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OBSERVING THE RENEWAL PERIOD IN AN ORGANIZATION THROUGH MATHEMATICAL MODEL

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Abstract

Renewal process is observed where an organization is subject to a sequence of K different shocks, and every organization needs to know the renewal time, before the organization reaches the maximum threshold level. The renewal function is derived using the Laplace transform of the Exponentiated Gamma Distribution, and its expected time is obtained explicitly with mathematical figures. This model will be suitable and alternate for analyzing the renewal time in an organization.

Key Words: Distribution, Expected, Organization, Renewal, Shock.

1. Introduction

Renewal defines the act of extending the time when something is effective or valid, renewal theory/model in the study of stochastic process forms an important constituent and is mostly used to examine the manpower models through recruitment. Fellar (1968), proposed significant contributions to renewal theory giving the paper a lead. Smith (1958), studied two dimensions of uncertainties in compound renewal process, i.e., the inter-arrival time of the event and magnitude are modeled as random variables. A comprehensive analysis of renewal models of manpower systems was delivered by Bartholomew (1982). Renewal process are applied to predict the loss of manpower of an institution in order to establish the institute to meet its demand for manpower under existing conditions (Sirvanci, 1984). In this study the authors observed the need of renewal to be done in an organization through mathematical shock model approach.

Literature review shows that the shock models in mathematical development are used for modelling certain reliablity systems set in a stochastic environment. Shanthikumar and Sumita (1984), studied a general shock model, where a system is exposed to a series of random shocks created by a renewal sequence. Recently, the expected time of employers in an institution was calculated by several researchers using mathematical model in stochastic process. Sathiyamorthy (1980), applied shock through cumulative damage model to explore the correlated inter-arrival times and also specified how renewal theory aids to decrease the cumulative damage. Parthasarathy et al. (2010), used Exponentiated Exponential Distribution to find out the grading system in an organization through stochastic modeling and revealed how the renewal process

$V_k(t)$ is been calculated in the organization. Pandiyan et al. (2010), discussed the use of mean and variance time to recruit manpower in an organization and found the importance of using renewal process in an organization. Through Three Parameter Generalized Exponential Distribution, Kannadasan et al. (2013), observed the recruitment time in an institution through the renewal theory; $V_k(t) = F_k(t) - F_{k+1}(t)$. Kalaivani et al. (2015), found the threshold time in an organization through Modified Weibull Distribution, deriving the expected time of employee recruitment. Ahmed Al Kuwaiti et.al (2016), considered Additive Weibull Distribution to predict the exit time of employee in an organization and observed the expected through shock model. These articles motivated the researchers to observe the renewal period in an organization through mathematical modeling using Exponentiated Gamma Distribution.

Further, some properties of a new family of distribution, namely Exponentiated Gamma Distribution which is an alternative to Gamma and Weibull distribution was studied by Gupta et al. (1998). The density functions of the Exponentiated Gamma Distribution can take different shapes, in this model the shape parameter $\lambda = 1$ is fixed. Detail study about the Exponentiated Gamma Distribution and fixing the shape parameter, was done by Shawky and Bakoban (2008 and 2009).

2. Description of Exponentiated Gamma Distribution

Assuming that variables are independent of one another and the marginal distribution for each variable is the same, they are said to be independent and identically distributed (iid). The probability density function (p.d.f) of the Exponentiated Gamma Distribution, after fixing the shape parameter $\lambda = 1$, we get the below equation (1)

$$f(x) = \theta x e^{-x}\{1 - e^{-x}(x+1)\}^{\theta-1} \qquad \theta > 0 \qquad (1)$$

The Cumulative Distribution Function (c.d.f) is

$$F(x) = \{1 - e^{-x}(x+1)\}^{\theta} \theta > 0$$

The corresponding Survival Function is

$$\bar{H} = xe^{-x} + e^{-x}$$

2.1 Probability of the renewal period

Consider, $[X_i: i = 1,2,\ldots]$ random variables that are non-negative, iid the sequence X_i is called a renewal process. Expecting each of the random variable X_i, a finite renewal period with, mean μ. The renewal period is been observed when, X_i a continuous random variable representing the amount of damage/depletion produced to the organization on the i^{th} occasion, $i = 1,2,3,\ldots k$ and $X_i's$ are i.i.d and $X_i = X$ for all i. In general, the threshold Y follows Exponentiated Gamma Distribution with parameter θ. It can be presented as;

$$P(X_i < Y) = \int_0^\infty g(.)[xe^{-x} + e^{-x}]$$

$$= \int_0^\infty g_k(.)\, xe^{-x}dx + \int_0^\infty g_k(.)e^{-x}dx \qquad (2)$$

where: $g(.)$: The p.d.f of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

2.2 Accessing the Renewal Process

Renewal process is determined by means of $f(.)$; the p.d.f of X_i. Related with a random variable $V_k(t)$ which denotes the number of renewals in the time interval $(0, t]$, $V_k(t)$ is named as renewal counting process (Parzen, 1962). We need to know the survival time of the total organization, by survival function which provides the probability that the cumulative threshold which fail only after time t, i.e. $S(t) = P[T > t]$, is been observed here.

$$= \sum_{k=0}^{\infty} \mathrm{P}\,\{\text{there are exact k decisions in } (0,\mathrm{t}] \\ * \mathrm{P}\,(\text{the total cumulative threshold } (0,\mathrm{t}]\}$$

It is also identifiedfrom renewal process that

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(X_i < Y)\,; where\ V_k(t) = F_k(t) - F_{k+1}(t)$$

where: $F_k(t)$: Probability that there are exact *'k'* policies decisions in $(0, \mathrm{t}]$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ P(X_i < Y)$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\, g_k^*(\lambda) - g_k^*(\lambda)$$

where: $g * (.)$: Laplace transform of $g(.)$

$g_k^*(.)$: Laplace transform of $\mathrm{g_k}(.)$

2.3 Renewal period of the Life Time: $\mathbf{L(T)}$

Data which measure, 'the length of time' until the occurrence of an event are named lifetime or survival data; i.e., $P(T < t) = L(t) = 1 - S(t)$. Taking Laplace transformation of $L(T)$, we get

$$= 1 - \left\{1 - [1 - g^*(\lambda)] f^*(s) \sum_{k=1}^{\infty} [f^*(s) g^*(\lambda)]^{k-1} - 1 \right. \\ \left. + [1 - g^*(\lambda) f^*(s)] \sum_{k=1}^{\infty} [f^*(s) g^*(\lambda)]^{k-1} \right\}$$

Using, convolution theorem for Laplace-Stieltjes transforms; $F_0(t) = 1$ and on simplification, it can be shown as,

$$= 1 + \frac{[1 - g^*(\lambda)] f^*(s)}{[1 - g^*(\lambda) f^*(s)]} - \frac{[1 - g^*(\lambda)] f^*(s)}{[1 - g^*(\lambda) f^*(s)]} \tag{3}$$

A random variable with inter arrival time $'c'$ which follows exponential distribution with parameter $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3) we get,

$$L^*(s) = 1 + \frac{[1 - g^*(\lambda)] \frac{c}{c+s}}{\left[1 - g^*(\lambda) \frac{c}{c+s}\right]} - \frac{[1 - g^*(\lambda)] \frac{c}{c+s}}{\left[1 - g^*(\lambda) \frac{c}{c+s}\right]}$$

$$= 1 + \frac{[1 - g^*(\lambda)] c}{[c + s - g^*(\lambda) c]} - \frac{[1 - g^*(\lambda)] c}{[c + s - g^*(\lambda) c]}$$

2.4 Expected Renewal time

When the organizations observe a shock they immediately try to replace by another suitable manpower by renewal. The expected value of the number of replacement observed up to sometime 't' is given in the below equation (5);

$$E(T) = -\frac{d}{ds} l^*(s) given s = 0$$

$$= \frac{[1 - g^*(\lambda)]c}{[1 - g^*(\lambda)]c^2} - \frac{[1 - g^*(\lambda)]c}{[1 - g^*(\lambda)]c^2} \quad (4)$$

where: $g^*(.) \sim \frac{\mu}{\mu+\lambda}$

on simplification of the above equation (4), we get

$$E(T) = \frac{\mu + \lambda}{c\lambda} - \frac{(\mu + \lambda)^2}{c[(\mu + \lambda)^2 + \mu]} \quad (5)$$

3. Conclusion

Figure 1 shows that when the parameter μ is constant and by increasing the inter-arrival time $'c'$, i.e., $c = 1,2, \ldots 10$, the time to renewal in an organization is been observed. The value of the expected time $E(T)$ is found to be decreasing, in all the case of the parameter value $\mu = 0.5, 1, 1.5, 2,2.5$. We observe, when there is a shock at the initial time $c = 1\ to\ 2$, the damage is more in all the constant (parameter μ) case, this is where the organization needs to access immediately and to check the renewal as the damage is more.

In Figure 2, the parameter λ is constant to check how the behavior of the expected time of the renewal acts, the inter-arrival time $'c'$ is been increased. The expected time $E(T)$ had the same behavior as found in Figure 1, which is a decreasing case for all the parameter value $\lambda = 0.5, 1, 1.5, 2,2.5$.

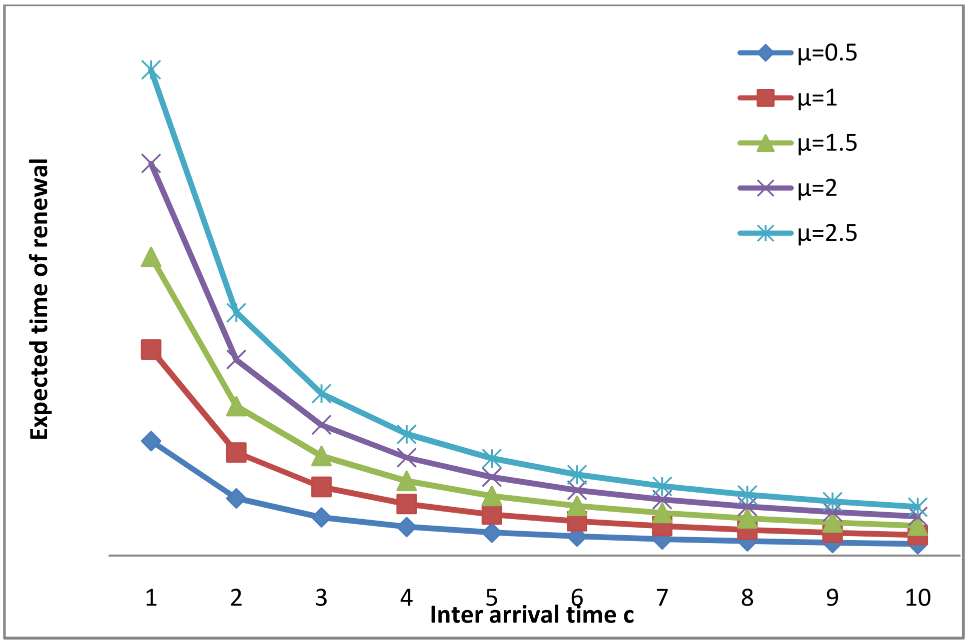

Figure 1

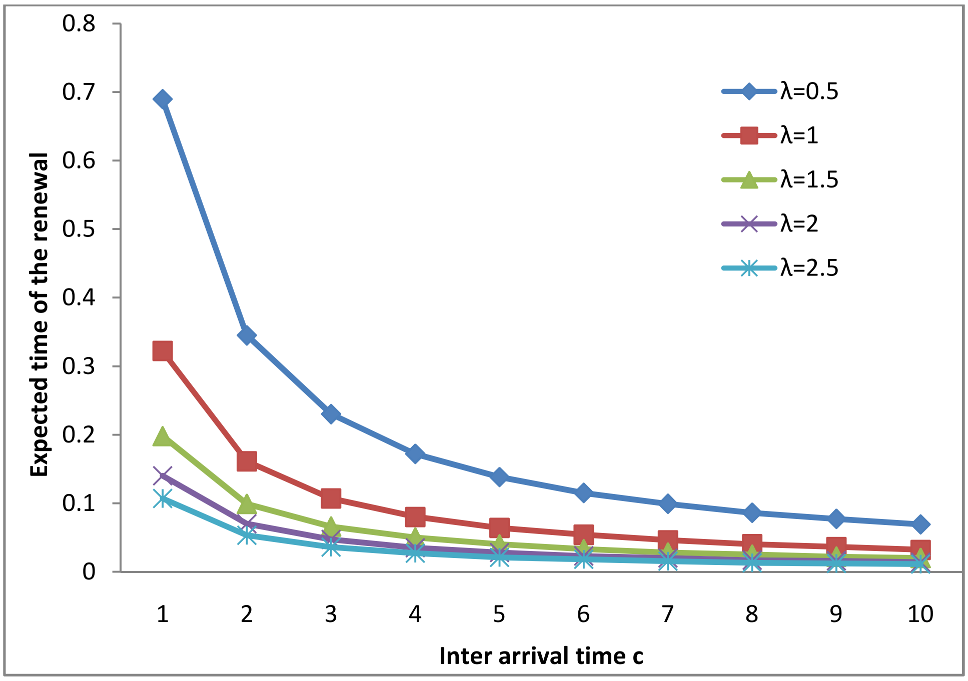

Figure 2

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