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COMPLEMENTARY COMPOUND LINDLEY POWER SERIES DISTRIBUTION WITH APPLICATION

***Adil Rashid, Zahoor Ahmad and T. R. Jan**
P.G Department of Statistics, University of Kashmir, Srinagar
E Mail: zahoor151@gmail.com; *adilstat@gmail.com; drtrjan@gmail.com
*Corresponding Author

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Abstract

The present paper will attempt at exploring the methods through which a new class of complementary compound lifetime distributions will be introduced. It is pertinent to mention that this new family of continuous lifetime distributions obtained after rigorous examination, study and analysis, will be called Complementary Compound Lindley Power Series (CCLPS) distribution. There are special cases of several lifetime distributions in the proposed class of distributions that are very flexible to accommodate different types of data sets. Needless to mention, the probability density function and hazard rate can take up different forms such as increasing, decreasing and upside down bathtub shapes which are reflected through graphs.

Key Words: Lindley Distribution, Power Series Distribution, Compounding and Order Statistics.

1. Introduction

Compounding of probability distributions as an area of research has been dealt since 1920. The pioneering work was done by Green Wood and Yule (1920) who established a relationship between Poisson distribution and a negative binomial distribution through compounding mechanism by treating the parameter in Poisson distribution as gamma variate. The compounding mechanism was exploited by a good corpus of researchers who significantly contributed in the construction of new distributionsboth in discrete as well as continuous versions.

Many examples can be cited in this regard. First, the efforts of Adil and Jan (2013, 2014a, 2014b, 2014c) materialized when they obtained many discrete compound distributions by compounding negative binomial, Consul and Geeta distribution with generalized beta distribution. Second, Zamani and Ismail (2010) created a new compound distribution by mixing negative binomial distribution with that of Lindley distribution. They exhibited with clinical precision that the resulting distributions can be employed efficiently to model the count data analysis. Third, Adil and Jan (2015a, 2016a) proposed two new competitive count data models, obtained by compounding Consul with Kumaraswamy distribution and by compounding negative binomial with Kumaraswamy distribution. The noteworthy part of these landmark researches is the fact that the application and potentiality of these new findings was rationalized in the sciences of mathematics and statistics. What was done the real world data sets from traffic, genetics and ecology was modelled on the above mentioned research.

Lifetime data modelling has evolved as a favorite area of research during the last decade with many continuous probability models like 'Exponential', 'Gamma' , 'Weibull' recurrently being used. But there's a red herring as these probability models cannot be efficiently employed to model the lifetime data that is bathtub in configuration besides having unimodal failure rates. To do away with this issue, the attention of researchers has shifted to compounding mechanism which assists in creating suitable, flexible and alternative models with the purpose of fixing with mathematical perfection the lifetime data of various types.

Let us take a system with N components, where N, the number of components is itself a discrete random variable with domain N =1,2,…, The lifetime of i^{th} component in this ste up can be portrayed by any one of the suitable lifetime distributions viz; exponential, gamma, Weibull, Lindley etc. And N, the discrete r.v may have any of the ascribed distribution such as geometric, zero truncated Poisson or power series distribution in general. The lifetime for this kind of system in series combination will be denoted by $Y = \min\{X_i\}_{i=1}^{N}$ and the lifetime for parallel combination will be denoted by $Y = \max\{X_i\}_{i=1}^{N}$.

Keeping the pressing demand in consideration as is evident in the above paragraphs; therefore, a substantial amount of work was done to construct new probability models to predict the lifetime of these kinds of systems. For, instance Adamadis and Loukas (1998), Kus (2007), Tahmasbi and Rezaei (2008) and Morais and Baretto Souza (2011) constructed several continuous lifetime distributions by compounding continuous distributions like Weibull, exponential, with classical discrete distributions. Similarly, Adil and Jan (2016b) constructed a new family of lifetime distribution by compounding Lindley distribution with that of power series distribution. Adil and Jan (2016c) proposed a generalized version of Lindley power series family of compound lifetime distributions which proved to be multifaceted, resourceful and all purpose because it not only generalizes most of the lifetime distributions but it also has some attractive desirable properties in terms of hazard rate and density functions.

The continuous lifetime distributions brought under discussion in the above paragraphs so far, however, addressed only the series aspect of the system. Therefore, try cannot be put to use in the parallel system. The present paper will discuss and analyze the parallel aspect of the problem with the assumption that there is no information as to which factor was responsible for the component failure. It is important to point out that only the maximum lifetime value is observed among all risks instead of minimum lifetime value among all risks as in Adil and Jan (2016b) and Morais & Barreto-Souza (2011). The present paper will place before the new distribution which is a counterpart of the Lindley power series distribution for which we have designated it as the complementary compound Lindley power series (CCLPS) distribution.

2. Construction of the Class

Suppose the lifetime of N independent and identically distributed random variables in parallel component system be defined by Lindley distribution with density function given by

$$g(x;\zeta)=\frac{\zeta^2}{\zeta+1}(1+x)e^{-\zeta x}, x>0, \zeta>0 \quad (1)$$

Here we are considering a problem in which the sample size i.e. N is itself a random variable following discrete power series distribution, truncated at zero with probability function

$$P(N=n)=\frac{c_n\eta^n}{\psi(\eta)}, \quad n=1,2,...$$

where a_n depends only on n, $\psi(\eta)=\sum_{n=1}^{\infty}c_n\eta^n$ and $\eta>0$ is such that $\psi(\eta)$ is finite.

It may be noted here, that a parallel system with N components ceases to function if the n^{th} component fails to perform. The below table, contains most of the classical discrete probability distributions for specific parametric function setting.

Distribution	c_n	$\psi(\eta)$	$\psi'(\eta)$	$\psi''(\eta)$	$\psi^{-1}(\eta)$	η
Poisson	$n!^{-1}$	$e^{\eta}-1$	e^{η}	e^{η}	$\log(\eta+1)$	$\eta\in(0,\infty)$
Logarithmic	n^{-1}	$-\log(1-\eta)$	$(1-\eta)^{-1}$	$(1-\eta)^{-2}$	$1-e^{-\eta}$	$\eta\in(0,1)$
Geometric	1	$\eta(1-\eta)^{-1}$	$(1-\eta)^{-2}$	$2(1-\eta)^{-3}$	$\eta(\eta+1)^{-1}$	$\eta\in(0,1)$
Binomial	$\binom{m}{n}$	$(\eta+1)^m-1$	$\eta(\eta+1)^{m-1}$	$\frac{m(m-1)}{(\eta+1)^{2-m}}$	$(\eta-1)^{1/m}-1$	$\eta\in(0,\infty)$

Table 1: Useful quantities of Some Power Series Distribution

Let $X_{(n)}=\max\{X_i\}_{i=1}^{N}$. The cumulative distribution function of $X_{(n)}$ Conditional on $N=n$ is given by

$$G_{X_{(n)}|N=n}(x)=(G(x))^n=\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^n$$

and

$$P\left(X_{(n)}\le x, N=n\right)=\frac{c_n\eta^n}{\psi(\eta)}\left(1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right)^n, \quad x>0,\ n\ge 1.$$

The distribution function proposed distribution is defined by the marginal cdf of $X_{(n)}$

$$F\left(x\right)=\sum_{n=1}^{\infty}\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n}\frac{c_{n}\,\eta^{n}}{\psi\left(\eta\right)}$$

$$=\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)},\ x>0 \qquad (2)$$

For convenience let us keep the symbolic notation for the proposed distribution with parameters ζ and η as $X\sim CCLPS\ (\zeta,\eta)$.

3. Density, Survival and Hazard Rate Function

The probability density function CCLPS distribution can be obtained by differentiating (2) both sides with respect to x .

$$f\left(x\right)=\frac{dF\left(x\right)}{dx}$$

$$f(x)=\frac{\zeta^{2}}{\zeta+1}\eta e^{-\zeta x}(1+x)\frac{\psi'\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)} \qquad (3)$$

$$S\left(x\right)=1-F(x)$$

$$S\left(x\right)=1-\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi\left(\eta\right)},\quad x>0$$

and the hazard function is

$$h(x)=\frac{f(x)}{S(x)}$$

$$h(x)=\frac{\frac{\zeta^{2}}{\zeta+1}\eta e^{-\zeta x}(1+x)\psi'\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi\left(\eta\right)-\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}\ ,x>0$$

Proposition 1: For $\eta\rightarrow0^{+}$, the proposed distribution tends to Lindley distribution.

Proof: From the cumulative distribution function of CCLPS distribution we have

$$\lim_{\eta\to 0^+} F(x) = \lim_{\eta\to 0^+} \frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}, \quad x>0$$

since

$$\psi(\eta) = \sum_{n=1}^{\infty} c_n \eta^n$$

$$\lim_{\eta\to 0^+} F(x) = \lim_{\eta\to 0^+} \frac{\sum_{n=1}^{\infty} c_n \left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]^n}{\sum_{n=1}^{\infty} c_n \eta^n}$$

On the implication of well known L' Hospital's rule, we get

$$\lim_{\eta\to 0^+} F(x) = \frac{c_1\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\} + \sum_{n=2}^{\infty} c_n n\eta^{n-1}\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}^n}{c_1 + \sum_{n=2}^{\infty} c_n n \eta^{n-1}}$$

$$= 1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}.$$

Hence the result.

Proposition 2: Show that by expressing the densities of proposed distribution as infinite linear combination of densities of n^{th} order statistics of Lindley distribution, the properties of CCLPS can be obtained from of Lindley distribution

$$f(x) = \sum_{n=1}^{\infty} P(N=n) g_n(x,n)$$

where $g_n(x,n) = \max(X_1, X_2, ..., X_n)$ is the n^{th} order statistics of Lindley distribution

Proof: Using the fact that

$$\psi'(\eta) = \sum_{n=1}^{\infty} n c_n \eta^{n-1}$$

The pdf of CCLPS distribution takes the form after using the above result

$$f(x)=\frac{\zeta^2}{\zeta+1}(1+x)e^{-\zeta x}\sum_{n=1}^{\infty}n\frac{c_n\eta^n}{\psi(\eta)}\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1}$$

$$f(x)=\sum_{n=1}^{\infty}P(N=n)g_n(x,n) \qquad (4)$$

where $g_n(x,n)=n\frac{\zeta^2}{\zeta+1}(1+x)e^{-\zeta x}\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1}$ is the n^{th} order statistics of Lindley distribution. Hence it is obvious and clear from the proof that properties of proposed distribution can be obtained from the Lindley distribution.

4. Moment Generating Function

The moment generating function of CCLPS is

$$M_X(t)=\sum_{n=1}^{\infty}P(N=n)M_{X_{(n)}}(t)$$

where $M_{X_{(n)}}(t)$ is the moment generating function of n^{th} order statistics of Lindley distribution

$$M_{X_{(n)}}(t)=\frac{\zeta^2}{\zeta+1}\int_0^{\infty}e^{tx}n\ (1+x)e^{-\zeta x}\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1}dx$$

$$=\frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^j\int_0^{\infty}(1+x)\left[\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1-j}e^{(t-\zeta)x}\,dx$$

$$=\frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^j\int_0^{\infty}(1+x)\left(1+\frac{\zeta x}{\zeta+1}\right)^{n-1-j}e^{-\zeta x(n-j-1)}e^{(t-\zeta)x}\,dx$$

$$=\frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\zeta+1}\right)^{n-j-k-1}\int_0^{\infty}x^{n-j-k-1}(1+x)e^{-((n-j)\zeta-t)x}\,dx$$

$$=\frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\zeta+1}\right)^{n-j-k-1}\left\{\frac{\Gamma(n-j-k)}{((n-j)\zeta-t)^{n-j-k}}+\frac{\Gamma(n-j-k+1)}{((n-j)\zeta-t)^{n-j-k+1}}\right\}$$

and it follows that

$$M_X(t) = \frac{\zeta^2}{\zeta+1}\sum_{n=1}^{\infty}\frac{nc_n\eta^n}{\psi(\eta)}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\lambda+1}\right)^{n-j-k-1}$$
$$\left\{\frac{\Gamma(n-j-k)}{\left((n-j)\zeta-t\right)^{n-j-k}}+\frac{\Gamma(n-j-k+1)}{\left((n-j)\zeta-t\right)^{n-j-k+1}}\right\}$$

using the proposition (4), the k^{th} moment of CCLPS distribution about origin is

$$E(X^k) = \sum_{n=1}^{\infty}P(N=n)\int_0^{\infty}x^k g_n(x)\,dx$$
$$E(X^k) = \sum_{n=1}^{\infty}P(N=n)E\left(X_{(n)}{}^k\right)$$

Now consider

$$E\left(X_{(n)}{}^k\right) = \int_0^{\infty}x^r n\frac{\zeta^2}{\zeta+1}(1+x)e^{-\lambda x}\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1}dx$$
$$= \frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^j\int_0^{\infty}x^r(1+x)\left[\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]^{n-1-j}e^{-\zeta x}\,dx$$
$$= \frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\binom{n-1}{j}(-1)^j\int_0^{\infty}(1+x)x^r\left(1+\frac{\zeta x}{\zeta+1}\right)^{n-1-j}e^{-\zeta x(n-j-1)-\zeta x}\,dx$$
$$= \frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\zeta+1}\right)^{n-j-k-1}$$
$$\int_0^{\infty}x^{n-j-k+r-1}(1+x)e^{-(n\zeta-j\zeta)x}\,dx$$
$$= \frac{n\zeta^2}{\zeta+1}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\zeta+1}\right)^{n-j-k-1}$$
$$\left\{\frac{\Gamma(n-j-k+r)}{\left((n-j)\zeta\right)^{n-j-k+r}}+\frac{\Gamma(n-j-k+r+1)}{\left((n-j)\zeta t\right)^{n-j-k+r+1}}\right\}$$

Hence we get

$$E(X^r) = \frac{\zeta^2}{\zeta+1}\sum_{n=1}^{\infty}\frac{nc_n\eta^n}{\psi(\eta)}\sum_{j=0}^{n-1}\sum_{k=0}^{n-j-1}\binom{n-1}{j}\binom{n-j-1}{k}(-1)^j\left(\frac{\zeta}{\zeta+1}\right)^{n-j-k-1}$$
$$\left\{\frac{\Gamma(n-j-k+r)}{\left((n-j)\zeta\right)^{n-j-k+r}}+\frac{\Gamma(n-j-k+r+1)}{\left((n-j)\zeta t\right)^{n-j-k+r+1}}\right\} \quad (5)$$

5. Order Statistics and Their Moments

Order statistics are frequently used in statistics theory and practice because they play a decisive role life testing analysis.

Let $X_1, X_2, ..., X_n$ be a random sample from CCLPS distribution and $X_{1:n}, X_{2:n}, ..., X_{1:n}$ denote the corresponding order statistics. The pdf of i^{th} order statistics say $X_{i:n}$ is given by

$$f_{i:n}(x) = \frac{n!\,f(x)}{(n-i)!(i-1)!}\left[\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}\right]^{i-1}$$

$$\left[1-\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}\right]^{n-i} \qquad (6)$$

Since, it is known that

$$f(x)[F(x)]^{k+i-1} = \left(\frac{1}{k+i}\right)\frac{d}{dx}[F(x)]^{k+i}$$

On the implementation of above fact, one can write the cdf of proposed distribution as

$$F_{i:n}(x) = \frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{n-i}{k}(-1)^k}{(k+i)}\left[\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}\right]^{k+i} \qquad (7)$$

on the other hand (7) can be written as

$$F_{i:n}(x) = 1-\frac{n!}{(n-i)!(i-1)!}\sum_{k=0}^{n-i}\frac{\binom{i-1}{k}(-1)^k}{(k+n-i+1)}\left[1-\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}\right]^{k+n-i+1}$$

consequently, upon using the formula due to due to Barakat and Abdelkadir (2004), the r^{th} moment of i^{th} order statistics

$$E\left(X_{i:n}^{r}\right) = r\sum_{k=n-i+1}^{n}(-1)^{k-n+i-1}\binom{k-1}{n-i}\binom{n}{k}\int_0^\infty x^{r-1}\left(1-\frac{\psi\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta\right]}{\psi(\eta)}\right)^k dx$$

where $r = 1,2,3...$ and $i = 1,2,...,n$

6. Estimation of Parameters

Let $X_1,...,X_N$ be a random sample with observed values $x_1,...x_n$ from CCLPS (ζ , η) distribution. The log-likelihood function will be

$$l_n = l_n\,(x,\Theta) = 2n\log\zeta + n\log\eta + \sum_{i=1}^{n}\log(1+x_i) - \zeta\sum_{i=1}^{n}x_i - n\log C(\eta)$$

$$-n\log\,(\zeta+1) + \sum_{i=1}^{n}\log\left\{\psi'\left[\eta\left(1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right)\right]\right\}$$

The corresponding differential derivatives are

$$\frac{\partial l_n}{\partial\zeta} = \frac{2n}{\zeta} - \sum_{i=1}^{n}x_i - \frac{n}{\zeta+1} + \sum_{i=1}^{n}\frac{\psi^n\left[\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta\right]}{\psi^n\left[\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta\right]}$$

$$\frac{\partial}{\partial\zeta}\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta$$

$$\frac{\partial l_n}{\partial\eta} = \frac{n}{\eta} - \frac{n\psi'(\eta)}{\psi(\eta)} + \sum_{i=1}^{n}\frac{\psi^n\left[\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta\right]}{\psi^n\left[\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta\right]}$$

$$\frac{\partial}{\partial\eta}\left\{1-\left(1+\frac{\zeta x_i}{\zeta+1}\right)e^{-\zeta x_i}\right\}\eta$$

The maximum likelihood estimate of unknown parameters shall be obtained numerically by using software such as R.

7. Consequences of the Proposed Model on Specific Parametric Function

In this section, we will learn about the consequences of proposed model by simply using the different results for $\psi(\eta)$ and $\psi'(\eta)$ given in table (i) in the proposed model.

7.1 Compound Complementary Lindley Poisson Distribution (CCLPD)

For $\psi(\eta) = e^{\eta} - 1$ and $\psi'(\eta) = e^{\eta}$ power series distribution reduces to Poisson distribution. Hence the distribution function, probability function and hazard rate function of compound complementary Lindley Poisson is

$$F(x) = \frac{e^{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta} - 1}{e^{\eta} - 1}, \quad x > 0 .$$

$$f(x) = \frac{\zeta^2}{\zeta+1}\eta(1+x)e^{-\zeta x}\, e^{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}(e^{\eta}-1)^{-1}$$

$$S(x) = 1 - \frac{e^{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta} - 1}{e^{\eta} - 1} \text{ and}$$

$$h(x) = \frac{\zeta^2\eta(1+x)e^{-\zeta x}e^{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}}{(\zeta+1)\left(e^{\eta} - e^{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}\right)}$$

For $x, \zeta > 0$, $0 < \eta < \infty$ respectively

k^{th} moment of a random variable following CCLPD becomes by taking $c_n = n!^{-1}$ and $\psi(\eta) = e^{\eta} - 1$ in (5)

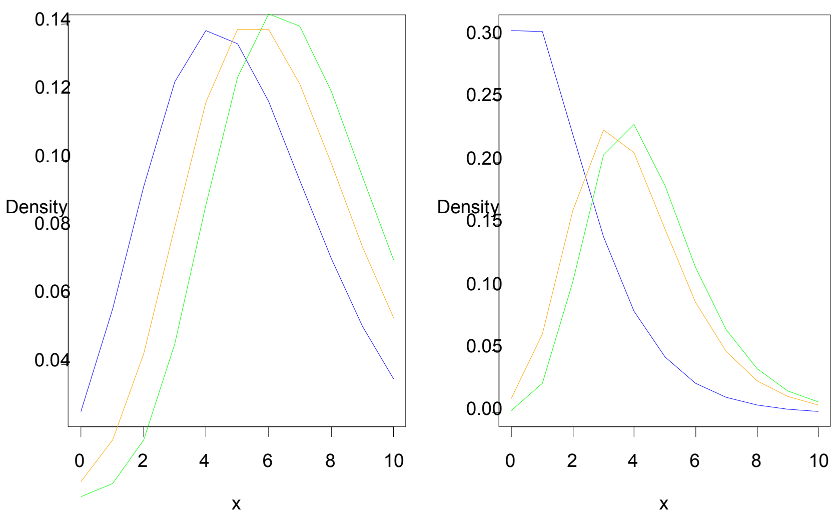

Fig .1: Graphs showing the flexibilty of density of CCLPD for some selected values of parameters

7.2 Compound Complementary Lindley Logarithmic Distribution (CCLLD)

For $\psi(\eta) = -\log(1-\eta)$ and $\psi'(\eta) = (1-\eta)^{-1}$ PSD reduces to logarithmic distribution. Hence one can get the distribution function, probability function and hazard rate function of CCLLD from CCLPS

$$F(x) = \frac{\log\left\{1-\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta\right\}}{\log(1-\eta)}, \quad x > 0$$

$$f(x) = \frac{\frac{\zeta^2}{\zeta+1}\eta e^{-\zeta x}(1+x)}{\left\{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta - 1\right\}l\,og(1-\eta)}$$

$$S(x) = 1 - \frac{\log\left\{1-\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta\right\}}{\log(1-\eta)}$$

$$h(x) = \frac{\zeta^2\eta(1+x)e^{-\zeta x}}{(\zeta+1)\left[\left\{\eta\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\right\}-1\right]\left[\log(1-\eta)-\log\left\{1-\eta\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\right\}\right]}$$

for $x, \zeta > 0$ and $0 < \eta < 1$ respectively.

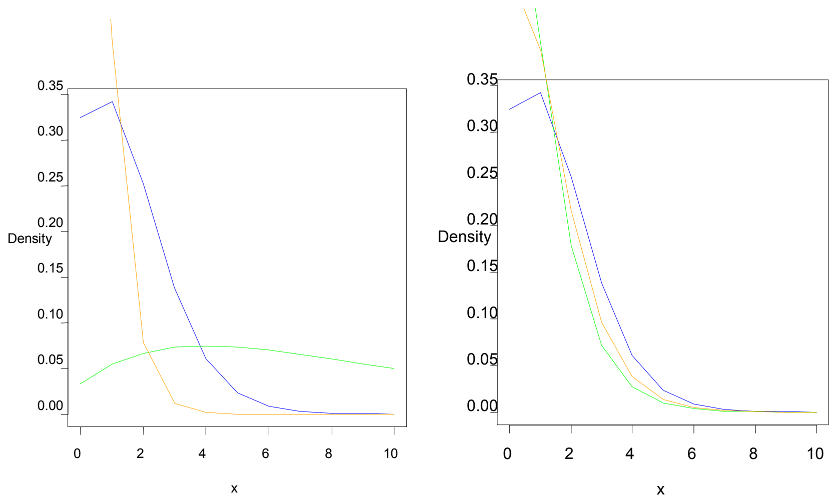

Fig .2: Graphs showing the flexibilty of density of CCLLD for some selected values of ζ and η

The k^{th} moment CCLLD can be determined by taking $c_n = n^{-1}$ and $C(\eta) = -\log(1-\eta)$ in (5).

7.3 Compound Complementary Lindley Geometric Distribution (CCLGD)

For $\psi(\eta) = \eta(1-\eta)^{-1}$ and $\psi'(\eta) = (1-\eta)^{-2}$ PSD reduces to Geometric distribution. Therefore it is obvious that one can get the distribution function, probability function and hazard rate function of CCLGD from CCLPS.

$$F(x) = \frac{\left[1-\left(1+\frac{\zeta x}{\lambda+1}\right)e^{-\zeta x}\right](1-\eta)}{1-\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}, x > 0$$

$$f(x) = \frac{\zeta^2}{\zeta+1}(1-\eta)(1+x)e^{-\zeta x}\left[1-\eta\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\right]^{-2}$$

$$S(x) = 1-\frac{\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right](1-\eta)}{1-\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}$$

$$= \frac{\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}}{1-\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta}$$

$$h(x) = \frac{\frac{\zeta^2}{\zeta+1}(1-\eta)(1+x)e^{-\zeta x}\left[1-\eta\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\right]^{-1}}{\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}}$$

for $x, \zeta, 0 < \eta < 1$ respectively.

By taking $c_n = 1$ and $\psi(\eta) = \eta(1-\eta)^{-1}$ in (5) we shall obtain the k^{th} moment of CCLGD.

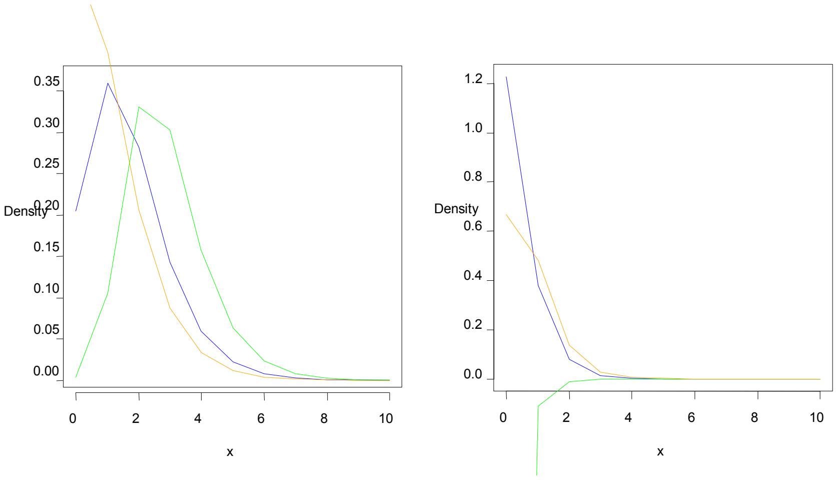

Fig .3: Graphs showing the flexibilty of density of CCLGD for some selected values of parameters ζ and η

7.4 Compound Complementary Lindley Binomial Distribution (CCLBD)

For $\psi(\eta)=(\eta+1)^m-1$ PSD reduces to binomial distribution. Therefore, we have the distribution function, probability function and hazard rate function of CCLBD

$$F(x)=\frac{\left\{\left(\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta+1\right)\right\}^{m}-1}{(\eta+1)^{m}-1},\quad x>0$$

$$f(x)=\frac{\zeta^2 m}{\zeta+1}(1+x)\eta e^{-\zeta x}\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta+1\right]^{m-1}\left[(\eta+1)^m-1\right]^{-1}$$

$$S(x)=1-\frac{\left\{\left(\left[1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right]\eta+1\right)\right\}^{m}-1}{(\eta+1)^{m}-1}$$

$$h(x)=\frac{\frac{\zeta^2 m}{\zeta+1}(1+x)\eta e^{-\zeta x}\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta+1\right]^{m-1}}{(\eta+1)^m-\left[\left\{1-\left(1+\frac{\zeta x}{\zeta+1}\right)e^{-\zeta x}\right\}\eta+1\right]^{m}}$$

respectively for $x,\zeta>0$ and $0<\eta<\infty$. The expression for the k^{th} moment of a random variable following CCLBD can be determined from (5).

8. Application

In this section, possible exploration of the potentiality of the proposed family of complementary compound distributions is made by fitting it to model real life vinyl chloride set obtained from clean up gradient monitoring wells.

5.1	1.2	1.3	0.6	0.5	2.4	0.5	1.1	8.0	0.8
0.4	0.6	0.9	0.4	2.0	0.5	5.3	3.2	2.7	2.9
2.5	2.3	1.0	0.2	0.1	0.1	1.8	0.9	2.0	4.0
6.8	1.2	0.4	0.2						

Table 2: Vinyl chloride data from clean upgradient ground-water monitoring wells in (μg/L)

Our aim is to fit this data by the proposed family of compound distributions. The MLE of unknown parameters and Akaike information criterion (AIC) and Bayesian information criterion (BIC) of the fitted distribution is given in table (3).

Model	MLE	AIC	BIC
LG	$\hat{\zeta}$ = 9.705721e-01 , $\hat{\eta}$ = 4.983673e-09	118.32	121.38
LP	$\hat{\zeta}$ = 8.203112e-01, $\hat{\eta}$ = 1.469566e-06	116.60	119.66
LL	$\hat{\zeta}$ = 8.096320e-01, $\hat{\eta}$ = 1.649876e-06	116.62	119.69

Table 3: Analysis of model fitting

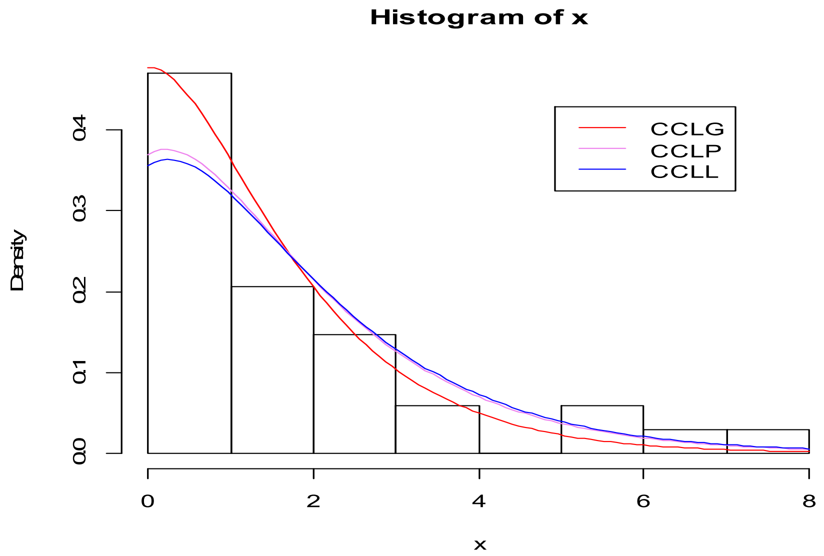

Fig 4: Fitting of CCLP, CCLG, CCLL to the vinyl chloride data

9. Conclusion

In our humble effort, we constructed a new class of compound complementary lifetime distribution by blending Lindley distribution with that of power series distribution. Furthermore, we also discussed and analyzed some special cases of a particular class of distributions that are very flexible in terms of density and hazard rate functions. Mathematical properties such as moments, order statistics and parameter estimation through MLE of the proposed class has also been discussed. Finally the potentiality of the CCLPS class has been illustrated by fitting it to model some real life data set. It is quite clear and evident from the above analysis that according to the AIC and BIC all the sub models of CCLPS family of compound distributions perform excellently well but among them Lindley Poisson is the wonderful competitor since it has lowest AIC and BIC which is also corroborated by the graphical analysis in the fig (4). With all professional command, the practitioners are advised to use one of our models to fit lifetime data so as to arrive at productive results.

The future course of work will be on version on complementary version of generalized compound Lindley powers series distribution.

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