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# NEW EXPONENTIAL-RATIO TYPE ESTIMATORS OF POPULATION MEAN IN TWO- PHASE SAMPLING USING NO INFORMATION CASE ON AUXILIARY VARIABLES

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## Abstract

In this paper exponential ratio type estimators have been proposed for population mean using two phase equal probability sampling with single and two auxiliary variables in case of no information. The proposed estimators are compared with the estimators given by Sukhatme (1962), Singh and Vishwakarma (2007) and Hanif et al. (2009). It is found that the proposed estimators are relatively more efficient than these estimators for all types of populations.

**Key Words:** Two Phase Sampling, Efficient, Bias, Auxiliary Variable, No Information

## 1. Introduction

The information on auxiliary variable has been extensively used in survey sampling in order to achieve precision of the estimator both in single and two-phase sampling. Graunt (1662), Laplace (1820), Neyman (1938) and Cochran (1940) are considered as the pioneers of using auxiliary information in equal probability sampling. The auxiliary information is sometimes completely available, sometimes partially available and sometimes no information is available at all. In case of no information with single auxiliary variable Sukhatme (1962) suggested a ratio and product type estimator for estimating the population mean. Singh and Vishwakarma (2007) proposed exponential ratio product type estimators in case of no information on single auxiliary variable in two phase sampling. Hanif et al (2009) suggested ratio and product type estimators in multivariate multiple auxiliary variables for population means.

Let us consider the finite population of size N and let $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ are the population means of the variables $y$, $x$ and $z$ respectively. The sample of size $n_1$ at the first phase is drawn from the population and we assume $\bar{x}_1 = \sum_{i=1}^{n_1} \frac{x_i}{n_1}$, $\bar{y}_1 = \sum_{i=1}^{n_1} \frac{y_i}{n_1}$ and $\bar{z}_1 = \sum_{i=1}^{n_1} \frac{z_i}{n_1}$ are the sample means of variable x, y and z respectively for the first phase sample, whereas

$\bar{x}_2 = \sum_{i=1}^{n_2} \frac{x_i}{n_2}$, $\bar{y}_2 = \sum_{i=1}^{n_2} \frac{y_i}{n_2}$ and $\bar{z}_2 = \sum_{i=1}^{n_2} \frac{z_i}{n_2}$ are the means of variable x ,y and z respectively for the sample of size $n_2$ obtained at second phase.

$\theta_1 = \frac{1-f}{n_1}$, $\theta_2 = \frac{1-f}{n_2}$,$\mu_{rsl} = \frac{1}{N}\sum_{i=1}^{N}(Y_i - \bar{Y})^r(X_i - \bar{X})^s(Z_i - \bar{Z})^l$,

$C_X = \frac{\mu_{020}}{\bar{X}}$,$C_Y = \frac{\mu_{200}}{\bar{Y}}$, $C_Z = \frac{\mu_{002}}{\bar{Z}}$,$e_{0(1)} = \bar{e}_{y_1} = \frac{\bar{y}_1-\bar{Y}}{\bar{Y}}$,$e_{1(1)} = \bar{e}_{x_1} = \frac{\bar{x}_1-\bar{X}}{\bar{X}}$,
$e_{0(2)} = \bar{e}_{y_2} = \frac{\bar{y}_2-\bar{Y}}{\bar{Y}}$,$e_{1(2)} = \bar{e}_{x_2} = \frac{\bar{x}_2-\bar{X}}{\bar{X}}$ ,$e_{2(1)} = \bar{e}_{z_1} = \frac{\bar{z}_1-\bar{Z}}{\bar{Z}}$and

$$e_{2(2)} = \bar{e}_{z_2} = \frac{\bar{z}_2 - \bar{Z}}{\bar{Z}}$$

Few populations are considered in this study the details of which are given in Table (1.1).

| **Parameters** | ***Population 1** | ***Population 2** | ****Population 3** | **#Population 4** | **#*Population 5** |
|---|---|---|---|---|---|
| | X:ANB<br>Y:AFS<br>Z: ANN | X: ANB<br>Y:ALS<br>Z:AFS | X : GNRP<br>Y: AMP<br>Z: AMMS | X: AMMS<br>Y: AMP<br>Z: GNRP | X: NOC<br>Y: NI<br>Z: TC |
| N | 113 | 113 | 36 | 36 | 788 |
| $\bar{Z}$ | 252.1681 | 43.1593 | 2.6639 | 18.5 | 2799.956 |
| $\rho_{xy}$ | 0.7835 | 0.4093 | -0.3877 | -0.1885 | 0.1961 |
| $\rho_{xz}$ | 0.7945 | 0.7945 | 0.07256 | 0.07256 | 0.1824 |
| $\rho_{yz}$ | 0.9155 | 0.3555 | -0.1885 | -0.3877 | 0.7267 |
| $\mu_{200}$ | 231.0662 | 3.6537 | 0.02656 | 0.02656 | 31.3002 |
| $\mu_{020}$ | 37188.3 | 37188.3 | 28389.71 | 0.06989 | 0.06154 |
| $\mu_{002}$ | 19394.85 | 231.0662 | 0.06989 | 28389.71 | 44759585 |
| $\mu_{110}$ | 1658.645 | 150.8594 | -10.64885 | -0.008122 | 0.2722 |
| $\mu_{011}$ | 2329.049 | 2329.049 | 3.23249 | 3.23249 | 302.7662 |
| $\mu_{101}$ | 24587.07 | 10.3304 | -0.008122 | -10.64885 | 27202.27 |
| $\mu_{400}$ | 133611.3 | 139.951 | 0.0022 | 0.0022046 | 12395.06 |
| $\mu_{120}$ | 215483.2 | 27721.26 | 1232.206 | -0.002430 | 0.31154 |
| $\mu_{030}$ | 3625817 | 9625804 | 1199329 | 0.0048496 | 0.07685 |

**Table 1.1: Description of the populations**

*source: Applied Linear Statistical ModelsPg 1348 set 1
**source: Applied Linear Statistical Models Pg 1348 set 3
#source: Applied Linear Statistical Models Pg 1350 set3
#*source: Applied Linear Statistical Models Pg 1352 set 9

## 2. Existing Estimators in Two- Phase Sampling with No Information

Existing estimators in two phase sampling with no information, single and two auxiliary variables are given in sections 2.1 and 2.2 respectively.

## 2.1 Single Auxiliary Variable

Sukhatme (1962) used auxiliary variable in his ratio and product type estimator for two phase sampling. Given estimator has much wider applicability as this estimator does not require any population information except its size. The estimator is as:

$$t_{Sr(1)} = \frac{\bar{y}_2}{\bar{x}_2}\bar{x}_1 \tag{2.1.1}$$

and

$$t_{Sp(1)} = \frac{\bar{y}_2}{\bar{x}_1}\bar{x}_2 \tag{2.1.2}$$

the mean square errors of (2.1.1) and (2.1.2) are

$$MSE\left(t_{Sr(1)}\right) = \bar{Y}^2\left[\theta_2 C_y^2 + (\theta_2 - \theta_1)\left(C_x^2 - 2C_x C_y \rho_{xy}\right)\right] \tag{2.1.3}$$

$$MSE\left(t_{Sp(1)}\right) = \bar{Y}^2\left[\theta_2 C_y^2 + (\theta_2 - \theta_1)\left(C_x^2 + 2C_x C_y \rho_{xy}\right)\right] \tag{2.1.4}$$

Singh and Vishwakarma (2007) suggested exponential ratio and product type estimator in two- phase sampling using single auxiliary variable as

$$t_{SVr(1)} = \bar{y}_2 exp\left(\frac{\bar{x}_1 - \bar{x}_2}{\bar{x}_1 + \bar{x}_2}\right) \tag{2.1.5}$$

and

$$t_{SVp(1)} = \bar{y}_2 exp\left(\frac{\bar{x}_2 - \bar{x}_1}{\bar{x}_1 + \bar{x}_2}\right) \tag{2.1.6}$$

mean square errors of (2.1.5) and (2.1.6) are

$$MSE\left(t_{SVr(1)}\right) = \mu_{200}\left[\theta_2 + \frac{C_x(\theta_2 - \theta_1)}{4C_y}\left\{\frac{C_x}{C_y} - 4\rho_{xy}\right\}\right] \tag{2.1.7}$$

$$MSE\left(t_{SVp(1)}\right) = \mu_{200}\left[\theta_2 + \frac{C_x(\theta_2 - \theta_1)}{4C_y}\left\{\frac{C_x}{C_y} + 4\rho_{xy}\right\}\right] \tag{2.1.8}$$

## 2.2. Two Auxiliary Variables

Hanif et al. (2009) suggested ratio and product type estimators in multivariate multiple auxiliary variables for population means

$$t_{Hr(2)} = \bar{y}_2 \frac{\bar{x}_1}{\bar{x}_2}\frac{\bar{z}_1}{\bar{z}_2} \tag{2.2.1}$$

and

$$t_{Hp(2)} = \bar{y}_2 \frac{\bar{x}_2}{\bar{x}_1}\frac{\bar{z}_2}{\bar{z}_1} \tag{2.2.2}$$

the mean square error of (2.2.1) and (2.2.2) are as

$$MSE\left(t_{Hr(2)}\right) = \theta_2 \bar{Y}^2 C_y^2 + (\theta_2 - \theta_1)\bar{Y}^2\left(C_x^2 + C_z^2 - 2C_y C_x \rho_{xy} - 2C_y C_z \rho_{yz} + 2C_x C_z \rho_{xz}\right) \tag{2.2.3}$$

$$MSE\left(t_{Hp(2)}\right) = \theta_2 \bar{Y}^2 C_y^2 + (\theta_2 - \theta_1)\bar{Y}^2\left(C_x^2 + C_z^2 + 2C_y C_x \rho_{xy} + 2C_y C_z \rho_{yz} + 2C_x C_z \rho_{xz}\right) \tag{2.2.4}$$

## 3. Proposed Estimator in Two- Phase Sampling with No Information

We propose exponential ratio type estimator of population mean in two-phase sampling with no information using single and two auxiliary variables.

### 3.1 Single Auxiliary Variable

The new exponential ratio type estimator of population mean in two-phase sampling with one auxiliary variable using no information case is proposed as

$$t_{HK(1)} = \bar{y}_2 exp\left[\alpha\left\{\frac{\bar{x}_1^{\frac{1}{h}} - \bar{x}_2^{\frac{1}{h}}}{\bar{x}_1^{\frac{1}{h}} + (a-1)\bar{x}_2^{\frac{1}{h}}}\right\}\right] \tag{3.1.1}$$

where $\alpha$, $a$ and $h$ are real constants.

Expanding (3.1.1) in terms of $e_0$ $and$ $e_1$we get

$$t_{HK(1)} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\alpha\left\{\frac{\bar{X}^{\frac{1}{h}}\left(1 + e_{1(1)}\right)^{\frac{1}{h}} - \bar{X}^{\frac{1}{h}}\left(1 + e_{1(2)}\right)^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}}\left(1 + e_{1(1)}\right)^{\frac{1}{h}} + (a-1)\bar{X}^{\frac{1}{h}}\left(1 + e_{1(2)}\right)^{\frac{1}{h}}}\right\}\right]$$

$$t_{HK(1)} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\frac{\alpha}{ah}\left(\frac{e_{1(1)}}{h} - \frac{e_{1(2)}}{h}\right)\left(1 + \frac{e_{1(1)}}{ah} + \frac{e_{1(2)}}{h} - \frac{e_{1(2)}}{ah}\right)^{-1}\right] \tag{3.1.2}$$

Expanding $\left(1 + \frac{e_{1(1)}}{ah} + \frac{e_{1(2)}}{h} - \frac{e_{1(2)}}{ah}\right)^{-1}$ up to order one, we get

$$t_{HK(1)} = \bar{Y}\left(1 + e_{o(2)}\right)exp\left[\frac{\alpha}{ah}\left(\frac{e_{1(1)}}{h} - \frac{e_{1(2)}}{h}\right)\right]$$

Expanding the exponential function up to order one and taking expectation we have

$$Bias\left(t_{HK(1)}\right) = \bar{Y}\left[-\frac{w^2\theta_1\mu_{020}}{2h^4\bar{X}^2}(\theta_1 - \theta_2) + \frac{w\mu_{110}}{h^2\bar{X}\bar{Y}}(\theta_1 - \theta_2)\right]$$

where$w = \frac{\alpha}{a}$ and $w_{opt} = \frac{h^2\bar{X}\mu_{110}}{\bar{Y}\mu_{020}}$ so that

$$Bias\left(t_{HK(1)}\right)_{opt} = \frac{\mu_{110}^2}{2h^2\bar{Y}^2}(\theta_1 - \theta_2) \tag{3.1.3}$$

For mean square error expand (3.1.2)up to of order one andtake expectations so that

$$MSE\left(t_{HK(1)}\right) = \bar{Y}^2\left[\frac{\theta_2\mu_{200}}{\bar{Y}^2} + \frac{w^2\theta_1\mu_{020}}{h^4\bar{X}^2} + \frac{w^2\theta_2\mu_{020}}{h^4\bar{X}^2} + \frac{2w\theta_1\mu_{110}}{h^2\bar{X}\bar{Y}} - \frac{2w\theta_2\mu_{110}}{h^2\bar{X}\bar{Y}} - \frac{2w^2\theta_1\mu_{020}}{h^4\bar{X}^2}\right] \tag{3.1.4}$$

equation(3.1.4) is minimum with $w = \frac{h\bar{X}\mu_{110}}{\bar{Y}\mu_{020}}$ so that

$$MSE\left(t_{HK(1)}\right)_{min} = \theta_2\mu_{200} - (\theta_1 - \theta_2)\frac{\mu_{110}^2}{h^2\mu_{020}} + (\theta_1 - \theta_2)\frac{2\mu_{110}^2}{h\mu_{020}} \tag{3.1.5}$$

### 3.2. Two Auxiliary Variables

In case of two auxiliary variables we propose the estimator as

$$t_{HK(2)} = \bar{y}_2 exp\left[\alpha\left\{\frac{\bar{x}_1^{\frac{1}{h}}-\bar{x}_2^{\frac{1}{h}}}{\bar{x}_1^{\frac{1}{h}}+(a-1)\bar{x}_2^{\frac{1}{h}}}\right\}\right] exp\left[\beta\left\{\frac{\bar{z}_1^{\frac{1}{h}}-\bar{z}_2^{\frac{1}{h}}}{\bar{z}_1^{\frac{1}{h}}+(b-1)\bar{z}_2^{\frac{1}{h}}}\right\}\right] \tag{3.2.1}$$

where $\beta$ and $b$ are real constants.

$$t_{HK(2)} = \bar{Y}\left(1+e_{o(2)}\right)exp\left[\alpha\left\{\frac{\bar{X}^{\frac{1}{h}}\left(1+e_{1(1)}\right)^{\frac{1}{h}}-\bar{X}^{\frac{1}{h}}\left(1+e_{1(2)}\right)^{\frac{1}{h}}}{\bar{X}^{\frac{1}{h}}\left(1+e_{1(1)}\right)^{\frac{1}{h}}+(a-1)\bar{X}^{\frac{1}{h}}\left(1+e_{1(2)}\right)^{\frac{1}{h}}}\right\}\right].$$

$$exp\left[\beta\left\{\frac{\bar{Z}^{\frac{1}{h}}\left(1+e_{2(1)}\right)^{\frac{1}{h}}-\bar{Z}^{\frac{1}{h}}\left(1+e_{2(2)}\right)^{\frac{1}{h}}}{\bar{Z}^{\frac{1}{h}}\left(1+e_{2(1)}\right)^{\frac{1}{h}}+(b-1)\bar{Z}^{\frac{1}{h}}\left(1+e_{2(2)}\right)^{\frac{1}{h}}}\right\}\right]$$

$$t_{HK(2)} = \bar{Y}\left(1+e_{o(2)}\right)exp\left[\frac{\alpha}{a}\left(\frac{e_{1(1)}}{h}-\frac{e_{1(2)}}{h}\right)\left(1+\frac{e_{1(1)}}{ah}+\frac{e_{1(2)}}{h}-\frac{e_{1(2)}}{ah}\right)^{-1}\right].$$

$$exp\left[\frac{\beta}{b}\left(\frac{e_{2(1)}}{h}-\frac{e_{2(2)}}{h}\right)\left(1+\frac{e_{2(1)}}{bh}+\frac{e_{2(2)}}{h}-\frac{e_{2(1)}}{bh}\right)^{-1}\right]$$

The first order approximation gives

$$t_{HK(2)} = \bar{Y}\left(1+e_{o(2)}\right)exp\left[w\left(\frac{e_{1(1)}}{h}-\frac{e_{1(2)}}{h}\right)\right]exp\left[d\left(\frac{e_{2(1)}}{h}-\frac{e_{2(2)}}{h}\right)\right] \tag{3.2.2}$$

Expanding the exponential of (3.2.2) upto of order one we get

$$t_{HKr(2)} = \bar{Y}\left(1+e_{o(2)}\right)\left[1+w\left(\frac{e_{1(1)}}{h}-\frac{e_{1(2)}}{h}\right)+w^2\left(\frac{e_{1(1)}}{h}-\frac{e_{1(2)}}{h}\right)^2\right]\left[1 + d\left(\frac{e_{2(1)}}{h}-\frac{e_{2(2)}}{h}\right)+d^2\left(\frac{e_{2(1)}}{h}-\frac{e_{2(2)}}{h}\right)^2\right]$$

On simplifying and retaining terms of order one and then adding and subtracting $\bar{Y}$we get

$$t_{HK(2)} - \bar{Y} = \bar{Y}\left[\frac{we_{1(1)}}{h}-\frac{we_{1(2)}}{h}+\frac{w^2}{2}\frac{e_{1(1)}^2}{h^2}+\frac{de_{o(2)}e_{2(1)}}{h}+e_{o(2)}-\frac{we_{o(2)}e_{1(2)}}{h} + \frac{w^2}{2}\frac{e_{1(2)}^2}{h^2}-\frac{w^2e_{1(1)}e_{1(2)}}{h^2}+\frac{de_{2(1)}}{h}-\frac{de_{2(2)}}{h}-\frac{de_{o(2)}e_{2(2)}}{h} + \frac{we_{o(2)}e_{1(1)}}{h}+\frac{d^2}{2}\frac{e_{2(1)}^2}{h^2}+\frac{d^2}{2}\frac{e_{2(2)}^2}{h^2}-\frac{d^2e_{1(1)}e_{2(2)}}{h^2}\right]$$

Applying expectation we have

$$Bias(t_{HK(2)}) = \bar{Y}\left[-\frac{w^2\mu_{o2o}(\theta_1-\theta_2)}{2h^2\bar{X}^2} + \frac{d^2\mu_{oo2}(\theta_1-\theta_2)}{2h^2\bar{Z}^2} - \frac{d\mu_{1o1}(\theta_1-\theta_2)}{h\bar{Y}\bar{Z}} - \frac{w\mu_{11o}}{h\bar{X}\bar{Y}}(\theta_2-\theta_1)\right] \tag{3.2.3}$$

(3.2.3) is minimum with $w = \frac{h\bar{X}\mu_{11o}}{\bar{Y}\mu_{o2o}}$ and $d = \frac{h\bar{Z}\mu_{1o1}}{\bar{Y}\mu_{oo2}}$ so that

$$Bias(t_{HK(2)})_{min} = \bar{Y}\left[-\frac{\mu_{11o}^2}{2\bar{Y}^2\mu_{o2o}}(\theta_2-\theta_1) - \frac{\mu_{1o1}^2}{2\bar{Y}^2\mu_{o2o}}(\theta_2-\theta_1)\right] \tag{3.2.4}$$

For mean square error consider (3.2.2) and expand the exponential function upto order one so that

$$t_{HK(2)} = \bar{Y}(1+e_{o(2)})\left(1+\frac{we_{1(1)}}{h}-\frac{we_{1(2)}}{h}\right)\left(1+\frac{de_{2(1)}}{h}-\frac{de_{2(2)}}{h}\right)$$

and

$$t_{HK(2)} - \bar{Y} = \bar{Y}\left[e_{o(2)} + \frac{we_{1(1)}}{h} - \frac{we_{1(2)}}{h} + \frac{de_{2(1)}}{h} - \frac{de_{2(2)}}{h}\right]$$

squaring and applying expectation we get

$$MSE(t_{HK(2)}) = \bar{Y}^2\left[\theta_2\frac{\mu_{2oo}}{\bar{Y}^2} + \frac{w^2\theta_1\mu_{o2o}}{h^2\bar{X}^2} + \frac{w^2\theta_2\mu_{o2o}}{h^2\bar{X}^2} + \frac{d^2\theta_1\mu_{oo2}}{h^2\bar{Z}^2} + \frac{d^2\theta_2\mu_{oo2}}{h^2\bar{Z}^2} + \frac{2wd\mu_{o11}(\theta_2-\theta_1)}{h^2\bar{X}\bar{Z}} + \frac{2w\mu_{11o}(\theta_1-\theta_2)}{h\bar{X}\bar{Y}} + \frac{2d\mu_{1o1}(\theta_1-\theta_2)}{h\bar{Y}\bar{Z}} + \frac{2wd\theta_1\mu_{o11}(\theta_1-\theta_2)}{h^2\bar{X}\bar{Z}}\right] \tag{3.2.5}$$

equation(3.2.5) is minimum with

$$w = \frac{h\bar{X}\mu_{11o}(\theta_2-\theta_1)}{\bar{Y}\mu_{o2o}(\theta_1+\theta_2)} \quad d = \frac{h\bar{Z}\mu_{1o1}(\theta_2-\theta_1)}{\bar{Y}\mu_{oo2}(\theta_1+\theta_2)}$$

$$MSE(t_{HK(2)})_{min} = \theta_2\mu_{2oo} - \frac{\mu_{11o}^2(\theta_2-\theta_1)^2}{\mu_{o2o}(\theta_1+\theta_2)} - \frac{\mu_{1o1}^2(\theta_2-\theta_1)^2}{\mu_{oo2}(\theta_1+\theta_2)} \tag{3.2.6}$$

## 4. Numerical Study

In this section numerical comparison between the proposed estimators and existing estimators using two phase sampling with single and two auxiliary variables of no information has been made for each of the population described in Table (1.1).The sample of size $n_1$ at the first phase is taken equal to 60% of the total sample size n and sample of size $n_2$ at phase two is taken 67% of $n_1$. Tables 4.1 – 4.5 show the MSE's of the estimators discussed in sections 2 and 3 for each population with no information on single and two auxiliary variables.

The study of the Table 4.1 - 4.5 shows that in case of single auxiliary variable with no information the proposed estimator (3.1.1) is highly efficient than the existing estimators. In case of two auxiliary variables with no information case,it is observed that the proposed estimator (3.2.1) is highly efficient than the existing estimators for each of the population under study.

Khan (2016) has shown that in two-phase sampling with no information single auxiliary case the estimator $t_{HK(1)}$ is relatively more efficient than the estimators of Sukhatme (1962) and Singh and Vishwakarma (2007). In case of two auxiliary variables the estimator $t_{HK(2)}$ is more efficient than the estimator given by Hanif et al (2009). Here the values of the constants may be $\alpha = 1$,$1.0 \leq h \leq 2.0$, $5.0 \leq a \leq 6.0$, $1.0 \leq b \leq 1.5$ and $5.0 \leq \beta \leq 6.0$.

| | | **No Information** | | | | |
|---|---|---|---|---|---|---|
| | | **Single auxiliary variable** | | | **Two auxiliary variables** | |
| $n_1$ | $n_2$ | $MSE(t_{SUr(1)})$ | $MSE(t_{SVr(1)})$ | $MSE(t_{HK(1)})_{min}$ | $MSE(t_{Hr(2)})$ | $MSE(t_{HK(2)})_{min}$ |
| 12 | 8 | 39.0102 | 21.5767 | 21.4360 | 142.0896 | 24.2182 |
| 18 | 12 | 25.3252 | 13.7029 | 13.6091 | 94.0448 | 15.3788 |
| 30 | 20 | 14.3772 | 7.4038 | 7.3475 | 55.6089 | 8.2908 |

**Table 4.1: Mean square error and relative efficiency in case of no information for Population 1**

| | | **No Information** | | | | |
|---|---|---|---|---|---|---|
| | | **Single auxiliary variable** | | | **Two auxiliary variables** | |
| $n_1$ | $n_2$ | $MSE(t_{SUr(1)})$ | $MSE(t_{SVr(1)})$ | $MSE(t_{HK(1)})_{min}$ | $MSE(t_{Hr(2)})$ | $MSE(t_{HK(2)})_{min}$ |
| 12 | 8 | 2.2118 | 0.7510 | 0.4053 | 4.1606 | 0.4146 |
| 18 | 12 | 1.4637 | 0.4899 | 0.2594 | 2.7629 | 0.2653 |
| 30 | 20 | 0.8653 | 0.2810 | 0.1427 | 1.6448 | 0.1458 |

**Table 4.2: Mean square error and relative efficiency in case of no information for Population 2**

| | | **No Information** | | | | |
|---|---|---|---|---|---|---|
| | | **Single auxiliary variable** | | | **Two auxiliary variables** | |
| $n_1$ | $n_2$ | $MSE(t_{SUp(1)})$ | $MSE(t_{SVp(1)})$ | $MSE(t_{HK(1)})_{min}$ | $MSE(t_{Hp(2)})$ | $MSE(t_{HK(2)})_{min}$ |
| 6 | 4 | 0.0802 | 0.0218 | 0.0057 | 0.0862 | 0.0058 |
| 12 | 8 | 0.0397 | 0.0105 | 0.0025 | 0.0427 | 0.0025 |
| 18 | 12 | 0.0282 | 0.0068 | 0.0014 | 0.0282 | 0.0014 |

**Table 4.3: Mean square error and relative efficiency in case of no information for Population 3**

| | | **No Information** | | | | |
|---|---|---|---|---|---|---|
| | | **Single auxiliary variable** | | | **Two auxiliary variables** | |
| $n_1$ | $n_2$ | $MSE(t_{SUp(1)})$ | $MSE(t_{SVp(1)})$ | $MSE(t_{HK(1)})_{min}$ | $MSE(t_{Hp(2)})$ | $MSE(t_{HK(2)})_{min}$ |
| 6 | 4 | 0.0092 | 0.0064 | 0.0058 | 0.0862 | 0.0058 |
| 12 | 8 | 0.0042 | 0.0028 | 0.0026 | 0.0427 | 0.0025 |
| 18 | 12 | 0.0026 | 0.0016 | 0.0015 | 0.0282 | 0.0014 |

**Table 4.4: Mean square error and relative efficiency in case of no information for Population 4**

| | | **No Information** | | | | |
|---|---|---|---|---|---|---|
| | | **Single auxiliary variable** | | | **Two auxiliary variables** | |
| $n_1$ | $n_2$ | $MSE(t_{SUr(1)})$ | $MSE(t_{SVr(1)})$ | $MSE(t_{HK(1)})_{min}$ | $MSE(t_{Hr(2)})$ | $MSE(t_{HK(2)})_{min}$ |
| 60 | 40 | 3.8527 | 1.4268 | 0.7353 | 4.8438 | 0.7113 |
| 180 | 120 | 1.2577 | 0.4491 | 0.2186 | 1.5881 | 0.2091 |
| 300 | 200 | 0.7388 | 0.2536 | 0.1153 | 0.9370 | 0.1083 |

**Table 4.5: Mean square error and relative efficiency in case of no information for Population 5**

## 5. Conclusion

The proposed estimator $t_{HK(1)}$in two-phase sampling with single auxiliary variable is more efficient than the estimators $t_{SUr(1)}$and $t_{SVr(1)}$for all types of populations under study. In two-phase sampling with two auxiliary variables, the proposed estimator $t_{HK(2)}$is more efficient than the estimator $t_{Hr(2)}$. Two phase sampling should increase the efficiency of the estimator. The present study reveals that in case where the correlation between study variable and one of the auxiliary variables in the population is small, the two-phase sampling increases the efficiency of the proposed estimator. But in case where the correlations between the study variable and both the auxiliary variables are large, the two- phase sampling does not increase the efficiency of the proposed estimator.

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