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ESTIMATION OF POPULATION MEAN USING TWO AUXILIARY VARIABLES IN STRATIFIED RANDOM SAMPLING

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Abstract

This article discusses the problem of estimation of population mean in stratified sampling using information on two auxiliary variables. The expressions for the mean square error of the proposed estimator have been derived up to the first order of approximation and are compared with the existing estimators. Also, an empirical study has been carried out in order to show that the proposed estimator turns out to be more efficient than the existing estimators and for this we have considered real data sets.

Key Words: Study Variable, Auxiliary Variables, Stratified Random Sampling, Mean Squared Error, Efficiency, Bias.

1. Introduction

In survey sampling, it is always advantageous to use the information available on the auxiliary variable which is highly correlated with the variable of interest. The use of auxiliary information increases the precision of the estimators used for estimating the unknown population parameters. Several authors have used auxiliary information on auxiliary variable in the estimation of population parameters like Srivastava and Jhajj (1981), Bahl and Tuteja (1991), Singh and Vishwakarma (2007), Sahai and Ray (1980), Srivastava and Jhajj (1983), Srivastava (1971), Swain (1970) and Perri (2007).

Here we have tried to incorporate the use of auxiliary information in stratified random sampling. Several authors like Haq and Shabbir (2013), Shabbir and Gupta (2006), Kadilar and Cingi (2003) have proposed estimators in stratified random sampling using information on a single auxiliary variable. It is seen that many a times instead of using information on a single auxiliary variable, we have information on two auxiliary variables like Tailor et al. (2012) suggested a ratio-cum-product estimator of population mean in stratified random sampling using two auxiliary variables. Koyuncu and Kadilar (2009) proposed a family of estimators of population mean using two auxiliary variables in stratified random sampling. Furthermore Verma et al. (2015) have given some families of estimators using two auxiliary variables in stratified random sampling. Likewise Singh and Kumar (2012) have proposed improved estimators of population mean using two auxiliary variables in stratified random sampling. Through this paper the problem of estimation of finite population mean in stratified random sampling using information on two auxiliary variables has been discussed.

Consider a finite population $P = (P_1, P_2, ..., P_N)$ of size N is divided into L strata of size $N_h (h = 1,2,......,L)$ such that there are N_h units in the h^{th} stratum and $N = \sum_{h=1}^{L} N_h$

Let Y be the study variable and X and Z be the auxiliary variables taking values y_{ni}, x_{ni} and z_{ni} $(h = 1,2,......,L)$, $(i = 1,2,......., N_h)$ on the i^{th} unit of the h^{th} stratum.

Now we define

$\bar{Y}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} y_{hi}$ h^{th} stratum mean for the study variable Y

$\bar{X}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} x_{hi}$ h^{th} stratum mean for the study variable X

$\bar{Z}_h = \frac{1}{N_h} \sum_{i=1}^{N_h} z_{hi}$ h^{th} stratum mean for the study variable Z

$\bar{Y} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} y_{ni} = \frac{1}{N} \sum_{h=1}^{L} N_h \bar{Y}_h = \sum_{h=1}^{L} W_h \bar{Y}_h$: Population mean of the study variable Y

$\bar{X} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} x_{ni} = \sum_{h=1}^{L} W_h \bar{X}_h$: Population mean of the auxiliary variable X

$\bar{Z} = \frac{1}{N} \sum_{h=1}^{L} \sum_{i=1}^{N_h} z_{ni} = \sum_{h=1}^{L} W_h \bar{Z}_h$: Population mean of the auxiliary variable Z

$\bar{y}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} y_{ni}$: Sample mean for the study variable Y for h^{th} stratum

$\bar{x}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} x_{ni}$: Sample mean for the study variable X for h^{th} stratum

$\bar{z}_h = \frac{1}{n_h} \sum_{i=1}^{n_h} z_{ni}$: Sample mean for the study variable Z for h^{th} stratum

$W_h = \frac{N_h}{N}$: Stratum weight for the h^{th} stratum

$f_h = \frac{n_h}{N_h}$: Sampling fraction

$$l_h = \frac{1-f_h}{n_h}$$

Also, $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$, $\bar{x}_{st} = \sum_{h=1}^{L} W_h \bar{x}_h$ and $\bar{z}_{st} = \sum_{h=1}^{L} W_h \bar{z}_h$ are the usual unbiased estimators of the population mean $\bar{Y}$, $\bar{X}$ and $\bar{Z}$ in stratified random sampling.

To obtain the MSE's let us define

$$\epsilon_0 = \frac{\bar{y}_{st} - \bar{Y}}{\bar{Y}} , \ \epsilon_1 = \frac{\bar{x}_{st} - \bar{X}}{\bar{X}} \text{ and } \epsilon_2 = \frac{\bar{z}_{st} - \bar{Z}}{\bar{Z}}$$

Using these notations,

$$E(\epsilon_0) = E(\epsilon_1) = E(\epsilon_2) = 0$$

$$V_{rst} = \sum_{h=1}^{L} W_h^{r+s+t} \frac{E[(\bar{y}_h - \bar{Y}_h)^r (\bar{x}_h - \bar{X}_h)^s (\bar{z}_h - \bar{Z})^t]}{\bar{Y}^r \bar{X}^s \bar{Z}^t} \tag{1.1}$$

Using (1.1) we can write:-

$$E(\epsilon_0^2) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{yh}^2}{\bar{Y}^2} = V_{200}$$

$$E(\epsilon_1^2) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{xh}^2}{\bar{X}^2} = V_{020}$$

$$E(\epsilon_2^2) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{zh}^2}{\bar{Z}^2} = V_{002}$$

$$E(\epsilon_0 \epsilon_1) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{xyh}}{\bar{X}\bar{Y}} = V_{110}$$

$$E(\epsilon_1 \epsilon_2) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{xzh}}{\bar{X}\bar{Z}} = V_{011}$$

$$E(\epsilon_0 \epsilon_2) = \frac{\sum_{h=1}^{L} W_h^2 l_h S_{yzh}}{\bar{Y}\bar{Z}} = V_{101}$$

Where $S_{yh}^2 = \sum_{i=1}^{N_h} \frac{(y_{hi} - \overline{Y}_h)^2}{N_h - 1}$, $S_{xh}^2 = \sum_{i=1}^{N_h} \frac{(x_{hi} - \overline{X}_h)^2}{N_h - 1}$, $S_{zh}^2 = \sum_{i=1}^{N_h} \frac{(z_{hi} - \overline{Z}_h)^2}{N_h - 1}$

$S_{xyh} = \sum_{i=1}^{N_h} \frac{(x_{hi} - \overline{X}_h)(y_{hi} - \overline{Y}_h)}{N_h - 1}$, $S_{zyh} = \sum_{i=1}^{N_h} \frac{(z_{hi} - \overline{Z}_h)(y_{hi} - \overline{Y}_h)}{N_h - 1}$,

2. Estimators available in literature

In this section, we consider several estimators of the finite population mean that are available in the sampling literature. The variance and mean squared error's (MSE's) of all the estimators considered here are obtained under the first order of approximation.

- The usual unbiased estimator of the population mean in stratified random sampling is defined as:-

$$\overline{y}_{st} = \sum_{h=1}^{l} W_h \overline{y}_h \tag{2.1}$$

- The notations used here have been used by Dayal (1980). Other useful references for stratified sampling are Cochran (1977, chapter-5), Reddy (1978).

Variance of the estimator $\overline{y}_{st}$ is defined as:

$$V(\overline{y}_{st}) = \sum_{h=1}^{L} W_h^2 1_h S_{yh}^2 \tag{2.2}$$

- Koyuncu and Kadilar (2009) suggested different ratio type estimators for population mean $\overline{Y}$ utilizing information on known value of population mean $\overline{X}$ and $\overline{Z}$ of auxiliary variables X and Z as:-

$$\overline{y}_1 = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right)\left(\frac{\overline{Z}}{\overline{z}_{st}}\right) \tag{2.3}$$

and

$$\overline{y}_2 = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right)\left(\frac{\overline{z}_{st}}{\overline{Z}}\right) \tag{2.4}$$

The mean square error of the estimator $\overline{y}_1$ is given by:

$$MSE(y_1) = \overline{Y}^2\left(V_{200} + V_{020} + V_{002} - 2V_{110} + 2V_{011} - 2V_{101}\right) \tag{2.5}$$

The mean square error of the estimator y_2 is given by:-

$$MSE(y_2) = \overline{Y}^2\left(V_{020} + V_{200} + V_{002} - 2V_{110} - 2V_{011} + 2V_{101}\right) \tag{2.6}$$

When we have information on two auxiliary variables then the usual regression estimator is defined as:

$$\overline{y}_{lr} = \overline{y}_{st} + b_1\left(\overline{X} - \overline{x}_{st}\right) + b_2\left(\overline{Z} - \overline{z}_{st}\right) \tag{2.7}$$

The mean square error of the estimator $\bar{y}_{lr}$ is given by:-

$$MSE(\bar{y}_{lr}) = \sum_{h=1}^{L} W_h^2 l_h S_{yh}^2 \left(1 - \rho_{yxh}^2 - \rho_{yzh}^2 + 2\rho_{yxhh}\rho_{yzh}\rho_{xzhh}\right) \quad (2.8)$$

3. Proposed estimator

For estimating unknown population mean $\overline{Y}$ of the study variable we propose an estimator as follows:

$$t = w_0 t_0 + w_1 t_1 + w_2 t_2 \quad (3.1)$$

Where, $t_0 = \bar{y}_{st}$, $t_1 = \bar{y}_{st}\left(\frac{\overline{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\overline{Z}}\right)$ and

$$t_2 = \bar{y}_{st} \exp\left(\frac{\overline{X} - \bar{x}_{st}}{\overline{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{z}_{st} - \overline{Z}}{\bar{z}_{st} + \overline{Z}}\right)$$

Here, $t_o, t_1, t_2 \in W$

Here W denotes the set of all possible estimators for estimating the population mean $\overline{Y}$. By definition, the set W is a linear variety if:

$$t = \sum_{i=0}^{2} w_i t_i \in W \quad (3.2)$$

Such that $\sum_{i=0}^{2} w_i = 1$ and $w_i \in R$ (3.3)

Where, $w_i (i = 0,1,2)$ denotes the constants used for reducing the bias in the class of estimators.

The form of the estimator defined in equation (3.1) has been so taken; so that it comes out to be an unbiased estimator for the population mean $\overline{Y}$. And the technique utilized here is the technique of "Filtration of Bias".

It is one of the methods used to remove Bias from ratio and product type estimators. Some other methods used to remove bias are Quenouille's method, interpenetrating sampling method etc. [Singh (2003)].

Expressing the estimator t in terms of $\in$'s we get

$$t = \overline{Y}(1+\in_0)\left[w_0 + w_1(1+\in_1)^{-1}(1+\in_2) + w_2 \exp\left\{-\in_1 (2+\in_1)^{-1}\right\}\exp\left\{\in_2 (2+\in_2)^{-1}\right\}\right] \quad (3.4)$$

By expanding the above equation (3.4) and keeping terms only up to order one in $\in$'s, we can write

$$t=\overline{Y}(1+\in_0)\left[1-\in_1\left(w_1+\frac{w_2}{2}\right)+\in_2\left(w_1+\frac{w_2}{2}\right)+\in_1^2\left(w_1+\frac{3}{8}w_2\right)-\frac{1}{8}w_2\in_2^2-\in_1\in_2\left(w_1+\frac{w_2}{4}\right)\right] \tag{3.5}$$

Now, subtracting $\overline{Y}$ from both the sides of equation (3.5) and then taking expectation of both sides, the bias of the estimator t is obtained up to the first order of approximation as:

$$Bias(t)=\overline{Y}\left[(V_{101}-V_{110})\left(w_1+\frac{w_2}{2}\right)+V_{020}\left(w_1+\frac{3}{8}w_2\right)-\frac{1}{8}w_2V_{002}-\left(w_1+\frac{w_2}{4}\right)V_{011}\right] \tag{3.6}$$

Ignoring 1st and higher order terms in (3.5) we get

$$t-\overline{Y}=\overline{Y}\left[\in_0+\left(w_1+\frac{w_2}{2}\right)(\in_2-\in_1)(1+\in_0)\right] \tag{3.7}$$

Squaring both the sides and then taking expectation we get

$$MSE(t)=\overline{Y}^2\left[V_{200}+Q^2\{V_{002}+V_{020}-2V_{011}\}+2Q\{V_{101}-V_{110}\}\right] \tag{3.8}$$

Where $w_1+\frac{w_2}{2}=Q$ (3.9)

The MSE of the estimator t is minimum when

$$Q=\frac{-(V_{101}-V_{110})}{(V_{002}+V_{020}-2V_{011})} \tag{3.10}$$

Putting this value of Q in equation (3.8), we get the minimum value for the MSE of the estimator t which is by,

$$\min MSE(t)=\overline{Y}^2\left[V_{200}-\frac{(V_{101}-V_{110})^2}{(V_{002}+V_{020}-2V_{011})}\right] \tag{3.11}$$

From equation (3.3) and (3.9) there are two equations and three unknown. It is not possible to find the unique values for w_i's, $i=0,1,2$. In order to get unique values of w_i's, we impose the linear restriction as,

$$\sum_{i=0}^{2}w_iB(t_i)=0 \tag{3.12}$$

Where $B(t_i)$ (i=0, 1, 2) denotes the Bias in the i^{th} estimator.

Equations (3.3), (3.9) and (3.12) can be written in the matrix form as,

$$\begin{bmatrix}1 & 1 & 1\\ 0 & 1 & 1/2\\ B(t_0) & B(t_1) & B(t_2)\end{bmatrix}\begin{bmatrix}w_0\\ w_1\\ w_2\end{bmatrix}=\begin{bmatrix}1\\ Q\\ 0\end{bmatrix} \tag{3.13}$$

Solving (3.13) we get the unique values of w_0, w_1 and w_2 as,

$$w_0 = A + \frac{(1-A)B}{2C-D}$$
$$w_1 = (1-A) + \frac{(1-A)B}{2C-D}$$
$$w_2 = \frac{-2(1-A)B}{2C-D}$$

Here,

$$A = \left[1 + \frac{V_{101} - V_{110}}{V_{002} + V_{020} - 2V_{011}}\right]$$
$$(1-A) = Q = -\frac{(V_{101} - V_{110})}{V_{002} + V_{020} - 2V_{011}}$$
$$B = (V_{020} - V_{110} - V_{011} + V_{101})$$
$$C = \left(\frac{3}{8}V_{020} - \frac{1}{8}V_{002} - \frac{1}{4}V_{011} - \frac{1}{2}V_{110} + \frac{1}{2}V_{101}\right)$$
$$D = (V_{020} - V_{110} - V_{011} + V_{101})$$

4. Empirical study

To examine the merits of the proposed estimator over the other existing estimators at optimum conditions, we have considered two natural population data sets from the literature. The source of population is given below:

Population 1 [Source: Koyuncu and Kadilar (2009)]
Y: Number of teachers,
X: Number of students,
Z: Number of classes in both primary and secondary school

$N_1 = 127$ $N_2 = 117$ $N_3 = 103$ $N_4 = 170$ $N_5 = 205$ $N_6 = 201$ $n_1 = 31$
$n_2 = 21$ $n_3 = 29$ $n_4 = 38$ $n_5 = 22$ $n_6 = 39$ $\bar{Y}_1 = 703.74$ $\bar{Y}_2 = 413$
$\bar{Y}_3 = 573.17$ $\bar{Y}_4 = 424..66$ $\bar{Y}_5 = 267.03$ $\bar{Y}_6 = 393.84$ $\bar{X}_1 = 20804.59$
$\bar{X}_2 = 9211.79$ $\bar{X}_3 = 14309.30$ $\bar{X}_4 = 9478.85$ $\bar{X}_5 = 5569.95$
$\bar{X}_6 = 12997.59$ $\bar{Z}_1 = 498.28$ $\bar{Z}_2 = 318.33$ $\bar{Z}_3 = 431.36$ $\bar{Z}_4 = 311.32$
$\bar{Z}_5 = 227.20$ $\bar{Z}_6 = 313.71$ $S_{y1} = 888.835$ $S_{y2} = 644.922$ $S_{y3} = 1033.467$
$S_{y4} = 810.585$ $S_{y5} = 403.654$ $S_{y6} = 711.723$ $S_{x1} = 30486.751$
$S_{x2} = 15180.760$ $S_{x3} = 27549.697$ $S_{x4} = 18218.931$ $S_{x5} = 8497.776$
$S_{x6} = 23094.141$ $S_{z1} = 555.5816$ $S_{z2} = 365.4576$ $S_{z3} = 612.9509$

$S_{z4} = 458.0282$ $S_{z5} = 260.8511$ $S_{z6} = 397.0481$ $S_{yx1} = 25237153.52$
$S_{yx2} = 9747942.85$ $S_{yx3} = 28294397.04$ $S_{yx4} = 14523885.53$
$S_{yx5} = 3393591.75$ $S_{yx6} = 15864573.97$ $S_{yz1} = 480688.2$
$S_{yz2} = 230092.8$ $S_{yz3} = 623019.3$ $S_{yz4} = 364943.4$ $S_{yz5} = 101539$
$S_{yz6} = 277696.1$ $S_{xz1} = 15914648$ $S_{xz2} = 5379190$ $S_{xz3} = 16490674.56$
$S_{xz4} = 8041254$ $S_{xz5} = 2144057$ $S_{xz6} = 8857729$ $\rho_{yx1} = 0.936$
$\rho_{yx2} = 0.996$ $\rho_{yx3} = 0.994$ $\rho_{yx4} = 0.983$ $\rho_{yx5} = 0.989$ $\rho_{yx6} = 0.965$
$\rho_{yz1} = 0.978$ $\rho_{yz2} = 0.976$ $\rho_{yz3} = 0.983$ $\rho_{yz4} = 0.982$ $\rho_{yz5} = 0.964$
$\rho_{yz6} = 0.982$

Population 2 (Source: Murthy (1967))
Y: Output
X: Fixed Capital
Z: Number of workers

$N = 10$, $n = 5$, $n_1 = 2$, $n_2 = 3$, $N_1 = 5$, $N_2 = 5$, $\bar{Y}_1 = 1925.80$, $\bar{Y}_2 = 315.60$ $\bar{X}_1 = 214.40$, $\bar{X}_2 = 333.80$, $\bar{Z}_1 = 51.80$, $\bar{Z}_2 = 60.60$, $S_{y1} = 615.92$, $S_{y2} = 340.38$, $S_{x1} = 74.87$ $S_{x2} = 66.35$
$S_{z1} = 0.75$, $S_{z2} = 4.84$, $S_{yx1} = 39360.68$, $S_{yx2} = 22356.50$,
$S_{yz1} = 411.16$, $S_{yz2} = 1536.24$, $S_{zx1} = 38.08$, $S_{zx2} = 287.92$

ESTIMATORS	VARIANCE/MSE's	PRE with respect to $\bar{y}_{st}$
$\bar{y}_{st}$	2228.52	100
$\bar{y}_1$	1613.59	138.10
$\bar{y}_2$	1489.095	149.65
$\bar{y}_{lr}$	2072.674	107.51
T	0.0296	309.7493

Table 1: MSE's and Percent Relative Efficiencies (PRE's) of the estimators w. r. to $\bar{y}_{st}$

ESTIMATORS	VARIANCE/MSE'S	PRE with respect to $\bar{y}_{st}$
$\bar{y}_{st}$	32313.7599	100
$\bar{y}_1$	10647.1302	303.4974
$\bar{y}_2$	13130.03	246.1058
$\bar{y}_{lr}$	17611.89	183.477
T	8948.4080	361.1118

Table 2: MSE's and Percent Relative Efficiencies (PRE's) of the estimators w. r. to $\bar{y}_{st}$

The Percent Relative Efficiencies (PRE's) of the estimators with respect to the usual unbiased estimator $\bar{y}_{st}$ are obtained from the following mathematical formula.

$$PRE\,(ESTIMATOR) = \frac{MSE\,(ESTIMATOR)}{MSE(\bar{y}_{st})} \times 100$$

5. Conclusion

In this paper, we have proposed an estimator for the population mean in stratified random sampling utilizing information on two auxiliary variables. The MSE of the proposed estimator has been derived up to first order of approximation. Furthermore, we have used empirical approach for comparing the efficiency of the proposed estimator with other estimators for which we have used known natural population datasets, see Murthy (1967) and Koyuncu and Kadilar (2009). The results have been shown above in the Table 1 and Table 2. From both the tables, it is clear that the proposed estimator turns out to be more efficient as compared to the existing estimators because of smaller value of MSE and higher value of PRE. So it is clearly more desirable to use the proposed estimator in practical surveys.

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