Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online): 2229-5666
Vol. 10, Issue 2 (2017): 105-115

CUSUM CONTROL CHARTS FOR TRUNCATED NORMAL DISTRIBUTION UNDER INSPECTION ERROR

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Received June 03, 2017
Modified November 23, 2017
Accepted December 09, 2017

Abstract

The cumulative sum (CUSUM) chart is commonly used for detecting small or moderate shifts in the fraction of defective manufactured items. However, its construction relies on the error-free inspection assumption, which can seldom be met in practice. This article has studied the effect of inspection error on the parameter of the CUSUM chart and discussed the construction of CUSUM chart in the presence of truncated normal distribution with inspection error. Expression for the parameter of the CUSUM chart has also been derived.

Key Words: Truncated Normal Distribution, Inspection Error, Average Run Length, Lead Distance and Angle of Mask.

1. Introduction

The normal distribution is a common model of randomness. Unlike the uniform distribution, it proposes a most probable value which is also the mean, while other values occur with a probability that decreases in a regular way with distance from the mean. This behavior is mathematically very satisfying, and has an easily observed correspondence with many physical processes. One drawback of the normal distribution, however, is that it supplies a positive probability density to every value in the range ($-\infty$; $+\infty$), although the actual probability of an extreme event will be very low. In many cases, it is desired to use the normal distribution to describe the random variation of a quantity that, for physical reasons, must be strictly positive. A mathematically defensible way to preserve the main features of the normal distribution while avoiding extreme values involves the truncated normal distribution, in which the range of definition is made finite at one or both ends of the interval. Nelson (1990) stresses the distinction between truncation and censoring. Chang (1990) and Schneider (1986) contain some basic results concerning the properties of the truncated normal distribution.

Inspections are performed in virtually every production system. Their purpose is to verify that the production operations were carried out properly and that the production output meets the expectations of the customer. Inspection operations can often be seen as procedures used to classify a product unit into two or more classes according to its conformance to a given set of requirements. Raz and Thomas (1990) pointed the fact that inspection operations can be unreliable, resulting in classification errors. These errors have both cost and quality implications. One common approach to deal with inspection errors is to introduce redundancy into the inspection procedure by carrying out multiple inspections – either identical or unique – on each product unit and to base the classification decision on their combined results. Jamkhaneh, E. B. ; Gildeh , B. S. and Yari, G. (2011) showed the effect of Inspection error on single sampling plans with fuzzy parameters. Gal1, I. B.; Herer, Y. T. and Raz, T. (2002) used inspection errors for Self-correcting inspection procedure.

In statistical quality control the cumulative sum control charts (CUSUM Charts) have found importance as a parallel process control technique to the well-known Shewhart control charts. An alternative method for testing statistical hypothesis parallel to Neyman's theory is the popular sequential probability ratio test (SPRT) due to Wald (1942). Page (1954, 1961) suggested the cumulative sum charts which are more effective than Shewhart control charts in detecting small and moderate size departures from a simple acceptable quality level. Johnsom and Leone (1962) considered mathematical procedure for construction of CUSUM control chart for Poisson variable using the relationship between Wald's Sequential Probability Ratio Test (SPRT) and CUSUM on the assumption that the probability of the second kind of error is small. They used this relationship to construct CUSUM charts for the mean and standard deviation of a normal distribution. Singh et al. (2002) showed the effect of inspection error on CUSUM chart for proportion. Singh and Sayyed (2001) constructed CUSUM chart for Poisson variable under inspection error. Kantam and Rao (2006) studied the cumulative sum control chart for log-logistic distributional Statistics. Ryu et al. (2010) used ARL based performance measure and proposed a method to optimally design a CUSUM chart based on expected weighted run length. Grigg and Spiegelhalter (2008) developed an empirical approximation to the null steady –state distribution of CUSUM. Chakraborty and Khurshid (2011) constructed one-sided cumulative sum (CUSUM) control chart for the zero-truncated binomial distribution. Sankle et al. (2012) introduced CUSUM chart for truncated normal distribution under measurement error. Sayyed and Singh (2015) pointed out the effect of inspection error on CSCC chart for Binomial parameter.

2. Truncated CUSUM Chart Parameters

Let X be the true value of the variable which is distributed as

$$f(x, \mu, \sigma_p) = \begin{cases} \frac{c(\mu,\sigma_p)}{\sigma_p\sqrt{2\pi}} \exp\left[\frac{-1}{2}\left(\frac{x-\mu}{\sigma_p}\right)^2\right] & ; as \;\; a \le x \le b \\ 0 & ; otherwise \end{cases} \tag{2.1}$$

Where

$$c(\mu, \sigma_p) = \frac{1}{\Phi\left(\frac{b-\mu}{\sigma_p}\right) - \Phi\left(\frac{a-\mu}{\sigma_p}\right)} \tag{2.2}$$

and a and b are the points of intersection with

$$\Phi(t) = \int_{-\infty}^{t} (2\pi)^{\frac{-1}{2}} \exp\left(\frac{-1}{2} y^2\right) dy \tag{2.3}$$

If $x_1, x_2, \ldots, x_m$ are m independent random variables whose pdf is given by (2.1). The likelihood ratio of the hypothesis $H_0 : \mu = \mu_0$ **vs** $H_1 : \mu = \mu_1$ is given by

$$\frac{f(x_1,x_2 \ldots x_m \mid \mu_1,\sigma_p)}{f(x_1,x_2 \ldots x_m \mid \mu_0,\sigma_p)} = m \log R + \frac{(\mu_1-\mu_0)}{\sigma_p^2}\left\{\sum_{i=1}^{m} x_i - \frac{m(\mu_1+\mu_0)}{2}\right\} \quad (2.4)$$

Where

$$R = \frac{c(\mu_1, \sigma_p)}{c(\mu_0, \sigma_p)}$$

The continuation region of SPRT discriminating between the two hypotheses, H_0: $\mu = \mu_0$ v_s H_1: $\mu = \mu_1$ is given by

$$\frac{\sigma_p^2}{(\mu_1-\mu_0)} \log\left(\frac{\beta}{1-\alpha}\right) + m\left[\frac{(\mu_1+\mu_0)}{2} - \frac{\sigma_p^2}{(\mu_1-\mu_0)}\log R\right] \le x_i \le \frac{\sigma_p^2}{(\mu_1-\mu_0)} \log\left(\frac{1-\beta}{\alpha}\right) + m\left[\frac{(\mu_1+\mu_0)}{2} - \frac{\sigma_p^2}{(\mu_1-\mu_0)}\log R\right] \quad (2.5)$$

Foe every small value of β above equation reduces to

$$x_i \le \frac{-\sigma_p^2}{(\mu_1-\mu_0)} \log \alpha + m\left[\frac{(\mu_1+\mu_0)}{2} - \frac{\sigma_p^2}{(\mu_1-\mu_0)}\log R\right] \quad (2.6)$$

A narrow V-mask will detect change more quickly but it will give more frequent false alarms. On the other hand, we would reduce the frequency of false alarms by widening the angle of mask, but the average run length for real changes would be increased.

Under the inspection error the parameter of the mask, namely the angle of the mask Φ and the lead distance d are given by

$$\tan\phi = \frac{1}{(\mu_1-\mu_0)}\left[\frac{(\mu_1^2-\mu_0^2)}{2} - \sigma_p^2 \log R\right] \quad (2.7)$$

$$R = \frac{c(\mu_1, \sigma_p)}{c(\mu_0, \sigma_p)}$$

and

$$d = \frac{-\sigma_p^2 \log\alpha}{\frac{(\mu_1^2-\mu_0^2)}{2} - \sigma_p^2 \log R} \quad (2.8)$$

For ARL we consider the situation where the true mean θ has shifted from θ_0 to θ_1. Foe every small value of α_1 the ARL that is expected number of observation before the change from θ_0 to θ_1 is detected (Mood and Grabill 1963) is approximately

$$\left(-\sigma_p^2 \log\alpha\right) E_{\mu_1}^{-1} \quad (2.9)$$

Where

$$E_{\mu 1} = E\left[\log \frac{f(x \mid \mu_1,\sigma_p)}{f(x \mid \mu_0,\sigma_p)} \mid \mu_1, \sigma_p\right] = \sigma_p^2 \log R + \frac{\left(\mu_1^2-\mu_0^2\right)^2}{2} + \frac{(\mu_1-\mu_0)c(\mu_1,\sigma_p)}{\sqrt{2\pi}}\left[\exp\left\{\frac{-1}{2}\left(\frac{a-\mu_1}{\sigma_p}\right)^2\right\} - \exp\left\{\frac{-1}{2}\left(\frac{b-\mu_1}{\sigma_p}\right)^2\right\}\right] \quad (2.10)$$

3. Truncated CUSUM Chart Parameters under Inspection Error

Let r be the probability that nonconformity is correctly noted by the inspector. We note that r is assumed to be a constant over different values of μ. In one study reported by Harris and Chaney (1969) r varies from 0.58 to 0.80 where as μ varies from 0.0025 to 0.16. However, this variability in r also included the variability among different inspectors, since different inspectors were used for different values of μ.Therefore, the assumption of a constant r does not seen too be seriously. Especially noting the large spread in values of μ. If μ_f is the average no. of false alarms per part and μ is the true average no. of true non conformities per part and μ' is the average no. per part observed by the inspector, then

$$\mu' = r\mu + \mu_f$$

Every effort should be made to estimate both type of errors i. e. r close to one and $\mu_f$close to zero. If μ is the target value then the control limits and ARL under inspection error is obtained by the following formulae

$$\phi = tan^{-1}\left\{\frac{1}{(\mu_1'-\mu_0')}\left[\frac{(\mu_1'^2-\mu_0'^2)}{2} - \sigma_p^2 \log R\right]\right\} \quad (3.1)$$

$$R = \frac{c(\mu_1' , \sigma_p)}{c(\mu_0' , \sigma_p)}$$

and

$$d = \frac{-\sigma_p^2 \, log\alpha}{\frac{(\mu_1'^2-\mu_0'^2)}{2} - \sigma_p^2 \log R} \quad (3.2)$$

ARL is

$$\left(-\sigma_p^2 \, log\alpha\right) E_{\mu'_1}^{-1} \quad (3.3)$$

Where

$$E_{\mu 1} = E\left[log \frac{f(x|\,\mu'_1, \sigma_p)}{f\left(x|\,\mu'_0, \sigma_p\right)} \,|\, \mu'_1, \sigma_p\right] =$$

$$\sigma_p^2 \, logR + \frac{\left(\mu_1'^2-\mu_0'^2\right)^2}{2} + \frac{(\mu'_1-\mu'_0)c\,(\mu'_1, \sigma_p)}{\sqrt{2\pi}}\left[exp\left\{\frac{-1}{2}\left(\frac{a-\mu'_1}{\sigma_p}\right)^2\right\} - exp\left\{\frac{-1}{2}\left(\frac{b-\mu'_1}{\sigma_p}\right)^2\right\}\right] \quad (3.4)$$

4. Tabulation of Results and Conclusions

To illustrate the effect on the dimensions of the mask on truncated CSCC under the inspection error the values of the angle of the mask, lead distance and ARL for the mean chart have been tabulated in Table -1 , Table-2 and Table-3 for the different values of inspection rates (1,0),(1,2) (0.8,2) and (0.8,0) and the truncation points ± 2.5 , ± 2.25 , ± 2 , (0,2.5) and (0,2).The values of α are taken to be 0.05,0.025,0.01,0.005 and 0.001.

It is seen from Table-1 that the angle of the mask increases as the range of truncation increased and is bigger in case of one sided truncation. Angle of the mask increases as the type I error decreases and the angle of the mask decreases as the ratio μ_0/μ_1 decreases. As the error term increase the angle of the mask also increases. From Table-2 and Table -3 it is evident that for fixed α the values of d and ARL decreases as the difference $\mu_1 - \mu_0$ increases that's mean as the error rates increases the value of lead distance and ARL increases. Lead distance and ARL is higher in case when average no. of false alarms per part is null as compare error free case.

Truncation Point	**μ_0**	**μ_1**	**A**	**(r,μ_f)**			
				(1,0)	**(1,2)**	**(0.8,2)**	**(0.8,0)**
(-2.5,2.5)	0.4	0.43	0.05	20.507	53.377	53.123	16.725
	0.4	0.46	0.025	21.165	53.42	53.162	17.285
	0.4	0.49	0.01	21.815	53.462	53.199	17.84
	0.4	0.52	0.005	22.457	53.503	53.236	18.39
	0.4	0.55	0.001	23.089	53.543	53.273	18.936
	0.3	0.43	0.05	18.251	53.226	52.99	14.825
	0.3	0.46	0.025	18.933	53.272	53.03	15.398
	0.3	0.49	0.01	19.607	53.316	53.069	15.966
	0.3	0.52	0.005	20.273	53.359	53.108	16.53
	0.3	0.55	0.001	20.931	53.402	53.146	17.09
	0.2	0.43	0.05	15.913	53.064	52.848	12.88
	0.2	0.46	0.025	16.616	53.112	52.89	13.464
	0.2	0.49	0.01	17.312	53.158	52.931	14.045
	0.2	0.52	0.005	18.001	53.204	52.972	14.622
	0.2	0.55	0.001	18.682	53.249	53.011	15.194
(-2.25,2.25)	0.4	0.43	0.05	19.27	51.956	51.663	15.699
	0.4	0.43	0.025	19.893	52.005	51.707	16.227
	0.4	0.43	0.01	20.508	52.054	51.75	16.75
	0.4	0.43	0.005	21.116	52.101	51.793	17.269
	0.4	0.43	0.001	21.716	52.148	51.835	17.784
	0.3	0.43	0.05	17.138	51.782	51.51	13.909
	0.3	0.43	0.025	17.781	51.834	51.556	14.448
	0.3	0.43	0.01	18.417	51.885	51.601	14.983
	0.3	0.43	0.005	19.046	51.935	51.646	15.514
	0.3	0.43	0.001	19.668	51.984	51.689	16.041
	0.2	0.43	0.05	15.913	53.064	52.848	12.88
	0.2	0.43	0.025	16.616	53.112	52.89	13.464
	0.2	0.43	0.01	17.312	53.158	52.931	14.045
	0.2	0.43	0.005	18.001	53.204	52.972	14.622
	0.2	0.43	0.001	18.682	53.249	53.011	15.194

	0.4	0.43	0.05	17.56	49.882	49.541	14.28
	0.4	0.43	0.025	18.135	49.94	49.592	14.764
	0.4	0.43	0.01	18.702	49.997	49.642	15.244
	0.4	0.43	0.005	19.264	50.052	49.692	15.72
(-2,2)	0.4	0.43	0.001	19.818	50.107	49.74	16.193
	0.3	0.43	0.05	15.599	49.679	49.364	12.641
	0.3	0.43	0.025	16.19	49.739	49.417	13.134
	0.3	0.43	0.01	16.774	49.799	49.469	13.624
	0.3	0.43	0.005	17.353	49.857	49.521	14.11
	0.3	0.43	0.001	17.925	49.915	49.571	14.592
	0.2	0.43	0.05	13.575	49.462	49.178	10.97
	0.2	0.43	0.025	14.181	49.525	49.233	11.47
	0.2	0.43	0.01	14.781	49.588	49.287	11.968
	0.2	0.43	0.005	15.375	49.649	49.34	12.463
	0.2	0.43	0.001	15.964	49.709	49.392	12.954
	0.4	0.43	0.05	41.431	53.598	53.396	40.591
	0.4	0.43	0.025	41.582	53.632	53.426	40.714
	0.4	0.43	0.01	41.732	53.666	53.456	40.836
	0.4	0.43	0.005	41.882	53.7	53.485	40.958
(0,2.5)	0.4	0.43	0.001	42.032	53.732	53.514	41.081
	0.3	0.43	0.05	40.929	53.477	53.292	40.184
	0.3	0.43	0.025	41.082	53.513	53.323	40.308
	0.3	0.43	0.01	41.235	53.549	53.354	40.431
	0.3	0.43	0.005	41.387	53.584	53.384	40.555
	0.3	0.43	0.001	41.539	53.618	53.414	40.678
	0.2	0.43	0.05	40.425	53.35	53.183	39.776
	0.2	0.43	0.025	40.581	53.388	53.215	39.901
	0.2	0.43	0.01	40.736	53.425	53.247	40.026
	0.2	0.43	0.005	40.89	53.462	53.279	40.151
	0.2	0.43	0.001	41.045	53.497	53.31	40.276
	0.4	0.43	0.05	39.069	50.411	50.168	38.36
	0.4	0.43	0.025	39.196	50.454	50.204	38.463
	0.4	0.43	0.01	39.322	50.495	50.24	38.566
	0.4	0.43	0.005	39.449	50.537	50.275	38.669
	0.4	0.43	0.001	39.575	50.577	50.31	38.772
	0.3	0.43	0.05	38.644	50.266	50.045	38.015
(0,2)	0.3	0.43	0.025	38.773	50.309	50.082	38.119
	0.3	0.43	0.01	38.901	50.353	50.118	38.224
	0.3	0.43	0.005	39.029	50.395	50.154	38.327
	0.3	0.43	0.001	39.157	50.437	50.19	38.431
	0.2	0.43	0.05	38.217	50.114	49.917	37.67
	0.2	0.43	0.025	38.347	50.159	49.955	37.775
	0.2	0.43	0.01	38.477	50.203	49.993	37.88
	0.2	0.43	0.005	38.607	50.247	50.03	37.985
	0.2	0.43	0.001	38.736	50.291	50.066	38.089

Table 1: Angle of the Mask for different Truncation Points (a, b) under Inspection Error

Truncation Point	μ_0	μ_1	A	(r,μ_f)			
				(1,0)	(1,2)	(0.8,2)	(0.8,0)
(-2.5,2.5)	0.4	0.43	0.05	266.6536	59.77988	76.20867	415.1839
	0.4	0.43	0.025	158.5682	36.68101	46.78416	246.8266
	0.4	0.43	0.01	127.6243	30.42625	38.82464	198.6025
	0.4	0.43	0.005	106.6252	26.16803	33.40603	165.8742
	0.4	0.43	0.001	107.7966	27.20513	34.74504	167.6426
	0.3	0.43	0.05	69.82636	13.95879	17.76402	108.798
	0.3	0.43	0.025	67.15389	13.91682	17.71969	104.6058
	0.3	0.43	0.01	67.97114	14.58001	18.57332	105.8484
	0.3	0.43	0.005	65.12261	14.43791	18.40113	101.3816
	0.3	0.43	0.001	72.14407	16.5094	21.05102	112.2763
	0.2	0.43	0.05	45.66678	7.988213	10.14615	71.19073
	0.2	0.43	0.025	47.51938	8.670071	11.01824	74.05877
	0.2	0.43	0.01	50.91451	9.669414	12.29478	79.32726
	0.2	0.43	0.005	50.91778	10.0465	12.78082	79.30815
	0.2	0.43	0.001	58.31954	11.9342	15.18985	90.80764
(-2.25,2.25)	0.4	0.43	0.05	285.3539	66.1661	84.27072	443.9238
	0.4	0.43	0.025	169.7189	40.6062	51.74065	263.9435
	0.4	0.43	0.01	136.625	33.68751	42.94379	212.4018
	0.4	0.43	0.005	114.1678	28.97743	36.95533	177.4229
	0.4	0.43	0.001	115.4463	30.13054	38.44179	179.3394
	0.3	0.43	0.05	74.68752	15.44133	19.63388	116.2929
	0.3	0.43	0.025	71.84197	15.39749	19.58769	111.8251
	0.3	0.43	0.01	72.73019	16.13392	20.53423	113.1677
	0.3	0.43	0.005	69.69629	15.97927	20.34671	108.4064
	0.3	0.43	0.001	77.22725	18.27485	23.28001	120.0728
	0.2	0.43	0.05	48.82903	8.831432	11.20858	76.07776
	0.2	0.43	0.025	50.81914	9.586938	12.1738	79.15208
	0.2	0.43	0.01	54.46051	10.6938	13.58619	84.7936
	0.2	0.43	0.005	54.47507	11.11272	14.12533	84.78449
	0.2	0.43	0.001	62.40723	13.20296	16.79018	97.09168
(-2,2)	0.4	0.43	0.05	315.3421	74.76902	95.17468	490.2811
	0.4	0.43	0.025	187.5787	45.89018	58.44033	291.5306
	0.4	0.43	0.01	151.0225	38.07474	48.50837	234.6225
	0.4	0.43	0.005	126.2166	32.75426	41.74731	196.0027
	0.4	0.43	0.001	127.649	34.06071	43.42997	198.1394
	0.3	0.43	0.05	82.50852	17.44322	22.168	128.4085
	0.3	0.43	0.025	79.37515	17.39542	22.11774	123.4855
	0.3	0.43	0.01	80.36739	18.22919	23.1885	124.9793
	0.3	0.43	0.005	77.02589	18.05618	22.97866	119.7324
	0.3	0.43	0.001	85.36162	20.65206	26.29356	132.6308
	0.2	0.43	0.05	53.9289	9.972877	12.65148	83.99016
	0.2	0.43	0.025	56.1341	10.82715	13.74216	87.3916
	0.2	0.43	0.01	60.1645	12.07843	15.33786	93.62877
	0.2	0.43	0.005	60.18921	12.55284	15.94788	93.62757
	0.2	0.43	0.001	68.96381	14.91541	18.95818	107.2291
	0.4	0.43	0.05	109.2372	58.72769	74.61625	141.1904
	0.4	0.43	0.025	66.85158	36.05498	45.83023	86.50813
	0.4	0.43	0.01	55.30331	29.92262	38.05205	71.64838
	0.4	0.43	0.005	47.43341	25.74786	32.75719	61.52458

(0,2.5)	0.4	0.43	0.001	49.17604	26.78129	34.08627	63.85953
	0.3	0.43	0.05	25.71866	13.68571	17.36054	33.11203
	0.3	0.43	0.025	25.57514	13.65255	17.3267	32.96676
	0.3	0.43	0.01	26.72329	14.31124	18.17106	34.48803
	0.3	0.43	0.005	26.39171	14.17946	18.0119	34.10085
	0.3	0.43	0.001	30.09565	16.22237	20.61608	38.93316
	0.2	0.43	0.05	14.83004	7.814176	9.895346	19.01884
	0.2	0.43	0.025	16.05533	8.486551	10.75215	20.61538
	0.2	0.43	0.01	17.85978	9.470523	12.00464	22.96032
	0.2	0.43	0.005	18.50744	9.845649	12.48607	23.82194
	0.2	0.43	0.001	21.92589	11.70225	14.84747	28.25635
(0,2)	0.4	0.43	0.05	119.9329	72.61986	92.01326	154.1311
	0.4	0.43	0.025	73.47457	44.60494	56.53887	94.5148
	0.4	0.43	0.01	60.84681	37.03557	46.96224	78.34467
	0.4	0.43	0.005	52.24395	31.88296	40.44368	67.33092
	0.4	0.43	0.001	54.22177	33.17749	42.1012	69.94505
	0.3	0.43	0.05	28.14066	16.89527	21.37799	36.05034
	0.3	0.43	0.025	28.01324	16.86262	21.34533	35.92179
	0.3	0.43	0.01	29.30208	17.6847	22.39486	37.61065
	0.3	0.43	0.005	28.96962	17.53017	22.20783	37.21956
	0.3	0.43	0.001	33.07117	20.06519	25.42897	42.52966
	0.2	0.43	0.05	16.17311	9.62999	12.16732	20.65274
	0.2	0.43	0.025	17.52795	10.46394	13.22659	22.40493
	0.2	0.43	0.01	19.51877	11.68303	14.77366	24.97419
	0.2	0.43	0.005	20.24842	12.15178	15.37265	25.93312
	0.2	0.43	0.001	24.01453	14.45024	18.2876	30.78652

Table 2: Lead Distance for Mean for different Truncation Points (a, b) under Inspection Error

Truncation Point	**μ_0**	**μ_1**	**A**	**(r,μ_f)**			
				(1,0)	**(1,2)**	**(0.8,2)**	**(0.8,0)**
(-2.5,2.5)	0.4	0.43	0.05	227.3715	74.32166	89.33626	353.4115
	0.4	0.43	0.025	135.0868	46.77862	55.69083	210.0127
	0.4	0.43	0.01	108.6142	39.89782	46.98156	168.8946
	0.4	0.43	0.005	90.6397	35.37885	41.14275	140.9782
	0.4	0.43	0.001	91.52072	38.03856	43.60812	142.3852
	0.3	0.43	0.05	59.27161	17.36265	20.73138	92.29594
	0.3	0.43	0.025	56.95104	17.74212	20.98659	88.70091
	0.3	0.43	0.01	57.58454	19.09556	22.34748	89.70806
	0.3	0.43	0.005	55.10777	19.47696	22.51791	85.87086
	0.3	0.43	0.001	60.97213	23.00741	26.23221	95.03408
	0.2	0.43	0.05	38.60103	9.815693	11.70945	60.19526
	0.2	0.43	0.025	40.12991	10.90372	12.89413	62.59277
	0.2	0.43	0.01	42.95221	12.47321	14.60404	67.01049
	0.2	0.43	0.005	42.90506	13.32523	15.42574	66.95382
	0.2	0.43	0.001	49.07925	16.32	18.6499	76.60944

(-2.25,2.25)	0.4	0.43	0.05	220.0917	90.9534	106.1048	340.7783
	0.4	0.43	0.025	130.8094	58.76688	67.20312	202.5719
	0.4	0.43	0.01	105.2116	51.65833	57.6926	162.9618
	0.4	0.43	0.005	87.82933	47.4435	51.50694	136.067
	0.4	0.43	0.001	88.71113	53.16066	55.77482	137.4646
	0.3	0.43	0.05	57.17864	22.18172	25.26383	88.77029
	0.3	0.43	0.025	54.95904	23.34486	26.01327	85.33974
	0.3	0.43	0.01	55.58863	25.99504	28.22277	86.33499
	0.3	0.43	0.005	53.21427	27.58649	29.03117	82.66611
	0.3	0.43	0.001	58.89454	34.14734	34.6034	91.51279
	0.2	0.43	0.05	37.12099	12.98738	14.57611	57.75856
	0.2	0.43	0.025	38.60401	14.89047	16.33754	60.07763
	0.2	0.43	0.01	41.33187	17.66608	18.86754	64.33685
	0.2	0.43	0.005	41.29864	19.69112	20.36141	64.30057
	0.2	0.43	0.001	47.25478	25.35412	25.20967	73.59334
(-2,2)	0.4	0.43	0.05	210.9755	120.5302	133.6799	324.9536
	0.4	0.43	0.025	125.483	82.23594	87.28764	193.2788
	0.4	0.43	0.01	101.0004	77.43804	77.62711	155.5762
	0.4	0.43	0.005	84.37412	77.79865	72.23059	129.9747
	0.4	0.43	0.001	85.28155	98.51488	82.15345	131.3841
	0.3	0.43	0.05	54.594	32.68012	33.75035	84.39413
	0.3	0.43	0.025	52.51119	37.04827	36.04585	81.17892
	0.3	0.43	0.01	53.14907	45.44169	40.81305	82.17172
	0.3	0.43	0.005	50.91324	54.99152	44.15583	78.72325
	0.3	0.43	0.001	56.38543	82.34286	55.92171	87.19548
	0.2	0.43	0.05	35.30914	21.33984	20.61224	54.75368
	0.2	0.43	0.025	36.74408	26.93968	24.10464	56.98356
	0.2	0.43	0.01	39.36609	36.35395	29.26012	61.05657
	0.2	0.43	0.005	39.35959	48.6391	33.5092	61.05474
	0.2	0.43	0.001	45.06442	83.27512	44.58763	69.91516
(0,2.5)	0.4	0.43	0.05	89.22489	72.15262	86.55645	117.1087
	0.4	0.43	0.025	54.60567	45.4714	54.02195	71.65148
	0.4	0.43	0.01	45.18806	38.82982	45.62583	59.27439
	0.4	0.43	0.005	38.78224	34.47121	39.99961	50.858
	0.4	0.43	0.001	40.24404	37.10262	42.44156	52.74497
	0.3	0.43	0.05	21.31471	16.90035	20.13361	27.89392
	0.3	0.43	0.025	21.17769	17.29214	20.40569	27.71357
	0.3	0.43	0.01	22.11717	18.63431	21.75389	28.93976
	0.3	0.43	0.005	21.83879	19.02892	21.94416	28.57035
	0.3	0.43	0.001	24.90703	22.50341	25.59129	32.5764
	0.2	0.43	0.05	12.5056	9.583975	11.40174	16.30362
	0.2	0.43	0.025	13.51438	10.66035	12.57021	17.62259
	0.2	0.43	0.01	15.01181	12.21024	14.25355	19.57777
	0.2	0.43	0.005	15.53965	13.06015	15.07235	20.26717
	0.2	0.43	0.001	18.39671	16.01403	18.24237	23.99283
	0.4	0.43	0.05	99.34995	113.295	125.9201	128.969
	0.4	0.43	0.025	60.87125	77.15463	82.19727	78.97012
	0.4	0.43	0.01	50.43205	72.43375	73.05549	65.38103
	0.4	0.43	0.005	43.33526	72.42014	67.90704	56.13636
	0.4	0.43	0.001	45.02503	90.9772	77.11419	58.27458
	0.3	0.43	0.05	23.49688	30.69831	31.83385	30.5066

(0,2)	0.3	0.43	0.025	23.36993	34.70443	33.9848	30.33152
	0.3	0.43	0.01	24.43256	42.37395	38.44899	31.69702
	0.3	0.43	0.005	24.15141	50.89178	41.5448	31.31596
	0.3	0.43	0.001	27.57564	75.16943	52.51211	35.73432
	0.2	0.43	0.05	13.66965	20.07529	19.4913	17.72521
	0.2	0.43	0.025	14.78604	25.25523	22.78387	19.17206
	0.2	0.43	0.01	16.44001	33.88116	27.63388	21.31362
	0.2	0.43	0.005	17.0347	44.85624	31.60359	22.07934
	0.2	0.43	0.001	20.18693	75.15477	41.963	26.15642

Table 3: ARL for mean for different Truncation points (a, b) and Inspection Error

Since angle of mask is grater for one sided truncation as compare to symmetrical truncation it shows that it will detect change more quickly and give more frequently false alarms as compare to ARL.

This paper indicated that both types of error seriously affect the control limits d and ARL from that which would be obtained under error from inspection. This statement is especially true when one is setting up CSCC for truncated data.

Acknowledgement

The author is grateful to the Chief Editor and Reviewers cogent comments that have eventually improved the shape of the paper.

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