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AN IMPROVED CLASS OF RATIO ESTIMATORS FOR ESTIMATING POPULATION MEAN USING AUXILIARY INFORMATION IN SURVEY SAMPLING

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Abstract

The use of ancillary information in sample survey is to get gain in precision of estimates. However, various estimators have been developed using other conventional locations parameters, in the present paper we have proposed some new estimators by using the ancillary information of decile mean, second quartile and quartile deviation with other measures of population such as skewness, coefficient of correlation, coefficient of variation of the concomitant variable. The performance linked among the anticipated estimators are determined by MSE (mean square error) and Bias and compare by means of usual ratio estimator by Cochran (1940) and with existing estimators proposed via Abid *et al*. in (2016a, 2016b). With this evaluation we initiate with the aim of that our anticipated estimators are proficient set of estimators than the ratio estimator by Cochran (1940) and the existing estimators via Abid *et al*. in (2016a, 2016b). Numerical study is provided to hold up the theoretical results.

Key Words: Decile Mean; Median; Quartile Deviation; coefficient of Skewness, correlation coefficient; coefficient of variation; Bias; Efficiency; Mean Square Error.

1. Introduction

The use of ancillary information is employed either at design or estimation stage or at both stages in order to obtain the proficient estimators. So in the estimation theory increase in precision is achieved by taking the advantage of correlation between the response and the concomitant variable and utilizing the ancillary information by various method of estimation such as ratio, regression and product. The basic assumptions of these methods are that when the line of passes through the origin and the correlation between the response and the ancillary variable is positive and high, in that case ratio method of estimation gives better results than the others (Cochran, 1940), but if the line of regression does not pass through origin but makes an intercept and the correlation is also high between the response and the ancillary variable, then regression method of estimation gives better results than the others (Okafor, 2000), (Murthy, 1967), (Perri, 2005), (Muhammad *et al.*, 2009), but when the correlation between the response and the ancillary variable is negative and high, in that (Robson, 1957) proposed the product estimator and suggests that this estimators gives better results when this situation occurs. After obtain the proficient estimators, among those whose

variance is minimum is regarded as most proficient estimators and also among the proficient estimators whose bias is too much lower than others then the estimators is considered as good estimator. Rajesh *et al.* 2011 courted like that the estimator with minimum absolute bias is regarded as a better estimator among others in the class. So when different parameters of the ancillary variable is known, number of modifications to the usual ratio, regression, Usual product have been suggested by various authors such as Subramani and Kumarapandiyan (2012), also Abid *et al.* (2016) suggests some new modified ratio estimators in simple random sampling and Subzar *et al.* (2016) also had taken initiative by proposed modified ratio estimators in simple random sampling.
Keeping above in view, the present study was carried out to propose some new modified ratio estimators for population mean using the above mentioned ancillary information.

To increase valuable evaluation, we recapitulate underneath several existing estimators, their biases and mean square errors.
Consider a finite population $U_N = \{U_{N1}, U_{N2}, U_{N3}, ..., U_{NN}\}$ of N_N distinct and identifiable units. Let Y_N be the study variable with value Y_{Ni} measured of U_{Ni} , $i = 1,2,3,..., N_N$ giving a vector $Y = \{Y_1, Y_2, Y_3, ..., Y_N\}$. The objective is to estimate population mean $\overline{Y}_N = \frac{1}{N_N}\sum_{i=1}^{N_N} Y_{Ni}$ on the basis of a random sample.
Before discussing about proposed estimators, we will point out the estimators in fiction with the notations prearranged in the next section.

N_N	Size of the population
n_n	Size of the sample
$f_n = N_N / n_n$	Fraction of sampling
Y_N	Response variable
X_N	Ancillary variable
$\overline{Y}_N, \overline{X}_N$	Population mean of the Response and the Ancillary variable
$\overline{y}_n, \overline{x}_n$	Sample means of the Response and the Ancillary variable
y, x	Sample totals of the Response and the Ancillary variable
S_{x_n}, S_{y_n}	Population standard deviations of the Response and the Ancillary variable
$S_{y_n x_n}$	Population covariance between Response and the Ancillary variables
C_{x_n}, C_{y_n}	Population coefficient of variation of the Response and the Ancillary variable
ρ_{xy}	Population correlation coefficient

$B(.)$ Bias of the estimator

$MSE(.)$ Mean square error of the estimator

$\hat{\bar{Y}}_{Ni}$ Existing modified ratio estimator of $\bar{Y}_{Ni}$

$\hat{\bar{Y}}_{Npj}$ Proposed modified ratio estimator of $\bar{Y}_{Npj}$

Q_{2N} Second Quartile of ancillary variable

β_2 Population kurtosis

β_1 Population skewness

$DM_N = (D_{N1} + D_{N2} + D_{N3} + \cdots + D_{N9})/9$ Decile Mean

$QD_N = (Q_{N3} - Q_{N1})/2$ Population quartile deviation

$HL_N = median\ ((X_{Nj} + X_{Nk})/2, 1 \le j \le k \le N_N$ Hodges-Lehmann estimator

$MR_N = (X_{N1} + X_{NN})/2$ Population mid-range

$TM_N = (Q_{N1} + 2Q_{N2} + Q_{N3})/4$ Population tri- mean

$G_N = \frac{4}{N_N - 1}\sum_{I=1}^{N_N}\left(\frac{2i - N_N - 1}{2N_N}\right)X_{N(i)}$ Gini's Mean Difference

$D_N = \frac{2\sqrt{\lambda}}{N_N(N_N - 1)}\sum_{I=1}^{N_N}\left(i - \frac{N_N + 1}{2}\right)X_{N(i)}$ Downtown's method

$S_{pwN} = \frac{\sqrt{\lambda}}{N_N^2}\sum_{I=1}^{N_N}(2i - N_N - 1)X_{N(i)}$ Probability Weighted Moments

Subscript

i For existing estimators

j For proposed estimators

From the notations given above, the mean ratio estimator for estimating the population mean $\bar{Y}_N$ of the study variable Y_N is given as

$$\hat{\bar{Y}}_{Nr} = \frac{\bar{y}_n}{\bar{x}_n}\bar{X}_N = \hat{R}\bar{X}_N,$$

Where $\hat{R} = \frac{\bar{y}_n}{\bar{x}_n} = \frac{y_n}{x_n}$ is the estimate of $R = \frac{\bar{Y}_N}{\bar{X}_N} = \frac{Y_N}{X_N}$.

The bias, constant and the mean square error of the mean ratio estimator is given by

$$B(\hat{\hat{\bar{Y}}}_{Nr}) = \frac{(1-f_n)}{n_n}\frac{1}{\bar{X}_N}(RS_{x_n}^2 - \rho_{xy}S_{x_n}S_{y_n}),\ \ R = \frac{\bar{Y}_N}{\bar{X}_N},$$

$$MSE(\hat{\hat{\bar{Y}}}_{Nr}) = \frac{(1-f_n)}{n_n}(S_{y_n}^2 + R^2S_{x_n}^2 - 2R\rho_{xy}S_{x_n}S_{y_n}).$$

The ratio estimator mentioned above is employed to obtain more precision in estimates of the finite population mean and evaluated with sample mean estimator as the response and the ancillary variables are positively correlated.

2. Existing Estimators

Abid et al (2016 a) proposed some ratio estimators for finite population mean $\bar{Y}_N$ in simple random sampling using Tri-mean, Mid-range and Hodges Lehmann with correlation coefficient and coefficient of variation as supplementary information are as under

$$\hat{\hat{\bar{Y}}}_{N1} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n + TM_N)}(\bar{X} + TM_N),$$

$$\hat{\hat{\bar{Y}}}_{N2} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{x_n} + TM_N)}(\bar{X}_N C_{x_n} + TM_N)$$

$$\hat{\hat{\bar{Y}}}_{N3} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n \rho_{xy} + TM_N)}(\bar{X}_N \rho_{xy} + TM_N),$$

$$\hat{\hat{\bar{Y}}}_{N4} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n + MR_N)}(\bar{X}_N + MR_N),$$

$$\hat{\hat{\bar{Y}}}_{N5} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{x_n} + MR_N)}(\bar{X}_N C_{x_n} + MR_N),$$

$$\hat{\hat{\bar{Y}}}_{N6} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n \rho_{xy} + MR_N)}(\bar{X}_N \rho_{xy} + MR_N)$$

$$\hat{\hat{\bar{Y}}}_{N7} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n + HL_N)}(\bar{X}_N + HL_N),$$

$$\hat{\hat{\bar{Y}}}_{N8} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n C_{x_n} + HL_n)}(\bar{X} C_x + HL_N),$$

$$\hat{\hat{\bar{Y}}}_{N9} = \frac{\bar{y}_n + b(\bar{X}_N - \bar{x}_n)}{(\bar{x}_n \rho_{xy} + HL_N)}(\bar{X}_N \rho_{xy} + HL_N)$$

The biases, related constants and the mean square error (MSE) for Abid *et al* (2016 a) estimators are respectively given by

$$B(\hat{\bar{Y}}_{N1})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_1^2,\quad R_1=\frac{\bar{Y}_N}{(\bar{X}_N+TM_N)}\quad MSE(\hat{\bar{Y}}_{N1})=\frac{(1-f_n)}{n_n}(R_1^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N2})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_2^2,\quad R_2=\frac{\bar{Y}_N C_{x_n}}{(\bar{X}_N C_{x_n}+TM_N)}\quad MSE(\hat{\bar{Y}}_{N2})=\frac{(1-f_n)}{n_n}(R_2^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N3})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_3^2,\quad R_3=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+TM_N)}\quad MSE(\hat{\bar{Y}}_{N3})=\frac{(1-f_n)}{n_n}(R_3^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N4})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_4^2,\quad R_4=\frac{\bar{Y}_N}{(\bar{X}_N+MR_N)}\quad MSE(\hat{\bar{Y}}_{N4})=\frac{(1-f_n)}{n_n}(R_4^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N5})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_5^2,\quad R_5=\frac{\bar{Y}_N C_{x_n}}{(\bar{X}_N C_{x_n}+MR_N)}\quad MSE(\hat{\bar{Y}}_{N5})=\frac{(1-f_n)}{n_n}(R_5^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N6})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_6^2,\quad R_6=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+MR_N)}\quad MSE(\hat{\bar{Y}}_{N6})=\frac{(1-f_n)}{n_n}(R_6^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N7})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_7^2,\quad R_7=\frac{\bar{Y}_N}{(\bar{X}_N+HL_N)}\quad MSE(\hat{\bar{Y}}_{N7})=\frac{(1-f_n)}{n_n}(R_7^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N8})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_8^2,\quad R_8=\frac{\bar{Y}_N C_{x_n}}{(\bar{X}_N C_{x_n}+HL_N)}\quad MSE(\hat{\bar{Y}}_{N8})=\frac{(1-f_n)}{n_n}(R_8^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{N9})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_9^2,\quad R_9=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+HL_N)}\quad MSE(\hat{\bar{Y}}_{N9})=\frac{(1-f_n)}{n_n}(R_9^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_9)=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2,\quad R_9=\frac{\bar{Y}\rho}{(\bar{X}\rho+HL)}\quad MSE(\hat{\bar{Y}}_9)=\frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2))$$

Abid *et al* (2016 b) also proposed some new modified ratio estimators in srs using Gini's mean difference, Downton's method and probability weighted moments with correlation coefficient and coefficient of variation are as under

$$\hat{\bar{Y}}_{N10}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n+G_N)}(\bar{X}_N+G_N),$$

$$\hat{\bar{Y}}_{N11}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\rho_{xy}+G_N)}(\bar{X}_N\rho_{xy}+G_N)$$

$$\hat{\bar{Y}}_{N12}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n C_{x_n}+G_N)}(\bar{X}_N C_{x_n}+G_N),$$

$$\bar{\bar{\hat{Y}}}_{N13}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n+D_N)}(\bar{X}_N+D_N),$$

$$\bar{\bar{\hat{Y}}}_{N14}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\rho_{xy}+D_N)}(\bar{X}_N\rho_{xy}+D_N),$$

$$\bar{\bar{\hat{Y}}}_{N15}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_nC_{x_n}+D_N)}(\bar{X}_NC_{x_n}+D_N)$$

$$\bar{\bar{\hat{Y}}}_{N16}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n+S_{pwN})}(\bar{X}_N+S_{pwN}),$$

$$\bar{\bar{\hat{Y}}}_{N17}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\rho_{xy}+S_{pwN})}(\bar{X}_N\rho_{xy}+S_{pwN}),$$

$$\bar{\bar{\hat{Y}}}_{N18}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_nC_{x_n}+S_{pwN})}(\bar{X}_NC_{x_n}+S_{pwN})$$

The biases, related constants and the mean square error (MSE) for Abid *et al* (2016 b) estimators are respectively given by

$$B(\bar{\bar{\hat{Y}}}_{N10})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{10},\quad R_{10}=\frac{\bar{Y}_N}{(\bar{X}_N+G_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N10})=\frac{(1-f_n)}{n_n}(R^2_{10}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy}))$$

$$B(\bar{\bar{\hat{Y}}}_{N11})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{11},\quad R_{11}=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+G_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N11})=\frac{(1-f_n)}{n_n}(R^2_{11}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy}))$$

$$B(\bar{\bar{\hat{Y}}}_{N12})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{12},\quad R_{12}=\frac{\bar{Y}_NC_{x_n}}{(\bar{X}_NC_{x_n}+G_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N12})=\frac{(1-f_n)}{n_n}(R^2_{12}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy})),$$

$$B(\bar{\bar{\hat{Y}}}_{N13})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{13},\quad R_{13}=\frac{\bar{Y}_N}{(\bar{X}_N+D_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N13})=\frac{(1-f_n)}{n_n}(R^2_{13}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy})),$$

$$B(\bar{\bar{\hat{Y}}}_{N14})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{14},\quad R_{14}=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+D_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N14})=\frac{(1-f_n)}{n_n}(R^2_{14}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy}))$$

$$B(\bar{\bar{\hat{Y}}}_{N15})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{15},\quad R_{15}=\frac{\bar{Y}_NC_{x_n}}{(\bar{X}_NC_{x_n}+D_N)}\quad MSE(\bar{\bar{\hat{Y}}}_{N15})=\frac{(1-f_n)}{n_n}(R^2_{15}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy})),$$

$$B(\bar{\bar{\hat{Y}}}_{N16})=\frac{(1-f_n)}{n_n}\frac{s^2_{x_n}}{\bar{Y}_N}R^2_{16},\quad R_{16}=\frac{\bar{Y}_N}{(\bar{X}_N+S_{pwN})}\quad MSE(\bar{\bar{\hat{Y}}}_{N16})=\frac{(1-f_n)}{n_n}(R^2_{16}S^2_{x_n}+S^2_{y_n}(1-\rho^2_{xy})),$$

$$B(\hat{\bar{Y}}_{M17})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_{17}^2,\quad R_{17}=\frac{\bar{Y}_N\rho_{xy}}{(\bar{X}_N\rho_{xy}+S_{pwN})}\quad MSE(\hat{\bar{Y}}_{M17})=\frac{(1-f_n)}{n_n}(R_{17}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\hat{\bar{Y}}_{M18})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{Y}_N}R_{18}^2,\quad R_{18}=\frac{\bar{Y}_N C_{x_n}}{(\bar{X}_N C_{x_n}+S_{pwN})}\quad MSE(\hat{\bar{Y}}_{M18})=\frac{(1-f_n)}{n_n}(R_{18}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

3. Improved Ratio Estimators

So keeping the above estimators in view, we proposed an efficient ratio type estimators using the ancillary information of decile mean, quartile deviation, median with coefficient of skewness, correlation coefficient and coefficient of variation of concomitant variable as supplementary information and the proposed estimators are given below;

$$\hat{\bar{Y}}_{Np1}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n+\varphi_1)}(\bar{X}+\varphi_1),$$

$$\hat{\bar{Y}}_{Np2}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n+\varphi_2)}(\bar{X}_N+\varphi_2),$$

$$\hat{\bar{Y}}_{Np3}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\rho_{xy}+\varphi_1)}(\bar{X}_N\rho_{xy}+\varphi_1),$$

$$\hat{\bar{Y}}_{Np4}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\rho_{xy}+\varphi_2)}(\bar{X}_N\rho_{xy}+\varphi_2),$$

$$\hat{\bar{Y}}_{Np5}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n C_{x_n}+\varphi_1)}(\bar{X}_N C_{x_n}+\varphi_1),$$

$$\hat{\bar{Y}}_{Np6}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n C_{x_n}+\varphi_2)}(\bar{X}_N C_{x_n}+\varphi_2),$$

$$\hat{\bar{Y}}_{Np7}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\beta_1+\varphi_1)}(\bar{X}_N\beta_1+\varphi_1),$$

$$\hat{\bar{Y}}_{Np8}=\frac{\bar{y}_n+b(\bar{X}_N-\bar{x}_n)}{(\bar{x}_n\beta_1+\varphi_2)}(\bar{X}_N\beta_1+\varphi_2),$$ Where $\varphi_1=(DM_N\times Q_{2N})$ and $\varphi_2=(DM_N\times QD_N)$

The bias, related constant and the MSE for the first proposed estimator can be obtained by using the as Taylor series method defined as

$$h(\bar{x}_n,\bar{y}_n)\cong h(\bar{X}_N,\bar{Y}_N)+\frac{\partial h(c,d)}{\partial c}|_{\bar{X}_N,\bar{Y}_N}(\bar{x}_n-\bar{X}_N)+\frac{\partial h(c,d)}{\partial d}|_{\bar{X}_N,\bar{Y}_N}(\bar{y}_n-\bar{Y}_N)$$

(2.1)

Where $h(\bar{x}_n, \bar{y}_n) = \hat{R}_{p1}$ and $h(\bar{X}_N, \bar{Y}_N) = R.$

As shown in Wolter (1985), (2.1) can be applied to the proposed estimator in order to obtain MSE equation as follows:

$$\hat{R}_{p1} - R \cong \frac{\partial((\bar{y}_n + b(\bar{X}_N - \bar{x}_n))/(\bar{x}_n + \varphi_1)}{\partial x}|_{\bar{X}_N,\bar{Y}_N} (\bar{x}_n - \bar{X}_N) + \frac{\partial((\bar{y}_n + b(\bar{X}_N - \bar{x}_n))/(\bar{x}_n + \varphi_1)}{\partial y}|_{\bar{X}_N,\bar{Y}_N} (\bar{y}_n - \bar{Y}_N)$$

$$\cong -\left(\frac{\bar{y}_n}{(\bar{x}_n + \varphi_1)^2} + \frac{b(\bar{X}_N + \varphi_1)}{(\bar{x}_n + \varphi_1)^2}\right)|_{\bar{X}_N,\bar{Y}_N} (\bar{x}_n - \bar{X}_N) + \frac{1}{(\bar{x}_n + \varphi_1)}|_{\bar{X}_N,\bar{Y}_N} (\bar{y}_n - \bar{Y}_N)$$

$$E(\hat{R}_{p1} - R)^2 \cong \frac{(\bar{Y}_N + B(\bar{X}_N + \varphi_1))^2}{(\bar{X}_N + \varphi_1)^4} V(\bar{x}_n) - \frac{2(\bar{Y}_N + B(\bar{X}_N + \varphi_1))}{(\bar{X}_N + \varphi_1)^3} Cov(\bar{x}_n, \bar{y}_n) + \frac{1}{(\bar{X}_N + \varphi_1)^2} V(\bar{y}_n)$$

$$\cong \frac{1}{(\bar{X}_N + \varphi_1)^2} \left\{ \frac{(\bar{Y}_N + B(\bar{X}_N + \varphi_1))^2}{(\bar{X}_N + \varphi_1)^2} V(\bar{x}_n) - \frac{2(\bar{Y}_N + B(\bar{X}_N + \varphi_1)}{(\bar{X}_N + \varphi_1)} Cov(\bar{x}_n, \bar{y}_n) + V(\bar{y}_n) \right\}$$

Where $B = \frac{s_{x_n y_n}}{s_{x_n}^2} = \frac{\rho_{xy} s_{x_n} s_{y_n}}{s_{x_n}^2} = \frac{\rho_{xy} s_{y_n}}{s_{x_n}}$. Here we exclude this $(E(b) - B)$.

$$MSE(\bar{y}_{Np1}) = (\bar{X}_N + \varphi_1)^2 E(\hat{R}_{p1} - R)^2 \cong \frac{(\bar{Y}_N + B(\bar{X}_N + \varphi_1))^2}{(\bar{X}_N + \varphi_1)^2} V(\bar{x}_n) - \frac{2(\bar{Y}_N + B(\bar{X}_N + \varphi_1))}{(\bar{X}_N + \varphi_1)} Cov(\bar{x}_n, \bar{y}_n) + V(\bar{y}_n)$$

$$\cong \frac{\bar{Y}_N^2 + 2B(\bar{X}_N + \varphi_1)\bar{Y}_N + B^2(\bar{X}_N + \varphi_1)^2}{(\bar{X}_N + \varphi_1)^2} V(\bar{x}_n) - \frac{2\bar{Y}_N + 2B(\bar{X}_N + \varphi_1)}{(\bar{X}_N + \varphi_1)} Cov(\bar{x}_n, \bar{y}_n) + V(\bar{y}_n)$$

$$\cong \frac{(1 - f_n)}{n_n} \left\{ \left(\frac{\bar{Y}_N^2}{(\bar{X}_N + \varphi_1)^2} + \frac{2B\bar{Y}_N}{(\bar{X}_N + \varphi_1)} + B^2 \right) S_{x_n}^2 - \left(\frac{2\bar{Y}_N}{(\bar{X}_N + \varphi_1)} + 2B \right) S_{x_n y_n} + S_{y_n}^2 \right\}$$

$$\cong \frac{(1 - f_n)}{n_n} (R_{p1}^2 S_{x_n}^2 + 2BR_{p1} S_{x_n}^2 + B^2 S_{x_n}^2 - 2R_{p1} S_{x_n y_n} - 2B S_{x_n y_n} + S_{y_n}^2)$$

$$MSE(\bar{y}_{Np1}) \cong \frac{(1 - f_n)}{n_n} (R_{p1}^2 S_{x_n}^2 + 2R_{p1} \rho_{xy} S_{x_n} S_{y_n} + \rho_{xy}^2 S_{y_n}^2 - 2R_{p1} \rho_{xy} S_{x_n} S_{y_n} - 2\rho_{xy}^2 S_{y_n}^2 + S_{y_n}^2$$

$$\cong \frac{(1 - f_n)}{n_n} (R_{p1}^2 S_{x_n}^2 - \rho_{xy}^2 S_{y_n}^2 + S_{y_n}^2) \cong \frac{(1 - f_n)}{n_n} (R_{p1}^2 S_{x_n}^2 + S_{y_n}^2 (1 - \rho_{xy}^2))$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{Np1}) \cong \frac{(1 - f_n)}{n_n} \frac{S_{x_n}^2}{\bar{Y}_N} R_{p1}^2$$

Thus the bias and MSE of the proposed estimator is given below:

$$B(\bar{\bar{\hat{Y}}}_{Np1})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p1}^2,\quad R_{p1}=\frac{\bar{Y}_N}{\bar{X}_N+\varphi_1}\qquad MSE(\bar{\bar{\hat{Y}}}_{p1})=\frac{(1-f_n)}{n_n}(R_{p1}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

Similarly we can obtain the bias, constant and mean square error for the other proposed estimators and is given as follows

$$B(\bar{\bar{\hat{Y}}}_{Np2})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p2}^2,\quad R_{p2}=\frac{\bar{Y}_N}{\bar{X}_N+\varphi_2}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np2})=\frac{(1-f_n)}{n_n}(R_{p2}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\bar{\bar{\hat{Y}}}_{Np3})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p3}^2,\quad R_{p3}=\frac{\bar{Y}_N\rho_{xy}}{\bar{X}_N\rho_{xy}+\varphi_1}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np3})=\frac{(1-f_n)}{n_n}(R_{p3}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\bar{\bar{\hat{Y}}}_{Np4})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p4}^2,\quad R_{p4}=\frac{\bar{Y}_N\rho_{xy}}{\bar{X}_N\rho_{xy}+\varphi_2}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np4})=\frac{(1-f_n)}{n_n}(R_{p4}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\bar{\bar{\hat{Y}}}_{Np5})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p5}^2,\quad R_{p5}=\frac{\bar{Y}_NC_{x_n}}{\bar{X}_NC_{x_n}+\varphi_1}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np5})=\frac{(1-f_n)}{n_n}(R_{p5}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\bar{\bar{\hat{Y}}}_{Np6})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p6}^2,\quad R_{p6}=\frac{\bar{Y}_NC_{x_n}}{\bar{X}_NC_{x_n}+\varphi_2}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np6})=\frac{(1-f_n)}{n_n}(R_{p6}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2))$$

$$B(\bar{\bar{\hat{Y}}}_{Np7})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p7}^2,\quad R_{p7}=\frac{\bar{Y}_N\beta_1}{\bar{X}_N\beta_1+\varphi_1}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np7})=\frac{(1-f_n)}{n_n}(R_{p7}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

$$B(\bar{\bar{\hat{Y}}}_{Np8})=\frac{(1-f_n)}{n_n}\frac{s_{x_n}^2}{\bar{\bar{Y}}_N}R_{p8}^2,\quad R_{p8}=\frac{\bar{Y}_N\beta_1}{\bar{X}_N\beta_1+\varphi_2}\qquad MSE(\bar{\bar{\hat{Y}}}_{Np8})=\frac{(1-f_n)}{n_n}(R_{p8}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2)),$$

4. Efficiency Comparisons

In this part, the effectiveness situation meant for the anticipated ratio estimators have been derived algebraically
according to usual ratio estimator and existing ratio estimators in literature.

4.1. Evaluation with usual ratio estimator

The anticipated ratio estimators are more proficient than that of the usual ratio estimator if

$$MSE(\bar{\bar{\hat{Y}}}_{Npj})\le MSE(\bar{\bar{\hat{Y}}}_{Nr}),$$

$$\frac{(1-f_n)}{n_n}(R_{pj}^2S_{x_n}^2+S_{y_n}^2(1-\rho_{xy}^2))\le\frac{(1-f_n)}{n_n}(S_{y_n}^2+R^2S_{x_n}^2-2R\rho_{xy}S_{x_n}S_{y_n}),$$

$$R_{pj}^2S_{x_n}^2-\rho_{xy}^2S_{y_n}^2-R^2S_{x_n}^2+2R\rho_{xy}S_{x_n}S_{y_n}\le 0,$$

$$(\rho_{xy}S_{y_n}-RS_{x_n})^2-R_{pj}^2S_{x_n}^2\ge 0,$$

$$(\rho_{xy}S_{y_n}-RS_{x_n}+R_{pj}^2)(\rho_{xy}S_{y_n}-RS_{x_n}-R_{pj}S_{x_n})\ge 0.$$

Condition I: $(\rho_{xy} S_{y_n} - RS_{x_n} + R_{pj} S_{x_n}) \leq 0$ and $(\rho_{xy} S_{y_n} - RS_{x_n} - R_{pj} S_{x_n}) \leq 0$

After solving the condition I, we get

$$\left(\frac{RS_{y_n} - RS_{x_n}}{S_{x_n}} \right) \leq R_{pj} \leq \left(\frac{RS_{x_n} - \rho S_{y_n}}{S_{x_n}} \right).$$

Hence,

$$MSE(\hat{\bar{Y}}_{Npj}) \leq MSE(\hat{\bar{Y}}_{Nr}),$$

$$\left(\frac{\rho_{xy} S_{y_n} - RS_{x_n}}{S_{x_n}} \right) \leq R_{Npj} \leq \left(\frac{RS_{x_n} - \rho_{xy} S_{y_n}}{S_{x_n}} \right),$$

Or

$$\left(\frac{RS_{x_n} - \rho_{xy} S_{y_n}}{S_{x_n}} \right) \leq R_{pj} \leq \left(\frac{\rho_{xy} S_{y_n} - RS_{x_n}}{S_{x_n}} \right).$$ Where $j = 1,2,....,8.$

4.2 Comparisons with existing ratio estimators

Proposed Estimators are more efficient than the existing ones

$$MSE(\hat{\bar{Y}}_{Npj}) \leq MSE(\hat{\bar{Y}}_{Ni}),$$

$$\frac{(1-f_n)}{n_n}(R_{pj}^2 S_{x_n}^2 + S_{y_n}^2 (1-\rho_{xy}^2) \leq \frac{(1-f_n)}{n_n}(R_i^{\,2} S_{x_n}^2 + S_{y_n}^2 (1-\rho_{xy}^2)),$$

$$R_{pj}^2 S_{x_n}^2 \leq R_i^2 S_{x_n}^2,$$

$$R_{pj} \leq R_i,$$

Where $j = 1,2,....,8.$ and $i = 1,2,....,18.$

5. Numerical Study

The performances of the proposed ratio estimators are assessed and compared with the mentioned ratio estimators in Section 2 by using the data of the three natural populations.

For the population I and II we use the data of Singh and Chaudhary (1986) page 177 and for the population III we use the data of Murthy (1967) page 228.

We apply the proposed and existing estimators to these data sets and the data statistics of these populations are given in Table 1.

From Table 2, we observe that the proposed estimators are more proficient than all of the estimators in literature as their Bias, Constant and MSE are much lower than the usual and the existing estimators.

The PRE of the proposed estimators (*p*), with respective to the existing estimators (*e*), is computed by

$$PRE = \frac{MSE\, of\ Existing\ Estimator}{MSE\, of\ propoesd\, estimator} \times 100$$

These PRE values are given in Table 3, Table 4 and Table 5 for the population I, population II and population III respectively. From these tables, we reveal that suggested estimators perform better than the estimators taken into consideration in the present study.

6. Conclusion

Thus by incorporating the above mentioned auxiliary information we conclude that our suggested estimators are proficient than the usual and the estimators taken into consideration in the present study, as the mean square error as well as bias of the suggested estimators are lower than the usual and the estimators taken into consideration in the present study. For all the three populations under consideration in present study we have analysed that the suggested estimators third, fifth, fourth, seventh are more proficient than the other suggested estimators for population 1 and the suggested estimators sixth, third, fourth, fifth are more proficient than the other suggested estimators for population 2, and the suggested estimators fifth, third, sixth, first are more proficient than the other suggested estimators for population 3, whereas all the proposed estimators are more proficient than the usual and the existing estimators. Thus we strongly urge that our suggested estimators preferred over the usual and the estimators taken into consideration in the present study to be used in future for practical applications.

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Parameters	Population 1	Population 2	Population 3
N_N	34	34	80
n_n	20	20	20
$\bar{Y}_N$	856.4117	856.4117	5182.637
$\bar{X}_N$	208.8823	199.4412	1126.463
ρ_{xy}	0.4491	0.4453	0.941
S_{y_n}	733.1407	733.1407	1835.659
C_{y_n}	0.8561	0.8561	0.354193
S_{x_n}	150.5059	150.2150	845.610
C_{x_n}	0.7205	0.7531	0.7506772
β_2	0.0978	1.0445	-0.063386
β_1	0.9782	1.1823	1.050002
M_{dN}	150	142.5	757.5
TM_N	162.25	165.562	931.562
MR_N	284.5	320	1795.5
HL_N	190	184	1040.5
QD_N	80.25	89.375	588.125
G_N	155.446	162.996	901.081
D_N	140.891	144.481	801.381
S_{pwN}	199.961	206.944	791.364
DM_N	234.82	206.944	1150.7

Table 1: Characteristics of these populations

Estimators	**Population I**			**Population II**			**Population III**		
	Cons tant	**Bias**	**MSE**	**Const ant**	**Bias**	**MSE**	**Const ant**	**Bias**	**MSE**
$\hat{\bar{Y}}_{Nr}$	4.100	4.270	10539.2	4.294	4.940	10960.7	4.601	60.88	189775.1
$\hat{\bar{Y}}_{N1}$	2.308	2.900	11317.28	2.346	2.986	11429.08	2.518	32.81	184446.20
$\hat{\bar{Y}}_{N2}$	1.973	2.120	10649.40	2.043	2.263	10809.99	2.189	24.79	142903.20
$\hat{\bar{Y}}_{N3}$	1.502	1.229	9886.21	1.499	1.219	9915.81	2.449	31.03	175238.70
$\hat{\bar{Y}}_{N4}$	1.736	1.641	10239.11	1.649	1.475	10134.39	1.774	16.23	98755.61
$\hat{\bar{Y}}_{N5}$	1.418	1.096	9772.39	1.372	1.021	9745.79	1.473	11.23	72582.52
$\hat{\bar{Y}}_{N6}$	1.0167	0.563	9316.02	0.932 9	0.472	9275.87	1.708	15.10	92644.60
$\hat{\bar{Y}}_{N7}$	2.147	2.510	10983.77	2.233	2.706	11189.04	2.392	29.59	167778.60
$\hat{\bar{Y}}_{N8}$	1.812	1.788	10365.55	1.930	2.021	10602.02	2.063	22.01	128487.60
$\hat{\bar{Y}}_{N9}$	1.355	1.000	9690.50	1.398	1.060	9779.43	2.322	27.90	158990.7
$\hat{\bar{Y}}_{N10}$	2.350	3.009	11411.10	2.363	3.030	11465.43	2.552	33.71	189080.3
$\hat{\bar{Y}}_{N11}$	1.543	1.297	9944.57	2.059	2.300	10841.88	2.224	25.58	146971.8
$\hat{\bar{Y}}_{N12}$	2.016	2.215	10730.9	1.515	1.240	9937.20	2.483	31.91	179770.5
$\hat{\bar{Y}}_{N13}$	2.448	3.265	11630.0	1.635	1.450	10113.06	2.620	35.53	198518.7
$\hat{\bar{Y}}_{N14}$	1.638	1.462	10086.5	2.189	2.600	11097.24	2.362	28.87	164020.7
$\hat{\bar{Y}}_{N15}$	2.117	2.442	10925.4	2.490	3.360	11752.23	2.688	37.39	208187.2
$\hat{\bar{Y}}_{N16}$	2.094	2.389	10880.4	1.645	1.470	10129.01	2.635	35.91	200516.2
$\hat{\bar{Y}}_{N17}$	1.309	0.933	9633.54	2.200	2.630	11119.84	2.377	29.22	165857.4
$\hat{\bar{Y}}_{N18}$	1.760	1.688	10279.8	2.501	3.390	11777.26	2.702	37.78	210216.9
$\hat{\bar{Y}}_{Np1}$	0.0242	0.000	8834.43	0.029	0.001	8872.15	0.006	0.000	14471.76
$\hat{\bar{Y}}_{Np2}$	0.0450	0.001	8835.10	0.046	0.001	8872.74	0.008	0.000	14472.37
$\hat{\bar{Y}}_{Np3}$	0.011	0.000	8834.21	0.013	0.000	8871.84	0.006	0.000	14471.64

$\hat{\bar{Y}}_{Np4}$	0.020	0.000	8834.35	0.021	0.000	8871.96	0.007	0.000	14472.19
$\hat{\bar{Y}}_{Np5}$	0.017	0.000	8834.29	0.022	0.000	8871.98	0.004	0.000	14471.34
$\hat{\bar{Y}}_{Np6}$	0.033	0.001	8834.64	0.035	0.001	8872.32	0.006	0.000	14471.69
$\hat{\bar{Y}}_{Np7}$	0.024	0.000	8834.41	0.034	0.000	8872.30	0.006	0.000	14471.85
$\hat{\bar{Y}}_{Np8}$	0.044	0.001	8835.05	0.054	0.002	8873.12	0.008	0.000	14472.53

Table 2: The Statistical Analysis of the Estimators for these Populations

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$
$\hat{\bar{Y}}_{r}$	119.2969	119.2878	119.2999	119.298	119.2988	119.2941	119.2972	119.2885
$\hat{\bar{Y}}_{1}$	128.1043	128.0946	128.0930	128.1055	128.1062	128.1011	128.1045	128.0952
$\hat{\bar{Y}}_{2}$	120.5443	120.5352	120.5337	120.5454	120.5461	120.5413	120.5445	120.5358
$\hat{\bar{Y}}_{3}$	111.9055	111.897	111.8956	111.9065	111.9072	111.9027	111.9057	111.8976
$\hat{\bar{Y}}_{4}$	115.9001	115.8913	115.8899	115.9012	115.9018	115.8972	115.9003	115.8919
$\hat{\bar{Y}}_{5}$	110.6172	110.6088	110.6074	110.6182	110.6188	110.6144	110.6173	110.6093
$\hat{\bar{Y}}_{6}$	105.4513	105.4433	105.4420	105.4523	105.4529	105.4487	105.4515	105.4438
$\hat{\bar{Y}}_{7}$	124.3292	124.3198	124.3182	124.3303	124.3310	124.3261	124.3294	124.3203
$\hat{\bar{Y}}_{8}$	117.3313	117.3225	117.3209	117.3324	117.3331	117.3284	117.3315	117.3230
$\hat{\bar{Y}}_{9}$	109.6902	109.6819	109.6805	109.6912	109.6918	109.6875	109.6904	109.6824
$\hat{\bar{Y}}_{10}$	129.1663	129.1565	129.1549	129.1675	129.1682	129.1631	129.1665	129.1571
$\hat{\bar{Y}}_{11}$	112.5661	112.5576	112.5561	112.5671	112.5678	112.5633	112.5663	112.5581
$\hat{\bar{Y}}_{12}$	121.4669	121.4577	121.4561	121.4680	121.4687	121.4639	121.4670	121.4582
$\hat{\bar{Y}}_{13}$	131.6441	131.6341	131.6324	131.6453	131.6461	131.6408	131.6443	131.6347
$\hat{\bar{Y}}_{14}$	114.1727	114.1640	114.1626	114.1737	114.1744	114.1698	114.1728	114.1645

$\hat{\bar{Y}}_{15}$	123.6685	123.6591	123.6575	123.6696	123.6703	123.6654	123.6687	123.6597
$\hat{\bar{Y}}_{16}$	123.1591	123.1498	123.1482	123.1602	123.1609	123.1561	123.1593	123.1503
$\hat{\bar{Y}}_{17}$	109.0455	109.0372	109.0358	109.0465	109.0471	109.0428	109.0456	109.0377
$\hat{\bar{Y}}_{18}$	116.3607	116.3519	116.3504	116.3618	116.3624	116.3578	116.3609	116.3524

Table 3: PRE of the Proposed Estimators with the Estimators in Literature for population I

	$\hat{\bar{Y}}_{Np1}$	$\hat{\bar{Y}}_{Np2}$	$\hat{\bar{Y}}_{Np4}$	$\hat{\bar{Y}}_{Np5}$	$\hat{\bar{Y}}_{Np5}$	$\hat{\bar{Y}}_{Np6}$	$\hat{\bar{Y}}_{Np7}$	$\hat{\bar{Y}}_{Np8}$
$\hat{\bar{Y}}_{Nr}$	123.5405	123.5323	123.5448	123.5432	123.5429	123.5382	123.5384	123.527
$\hat{\bar{Y}}_{N1}$	128.8197	128.8112	128.8242	128.8225	128.8222	128.8318	128.8175	128.8056
$\hat{\bar{Y}}_{N2}$	121.8418	121.8338	121.8461	121.8445	121.8441	121.8532	121.8397	121.8285
$\hat{\bar{Y}}_{N3}$	111.7633	111.7559	111.7672	111.7657	111.7654	111.7738	111.7614	111.7511
$\hat{\bar{Y}}_{N4}$	114.2270	114.2194	114.2310	114.2295	114.2291	114.2377	114.2250	114.2145
$\hat{\bar{Y}}_{N5}$	109.8470	109.8397	109.8508	109.8494	109.8491	109.8573	109.8451	109.8350
$\hat{\bar{Y}}_{N6}$	104.5504	104.5435	104.5541	104.5527	104.5524	104.5602	104.5486	104.5390
$\hat{\bar{Y}}_{N7}$	126.1142	126.1058	126.1186	126.1169	126.1166	126.1260	126.112	126.1004
$\hat{\bar{Y}}_{N8}$	119.4978	119.4898	119.5019	119.5003	119.5000	119.5089	119.4957	119.4847
$\hat{\bar{Y}}_{N9}$	110.2262	110.2189	110.2300	110.2285	110.2282	110.2365	110.2243	110.2141
$\hat{\bar{Y}}_{N10}$	129.2294	129.2209	129.2340	129.2322	129.2319	129.2415	129.2272	129.2153
$\hat{\bar{Y}}_{N11}$	122.2013	122.1932	122.2055	122.2039	122.2036	122.2127	122.1992	122.1879
$\hat{\bar{Y}}_{N12}$	112.0044	111.9970	112.0083	112.0068	112.0065	112.0149	112.0025	111.9922
$\hat{\bar{Y}}_{N13}$	113.9866	113.9790	113.9906	113.9890	113.9887	113.9973	113.9846	113.9741
$\hat{\bar{Y}}_{N14}$	125.0795	125.0712	125.0839	125.0822	125.0818	125.0912	125.0773	125.0658
$\hat{\bar{Y}}_{N15}$	132.4620	132.4532	132.4667	132.4649	132.4645	132.4744	132.4598	132.4475

$\hat{\hat{Y}}_{N16}$	114.1664	114.1588	114.1703	114.1688	114.1685	114.1770	114.1644	114.1539
$\hat{\hat{Y}}_{N17}$	125.3342	125.3259	125.3386	125.3369	125.3366	125.3460	125.3321	125.3205
$\hat{\hat{Y}}_{N18}$	132.7442	132.7353	132.7488	132.7470	132.7466	132.7566	132.7419	132.7296

Table 4: PRE of the Proposed Estimators with the Estimators in Literature for population II

	$\hat{\hat{Y}}_{Np1}$	$\hat{\hat{Y}}_{Np2}$	$\hat{\hat{Y}}_{Np4}$	$\hat{\hat{Y}}_{Np5}$	$\hat{\hat{Y}}_{Np5}$	$\hat{\hat{Y}}_{Np6}$	$\hat{\hat{Y}}_{Np7}$	$\hat{\hat{Y}}_{Np8}$
$\hat{\hat{Y}}_{Nr}$	1311.348	1311.292	1311.359	1311.309	1311.386	1311.354	1311.34	1311.278
$\hat{\hat{Y}}_{N1}$	1274.525	1274.471	1274.535	1274.574	1274.562	1274.531	1274.517	1274.457
$\hat{\hat{Y}}_{N2}$	987.4628	987.4204	987.4702	987.5008	987.4909	987.4670	987.4562	987.4094
$\hat{\hat{Y}}_{N3}$	1210.901	1210.849	1210.911	1210.948	1210.936	1210.907	1210.893	1210.836
$\hat{\hat{Y}}_{N4}$	682.4024	682.3731	682.4075	682.4287	682.4219	682.4053	682.3979	682.3655
$\hat{\hat{Y}}_{N5}$	501.5461	501.5245	501.5498	501.5654	501.5604	501.5482	501.5427	501.5190
$\hat{\hat{Y}}_{N6}$	640.1753	640.1478	640.1800	640.1999	640.1935	640.1780	640.1710	640.1407
$\hat{\hat{Y}}_{N7}$	1159.352	1159.302	1159.361	1159.397	1159.385	1159.357	1159.344	1159.289
$\hat{\hat{Y}}_{N8}$	887.8508	887.8127	887.8574	887.8850	887.8761	887.8546	887.8449	887.8028
$\hat{\hat{Y}}_{N9}$	1098.628	1098.580	1098.636	1098.670	1098.659	1098.632	1098.620	1098.568
$\hat{\hat{Y}}_{N10}$	1306.547	1306.491	1306.557	1306.597	1306.584	1306.553	1306.538	1306.476
$\hat{\hat{Y}}_{N11}$	1015.577	1015.533	1015.584	1015.616	1015.606	1015.581	1015.570	1015.522
$\hat{\hat{Y}}_{N12}$	1242.216	1242.163	1242.226	1242.264	1242.252	1242.222	1242.208	1242.149
$\hat{\hat{Y}}_{N13}$	1371.767	1371.708	1371.777	1371.819	1371.806	1371.772	1371.757	1371.692
$\hat{\hat{Y}}_{N14}$	1133.385	1133.336	1133.393	1133.429	1133.417	1133.390	1133.377	1133.324
$\hat{\hat{Y}}_{N15}$	1438.576	1438.514	1438.587	1438.631	1438.617	1438.582	1438.566	1438.498
$\hat{\hat{Y}}_{N16}$	1385.569	1385.510	1385.580	1385.623	1385.609	1385.575	1385.560	1385.494

$\hat{\bar{Y}}_{N17}$	1146.077	1146.027	1146.085	1146.121	1146.109	1146.081	1146.069	1146.015
$\hat{\bar{Y}}_{N18}$	1452.601	1452.539	1452.612	1452.657	1452.643	1452.607	1452.592	1452.523

Table 5: PRE of the Proposed Estimators with the Estimators in Literature for population III