$

where

$$\upsilon_{Alp} = \bar{Y}^2[1 + b_0 + b_1 - 2b_{01}],\ \upsilon_{Blp} = \bar{X}^2 b_1,\ \upsilon_{Clp} = \bar{X}\bar{Y}[b_1 - b_{01}]$$

$$\upsilon_{Dlp} = \bar{Y}^2[1 + \frac{3b_1}{8} - \frac{b_{01}}{2},\ \upsilon_{Elp} = \bar{X}\bar{Y}\frac{b_1}{2}$$

which is minimum for

$$q_{lp1}^{opt} = \left[\frac{\upsilon_{Blp}\upsilon_{Dlp} - \upsilon_{Clp}\upsilon_{Elp}}{\upsilon_{Alp}\upsilon_{Blp} - \upsilon_{Clp}^2}\right] \text{ and } q_{lp2}^{opt} = \left[\frac{\upsilon_{Alp}\upsilon_{Elp} - \upsilon_{Clp}\upsilon_{Dlp}}{\upsilon_{Alp}\upsilon_{Blp} - \upsilon_{Clp}^2}\right]$$

$$MSE_{\min}(\hat{D}_{lp}) = \left[\bar{Y}^2 - \frac{\upsilon_{Blp}\upsilon_{Dlp}^2 + \upsilon_{Alp}\upsilon_{Elp}^2 - 2\upsilon_{Clp}\upsilon_{Dlp}\upsilon_{Elp}}{\upsilon_{Alp}\upsilon_{Blp} - \upsilon_{Clp}^2}\right] \tag{16}$$

Shabbir and Gupta (2011) introduced the following estimators for the estimation of population mean as follows

$$\hat{D}_{sg} = \left[q_{sg1}\bar{y} + q_{sg2}(\bar{X} - \bar{x})\right]\exp\left[\frac{A' - a'}{A' + a'}\right] \tag{17}$$

where $A' = \bar{X} + N\bar{X}$ and $a' = \bar{x} + N\bar{X}$.

By substituting A' and a', we can write $\hat{D}_{sg}$ as

$$\hat{D}_{sg} = \left[q_{sg1}\bar{y} + q_{sg2}(\bar{X}-\bar{x})\right]\exp\left[\frac{\bar{X}-\bar{x}}{2N\bar{X}+\bar{X}+\bar{x}}\right]$$

The MSE of $\hat{D}_{sg}$ is given by

$$MSE(\hat{D}_{sg}) = \bar{Y}^2 + q_{sg1}^2\upsilon_{Asg} + q_{sg2}^2\upsilon_{Bsg} + 2q_{sg1}q_{sg2}\upsilon_{Csg} - 2q_{sg1}\upsilon_{Dsg} - 2q_{sg2}\upsilon_{Esg}$$

where

$$\upsilon_{Asg} = \bar{Y}^2\left[1+b_0+\frac{b_1}{(N+1)^2}-\frac{2b_{01}}{(N+1)}\right],\ \upsilon_{Bsg} = \bar{X}^2 b_1,\ \upsilon_{Csg} = \bar{X}\bar{Y}\left[\frac{b_1}{(N+1)}-b_{01}\right]$$

$$\upsilon_{Dsg} = \bar{Y}^2[1+\frac{3b_1}{8(N+1)^2}-\frac{b_{01}}{2(N+1)},\ \upsilon_{Esg} = \bar{X}\bar{Y}\frac{b_1}{2(N+1)}$$

which is minimum for

$$q_{sg1}^{opt} = \left[\frac{\upsilon_{Bsg}\upsilon_{Dsg}-\upsilon_{Csg}\upsilon_{Esg}}{\upsilon_{Asg}\upsilon_{Bsg}-\upsilon_{Csg}^2}\right] \text{ and } q_{sg2}^{opt} = \left[\frac{\upsilon_{Asg}\upsilon_{Esg}-\upsilon_{Csg}\upsilon_{Dsg}}{\upsilon_{Asg}\upsilon_{Bsg}-\upsilon_{Csg}^2}\right]$$

$$MSE_{\min}(\hat{D}_{sg}) = \left[\bar{Y}^2 - \frac{\upsilon_{Bsg}\upsilon_{Dsg}^2+\upsilon_{Asg}\upsilon_{Esg}^2-2\upsilon_{Csg}\upsilon_{Dsg}\upsilon_{Esg}}{\upsilon_{Asg}\upsilon_{Bsg}-\upsilon_{Csg}^2}\right] \quad (18)$$

Grover and Kaur (2014) introduced the generalized form of Shabbir and Gupta (2011) estimator as follows

$$\hat{D}_{gk} = \left[q_{gk1}\bar{y} + q_{gk2}(\bar{X}-\bar{x})\right]\exp\left[\frac{\upsilon(\bar{X}-\bar{x})}{\upsilon(\bar{X}-\bar{x})+2\lambda}\right] \quad (19)$$

For ease in calculation they use $\upsilon = 1$ and $\lambda = -1$

The MSE of $\hat{D}_{gk}$ is given by

$$MSE(\hat{D}_{gk}) = \bar{Y}^2 + q_{gk1}^2\upsilon_{Agk} + q_{gk2}^2\upsilon_{Bgk} + 2q_{gk1}q_{gk2}\upsilon_{Cgk} - 2q_{gk1}\upsilon_{Dgk} - 2q_{gk2}\upsilon_{Egk}$$

where

$$\upsilon_{Agk} = \bar{Y}^2\left[1+b_0+4\theta_{gk}^2 b_1 - 4\theta_{gk}b_{01}\right],\ \upsilon_{Bgk} = \bar{X}^2 b_1,\ \upsilon_{Cgk} = \bar{X}\bar{Y}\left[2\theta_{gk}b_1 - b_{01}\right],$$

$$\upsilon_{Dgk} = \bar{Y}^2[1+\frac{3\theta_{gk}^2 b_1}{2}-\theta_{gk}b_{01}],\ \upsilon_{Egk} = \bar{X}\bar{Y}\theta_{gk}b_1,\ \theta_{gk} = \frac{\upsilon\bar{X}}{2(\upsilon\bar{X}+\lambda)}$$

which is minimum for

$$q_{gk1}^{opt} = \left[\frac{\upsilon_{Bgk}\upsilon_{Dgk}-\upsilon_{Cgk}\upsilon_{Egk}}{\upsilon_{Agk}\upsilon_{Bgk}-\upsilon_{Cgk}^2}\right] \text{ and } q_{gk2}^{opt} = \left[\frac{\upsilon_{Agk}\upsilon_{Egk}-\upsilon_{Cgk}\upsilon_{Dgk}}{\upsilon_{Agk}\upsilon_{Bgk}-\upsilon_{Cgk}^2}\right]$$

$$MSE_{\min}(\hat{D}_{gk}) = \left[\bar{Y}^2 - \frac{\upsilon_{Bgk}\upsilon_{Dgk}^2+\upsilon_{Agk}\upsilon_{Egk}^2-2\upsilon_{Cgk}\upsilon_{Dgk}\upsilon_{Egk}}{\upsilon_{Agk}\upsilon_{Bgk}-\upsilon_{Cgk}^2}\right] \quad (20)$$

Shabbir et al. (2014) introduced the following difference-cum-exponential estimator for the estimation of population mean as follows

$$\hat{D}_j = \left[\frac{\bar{y}}{2}\left\{\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)+\exp\left(\frac{\bar{x}-\bar{X}}{\bar{x}+\bar{X}}\right)\right\}+q_{j1}\bar{y}+q_{j2}(\bar{X}-\bar{x})\right]\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \quad (21)$$

The MSE of $\hat{D}_j$ is given by

$$MSE(\hat{D}_j) = \bar{Y}^2 + q_{j1}^2 \upsilon_{Aj} + q_{j2}^2 \upsilon_{Bj} + 2q_{j1}q_{j2}\upsilon_{Cj} - 2q_{j1}\upsilon_{Dj} - 2q_{j2}\upsilon_{Ej}$$

where

$$\upsilon_{Lj} = \bar{Y}^2\left[b_0 - b_{01} + \frac{1}{4}b_1\right],\ \upsilon_{Aj} = \bar{Y}^2\left[1 + b_0 + b_1 - 2b_{01}\right],\ \upsilon_{Bj} = \bar{X}^2 b_1,\ \upsilon_{Cj} = \bar{X}\bar{Y}\left[b_1 - b_{01}\right],$$

$$\upsilon_{Dj} = \bar{Y}^2[\frac{3b_{01}}{2} - b_0 - \frac{3}{4}b_1],\ \upsilon_{Ej} = \bar{X}\bar{Y}\left[b_{01} - \frac{1}{2}b_1\right]$$

which is minimum for

$$q_{j1}^{opt} = \left[\frac{\upsilon_{Bj}\upsilon_{Dj} - \upsilon_{Cj}\upsilon_{Ej}}{\upsilon_{Aj}\upsilon_{Bj} - \upsilon_{Cj}^2}\right] \text{ and } q_{j2}^{opt} = \left[\frac{\upsilon_{Aj}\upsilon_{Ej} - \upsilon_{Cj}\upsilon_{Dj}}{\upsilon_{Aj}\upsilon_{Bj} - \upsilon_{Cj}^2}\right]$$

$$MSE_{\min}(\hat{D}_j) = \left[\upsilon_{L_j} - \frac{\upsilon_{Bj}\upsilon_{Dj}^2 + \upsilon_{Aj}\upsilon_{Ej}^2 - 2\upsilon_{Cj}\upsilon_{Dj}\upsilon_{Ej}}{\upsilon_{Aj}\upsilon_{Bj} - \upsilon_{Cj}^2}\right] \tag{22}$$

Haq and Shabbir (2014) proposed three estimators for the estimation of population mean. Their first estimator is as follows

$$\hat{D}_{hsa} = \left[q_{hsa1}\frac{\bar{y}}{2}\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}}\right) + q_{hsa2}(\bar{X} - \bar{x})\right]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{23}$$

The MSE of $\hat{D}_{hsa}$ is given by

$$MSE(\hat{D}_{hsa}) = \bar{Y}^2 + q_{hsa1}^2\upsilon_{Ahsa} + q_{hsa2}^2\upsilon_{Bhsa} + 2q_{hsa1}q_{hsa2}\upsilon_{Chsa} - 2q_{hsa1}\upsilon_{Dhsa} - 2q_{hsa2}\upsilon_{Ehsa}$$

where

$$\upsilon_{Ahsa} = \bar{Y}^2\left[1 + b_0 + b_1 - 2b_{01}\right],\ \upsilon_{Bhsa} = \bar{X}^2 b_1,$$

$$\upsilon_{Chsa} = \bar{X}\bar{Y}\left[b_1 - b_{01}\right],\ \upsilon_{Dhsa} = \bar{Y}^2[1 + \frac{7}{8}b_1 - \frac{1}{2}b_{01}],\ \upsilon_{Ehsa} = \bar{X}\bar{Y}\left[\frac{1}{2}b_1\right]$$

which is minimum for

$$q_{hsa1}^{opt} = \left[\frac{\upsilon_{Bhsa}\upsilon_{Dhsa} - \upsilon_{Chsa}\upsilon_{Ehsa}}{\upsilon_{Ahsa}\upsilon_{Bhsa} - \upsilon_{Chsa}^2}\right] \text{ and } q_{hsa2}^{opt} = \left[\frac{\upsilon_{Ahsa}\upsilon_{Ehsa} - \upsilon_{Chsa}\upsilon_{Dhsa}}{\upsilon_{Ahsa}\upsilon_{Bhsa} - \upsilon_{Chsa}^2}\right]$$

$$MSE_{\min}(\hat{D}_{hsa}) = \left[\upsilon_{Lhsa} - \frac{\upsilon_{Bhsa}\upsilon_{Dhsa}^2 + \upsilon_{Ahsa}\upsilon_{Ehsa}^2 - 2\upsilon_{Chsa}\upsilon_{Dhsa}\upsilon_{Ehsa}}{\upsilon_{Ahsa}\upsilon_{Bhsa} - \upsilon_{Chsa}^2}\right] \tag{24}$$

Second estimator of Haq and Shabbir (2014) is as follows

$$\hat{D}_{hsb} = \left[q_{hsb1}\frac{\bar{y}}{2}\left\{\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}}\right)\right\} + q_{hsb2}(\bar{X} - \bar{x})\right]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right] \tag{25}$$

the MSE of $\hat{D}_{hsb}$ is given by

$$MSE(\hat{D}_{hsb}) = \bar{Y}^2 + q_{hsb1}^2\upsilon_{Ahsb} + q_{hsb2}^2\upsilon_{Bhsb} + 2q_{hsb1}q_{hsb2}\upsilon_{Chsb} - 2q_{hsb1}\upsilon_{Dhsb} - 2q_{hsb2}\upsilon_{Ehsb}$$

where

$$\upsilon_{Ahsb} = \bar{Y}^2\left[1 + b_0 + \frac{5}{4}b_1 - 2b_{01}\right],\ \upsilon_{Bhsb} = \bar{X}^2 b_1,\ \upsilon_{Chsb} = \bar{X}\bar{Y}\left[b_1 - b_{01}\right],$$

$$\upsilon_{Dhsb} = \bar{Y}^2[1 + \frac{1}{2}b_1 - \frac{1}{2}b_{01}],\ \upsilon_{Ehsb} = \bar{X}\bar{Y}\left[\frac{1}{2}b_1\right]$$

which is minimum for

$$q_{hsb1}^{opt} = \left[\frac{\upsilon_{Bhsb}\upsilon_{Dhsb} - \upsilon_{Chsb}\upsilon_{Ehsb}}{\upsilon_{Ahsb}\upsilon_{Bhsb} - \upsilon_{Chsb}^2} \right] \text{ and } q_{hsb2}^{opt} = \left[\frac{\upsilon_{Ahsb}\upsilon_{Ehsb} - \upsilon_{Chsb}\upsilon_{Dhsb}}{\upsilon_{Ahsb}\upsilon_{Bhsb} - \upsilon_{Chsb}^2} \right]$$

$$MSE_{\min}(\hat{D}_{hsb}) = \left[\upsilon_{Lhsb} - \frac{\upsilon_{Bhsb}\upsilon_{Dhsb}^2 + \upsilon_{Ahsb}\upsilon_{Ehsb}^2 - 2\upsilon_{Chsb}\upsilon_{Dhsb}\upsilon_{Ehsb}}{\upsilon_{Ahsb}\upsilon_{Bhsb} - \upsilon_{Chsb}^2} \right] \quad (26)$$

Third estimator of Haq and Shabbir (2014) is as follows

$$\hat{D}_{hsc} = \left[q_{hsc1} \frac{\bar{y}}{4}\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}} \right)\left\{ \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{x} + \bar{X}} \right) \right\} + q_{hsc2}(\bar{X} - \bar{x}) \right] \exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right] \quad (27)$$

The MSE of $\hat{D}_{hsc}$ is given by

$$MSE(\hat{D}_{hsc}) = \bar{Y}^2 + q_{hsc1}^2\upsilon_{Ahsc} + q_{hsc2}^2\upsilon_{Bhsc} + 2q_{hsc1}q_{hsc2}\upsilon_{Chsc} - 2q_{hsc1}\upsilon_{Dhsc} - 2q_{hsc2}\upsilon_{Ehsc}$$

where

$$\vartheta_{Ahsc} = \bar{Y}^2\left[1 + b_0 + \frac{9}{4}b_1 - 2b_{01} \right], \ \vartheta_{Bhsc} = \bar{X}^2 b_1, \ \vartheta_{Chsc} = \bar{X}\bar{Y}\left[b_1 - b_{01} \right]$$

$$\vartheta_{Dhsc} = \bar{Y}^2\left[1 + b_1 - \frac{1}{2}b_{01} \right], \ \vartheta_{Ehsc} = \bar{X}\bar{Y}\left[\frac{1}{2}b_1 \right]$$

This is minimum for

$$q_{hsc1}^{opt} = \left[\frac{\vartheta_{Bhsc}\vartheta_{Dhsc} - \vartheta_{Chsc}\vartheta_{Ehsc}}{\vartheta_{Ahsc}\vartheta_{Bhsc} - \vartheta_{Chsc}{}^2} \right] \text{ and } q_{hsc2}^{opt} = \left[\frac{\vartheta_{Ahsc}\vartheta_{Ehsc} - \vartheta_{Chsc}\vartheta_{Dhsc}}{\vartheta_{Ahsc}\vartheta_{Bhsc} - \vartheta_{Chsc}^2} \right]$$

$$MSE_{\min}(\hat{D}_{hsc}) = \left[\upsilon_{Lhsc} - \frac{\upsilon_{Bhsc}\upsilon_{Dhsc}^2 + \upsilon_{Ahsc}\upsilon_{Ehsc}^2 - 2\upsilon_{Chsc}\upsilon_{Dhsc}\upsilon_{Ehsc}}{\upsilon_{Ahsc}\upsilon_{Bhsc} - \upsilon_{Chsc}^2} \right] \quad (28)$$

In this paper, our objective is to construct a new family of estimators utilizing single auxiliary variable under simple random sampling scheme. Which provide much better estimate of population parameter $\bar{Y}$ from a number of estimators available in literature under certain efficiency conditions. Further, new family of estimators is also proposed for the case when study variate is sensitive and supplementary variate is non-sensitive.

2. Proposed Family of Estimators

We propose the following generalized family of estimators as follows

$$\hat{D}_N = \left[w_1\bar{y}\exp\left(\frac{\sqrt{\bar{X}} - \sqrt{\bar{x}}}{\sqrt{\bar{X}} + \sqrt{\bar{x}}} \right) + w_2 \right] \exp\left[\frac{\bar{X} - \bar{x}}{2\xi\bar{X} - \bar{X} + \bar{x}} \right] \quad (29)$$

where w_1, w_2 are any constants and ξ is known value of auxiliary variable or any suitable scalar. We can generate some new estimators from (29) for different value of ξ as given in Table 2.

To find the bias of $\hat{D}_N$ we can rewrite the (29) with the e terms as given by:

$$\hat{D}_N - \bar{Y} = w_1\bar{Y}\left\{1+e_0-k_1e_1-k_1e_0e_1+2k_2e_1^2\right\}+w_2\left\{1-\frac{e_1}{2\xi}+\frac{3e_1^2}{8\xi^2}\right\}-\bar{Y}$$

where $k_1=\frac{1}{2\xi}+\frac{1}{4},\ k_2=\frac{3}{8\xi^2}+\frac{1}{8\xi}+\frac{5}{32}$

Estimator	ξ
$\hat{D}_{N1}=\left[w_1\bar{y}_{exp}\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$	1
$\hat{D}_{N2}=\left[w_1\bar{y}_{exp}\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]\exp\left[\frac{\bar{X}-\bar{x}}{2C_x\bar{X}-\bar{X}+\bar{x}}\right]$	C_x
$\hat{D}_{N3}=\left[w_1\bar{y}_{exp}\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]\exp\left[\frac{\bar{X}-\bar{x}}{2\beta_2(x)\bar{X}-\bar{X}+\bar{x}}\right]$	$\beta_2(x)$
$\hat{D}_{N4}=\left[w_1\bar{y}_{exp}\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]\exp\left[\frac{\bar{X}-\bar{x}}{2\left(1-\frac{n}{N}\right)\bar{X}-\bar{X}+\bar{x}}\right]$	$\left(1-\frac{n}{N}\right)$

Table 2: Some family members of proposed class

The bias of $\hat{D}_N$ is given by

$$B(\hat{D}_N)=w_1\bar{Y}\left\{1-k_1b_{01}+k_2b_1\right\}w_2\left\{1+\frac{3b_1}{8\xi^2}\right\}-\bar{Y}$$

The MSE of $\hat{D}_N$ is

$$MSE\left(\hat{D}_N\right)=\bar{Y}^2+w_1^2\phi_{A1}+w_2^2\phi_{B1}+w_1w_2\phi_{C1}+w_1\phi_{D1}+w_2\phi_{E1},$$

where

$\phi_{A1}=\bar{Y}^2\left\{1+b_0+(k_1^2+2k_2)b_1-4k_1b_{01}\right\},\ \phi_{B1}=1+\frac{b_1}{\xi^2}$

$\phi_{C1}=2\bar{Y}\left\{1+\left(\frac{3}{8\xi^2}+\frac{k_1}{2\xi}+k_2\right)b_1-\left(\frac{1}{2\xi}+k_1\right)b_{01}\right\},\ \phi_{D1}=-2\bar{Y}^2\left\{1+k_2b_1-2k_1b_{01}\right\}$

$\phi_{E1}=-2\bar{Y}\left\{1+\frac{3b_1}{8\xi^2}\right\}$

By minimizing MSE of $\hat{D}_N$, we get the optimum values of w_1, w_2 i.e.

$$w_1^{opt} = \left[\frac{-2\phi_{B1}\phi_{D1} + \phi_{C1}\phi_{E1}}{4\phi_{A1}\phi_{B1} - \phi_{C1}^{\;2}}\right] \text{ and } w_2^{opt} = \left[\frac{\phi_{C1}\phi_{D1} - 2\phi_{A1}\phi_{E1}}{4\phi_{A1}\phi_{B1} - \phi_{C1}^{\;2}}\right]$$

Hence, minimum mean square error of $\hat{D}_N$, i.e.

$$MSE_{min}\left(\hat{D}_N\right) = \left[\bar{Y}^2 - \frac{\phi_{B1}\phi_{D1}^{\;2} + \phi_{A1}\phi_{E1}^{\;2} - \phi_{C1}\phi_{D1}\phi_{E1}}{4\phi_{A1}\phi_{B1} - \phi_{C1}^{\;2}}\right] \tag{30}$$

3. In Case of Sensitive Study Variable

In Section 2 we have elaborated the issue of estimating the finite population mean$\bar{Y}$ of the study variable Y assuming that auxiliary variable X, highly correlated with Y, isknown in advance. In simple words we can say that both the study and the supplementary variables are directly observable. But sample surveys like habitual tax evasion, reckless driving, indiscriminate gambling, abortion etc; contain sensitive questions. In such type of sample surveys, we can't get a truthful direct response of our concerned sensitive questions. Warner (1965) developed the Randomized Response Technique (RRT) for solving this issue. Let we take S as a scrambling variable which is independent of Y and X. Further, respondent provide a scrambled response for Y given by $Z = Y + S$ and non-sensitive response for X. So $\bar{Z}$is the population mean of the scrambled variable. Let us define $e_{0'} = \dfrac{\bar{z} - \bar{Z}}{Z}$ such that

$$E\left(e_{0'}^2\right) = \left(\frac{1-f}{n}\right)C_x^{\;2} = b_{0'}$$

$$E\left(e_{0'}e_1\right) = \left(\frac{1-f}{n}\right)\rho_{xz}C_zC_x = b_{0'1}$$

where C_z is the coefficient of variation of variable $Z$and ρ_{xz} is the correlation between X and Z.The variance of ordinary estimator $\bar{Z}$of sensitive study variable is given by

$$\hat{\mathfrak{d}}_0 = V\left(\bar{z}\right) = \bar{Z}^2 b_{0'} \tag{31}$$

Sousa et al. (2010) introduced $\hat{\mathfrak{d}}_r$ in case of scrambled response as follows

$$\hat{\mathfrak{d}}_r = \bar{Z}\left[\frac{\bar{X}}{\bar{x}}\right] \tag{32}$$

The MSE of $\hat{\mathfrak{d}}_r$ is

$$MSE\left(\hat{\mathfrak{d}}_r\right) = \bar{Z}^2\left[b_{0'} + b_1 - 2b_{0'1}\right] \tag{33}$$

Gupta et al. (2012) developed $\hat{\mathfrak{d}}_{reg}$ in case of scrambled response is given by

$$\hat{\mathfrak{d}}_{reg} = \bar{z} + b\left(\bar{X} - \bar{x}\right) \tag{34}$$

The MSE of $\hat{\mathfrak{d}}_{reg}$ is

$$MSE\left(\hat{\delta}_{reg}\right)=\bar{Z}^2\left[b_{0'1}-\frac{b_{0'1}^2}{b_1}\right] \quad (35)$$

Koyuncu et al. (2013) introduced $\hat{\delta}_{lp}$ in case of scrambled response as follows

$$\hat{\delta}_{lp}=[p_{lp1}\bar{z}+[p_{lp2}\left(\bar{X}-\bar{x}\right)]\exp[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}] \quad (36)$$

The MSE of $\hat{\delta}_{lp}$ is

$$MSE\left(\hat{\delta}_{lp}\right)=\bar{Z}^2+p_{lp1}{}^2v_{Alp}+p_{lp2}{}^2v_{Blp}+2p_{Alp}p_{Blp}v_{Clp}-2p_{lp1}v_{Dlp}-2p_{lp2}v_{Elp}$$

where

$$v_{Alp}=\bar{Z}^2\left[1+b_{0'}+b_1-2b_{0'1}\right],\ v_{Blp}=\bar{Z}^2b_1,\ v_{Clp}=\bar{X}\bar{Z}\left[b_1-b_{0'1}\right]$$

$$v_{Dlp}=\bar{Z}^2\left[1+\frac{3b_1}{8}-\frac{b_{0'1}}{2}\right],\ v_{Elp}=\bar{X}\bar{Z}\frac{b_1}{2}$$

which is minimum for

$$p_{lp1}^{opt}=\left[\frac{v_{Blp}v_{Dlp}-v_{Clp}v_{Elp}}{v_{Alp}v_{Blp}-v_{Clp}{}^2}\right],\ p_{lp2}^{opt}=\left[\frac{v_{Alp}v_{Elp}-v_{Clp}v_{Dlp}}{v_{Alp}v_{Blp}-v_{Clp}{}^2}\right]$$

$$MSE_{min}\left(\hat{\delta}_{lp}\right)=\left[\bar{Z}^2-\frac{v_{Blp}v_{Dlp}{}^2+v_{Alp}v_{Elp}{}^2-2v_{Clp}v_{Dlp}v_{Elp}}{v_{Alp}v_{Blp}-v_{Clp}{}^2}\right] \quad (37)$$

Taking motivation from these, let we adapt some major families of estimator from Section 1 for scrambled response, in the upcoming sub-section.

3.1 Adapted Estimators

By adapting Bahl and Tuteja (1991) we propose an exponential estimator as follows

$$\hat{\delta}_{bt}=\bar{z}\exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right] \quad (38)$$

The MSE of$\hat{\delta}_{bt}$ is given by

$$MSE\left(\hat{\delta}_{bt}\right)=\bar{Z}^2\left[b_{0'}+\frac{b_1}{4}-b_{0'1}\right] \quad (39)$$

By adapting Shabbir and Gupta (2011) we propose an exponential estimator for scrambled response as follows

$$\hat{\delta}_{sg}=\left[p_{sg1}\bar{z}+p_{sg2}\left(\bar{X}-\bar{x}\right)\right]\exp\left[\frac{A'-a'}{A'+a'}\right] \quad (40)$$

where $A'=\bar{X}+N\bar{X}$ and $a'=\bar{x}+N\bar{X}$

By substituting $\acute{A}$ and $\acute{a}$, we can write $\hat{\delta}_{sg}$ as

$$\hat{\delta}_{sg}=\left[p_{sg1}\bar{z}+p_{sg2}\left(\bar{X}-\bar{x}\right)\right]\exp\left[\frac{\bar{X}-\bar{x}}{2N\bar{X}+\bar{X}+\bar{x}}\right]$$

The MSE of $\hat{\delta}_{sg}$ is

$$MSE\left(\hat{\eth}_{sg}\right)=\bar{Z}^2+p_{sg1}{}^2v_{Asg}+p_{sg2}{}^2v_{Bsg}+2p_{Asg}p_{Bsg}v_{Csg}-2p_{sg1}v_{Dsg}-2p_{sg2}v_{Esg}$$

where

$$v_{Asg}=\bar{Z}^2\left[1+b_{0'}+\frac{b_1}{\left(N+1\right)^2}-\frac{2b_{0'1}}{\left(N+1\right)}\right],\ v_{Bsg}=\bar{X}^2b_1,$$

$$v_{Csg}=\bar{X}\bar{Z}\left[\frac{b_1}{\left(N+1\right)^2}-b_{0'1}\right],\ v_{Dsg}=\bar{Z}^2\left[1+\frac{3b_1}{8\left(N+1\right)^2}-\frac{b_{0'1}}{2\left(N+1\right)}\right],\ v_{Esg}=\bar{X}\bar{Z}\frac{b_1}{2\left(N+1\right)}$$

which is minimum for

$$p_{sg1}^{opt}=\left[\frac{v_{Bsg}v_{Dsg}-v_{Csg}v_{Esg}}{v_{Asg}v_{Bsg}-v_{Csg}{}^2}\right],\ p_{sg2}^{opt}=\left[\frac{v_{Asg}v_{Esg}-v_{Csg}v_{Dsg}}{v_{Asg}v_{Bsg}-v_{Csg}{}^2}\right]$$

$$MSE_{min}\left(\hat{\eth}_{sg}\right)=\left[\bar{Z}^2-\frac{v_{Bsg}v_{Dsg}{}^2+v_{Asg}v_{Esg}{}^2-2v_{Csg}v_{Dsg}v_{Esg}}{v_{Asg}v_{Bsg}-v_{Csg}{}^2}\right] \tag{41}$$

By adapting Grover and Kaur (2014) we propose an exponential estimator for scrambled response as follows

$$\hat{\eth}_{gk}=\left[p_{gk1}\bar{z}+p_{gk2}\left(\bar{X}-\bar{x}\right)\right]\exp\left[\frac{v\left(\bar{X}-\bar{x}\right)}{v\left(\bar{X}+\bar{x}\right)+2\lambda}\right] \tag{42}$$

For ease in calculation they use $v=1$ and $\lambda=-1$.
The MSE of $\hat{\eth}_{gk}$ is

$$MSE\left(\hat{\eth}_{gk}\right)=\bar{Z}^2+p_{gk1}{}^2v_{Agk}+p_{gk2}{}^2v_{Bgk}+2p_{Agk}p_{Bgk}v_{Cgk}-2p_{gk1}v_{Dgk}-2p_{gk2}v_{Egk}$$

where

$$v_{Agk}=\bar{Z}^2\left[1+b_{0'}+4\theta_{gk}{}^2b_1-4\theta_{gk}b_{0'1}\right],\ v_{Bgk}=\bar{X}^2b_1,\ v_{Cgk}=\bar{X}\bar{Z}\left[2\theta_{gk}b_1-b_{0'1}\right],$$

$$v_{Dgk}=\bar{Z}^2\left[1+\frac{3\theta_{gk}{}^2b_1}{2}-\theta_{gk}b_{0'1}\right],\ v_{Egk}=\bar{X}\bar{Z}\theta_{gk}b_1,\ \theta_{gk}=\frac{v\bar{X}}{2\left(v\bar{X}+\lambda\right)}$$

which is minimum for

$$p_{gk1}^{opt}=\left[\frac{v_{Bgk}v_{Dgk}-v_{Cgk}v_{Egk}}{v_{Agk}v_{Bgk}-v_{Cgk}{}^2}\right],\ p_{gk2}^{opt}=\left[\frac{v_{Agk}v_{Egk}-v_{Cgk}v_{Dgk}}{v_{Agk}v_{Bgk}-v_{Cgk}{}^2}\right]$$

$$MSE_{min}\left(\hat{\eth}_{gk}\right)=\left[\bar{Z}^2-\frac{v_{Bgk}v_{Dgk}{}^2+v_{Agk}v_{Egk}{}^2-2v_{Cgk}v_{Dgk}v_{Egk}}{v_{Agk}v_{Bgk}-v_{Cgk}{}^2}\right] \tag{43}$$

By adapting Shabbir et al. (2014) we propose an exponential estimator for scrambled response as follows

$$\hat{\eth}_j=\left[\frac{\bar{z}}{2}\left\{\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)+\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\}+p_{j1}\bar{z}+p_{j2}\bar{z}\left(\bar{X}-\bar{x}\right)\right]\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{44}$$

The MSE of $\hat{\eth}_j$ is

$$MSE\left(\hat{\eth}_j\right) = \bar{Z}^2 + p_{j1}{}^2 v_{Aj} + p_{j2}{}^2 v_{Bj} + 2p_{Aj}p_{Bj}v_{Cj} - 2p_{j1}v_{Dj} - 2p_{j2}v_{Ej}$$

where

$$v_{Lj} = \bar{Z}^2\left[b_{0'} - b_{0'1} + \frac{1}{4}b_1\right],\ v_{Aj} = \bar{Z}^2\left[1 + b_{0'} + b_1 - 2b_{0'1}\right],\ v_{Bj} = \bar{X}^2 b_1$$

$$v_{Cj} = \bar{X}\bar{Z}\left[b_1 - b_{0'1}\right],\ v_{Dj} = \bar{Z}^2\left[\frac{3}{2}b_{0'1} - b_{0'} - \frac{3}{4}b_{0'1}\right],\ v_{Ej} = \bar{X}\bar{Z}\left[b_{0'1} - \frac{1}{2}b_1\right]$$

which is minimum for

$$p_{j1}^{opt} = \left[\frac{v_{Bj}v_{Dj} - v_{Cj}v_{Ej}}{v_{Aj}v_{Bj} - v_{Cj}{}^2}\right],\ p_{j2}^{opt} = \left[\frac{v_{Aj}v_{Ej} - v_{Cj}v_{Dj}}{v_{Aj}v_{Bj} - v_{Cj}{}^2}\right]$$

$$MSE_{min}\left(\hat{\eth}_j\right) = \left[v_{Lj} - \frac{v_{Bj}v_{Dj}{}^2 + v_{Aj}v_{Ej}{}^2 - 2v_{Cj}v_{Dj}v_{Ej}}{v_{Aj}v_{Bj} - v_{Cj}{}^2}\right] \tag{45}$$

By adapting Haq and Shabbir (2014) we propose three exponential estimators for scrambled response. First proposed estimator is as follows

$$\hat{\eth}_{hsa} = \left[p_{hsa1}\frac{\bar{z}}{2}\left\{\exp\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}}\right)\right\} + p_{hsa2}\left(\bar{X} - \bar{x}\right)\right]\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{46}$$

The MSE of $\hat{\eth}_{hsa}$ is

$$MSE\left(\hat{\eth}_{hsa}\right) = \bar{Z}^2 + p_{hsa1}^2 v_{Ahsa} + p_{hsa2}^2 v_{Bhsa} + 2p_{hsa1}p_{hsa2}v_{Chsa} - 2p_{hsa1}v_{Dhsa} - 2p_{hsa2}v_{Ehsa}$$

Where

$$v_{Ahsa} = \bar{Z}^2\left[1 + b_{0'} + b_1 - 2b_{0'1}\right],\ v_{Bhsa} = \bar{X}^2 b_1,\ v_{Chsa} = \bar{X}\bar{Z}\left[b_1 - b_{0'1}\right]$$

$$v_{Dhsa} = \bar{Z}^2\left[1 + \frac{7}{8}b_1 - \frac{1}{2}b_{0'1}\right],\ v_{Ehsa} = \bar{X}\bar{Z}\left[\frac{1}{2}b_1\right]$$

which is minimum for

$$p_{hsa1}^{opt} = \left[\frac{v_{Bhsa}v_{Dhsa} - v_{Chsa}v_{Ehsa}}{v_{Ahsa}v_{Bhsa} - v_{Chsa}{}^2}\right],\ p_{hsa2}^{opt} = \left[\frac{v_{Ahsa}v_{Ehsa} - v_{Chsa}v_{Dhsa}}{v_{Ahsa}v_{Bhsa} - v_{Chsa}{}^2}\right]$$

$$MSE_{min}\left(\hat{\eth}_{hsa}\right) = \left[\bar{Z}^2 - \frac{v_{Bhsa}v_{Dhsa}{}^2 + v_{Ahsa}v_{Ehsa}{}^2 - 2v_{Chsa}v_{Dhsa}v_{Ehsa}}{v_{Ahsa}v_{Bhsa} - v_{Chsa}{}^2}\right] \tag{47}$$

Second proposed estimator by adapting Haq and Shabbir (2014) is as follows

$$\hat{\eth}_{hsb} = \left[p_{hsb1}\frac{\bar{z}}{2}\left\{\exp\left(\frac{\bar{X}}{\bar{x}} + \frac{\bar{x}}{\bar{X}}\right) + \exp\left(\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right)\right\} + p_{hsb2}\left(\bar{X} - \bar{x}\right)\right]\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \tag{48}$$

The MSE of $\hat{\eth}_{hsb}$ is

$$MSE\left(\hat{\eth}_{hsb}\right) = \bar{Z}^2 + p_{hsb1}{}^2 v_{Ahsb} + p_{hsb2}{}^2 v_{Bhsb} + 2p_{Ahsb}p_{Bhsb}v_{Chsb} - 2p_{hsb1}v_{Dhsb} - 2p_{hsb2}v_{Ehsb}$$

where

$$v_{Ahsb} = \bar{Z}^2\left[1+b_{0'}+\frac{5}{2}b_1-2b_{0'1}\right],\ v_{Bhsb} = \bar{X}^2 b_1\ ,\ v_{Chsb} = \bar{X}\bar{Z}\left[b_1-b_{0'1}\right],$$

$$v_{Dhsb} = \bar{Z}^2\left[1+\frac{1}{2}b_1-\frac{1}{2}b_{0'1}\right],\ v_{Ehsb} = \bar{X}\bar{Z}\left[\frac{1}{2}b_1\right]$$

which is minimum for

$$p_{hsb1}^{opt} = \left[\frac{v_{Bhsb}v_{Dhsb}-v_{Chsb}v_{Ehsb}}{v_{Ahsb}v_{Bhsb}-v_{Chsb}{}^2}\right],\ p_{hsb2}^{opt} = \left[\frac{v_{Ahsb}v_{Ehsb}-v_{Chsb}v_{Dhsb}}{v_{Ahsb}v_{Bhsb}-v_{Chsb}{}^2}\right]$$

$$MSE_{min}\left(\hat{\eth}_{hsb}\right) = \left[v_{Lhsb}-\frac{v_{Bhsb}v_{Dhsb}{}^2+v_{Ahsb}v_{Ehsb}{}^2-2v_{Chsb}v_{Dhsb}v_{Ehsb}}{v_{Ahsb}v_{Bhsb}-v_{Chsb}{}^2}\right] \quad (49)$$

Third proposed estimator by adapting Haq and Shabbir (2014) is as follow

$$\hat{\eth}_{hsc} = \left[p_{hsc1}\frac{\bar{z}}{4}\left\{[\exp\left(\frac{\bar{X}}{\bar{x}}+\frac{\bar{x}}{\bar{X}}\right)]exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)+\exp\left(\frac{\bar{x}-\bar{X}}{\bar{X}+\bar{x}}\right)\right\}+p_{hsc2}\left(\bar{X}-\bar{x}\right)\right]\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)$$

(50)

The MSE of $\hat{\eth}_{hsc}$ is given by

$$MSE\left(\hat{\eth}_{hsc}\right) = \bar{Z}^2+p_{hsc1}{}^2v_{Ahsc}+p_{hsc2}{}^2v_{Bhsc}+2p_{Ahsc}p_{Bhsc}v_{Chsc}-2p_{hsc1}v_{Dhsc}-2p_{hsc2}v_{Ehsc}$$

$$v_{A_{hsc}} = \bar{Z}^2\left[1+b_{o'}+\frac{9}{4}b_1-2b_{o'1}\right],\ v_{B_{hsc}} = \bar{X}^2 b_1,\ v_{C_{hsc}} = \bar{X}\bar{Z}\left[b_1-b_{o'1}\right],$$

$$v_{D_{hsc}} = \bar{Z}^2\left[1+b_1-\frac{1}{2}b_{o'1}\right],\ v_{E_{hsc}} = \bar{X}\bar{Z}\left[\frac{1}{2}b_1\right]$$

which is minimum for

$$p_{hsc1}^{opt} = \left[\frac{v_{B_{hsc}}v_{D_{hsc}}-v_{C_{hsc}}v_{E_{hsc}}}{v_{A_{hsc}}v_{B_{hsc}}-v_{C_{hsc}}^2}\right],\ p_{hsc2}^{opt} = \left[\frac{v_{A_{hsc}}v_{E_{hsc}}-v_{C_{hsc}}v_{D_{hsc}}}{v_{A_{hsc}}v_{B_{hsc}}-v_{C_{hsc}}^2}\right]$$

$$\mathcal{MSE}_{min}\left(\hat{\eth}_{hsc}\right) = \left[v_{L_{hsc}}-\frac{v_{B_{hsc}}v_{D_{hsc}}^2+v_{A_{hsc}}v_{E_{hsc}}^2-2v_{C_{hsc}}v_{D_{hsc}}v_{E_{hsc}}}{v_{A_{hsc}}v_{B_{hsc}}-v_{C_{hsc}}^2}\right] \quad (51)$$

3.2 Proposed Family of Estimators in Case of Sensitive Study Variable

We propose the following generalized family of estimators as follows:

$$\hat{\eth}_N = \left[w_1\bar{z}exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]exp\left[\frac{\bar{X}-\bar{x}}{2\xi\bar{X}-\bar{X}+\bar{x}}\right]. \quad (52)$$

where w_1, w_2 are any constants and ξ is known value of auxiliary variable or any suitable scalar. We can generate some new estimators from (52) for different value of ξ as given in Table 3.

Estimator	ξ
$\hat{\eth}_{N1}=\left[w_1\bar{z}exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]exp\left[\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right]$	1
$\hat{\eth}_{N2}=\left[w_1\bar{z}exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]exp\left[\frac{\bar{X}-\bar{x}}{2C_x\bar{X}-\bar{X}+\bar{x}}\right]$	C_x
$\hat{\eth}_{N3}=\left[w_1\bar{z}exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]exp\left[\frac{\bar{X}-\bar{x}}{2\beta_2(x)\bar{X}-\bar{X}+\bar{x}}\right]$	$\beta_2(x)$
$\hat{\eth}_{N4}=\left[w_1\bar{z}exp\left(\frac{\sqrt{\bar{X}}-\sqrt{\bar{x}}}{\sqrt{\bar{X}}+\sqrt{\bar{x}}}\right)+w_2\right]exp\left[\frac{\bar{X}-\bar{x}}{2\left(1-\frac{n}{N}\right)\bar{X}-\bar{X}+\bar{x}}\right]$	$\left(1-\frac{n}{N}\right)$

Table 3: Some family members of proposed class in case of sensitive study variable

The bias of $\hat{\eth}_N$ is

$$B\left(\hat{\eth}_N\right)=w_1\bar{Z}\left\{1-k_1b_{o'1}+k_2b_1\right\}+w_2\left\{1+\frac{3b_1}{8\xi^2}\right\}-\bar{Z}$$

The MSE of $\hat{\eth}_N$ is

$$\text{MSE}\left(\hat{\eth}_N\right)=\bar{Z}^2+w_1^2\phi_{A1}+w_2^2\phi_{B1}+w_1w_2\phi_{C1}+w_1\phi_{D1}+w_2\phi_{E1}$$

where

$$\phi_{A1}=\bar{Z}^2\left\{1+b_{o'}+\left(k_1^2+2k_2\right)b_1-4k_1b_{o'1}\right\},\ \phi_{B1}=1+\frac{b_1}{\xi^2},$$

$$\phi_{C1}=2\bar{Z}\left\{1+\left(\frac{3}{8\xi^2}+\frac{k_1}{2\xi}+k_2\right)b_1-\left(\frac{1}{2\xi}+k_1\right)b_{o'1}\right\},\ \phi_{D1}=-2\bar{Z}^2\left\{1+k_2b_1-k_1b_{o'1}\right\},$$

$$\phi_{E1}=-2\bar{Z}\left\{1+\frac{3b_1}{8\xi^2}\right\}$$

By minimizing MSE of $\hat{\eth}_N$, we get the optimum values of w_1, w_2 i.e

$$w_1^{opt}=\left[\frac{-2\phi_{B1}\phi_{D1}+\phi_{C1}\phi_{E1}}{4\phi_{A1}\phi_{B1}-\phi_{C1}^2}\right],\ w_2^{opt}=\left[\frac{\phi_{C1}\phi_{D1}-2\phi_{A1}\phi_{E1}}{4\phi_{A1}\phi_{B1}-\phi_{C1}^2}\right]$$

Hence, minimum mean square error of $\hat{\eth}_N$, i.e

$$\mathrm{MSE}_{\min}\left(\hat{\eth}_N\right)=\left[\bar{Z}^2-\frac{\phi_{B1}\phi_{D1}^2+\phi_{A1}\phi_{E1}^2-\phi_{C1}\phi_{D1}\phi_{E1}}{4\phi_{A1}\phi_{B1}-\phi_{C1}^2}\right] \tag{53}$$

4. Efficiency Comparison

Here we perform efficiency comparison for the proposed estimators by looking at the MSEs of the reviewed estimators as given below

Observation (1):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{r}}\right),$$

if

$$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\bar{Y}^2\left[1-\left(b_o+b_1-2b_{o1}\right)\right]>0$$

Observation (2):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{reg}}\right),$$

if

$$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\bar{Y}^2\left[1-\left(b_o-\frac{b_{01}^2}{b_1}\right)\right]>0$$

Observation (3):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{bt}}\right),$$

if

$$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\bar{Y}^2\left[1-\left(b_o+\frac{1}{4}b_1-b_{o1}\right)\right]>0$$

Observation (4):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{lp}}\right),$$

if

$$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\left[\frac{\vartheta_{B_{lp}}\vartheta_{D_{lp}}^2+\vartheta_{A_{lp}}\vartheta_{E_{lp}}^2-2\vartheta_{C_{lp}}\vartheta_{D_{lp}}\vartheta_{E_{lp}}}{\vartheta_{A_{lp}}\vartheta_{B_{lp}}-\vartheta_{C_{lp}}^2}\right]>0$$

Observation (5):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{sg}}\right),$$

if $$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\left[\frac{\vartheta_{B_{sg}}\vartheta_{D_{sg}}^2+\vartheta_{A_{sg}}\vartheta_{E_{sg}}^2-2\vartheta_{C_{sg}}\vartheta_{D_{sg}}\vartheta_{E_{sg}}}{\vartheta_{A_{sg}}\vartheta_{B_{sg}}-\vartheta_{C_{sg}}^2}\right]>0$$

Observation (6):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{gk}}\right),$$

if $$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\left[\frac{\vartheta_{B_{gk}}\vartheta_{D_{gk}}^2+\vartheta_{A_{gk}}\vartheta_{E_{gk}}^2-2\vartheta_{C_{gk}}\vartheta_{D_{gk}}\vartheta_{E_{gk}}}{\vartheta_{A_{gk}}\vartheta_{B_{gk}}-\vartheta_{C_{gk}}^2}\right]>0$$

Observation (7):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{j}}\right),$$

If $$\left\{\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\left[\frac{\vartheta_{B_j}\vartheta_{D_j}^2+\vartheta_{A_j}\vartheta_{E_j}^2-2\vartheta_{C_j}\vartheta_{D_j}\vartheta_{E_j}}{\vartheta_{A_j}\vartheta_{B_j}-\vartheta_{C_j}^2}\right]\right\}-\left[\vartheta_{L_j}-\bar{Y}^2\right]>0$$

Observation (8):

$$\mathrm{MSE}_{\min}\left(\hat{\mathrm{D}}_{\mathrm{N}}\right)<\mathrm{MSE}\left(\hat{\mathrm{D}}_{\mathrm{hs}}\right),$$

if $$\left[\frac{\Phi_{B1}\Phi_{D1}^2+\Phi_{A1}\Phi_{E1}^2-\Phi_{C1}\Phi_{D1}\Phi_{E1}}{4\Phi_{A1}\Phi_{B1}-\Phi_{C1}^2}\right]-\left[\frac{\vartheta_{B_{hs}}\vartheta_{D_{hs}}^2+\vartheta_{A_{hs}}\vartheta_{E_{hs}}^2-2\vartheta_{C_{hs}}\vartheta_{D_{hs}}\vartheta_{E_{hs}}}{\vartheta_{A_{hs}}\vartheta_{B_{hs}}-\vartheta_{C_{hs}}^2}\right]>0$$

From above observations we can argue that the new estimators perform better than all of the reviewed estimators. Also we can develop such type of efficiency conditions for the case of scrambled response.

5. Numerical Illustration

5.1 Real Data

For assessing the merits of the proposed class of estimators, we have assumed the four natural populations. The source and details of the populations, the description of the variates y and x are given as follows.

Population 1

We use the data set presented in Venables and Ripley (1999). We consider grams of potassium as (Y) and grams of sugars in one portion as (X). Descriptives of the population are $N = 65$, $\bar{Y} = 159.1197$, $\bar{X} = 10.05084$, $C_y = 1.133037$, $C_x = 0.5805722$, $\rho = 0.271$, $\beta_2(x) = 1.975671$ and $n = 20$.

Population 2

This data set is presented in Sarndal et al. (1992). We consider (P85) i.e. 1985 population in thousands as (Y) and (SS82) i.e. number of Social-Democratic seats in municipal council as (X). Descriptives of the population are $N = 284$, $\bar{Y} = 29.36268$,

$\bar{X} = 22.18662$, $C_y = 1.75586$, $C_x = 0.3267727$, $\rho = 0.474$, $\beta_2(x) = 3.411812$ and $n = 35$.

Population 3

Data set is mentioned in Cochran (1977). We consider number of placebo children as (Y) and number of paralytic polio cases in the not inoculated group as (X). Descriptives of the population are $N = 34$, $\bar{Y} = 2.588235$, $\bar{X} = 8.370588$, $C_y = 1.233278$, $C_x = 1.027981$, $\rho = 0.729$, $\beta_2(x) = 8.93249$ and $n = 7$.

Population 4

Data set is taken from Murthy (1967). We consider area under wheat in 1964 as (Y) and area under wheat in 1961 as (X). Descriptives of the population are $N = 34$, $\bar{Y} = 199.4412$, $\bar{X} = 747.5882$, $C_y = 0.7531797$, $C_x = 0.5938485$, $\rho = 0.904$, $\beta_2(x) = 2.808238$ and $n = 7$.

Using the above data sets we have calculated MSEs of all reviewed and suggested estimators. The PREs' are calculated using the equations are given by

$$\mathrm{PRE}(.) = \frac{\mathrm{MSE}\left(\hat{\mathrm{D}}_o\right)}{\mathrm{MSE}(.)} \times 100.$$

$$PRE(.) = \frac{MSE\left(\hat{\eth}_o\right)}{MSE(.)} \times 100.$$

We summarized the results in absence and presence sensitivity in Table 4 and Table 5. From these tables our proposed estimator $\hat{D}_{N3}$ gives the most efficient result for all data sets. When we compare our proposed estimators with the existing estimators in literature, we can say that proposed estimators perform better.

Est	Pop-1	Pop-2	Pop-3	Pop-4
$\hat{D}_0$	100	100	100	100
$\hat{D}_r$	101.62	116.56	208.57	510.93
$\hat{D}_{reg}$	107.97	129.12	213.43	548.51
$\hat{D}_{bt}$	107.95	108.66	176.67	226.02
$\hat{D}_{lp}$	112.75	136.93	238.50	561.77
$\hat{D}_{sg}$	112.42	136.84	230.69	554.95
$\hat{D}_{gk}$	114.73	137.29	349.23	599.75
$\hat{D}_j$	113.11	137.02	248.33	571.25
$\hat{D}_{hsa}$	152.05	121.18	369.76	283.60
$\hat{D}_{hsb}$	150.79	120.95	342.83	276.17
$\hat{D}_{hsc}$	114.54	137.37	301.91	637.69

$\hat{D}_{N1}$	1585.75	14334.39	1041.29	2470.60
$\hat{D}_{N2}$	553.59	1577.78	1092.01	929.37
$\hat{D}_{N3}$	6107.43	166327.60	72566.45	19009.78
$\hat{D}_{N4}$	774.94	11030.83	714.20	1588.94

Table 4: PREs of proposed and existing estimators in absence of sensitivity

Est.	Pop-1	Pop-2	Pop-3	Pop-4
$\hat{\delta}_{o}$	100	100	100	100
$\hat{\delta}_{r}$	101.68	116.59	152.12	513.64
$\hat{\delta}_{reg}$	108.01	129.17	165.24	551.47
$\hat{\delta}_{bt}$	107.98	108.67	155.01	226.43
$\hat{\delta}_{lp}$	112.79	136.97	187.62	564.76
$\hat{\delta}_{sg}$	112.45	136.88	181.57	557.90
$\hat{\delta}_{gk}$	114.77	137.33	265.21	603.10
$\hat{\delta}_{j}$	113.14	137.07	195.04	574.32
$\hat{\delta}_{hsa}$	151.99	121.19	502.92	284.48
$\hat{\delta}_{hsb}$	150.73	120.97	468.62	277.04
$\hat{\delta}_{hsc}$	114.58	137.42	232.31	641.48
$\hat{\delta}_{N1}$	1585.99	14317.17	818.61	2477.10
$\hat{\delta}_{N2}$	553.67	1575.88	857.74	931.78
$\hat{\delta}_{N3}$	6108.36	166128.01	55878.93	19060.61
$\hat{\delta}_{N4}$	775.06	11017.58	566.41	1593.09

Table 5: PREs of proposed and existing estimators in presence of sensitivity

5.2 Simulation Study

For assessing the performance of proposed and reviewed estimators, we perform a simulation study where two populations having size N = 1000 from a multivariate normal distribution with the mean of [Y, X] =[2,2] and different covariance matrices as given below

$$\sigma^2 = \begin{bmatrix} 10 & 3 \\ 3 & 2 \end{bmatrix} \text{ with } \rho = 0.68 \text{ and } \sigma^2 = \begin{bmatrix} 7 & 3 \\ 3 & 2 \end{bmatrix} \text{ with } \rho = 0.8 .$$

Further, we assume scramble variable S, which is normally distributed with mean equal to zero and standard deviation equal to 15% of the S_x i.e. standard deviation of X. Hence the reported response is given by Z = Y+S.

Note that numerical and simulated results of $\hat{D}_i$ for $(i = 1,2,...,26)$ are not provided in Table 4 and Table 5 because we already explained that these estimators can't provide much better results than regression. So we only provide the numerical and simulated results of $\hat{D}_{reg}$ and $\hat{\mathfrak{d}}_{reg}$ rather than $\hat{D}_i$. All the other PREs' are calculated and available in Table 4, Table 5 and Table 6.

Est.	Pop-1	Pop-2	Est.	Pop-1	Pop-2
$\hat{D}_0$	100	100	$\hat{\mathfrak{d}}_o$	100	100
$\hat{D}_r$	102.32	120.15	$\hat{\mathfrak{d}}_r$	101.11	1115.05
$\hat{D}_{reg}$	104.07	125.03	$\hat{\mathfrak{d}}_{reg}$	109.08	130.01
$\hat{D}_{bt}$	109.23	110.87	$\hat{\mathfrak{d}}_{bt}$	106.41	103.51
$\hat{D}_{lp}$	117.75	141.22	$\hat{\mathfrak{d}}_{lp}$	123.31	147.02
$\hat{D}_{sg}$	113.12	139.04	$\hat{\mathfrak{d}}_{sg}$	118.02	145.11
$\hat{D}_{gk}$	115.03	135.34	$\hat{\mathfrak{d}}_{gk}$	114.21	132.04
$\hat{D}_j$	102.01	141.12	$\hat{\mathfrak{d}}_j$	104.03	138.91
$\hat{D}_{hsa}$	131.15	127.29	$\hat{\mathfrak{d}}_{hsa}$	140.61	136.19
$\hat{D}_{hsb}$	147.09	128.03	$\hat{\mathfrak{d}}_{hsb}$	152.23	131.60
$\hat{D}_{hsc}$	124.04	148.07	$\hat{\mathfrak{d}}_{hsc}$	120.61	141.99
$\hat{D}_{N1}$	1470.15	4279.08	$\hat{\mathfrak{d}}_{N1}$	1328.03	4006.81
$\hat{D}_{N2}$	502.19	1420.21	$\hat{\mathfrak{d}}_{N2}$	471.28	1321.11
$\hat{D}_{N3}$	6009.43	6223.01	$\hat{\mathfrak{d}}_{N3}$	5756.23	6113.29
$\hat{D}_{N4}$	702.71	8676.21	$\hat{\mathfrak{d}}_{N4}$	631.89	8121.02

Table 6: PREs based on simulation in absence and presence of sensitivity

From Table 6, we can see that our proposed estimators are more efficient than the others for both simulated population.

6. Conclusion

In this article we propose a new family of estimators. Many more estimators can be generated in future through propose family. Theoretical properties such as bias and MSE are determined under first order of approximation. Moreover, theoretical results are also supported through four natural population and two simulated data sets. Proposed family of estimators is also discussed for the sensitivity issue along with its properties. Numerical illustration shows that proposed class is performing better not only for high correlated data sets but also suitable for the data having weak linear relationship for both sensitive and non-sensitive cases. In this way new proposed estimators are recommended for utilizing in real life applications.

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