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GENERAL MANPOWER – HYPER EXPONENTIAL MACHINE SYSTEM WITH MARKOVIAN PRODUCTION, GENERAL SALES AND GENERAL RECRUITMENT

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Abstract: In this paper, two manpower planning models with different recruitment patterns are studied. The machine attended by the manpower system has Hyper-exponential life time distribution. The manpower machine system fails when both of them are in failed state, when either manpower or machine alone is in failed state, the failed one is hired till the other also fails. Two models are treated. In model 1, when the system fails, the vacancies caused by employees' departures are filled up one by one. In model 2, recruitments are done one by one or in bulk pattern depending on the system operation time is within or exceeding a threshold. Using Laplace transform, the operation time, recruitment time of the employees, repair time of the machine and sales time of the products are studied. Their means and the covariance of the operation time and recruitment time are presented with numerical examples.

Mathematics Subject Classification: 91B70

Key Words: Departure and Recruitments, Failure and Repairs, Production and Sale Times, Joint Laplace Transforms.

1. Introduction

The models related to manpower planning have been studied by Grinold and Marshall [3]. One may refer to Bartholomew [1] for statistical approach. The methods to compute shortages (Resignations, Dismissals, Deaths) have been given by Lesson [6]. Hari Kumar, Sekar and Ramanarayanan [4] presented stochastic analysis of manpower levels affecting business with varying recruitment rate. Subramanian [14] has made an attempt to provide optimal policy for recruitment, training, promotion and shortages in manpower planning models with special provisions such as time bound promotions, cost of training and voluntary retirement schemes. Cumulative damage processes have been discussed by Esary et al. [2]. General production and sales system with SCBZ machine time and Markovian manpower have been discussed in depth by Madhusoodhanan, Sekar and Ramanarayanan [9]. Modified Erlang two phase system with general manpower has been discussed by Snehalatha.M., Sekar.P and Ramanarayanan.R [12]. For three characteristics system in manpower models one may refer to Mohan and Ramanarayanan [11]. Manpower System with Erlang departure and one by one recruitment is discussed by Hari Kumar [5]. For the study of semi Markov models in manpower planning, one may refer to Meclean [10]. General manpower-SCBZ machine system with Markovian production general sales and general recruitment have been discussed by Snehalatha.M., Sekar.P and Ramanarayanan.R [13]. General production and sales system with exponential machine time and manpower were discussed by Madhusoodhanan, Sekar and Ramanarayanan [7]. Madhusoodhanan.P., Sekar.P and Ramanarayanan.R discussed the general production and sales time with two-units system and manpower [8]. Markovian models designed for shortage and promotion in man power system have been discussed in depth by Vassiliou [15].

In this paper, a machine which produces products is considered. The machine has hyper exponential life time and the products are produced one at a time with inter-occurrence time distribution which is exponential. Employees attending the machine may depart with general inter-departure time distribution. The manpower system collapses with a probability when an employee departs and survives with a probability and continues operation. When the manpower machine system fails, sales, recruitments and repairs are undertaken. Two models are treated. In model 1, when the system fails, the vacancies caused by employees' departures are filled up one by one. In model 2 recruitments are done one by one or in bulk pattern depending on the system operation time within or exceeding a threshold. The joint Laplace transforms of the variables, their means and the covariance of operation time and recruitment time are obtained. Numerical cases are also treated.

2. Model 1

2.1 Assumptions

1. Inter departure times of employees are independent and identically distributed (iid) random variables with cdf F(x) and pdf f(x). The manpower system collapses with probability p when an employee departs and survives with probability q and continues operation.
2. The machine attended by the manpower system has hyper-exponential life time distribution with parameter 'λ_i' probability p_i, for $1 \leq i \leq m$.
3. The manpower machine system fails when both of them are in failed state, when either manpower or machine alone is in failed state, the failed one is hired till the other also fails.
4. When the system fails, the vacancies caused by employees' departures are filled up one by one with recruitment time V whose distribution function is V(y) and pdf is v(y).
5. The repair time R_i of the machine has cdf $R_i(z)$ and pdf $r_i(z)$ when the machine fails with rate a_i for $1 \leq i \leq m$.
6. When the manpower-machine system is in operation, products are produced m at a time with inter-production time as exponential with rate μ.
7. The sale time G_1 of a product has cdf G(w) and pdf g(w). The sales are done one by one.
8. When the manpower machine system fails, sales, recruitments and repairs are undertaken.

2.2 Analysis

To study the above model 1, the joint probability density function of four variables, namely ($X,\hat{V},\hat{R},\hat{S}$) where X is the operation time of the manpower-machine system which is the maximum of their life times, $\hat{V}$, the sum of recruitment times of employees to fill up the vacancies, $\hat{R}$ is the repair time of the machine which Ri when the failure rate is a_i and $\hat{S}$ is the total sales time of the products produced, We find the pdf of $(X,\hat{V},\hat{R},\hat{S})$ as follows.

$$f(x,y,z,w)=$$

$$\left\{\sum_{i=1}^{m} p_i\lambda_i e^{-\lambda_i x} r_i(z)\sum_{n=1}^{\infty} F_n(x)q^{n-1}pv_n(y)+\sum_{i=1}^{m} p_i(1-e^{-\lambda_i x})r_i(z)\sum_{n=1}^{\infty} f_n(x)q^{n-1}pv_n(y)\right\}$$

$$[\sum_{k=0}^{\infty} e^{-\mu x}\frac{(\mu x)^k}{k!}g_k(w)] \qquad (1)$$

The flower bracket has two terms. To consider the two cases that the machine fails when manpower system is in failed state and its recruitment and repairs are carried out. The second term considers the case that the manpower system when the machine is in failed state and the recruitments and repairs are carried out. The last bracket indicates that k-products are produced and sold. The suffix letter for $v_n(y)$ and for $g_k(w)$ indicates the convolution of v(y) and g(w) respectively. $F_n(x)$ is the n-fold Stiltjes convolution of cdf F(x) with itself and $f_n(x)$ is the n-fold convolution of f(x) with itself. The quadruple Laplace transform of f(x,y,z,w) is given by

$$f^*(\xi,\eta,\varepsilon,\delta)=\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\xi x-\eta y-\varepsilon z-\delta w} f(x,y,z,w)dxdydzdw \qquad (2)$$

Here * indicates Laplace transform using the structure of (1), the equation (2) reduces to the sum of two single integrals.

$$f^*(\xi,\eta,\varepsilon,\delta)=$$

$$\int_0^\infty e^{-\xi x-\mu x(1-g^*(\delta))}\sum_{i=1}^{m} p_i\lambda_i e^{-\lambda x} r_i^*(\varepsilon)\sum_{n=1}^{\infty} F_n(x)q^{n-1}pv^{*n}(\eta)dx$$

$$+\int_0^\infty e^{-\xi x-\mu x(1-g^*(\delta))}\sum_{i=1}^{m} p_i(1-e^{-\lambda_i x}) r_i^*(\varepsilon)\sum_{n=1}^{\infty} f_n(x)q^{n-1}pv^{*n}(\eta)dx \qquad (3)$$

We obtain

$$f^*(\xi,\eta,\varepsilon,\delta)$$
$$=\sum_{i=1}^{m} p_i\lambda_i r_i^*(\varepsilon)pv^*(\eta)\frac{f^*(\chi_i)}{\chi_i(1-qv^*(\eta)f^*(\chi_i))}$$
$$+\sum_{i=1}^{m} p_i r_i^*(\varepsilon)pv^*(\eta)\left[\frac{f^*(\chi)}{(1-qv^*(\eta)f^*(\chi))}-\frac{f^*(\chi_i)}{(1-qv^*(\eta)f^*(\chi_i))}\right] \tag{4}$$

Here

$$\chi_i = \xi+\mu(1-g^*(\delta))+\lambda_i \quad for\ 1\le i\le m$$
$$and\ \ \chi=\xi+\mu(1-g^*(\delta)) \tag{5}$$

We note that the Laplace transform of the pdf of the operation time X is

$$f^*(\xi,0,0,0)=\sum_{i=1}^{m} p_i p\left[\frac{f(\xi)}{(1-qf^*(\xi)}-\frac{\xi}{(\xi+\lambda_i)}\frac{f^*(\xi_i+\lambda_i)}{(1-qf^*(\xi_i+\lambda_i))}\right] \tag{6}$$

$$E(X)=-\frac{\partial}{\partial\xi}f^*(\xi,0,0,0)\,|\,\xi=0, gives$$

$$E(X)=\frac{E(F)}{p}+\sum_{i=1}^{m}\frac{p_i p}{\lambda_i}\frac{f^*(\lambda_i)}{(1-qf^*(\lambda_i))} \tag{7}$$

It is easy to obtain the Laplace transform of repair time pdf

$$f^*(0,0,\varepsilon,0)=\sum_{i=1}^{m} p_i v_i^*(\varepsilon)\, and$$

$$E(\hat{R})=-\frac{\partial}{\partial\varepsilon}f^*(0,0,\varepsilon,0)\ |\ \varepsilon=0\ \ gives$$

$$E(\hat{R})=\sum_{i=1}^{m} p_i E(V_i)$$

Now the Laplace transform of the pdf of sales time $\hat{S}$ *is*

$$f^*(0,0,0,\delta)=$$
$$\sum_{i=1}^{m} p_i p\left[\begin{array}{l}\dfrac{f^*(\mu(1-g^*(\delta)))}{1-qf^*(\mu(1-g^*(\delta)))}\\ -\dfrac{\mu(1-g^*(\delta))}{(\mu(1-g^*(\delta))+\lambda_i)}\dfrac{f^*(\lambda_i+\mu(1-g^*(\delta)))}{(1-qf^*(\lambda_i+\mu(1-g^*(\delta)))}\end{array}\right] \tag{9}$$

$$Now\ E(\hat{S}) = -\frac{\partial}{\partial \delta} f^*(0,0,0,\delta) \mid \delta = 0,\ then$$

$$E(\hat{S}) = E(G)E(F)\frac{\mu}{p} + \sum_{i=1}^{m} \frac{p_i}{\lambda_i} \frac{\mu p E(G) f^*(\lambda_i)}{(1 - q f^*(\lambda_i))} \tag{10}$$

The double Laplace transform of joint p.d.f. of $(X, \hat{V})$ is given by

$$f^*(\xi,\eta,0,0) =$$

$$\sum_{i=1}^{m} p_i p v^*(\eta)\left[\frac{f^*(\xi)}{(1 - q v^*(\eta) f^*(\xi))} - \frac{\xi}{\xi + \lambda_i} \frac{f^*(\xi + \lambda_i)}{(1 - q v^*(\eta) f^*(\xi + \lambda_i))}\right] \tag{11}$$

$$E(\hat{V}) = -\frac{\partial}{\partial \eta} f^*(\xi,\eta,0,0) \mid \xi = \eta = 0\ gives$$

$$E(\hat{V}) = \frac{E(V)}{p} \tag{12}$$

Now the product moment $E(X\hat{V})$ is given by

$$E(X\hat{V}) = -\frac{\partial^2}{\partial \xi \partial \eta} f^*(\xi,\eta,0,0) \mid \xi = \eta = 0$$

$$E(X\hat{V}) = \frac{E(V)E(F)}{p^2}(1 + q) + \sum_{i=1}^{m} \frac{p_i p}{\lambda_i} E(V) \frac{f^*(\lambda_i)}{(1 - q f^*(\lambda_i))^2} \tag{13}$$

Using the formula $Cov(X,\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$

and using equations (13), (12) and (7), $Cov(X,\hat{V})$ may be written.

3. Model 2

In this section, we treat the previous model 1 with all assumptions (1), (2), (3), (5), (6), (7) and (8) except the assumption (4) for manpower recruitment pattern.

3.1 Assumption for Manpower Recruitment

(i) When the operation time X is more than a threshold time U, the recruitments are all done together. It is assigned to an agent whose service time V_1, to fill up all vacancies, has cdf $V_1(y)$ and pdf $v_1(y)$.
(ii) When the operation time X is less than the threshold time U, the recruitments are done one by one and the recruitment time V_2 for each has Cdf $V_2(y)$ and pdf $v_2(y)$.
(iii) The threshold U has exponential distribution with parameter θ.

3.2 Analysis

Using the arguments given for model 1, the joint pdf of $(X, \hat{V}, \hat{R}, \hat{S})$ (Operation time, Recruitment time, Repair time of machine, Sales time) may be obtained as follows.

Note that the pdf f(x,y,z,w) is

$$f(x,y,z,w) =$$

$$\left\{ \begin{array}{l} \sum_{i=1}^{m} p_i \lambda_i e^{-\lambda_i x} r_i(z) \sum_{n=1}^{\infty} F_n(x) q^{n-1} p\left[(1-e^{-\theta x}) v_1(y) + e^{-\theta x} v_{2,n}(y)\right] \\ + \sum_{i=1}^{\infty} p_i (1-e^{-\lambda_i x}) r_i(z) \sum_{n=1}^{\infty} f_n(x) q^{n-1} p\left[(1-e^{-\theta x}) v_1(y) + e^{-\theta x} v_{2,n}(y)\right] \end{array} \right\}$$

$$[\sum_{k=0}^{\infty} e^{-\mu x} \frac{(\mu x)^k}{k!} g_k(w)] \qquad (14)$$

We use the same arguments given for model 1 for all the terms except the square brackets appearing in the terms. The first term in the square bracket means the threshold is less than the operation time calling for pdf $v_1(y)$ for sales together and the second term indicates that the threshold is more than the operation time and recruitments are done one by one with pdf $v_2(y)$. Here $v_{2,n}(y)$ is n-fold convolution of $v_2(y)$ with itself.

The Laplace transform of the pdf of four variables $(X, \hat{V}, \hat{R}, \hat{S})$ is given by

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w) dxdydzdw$$

This reduces using equation (15) to sum of two single integrals.

$$f^*(\xi,\eta,\varepsilon,\delta) =$$

$$\int_0^\infty e^{-\xi x - \mu x(1-g^*(\delta))} \sum_{i=1}^{m} p_i \lambda_i e^{-\lambda_i x} r_i^*(\varepsilon) \sum_{n=1}^{\infty} F_n(x) q^{n-1} p[(1-e^{-\theta x}) v_1^*(\eta) + e^{-\theta x} v_2^{*n}(\eta)] dx$$

$$+ \int_0^\infty e^{-\xi x - \mu x(1-g^*(\delta))} \sum_{i=1}^{m} p_i (1-e^{-\lambda_i x}) r_i^*(\varepsilon) \sum_{n=1}^{\infty} f_n(x) q^{n-1} p[(1-e^{-\theta x}) v_1^*(\eta) + e^{-\theta x} v_2^{*n}(\eta)] dx (15)$$

Also,

$$f^*(\xi,\eta,\varepsilon,\delta)$$

$$=\sum_{i=1}^{m} p_i p\lambda_i r_i^*(\varepsilon) v_1^*(\eta)\left\{\frac{f^*(\chi_i)}{\chi_i(1-qf^*(\chi_i))}-\frac{f^*(\chi_i+\theta)}{(\chi_i+\theta)(1-qf^*(\chi_i+\theta))}\right\}$$

$$+\sum_{i=1}^{m} p_i p\lambda_i r_i^*(\varepsilon) v_2^*(\eta)\frac{f^*(\chi_i+\theta)}{(\chi_i+\theta)(1-qv_2^*(\eta)f^*(\chi_i+\theta))}$$

$$+\sum_{i=1}^{m} p_i p r_i^*(\varepsilon) v_1^*(\eta)\left\{\begin{array}{l}\dfrac{f^*(\chi)}{(1-qf^*(\chi))}-\dfrac{f^*(\chi+\theta)}{(1-qf^*(\chi+\theta))}\\ -\dfrac{f^*(\chi_i)}{(1-qf^*(\chi_i))}+\dfrac{f^*(\chi_i+\theta)}{(1-qf^*(\chi_i+\theta))}\end{array}\right\}$$

$$+\sum_{i=1}^{m} p_i p r_i^*(\varepsilon) v_2^*(\eta)\left[\begin{array}{l}\dfrac{f^*(\chi+\theta)}{(1-qv_2^*(\eta)f^*(\chi+\theta))}\\ -\dfrac{f^*(\chi_i+\theta)}{(1-qv_2^*(\eta)f^*(\chi_i+\theta))}\end{array}\right] \qquad (16)$$

Here χ_i and χ are as defined in equation (5).
Since there is only change in the recruitment pattern of employees to fill up the manpower loss, $E(X), E(\hat{R})$ *and* $E(\hat{S})$ remain the same as those of model 1. The Laplace transform of $(X\hat{V})$ *is* $f^*(\xi,\eta,0,0)$ *is given by*

$$f^*(\xi,\eta,0,0)$$

$$=\sum_{i=1}^{m} p_i p v_1^*(\eta)\left\{\begin{array}{l}\dfrac{f^*(\xi)}{(1-qf^*(\xi))}-\dfrac{f^*(\xi+\theta)}{(1-qf^*(\xi+\theta))}\\ -\dfrac{f^*(\xi+\lambda_i)}{(1-qf^*(\xi+\lambda_i))}\left(\dfrac{\xi}{\xi+\lambda_i}\right)\\ +\dfrac{f^*(\xi+\lambda_i+\theta)}{(1-qf^*(\xi+\lambda_i+\theta))}\left(\dfrac{\xi+\theta}{\xi+\lambda_i+\theta}\right)\end{array}\right\}$$

$$+\sum_{i=1}^{m} p_i p v_2^*(\eta)\left\{\begin{array}{l}\dfrac{f^*(\xi+\theta)}{(1-qf^*(\xi+\theta)v_2^*(\eta))}\\ -\dfrac{f^*(\xi+\lambda_i+\theta)}{(1-qf^*(\xi+\lambda_i+\theta)v_2^*(\eta))}\left(\dfrac{\xi+\theta}{\xi+\lambda_i+\theta}\right)\end{array}\right\} \qquad (17)$$

$$E(\hat{V}) = -\frac{\partial}{\partial \eta} f^*(\xi,\eta,0,0) \mid \xi = \eta = 0 \ \ gives$$

$$E(\hat{V}) =$$

$$\sum_{i=1}^{m} p_i E(V_1)\left[1 - \frac{pf^*(\theta)}{(1-qf^*(\theta))} + \frac{pf^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))}\left(\frac{\theta}{\lambda_i+\theta}\right)\right]$$

$$+\sum_{i=1}^{m} p_i pE(V_2)\left[\frac{f^*(\theta)}{(1-qf^*(\theta))^2} - \frac{f^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}\left(\frac{\theta}{\lambda_i+\theta}\right)\right] \quad (18)$$

The product moment $E(X\hat{V})$ can be seen as

$$E(X\hat{V}) = \frac{\partial^2}{\partial\xi\partial\eta} f^*(\xi,\eta,0,0) \mid \xi = \eta = 0$$

After simplification it may be obtained as follows.

$$E(X\hat{V}) =$$

$$\sum_{i=1}^{m} p_i pE(V_1)\left[\begin{array}{l} \dfrac{E(F)}{p^2} + \dfrac{f^{*'}(\theta)}{(1-qf^*(\theta))^2} + \dfrac{f^*(\lambda_i)}{(1-qf^*(\lambda_i))}\left(\dfrac{1}{\lambda_i}\right) \\ -\dfrac{f^{*'}(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}\left(\dfrac{\theta}{\lambda_i+\theta}\right) \\ -\dfrac{f^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))}\left(\dfrac{\lambda_i}{(\lambda_i+\theta)^2}\right) \end{array}\right]$$

$$+\sum_{i=1}^{m} p_i pE(V_2)\left[\begin{array}{l} -\dfrac{f^{*'}(\theta)(1+qf^*(\theta))}{(1-qf^*(\theta))^3} \\ +\dfrac{f^{*'}(\lambda_i+\theta)(1+qf^*(\lambda_i+\theta))}{(1-qf^*(\lambda_i+\theta))^3}\left(\dfrac{\theta}{\lambda_i+\theta}\right) \\ +\dfrac{f^*(\lambda_i+\theta)}{(1-qf^*(\lambda_i+\theta))^2}\left(\dfrac{\lambda_i}{(\lambda_i+\theta)^2}\right) \end{array}\right] \quad (19)$$

Using the formula $Cov(X,\hat{V}) = E(X\hat{V}) - E(X)E(\hat{V})$

and using equations (7),(18) and (19) $Cov(X,\hat{V})$ may be written.

4. Numerical Examples

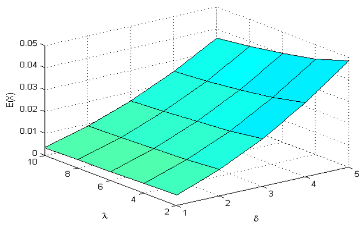
4.1 Numerical values for Model 1

Let the parameters and expected values of random variables stated in the assumptions of model 1 be assumed as follows:

(i) Expected departure time of employees E(F)= 10,
(ii) The probability of failure p=0.5,
(iii) Expected repair time of the machine E (R_i) =5, 10, 15, 20, 25.
(iv) Expected recruitment time E(V)=5
(v) The failure parameter is λ=2, 4, 6, 8, 10
(vi) The exponential density parameter is δ =1, 2, 3, 4, 5
(vii) The production time distribution parameter μ = 1
(viii) The expected sales time E (G) = 2.

λ/δ	1	2	3	4	5
2	40.0048	40.0121	40.0218	40.0339	40.04841
4	40.0044	40.0109	40.0196	40.0305	40.04359
6	40.004	40.0101	40.0181	40.0282	40.04033
8	40.0038	40.0095	40.017	40.0265	40.03788
10	40.0036	40.009	40.0162	40.0251	40.03591

Table (i): E(X)

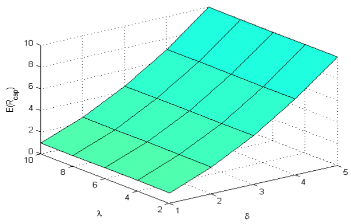


Graph (i): E(X)

From the table and graph it is observed that when λ increases the expected operation time E(X) decreases and when δ increases, E(X) increases.

λ/δ	1	2	3	4	5
2	1	2.5	4.5	7	10
4	1	2.5	4.5	7	10
6	1	2.5	4.5	7	10
8	1	2.5	4.5	7	10
10	1	2.5	4.5	7	10

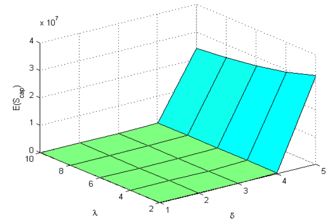
Table (ii): $E(\hat{R})$



Graph (ii): $E(\hat{R})$

From the table and graph it is observed that when both λ and δ increase, the expected repair time increases.

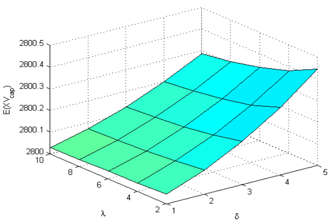
λ/δ	1	2	3	4	5
2	0.77464	63.1328	5052.17	404176	32334059
4	0.6974	56.838	4548.43	363876	29110121
6	0.64526	52.5887	4208.39	336672	26933797
8	0.60608	49.3952	3952.83	316228	25298248
10	0.57463	46.832	3747.71	299818	23985453



Table(iii): $E(\hat{S})$ **Graph (iii):** $E(\hat{S})$

From the table and graph it is observed that when λ increases the expected sales time decreases and when δ increases the expected sales time increases.

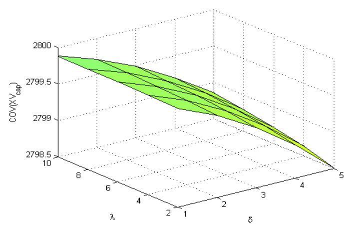
λ/δ	1	2	3	4	5
2	2800.04	2800.11	2800.2	2800.31	2800.445
4	2800.04	2800.09	2800.17	2800.27	2800.379
6	2800.03	2800.08	2800.15	2800.24	2800.338
8	2800.03	2800.08	2800.14	2800.22	2800.308
10	2800.03	2800.07	2800.13	2800.2	2800.286



Table(iv): $E(X,\hat{V})$ **Graph (iv):** $E(X,\hat{V})$

From the table and graph it is observed that when λ increases the expected product moment of operation time and recruitment time decreases and when δ increases the expected product moment of operation time and recruitment time increases.

λ/δ	1	2	3	4	5
2	2799.85	2799.63	2799.33	2798.96	2798.509
4	2799.86	2799.66	2799.39	2799.05	2798.636
6	2799.87	2799.68	2799.43	2799.11	2798.724
8	2799.88	2799.7	2799.46	2799.16	2798.793
10	2799.88	2799.71	2799.48	2799.19	2798.85



Table(v): $Cov(X,\hat{V})$ **Graph(v)** $Cov(X,\hat{V})$

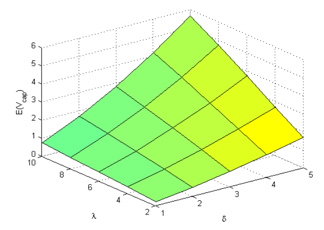
From the table and graph it is observed that when λ increases the covariance increases and when δ increases the covariance decreases.

4.2 Numerical values for Model 2

(i) Expected departure time of employees E(F)= 10,
(ii) The probability of failure q=0.75,
(iii) The machine attended by manpower with probability p_i=0.25, i=1, 2, 3….,10
(iv) Expected repair time of the machine E (R_i) =5, 10, 15, 20, 25.

(v) Expected recruitment time when recruitments are done together is $E(V_1)=2$
(vi) Expected recruitment time when recruitments are done one by one is $E(V_2)=5$
(vii)The failure parameter is λ=2, 4, 6, 8, 10
(viii)The exponential density parameter is δ =1, 2, 3, 4, 5
(ix)The production time distribution parameter μ = 1
(x)The expected sales time E (G) = 2
(xi) The threshold has exponential distribution with parameter θ=5

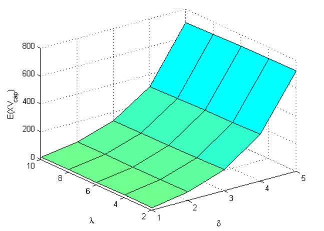
λ/δ	1	2	3	4	5
2	0.23446	0.51996	0.85649	1.24405	1.682639
4	0.37913	0.83781	1.37604	1.99382	2.691147
6	0.51327	1.14307	1.88938	2.75221	3.731569
8	0.63518	1.42666	2.37444	3.47851	4.738887
10	0.74459	1.68507	2.82147	4.15376	5.681958



Table(vi): $E(\hat{V})$ **Graph(vi):** $E(\hat{V})$

From the table and graph it is observed that when λ increases the expected recruitment time increases and when δ increases the expected recruitment time also increases.

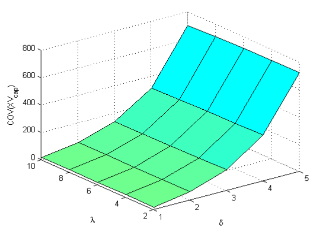
λ/δ	1	2	3	4	5
2	19.2711	61.904	151.61	337.213	723.3532
4	18.7221	60.3223	148.282	331.971	722.1826
6	18.2129	58.8416	145.085	326.472	717.722
8	17.7387	57.4521	142.024	320.863	710.9784
10	17.2975	56.1509	139.113	315.274	702.7486



Table(vii): $E(X,\hat{V})$ **Graph(vii):** $E(X,\hat{V})$

From the table and graph it is observed that when λ increases the expected product moment of operation and recruitment time decreases and when δ increases the expected product moment of operation time and recruitment time increases.

λ/δ	1	2	3	4	5
2	19.2699	61.8977	151.592	337.17	723.2717
4	18.7204	60.3132	148.255	331.91	722.0653
6	18.2108	58.83	145.051	326.395	717.5715
8	17.7363	57.4386	141.984	320.771	710.7989
10	17.2948	56.1358	139.067	315.17	702.5445



Table(viii): $Cov(X,\hat{V})$ **Graph(viii)** $Cov(X,\hat{V})$

From the table and graph it is observed that when λ increases the covariance decreases and when δ increases the covariance increases.

5. Conclusion

Manpower planning models with different recruitment patterns have been treated assuming the machine producing products having hyper exponential life time pattern. Repair times of the machine, recruitment times of employees and sales time of products have been assumed to have general distributions. In model 1, the vacancies caused by employee's departure are filled up one by one. It is observed that when the departure rate of employees increases, the expected operation time decreases. The product moment of operation time and recruitment time increases when the departure rate of employees increases. The covariance of operation time and recruitment time increases when the inter departure rate of employees increases. In model 2, when the operation time is more than a threshold, the recruitments are done altogether and when it is less than the threshold, the recruitments are done one by one. It is observed that when the inter departure rate of employees increases the expected recruitment time, the product moment of operation time and recruitment time, covariance of operation time and recruitment time decreases.

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