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OPTIMAL N-POLICY FOR UNRELIABLE SERVER QUEUE WITH IMPATIENT CUSTOMER AND VACATION INTERRUPTION

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Abstract

This study deals with animpatient behavior of the customers for unreliable server queue under N-policy and vacation interruption. N-policy states that the server is turned on only when there are 'N' or more customers encountered in the system. Further, we assume two types of vacation namely (i) working vacation and (ii) vacation interruptions. The working vacation queueing models are those wherein the server works at a lower rate rather than stopping the service completely during the dormant period. In vacation interruption, it is assumed that the server can come back to the normal working level immediately if he finds at least one customer waiting in the queue for its service. The server is subject to random breakdowns while rendering service to the customers. The broken-down and repair times of the server are assumed to be exponentially distributed. Using the recursive method, we obtain various performance measures of the concerned queueing system. Special cases are deduced in order to match our results with the existing results. Moreover, cost analysis of the model is also carried out. To examine the effect of different parameters on various performance indices, the numerical results are provided.

Key Words: Unreliable Server, N-Policy, Working Vacation, Vacation Interruption, Balking, Reneging, Average Cost, Queue Size.

1. Introduction

Queueing models have played a significant role in day-to-day as well as industrial congestion situations such as computer systems, communication networks, manufacturing systems, transportation systems, etc. In many queueing systems, when the server becomes unavailable for random duration of time and wants to utilize his time for doing some secondary task, then there is a queue, termed as queue with working vacation. In this investigation, we have introduced vacation interruption policy according to this during the vacation period, the server can return back to the normal working level if the large number of the customers accumulated in the system, which means the server may not accomplish a complete vacation. Many researchers have studied the vacation queueing models in different frame-works. Servi and Finn (2002) introduced a class of semi-vacation policy wherein the server does not stop service completely during a vacation but continues to serve the customers at lower rate. A $M^X/H_2/1$ queueing system with un-reliable server and vacations was considered by Sharma (2010). Working vacation queue with service interruption and mulit-optional repair was investigated by Jain et al. (2011). Moreover, Jain et al. (2013) dealt with an unreliable server queue with multiple optional services and multiple optional vacations. Very recently, Rajadurai et al. (2017) considered a single server queueing system with multiple working vacations and vacation interruption wherein an arriving customer may balk the system at some particular times.

Over recent decade, an increasing attention can be seen in queueing scenario to control the queue by applying the concept of N-policy. The concept of N-policy is most commonly used for controlling service. This has been widely accepted due to their applicability for modeling purpose of any production and manufacturing system as well as computer and telecommunication system. The so called N-policy states that whenever 'N' or more customers are present in the system, the sever is turned on and off when the system becomes empty. The concept of the N-policy was first introduced by Yadin and Naor (1963). Ke and Wang (2002) focused on a single removable server queueing system with finite capacity operating under N policy. They have provided a recursive approach for developing the steady-state probability distributions of the number of customers in the system. Jain et al. (2012) analyzed unreliable M/M/2/K queueing system under N and F-policy with multi optional phase repair and startup. Moreover, strategic behavior was examined by Wang et al. (2017) in an M/M/1 constant retrial queue with the N-policy.

With the advancement of technology, performance modeling is one of the important issues that affect the design, development, configuration and modification of any real time system. Unreliable server makes negative impact on performance of any system; therefore some measures are to be taken to maintain a desired grade of service. In many practical applications, the server may fail and requires repair (renewal). From a practical point of view, several authors have studied different models that describe the failure of a server and its renewal (repairs) as well as the rules of servicing a customer who finds a server in a broken down condition. In many waiting line systems, the role of servers is performed by mechanical/electronic devices, such as computers, pallets, ATM, traffic lights and many more; all of these are subject to accidental/random failures until the failed server is repaired, it may cause a halt in service. A queue with Bernoulli vacation and second optional service was studied by Jain et al. (2012) wherein breakdowns occur randomly at any instant while servicing the customers. The M/M/N repairable queueing system was considered by Lv and Li (2013).

Impatient behavior of the customer becomes the burning problem of private as well as government sector enterprises. Impatience is the most prominent feature during individuals want to experience service but need to queueing, in fact, we always feel anxious and impatient during waiting for a service in real life. The assumptions of impatience in the study of queueing situations make the model more realistic and flexible. Queue with reneging was firstly studied by Haight (1959). Further, Ancker and Gafarian (1963) analyzed queueing system with balking and reneging concept. A survey article on queueing system with impatient customer was provided by Wang et al. (2010).The balking behavior of customers in the single-server queue with generally distributed service and vacation times has been examined by Antonis et al. (2011). Recently, optimal balking strategies in single-server Markovian queues with multiple vacations and N-policy was provided by Sun et al. (2016).

In this investigation, the main objective of the research is to determine the queueing performance of unreliable server queue under N-policy with vacation interruption. Further, balking and reneging behavior of the customers are also taken into consideration. In this study, recursive method is used to determine the stationary system and queue lengths. The rest of this paper is organized as follows. In section 2, we give a description of the queueing model. In section 3, the governing equations of the model are constructed to formulate the model using the appropriate birth-death rates. In section 4, the recursive method is provided to obtain the steady state probabilities. Based on the performance analysis, in next section 5, we provide the various performance measures of the system and also formulate a cost model to determine the optimal threshold parameter. Moreover, section 6 is devoted for some special cases of the model. In next section 7, some numerical results are presented to demonstrate how the various parameters of the model influence the behavior of the system. Finally, conclusions are given in section 8.

2. Model Description

Consider an unreliable M/M/1/K/WV queue with vacation interruption under N-policy wherein the customers may balk or renege. The following notations and assumptions are used to formulate the queueing model:

Notations:

λ : Arrival rate
μ_v : Service rate during working vacation period
μ_b : Service rate during busy period
α : Failure rate
β : Repair rate
$b_i(1 \leq i \leq K-1)$: Joining probability
Ψ : Reneging parameter
N : Threshold Parameter
K : Finite capacity of the system
$P_{0,n}(0 \leq n \leq K)$: Steady state probability that there are 'n' customers in the system whenthe server is on working vacation state.
$P_{1,n}(1 \leq n \leq K)$: Steady state probability that there are 'n' customers in the system when the server is in busy state
$P_{2,n}(1 \leq n \leq K)$: Steady state probability that there are 'n' customers in the system when the server is under repair state.

Assumptions

- The server is turned on when 'N' or more customers are accumulated in the system and it turns off when the queue becomes empty.
- The customers arrive at service station singly according to the Poisson fashion with arrival rate λ. Upon arrival, a customer has a choice to join the queue with probability $b_n(0 \leq n \leq K-1)$ or may depart from the system forever with complementary probability $1-b_n$.Here, $b_0=1$ and $b_K=0$.

- The server provides the service to all arriving customers with rate μ_b on the first come first serve (FCFS) basis. After joining the queue, each customer will wait a certain length of time say 'T' for his turn in the queue. Thecustomers get impatient and leave the queue without receiving service. This time is exponentially distributed with mean 1/Ψ. As, arrival and departure of an impatient customerare independent. Therefore, the average rate of a reneged customer is (n-1) Ψ, $(1 \leq n \leq K)$.
- During the working vacation, if there are 'N' or more customers waiting in the queue then the server switches to normal working level with rate μ_b from rate μ_v. This means that the server may not accomplish the complete working vacation which known as vacation interruption. On the other hand, if the queue size is lesser than 'N' then server starts a new vacation. The vacation time is assumed to be exponentially distributed with mean $1/\theta$.
- During busy state, the server may breakdown with rate α. As the breakdown occur, it is immediately sent for repairing at repair facility with repair rate β. After repairing, the server renews and works properly as before failure.
- The inter arrival time, service time, repair time and vacation time are assumed to be exponentially distributed.

3. Governing Equations

In this section, we provide the governing equations of the queueing model using the Markov theory as follows:

***I.* For working vacation state**

$$\lambda b_0 P_{0,0} = \mu_v P_{0,1} + \mu_b P_{0,1} \quad (1)$$

$$(\lambda b_n + \mu_v + (n-1)\psi)P_{0,n} = (\mu_v + n\psi)P_{0,n+1} + \lambda b_{n-1}, \quad 1 \leq n \leq N-1 \quad (2)$$

$$(\lambda b_n + \mu_v + (n-1)\psi + \theta)P_{0,n} = \lambda b_{n-1}P_{0,n-1} + n\psi P_{0,n+1}, \quad N \leq n \leq K-1 \quad (3)$$

$$(\mu_v + (K-1)\psi + \theta)P_{0,K} = \lambda b_{K-1}P_{0,K-1} \quad (4)$$

II. For busy state

$$(\lambda b_n + \mu_b + (n-1)\psi + \alpha)P_{1,n} = (\mu_b + n\psi)P_{1,n+1} + \lambda b_{n-1}P_{1,n-1} + \beta P_{2,n}, \quad 1 \leq n \leq N-1 \quad (5)$$

$$(\lambda b_n + \mu_b + (n-1)\psi + \alpha)P_{1,n} = (\mu_b + n\psi)P_{1,n+1} + \lambda b_{n-1}P_{1,n-1} + \mu_v P_{0,n+1} + \theta P_{0,n} + \beta P_{2,i}, \; N \leq n \leq K-1 \quad (6)$$

$$(\mu_b + (K-1)\psi + \alpha)P_{1,K} = \lambda b_{K-1}P_{1,K-1} + \theta P_{0,K} + \beta P_{2,K} \quad (7)$$

III. For repair state

$$(\lambda b_n + \beta)P_{2,n} = \lambda b_{n-1}P_{2,n-1} + \alpha P_{1,n}, \quad 1 \leq n \leq K-1 \quad (8)$$

$$\beta P_{2,K} = \lambda b_{K-1}P_{2,K-1} + \alpha P_{1,K} \quad (9)$$

4. Recursive Method

Recursive method is a very powerful tool for solving the strategy that has few direct counterparts in the real world. That strategy, called recursion, is defined as any solution technique in which large problems are solved by reducing them to smaller problems of the same form. A general idea behind this is that if we have a complex situation that can be broken into basic structure as the original each time, this method calls itself; the situation becomes simpler until it converges to the simplest possible case. At this point, the recursive method will cease calling itself and the recursion terminates. Then the last recursive call will finish processing and return back. After that next to last recursive call finishes and soon until the first call of the method terminates and the process completes. Therefore, in this section, we apply recursive method for obtaining the steady state probabilities in the following theorem.

Theorem 1: The steady-state probabilities for working vacation, busy and repair states, respectively are given as:

$$P_{0,n} = \frac{\Theta_n}{\sum_{i=0}^{K}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 0 \le n \le \mathrm{K} \tag{10}$$

$$P_{1,n} = \frac{\xi_n}{\sum_{i=0}^{K}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 1 \le n \le K \tag{11}$$

$$P_{2,n} = \frac{\zeta_n}{\sum_{i=0}^{K}\Theta_i + \sum_{i=1}^{K}\xi_i + \sum_{i=1}^{K}\zeta_i}, \qquad 1 \le n \le K \tag{12}$$

where

$$\Theta_K = 1, \qquad \Theta_{K-1} = \frac{\theta + \mu_v + (K-1)\psi}{\lambda b_{K-1}},$$

$$\Theta_n = \frac{(\lambda b_{n+1} + \mu_v + \theta + n\psi)\Theta_{n+1} - (n+1)\psi\Theta_{n+2}}{\lambda b_n}, n = K-2, K-3, \ldots, N-1$$

$$\Theta_n = \frac{(\lambda b_{n+1} + \mu_v + n\psi)\Theta_{n+1} - (\mu_v + (n+1)\psi)\Theta_{n+2}}{\lambda b_n}, n = N-2, N-3, \ldots, 0,$$

$$\xi_0 = 0, \qquad \xi_1 = \frac{\lambda b_0 \Theta_0 - \mu_v \Theta_1}{\mu_b},$$

$$\xi_n = \frac{(\lambda b_{n-1} + \mu_b + (\mathrm{n}-2)\psi) + \alpha)\xi_{n-1} - \lambda b_{n-2}\xi_{n-2} - \beta\zeta_{n-1}}{\mu_b + (\mathrm{n}-1)\psi}, \qquad n = 2, 3, \ldots, N$$

$$\xi_n = \frac{(\lambda b_{n-1} + \mu_b + (\mathrm{n}-2)\psi) + \alpha)\xi_{n-1} - \lambda b_{n-2}\xi_{n-2} - \mu_v\Theta_n - \theta\Theta_{n-1} - \beta\zeta_{n-1}}{\mu_b + (\mathrm{n}-1)\psi},$$

$$n = N+1, N+2, \ldots, K$$

$$\zeta_0 = 0, \; \zeta_n = \frac{(\lambda b_{n+1} + \beta)\zeta_{n+1} - \alpha\xi_{n+1}}{\lambda b_n}, \qquad n = 1, 2, \ldots, K.$$

Proof: For proof, see Appendix A.I.

5. Performance Measures

In this section, we establish expressions for some important performance measures which are helpful for the system designer and analyst for developing concerned queueing system. These measures are as follows:

(I) The server's states: The long run probabilities of the server being in different states are obtained as follows:

- The probability of the server on working vacation state is
  $P(WV) = \sum_{n=0}^{K} P_{0,n}$ (13)
- The probability of the server on busy state is
  $P(B) = \sum_{n=1}^{K} P_{1,n}$ (14)
- The probability of the server under repair state is
  $P(R) = \sum_{n=1}^{K} P_{2,n}$ (15)

(II) Queue Size

- The average queue length is
  $EL_q = \sum_{n=1}^{K} (n-1)(P_{0,n} + P_{1,n} + P_{2,n})$ (16)
- The average system length is
  $EL_s = \sum_{n=1}^{K} n(P_{0,n} + P_{1,n} + P_{2,n})$ (17)
- The throughput is given as
  $TP = \sum_{n=1}^{K} \left(\mu_v P_{0,n} + \mu_b P_{1,n}\right)$ (18)
- The expected delay is obtained as
  $ED = \frac{EL_s}{TP}$ (19)
- The average balking rate is
  $BR = \sum_{n=1}^{K} \lambda(1-b_n)(P_{0,n} + P_{1,n} + P_{2,n})$ (20)
- The average reneging rate is given as
- $RR = \sum_{n=1}^{K} (n-1)\psi(P_{0,n} + P_{1,n} + P_{2,n})$ (21)
- The average rate of lost customer is calculated as

$AR_{LC} = BR + RR$(22)

(II) Waiting Time

- The average waiting time in the queue is
  $EW_q = \frac{EL_q}{\lambda}$ (23)
- The average waiting time in the system is calculated as

  $EW_s = \frac{EL_s}{\lambda}$ (24)

(III) Cost Function

System managers are always interested for minimizing the total average cost. For this purpose, we construct the cost function in terms of appropriate performance measures and corresponding cost elements for determining the optimal threshold parameter.

$TC = C_H EL_S + C_{WV} P(WV) + C_B P(B) + C_R P(R) + C_{LR}$ (25)

where

C_H: Holding cost per unit time per customer present in the system
C_{WV}: Cost per unit time during working vacation period
C_B: Cost per unit time during busy period
C_R: Cost per unit time under repair period
C_{LR} : Cost per unit time when the customers balks or reneges

6. Special Cases

In this section, we provide some special cases of our model. Some specific cases are follows:

Case 1: When $\boldsymbol{\alpha = 0, \beta = 0}$, N=1 and θ=0, our model facilitates results for M/M/1/K queue with balking, reneging and multiple working vacations which was studied by VijayaLaxmi et al. (2013).

Case 2: When $\boldsymbol{\alpha = 0, \beta = 0, \psi = 0, b_n = 1, 0 \leq n \leq K \text{ and } N = 1}$ then our model reduces to the model considered by Li and Tian (2007).

Case 3: When $\boldsymbol{\alpha = 0, \beta = 0}$, N=1 and μ_v=0, then the results are tally with the results analyzed by Yue et al. (2006).

7. Numerical Results

In this section, we have described various numerical results. For this purpose, the coding of the program has been done in mathematical software 'MATLAB'. Here, we illustrate the effect of system parameters on various performance measures such as probability of the server during working vacation, busy, repair states, average queue length (EL_q), throughput (TP), etc. The numerical results are summarized in Tables 1 & 2 and also visualized in Figs 1 (a) to 5(b). The default parameters for Tables 1 & 2 and Figs. 1(a) to 5(b) are chosen as λ=0.7, μ_v=0.5, μ_b=2, α=0.1, β=2, θ=0.01, ψ=1, b_1=0.1, b_2=0.2, b_3=0.3, b_4=0.4, b_5=0.5, N=3, K=5. Further, the various cost elements are chosen in form of set 1 as C_H=10, C_{WV}=2, C_B=25, C_R=10, C_{LR}=15 for determining the total average cost.

In Tables 1 and 2, we examine the effect of arrival rate, service rate, failure rate and repair rate respectively on various system characteristics. As, we increase the arrival rate (service rate), the decreasing (increasing) pattern is followed by P(WV) while P(B), P(R) and AR_{LC} increase (decrease). On increasing the values of α(β), P(B) and P(R) increase (decrease) whereas other parameters show the reverse effect. It is clear that P(WV), P(B) and AR_{LC} increase by increasing the values of ψ which is quite obvious. And reverse trend is followed by P(R) for increasing values of ψ.

λ	ψ=1				ψ=1.3			
	P(WV)	**P(B)**	**P(R)**	**AR_{LC}**	**P(WV)**	**P(B)**	**P(R)**	**AR_{LC}**
0.3	0.913	0.015	0.027	2.198	0.970	0.058	0.014	2.453
0.4	0.854	0.068	0.056	2.665	0.888	0.089	0.033	3.031
0.5	0.829	0.074	0.091	3.053	0.847	0.098	0.057	3.511
0.6	0.816	0.087	0.131	3.381	0.826	0.120	0.085	3.928
0.7	0.807	0.099	0.170	3.654	0.813	0.121	0.116	4.290

μ_b	P(WV)	P(B)	P(R)	AR_{LC}	P(WV)	P(B)	P(R)	AR_{LC}
1.0	0.768	0.066	0.165	3.489	0.769	0.126	0.112	4.061
1.5	0.793	0.036	0.169	3.594	0.794	0.090	0.115	4.207
2.0	0.807	0.021	0.170	3.654	0.813	0.069	0.116	4.290
2.5	0.815	0.012	0.171	3.694	0.826	0.055	0.117	4.345
3.0	0.820	0.007	0.172	3.721	0.835	0.046	0.118	4.385

Table 1: Effect on various performance measures by varying λ, μ_b andψ

	ψ=1				ψ=1.3			
α	P(WV)	P(B)	P(R)	AR_{LC}	P(WV)	P(B)	P(R)	AR_{LC}
0.1	0.807	0.021	0.170	3.654	0.813	0.069	0.116	4.29
0.2	0.804	0.021	0.173	3.645	0.810	0.069	0.120	4.27
0.3	0.802	0.021	0.175	3.635	0.806	0.069	0.124	4.26
0.4	0.799	0.021	0.178	3.625	0.802	0.068	0.128	4.24
0.5	0.797	0.022	0.180	3.616	0.798	0.068	0.132	4.23
β	P(WV)	P(B)	P(R)	AR_{LC}	P(WV)	P(B)	P(R)	AR_{LC}
1.0	0.804	0.021	0.173	3.645	0.810	0.069	0.120	4.277
2.0	0.807	0.021	0.170	3.654	0.813	0.069	0.116	4.290
3.0	0.808	0.021	0.170	3.657	0.815	0.069	0.115	4.295
4.0	0.808	0.021	0.169	3.659	0.815	0.069	0.114	4.297
5.0	0.808	0.020	0.169	3.660	0.816	0.069	0.114	4.299

Table 2: Effect on various performance measures by varying α, β and ψ.

Figs. 1(a) to 2(b) depict the behavior of EL_q for different values of ψ by varying λ, μ_b, α and β. It is seen from Figs. 1 (a-b) that on increasing the values of λ(μ_b), the average queue length increases (decreases), which is quite obvious. From Figs. 2(a-b), we note that the trend of EL_q is decreasing (increasing) slightly with the increasing values of α(β). Further, the decreasing trend is observed for EL_q with the increasing values of ψ.

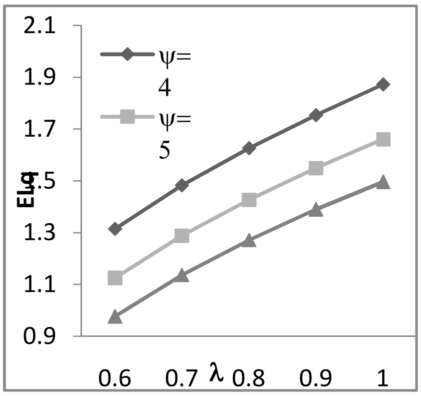

Fig. 1 (a): Effect of λ on EL_q for different values of ψ

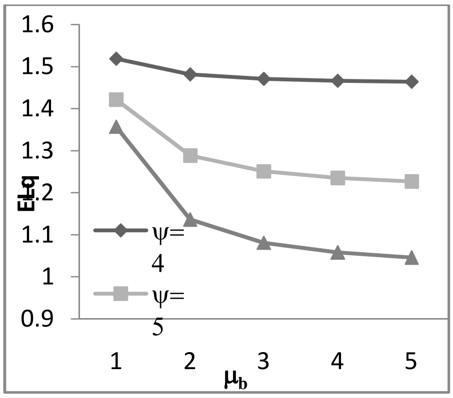

Fig. 1 (b): Effect of μ_b on EL_q for different values of ψ

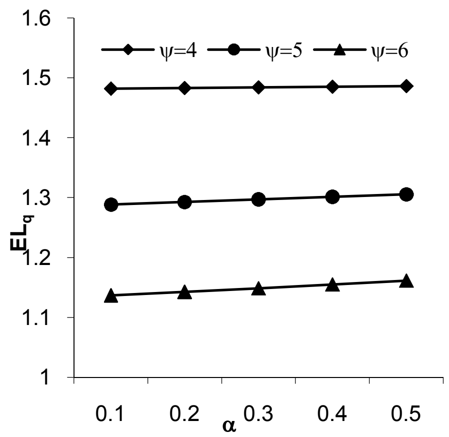

Fig. 2 (a): Effect of α on EL_q for different values of ψ

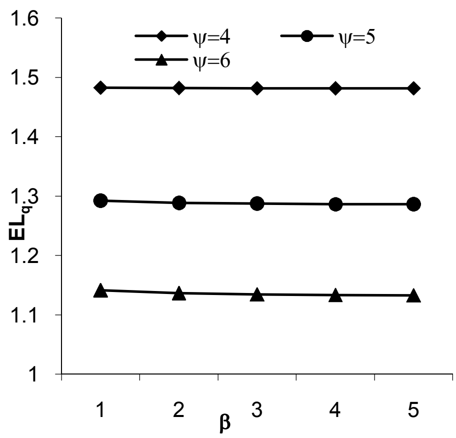

Fig. 2 (b): Effect of β on EL_q for different values of ψ

Figs. 3(a) to 4(b) visualize the trend of throughput (TP) by varying arrival rate, service rate, failure rate and repair rate, respectively for the different values of vacation rate. From Figs. 3(a-b) and 4(a-b), it can be seen easily that TP shows the decreasing pattern for the increasing values of λ, α and θ. The following trend is reversed for service rate and repair rate which tally with many real life situations.

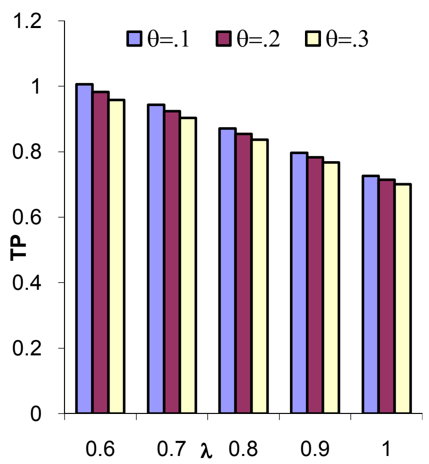

Fig. 3 (a): Effect of λ on TP for different values of θ

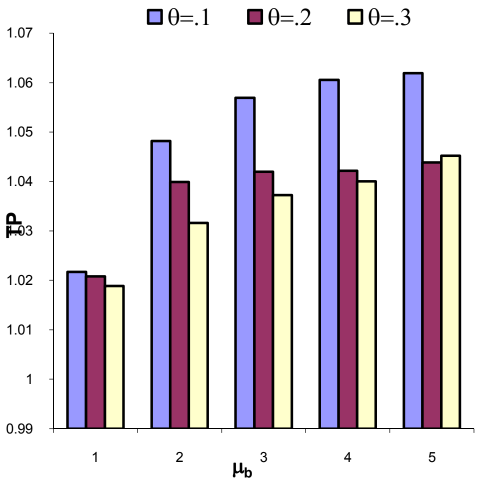

Fig. 3 (b): Effect of μ_b on TP for different values of θ

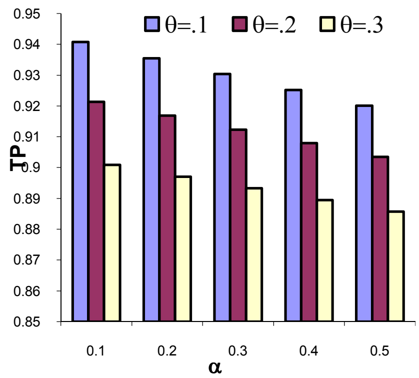

Fig. 4 (a): Effect of α on TP for different values of θ

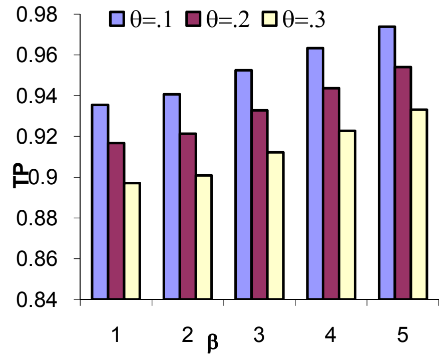

Fig. 4 (b): Effect of β on TP for different values of θ

Figs. 5 (a-d) exhibit the combined effect of arrival rate (λ), service rate (μ_b), failure rate (α), repair rate (β), along with threshold parameter (N) on total average cost (TC) for set 1 of different cost elements. It can be easily observed that TC first decreases and then increases sharply for increasing values of λ, μ_b, α and β for different cost elements.

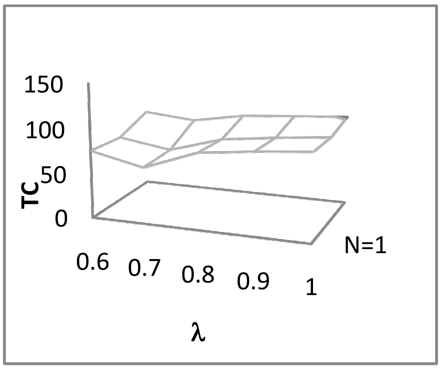

Fig. 5 (a): Effect of N on TC for Set 1 by varying λ

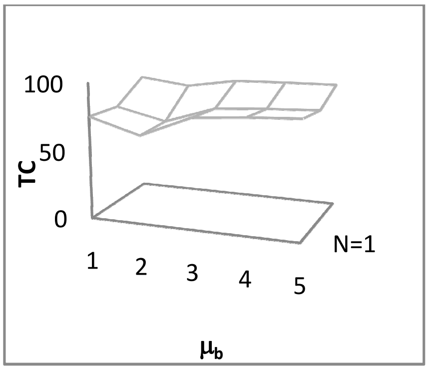

Fig. 5 (b): Effect of N on TC for set 1 by varying μ_b

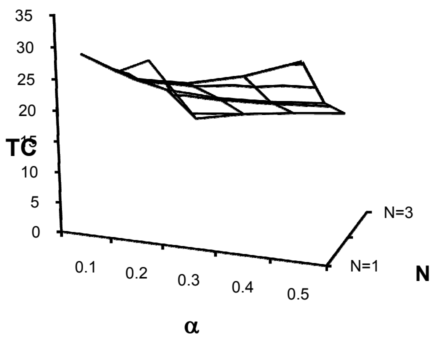

Fig. 5 (c): Effect of N on TC for Set 1 by varying α

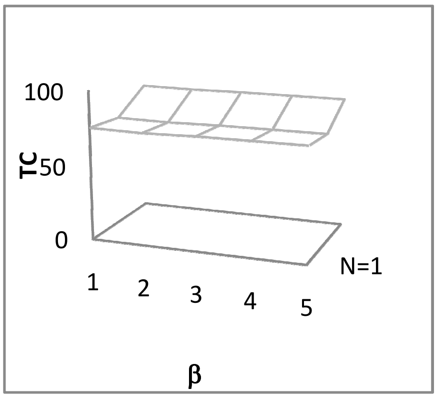

Fig. 5 (d): Effect of N on TC for set 1 by varying β

Overall, we conclude that the average queue length increases remarkably as arrival rate and breakdown rate increase. On the other hand, the service rate and repair rate of the server have played significant role in lowering down the average queue length. Moreover, throughput of the system increases rapidly as the service rate and repair rate increases during the busy state. On the basis of above results, we claim that the probability of loss customer increases as arrival rate of customer increases. In the case of vacation interruption, throughput of the concern system decreases which plays an important for controlling the queueing system. These observations are same what we expect in real time systems. The average total cost can be reduced to some extent by controlling threshold parameter.

8. Conclusion

In this paper, we have analyzed working vacation queue with vacation interruption and unreliable server under N-policy. The impatient behavior of the customers has been also incorporated in our study for making the model more realistic. The present investigation includes many features simultaneously such as (i) N-policy (ii) working vacation (iii) vacation interruption (iv) unreliable server (v) balking and (vi) reneging. Theseall realistic assumptions have not been taken together previously. Our results can be treated as performance evaluation tools for the concerned system which may be suited to many congestion situations arising in many practical applications encountered in computer and communication systems, distribution and service sectors, production and manufacturing systems, etc. The results have been obtained in the closed form using recursive approach. The numerical illustrations have been provided for validating the analytical results. The present investigation can be further extended using the concepts of bulk arrival, priority and startup rate but the analysis becomes more cumbersome.

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APPENDIX

On solving equations (1) - (9) recursively and using the normalization condition given in equation (A.1), the results of theorem 1 have been obtained easily.

$$\sum_{i=0}^{K} \Theta_i + \sum_{i=1}^{K} \xi_i + \sum_{i=1}^{K} \zeta_i = 1 \quad \text{(A.1)}$$