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EVALUATION OF RELIABILITY AND MTSF OF A PARALLEL SYSTEM WITH WEIBULL FAILURE LAWS

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Abstract

The performance of systems can be improved by providing appropriate structural design of the components. And, therefore, the basic concentration of the scholars is to identify the most suitable technique that can be used to improve the reliability of operating systems. The provision of series and parallel structures of the components in the systems has been considered as the effective one to maintain life of the systems for a considerable period with least possible costs. The reliability and mean time to system failure (MTSF) of such systems have been evaluated by the researchers with a common assumption that the failure rate of the components follows negative exponential distribution. But there are several systems in which components may have monotonic failure rates and in such cases Weibull distribution can be used due to its versatile character and relative simplicity. Here, the reliability and MTSF of a parallel system are obtained by considering Weibull failure laws. The behaviour of these measures has been examined for arbitrary values of the failure rates, operating time, shape parameter (β) and number of components. The values of these measures have also been evaluated for a special case of Weibull distribution i.e. Rayleigh distribution.

Key Words: Parallel System, Reliability, MTSF, Weibull Distribution, Rayleigh Distribution.

1. Introduction

Over the years, the main concentration of the reliability engineers and system designers is on the identification of best possible structure of the components that can be used in the systems for providing better services to the users with overall least operating cost. And, up to some extent they succeeded in it. As a result of which, we have several structures of the components in the systems which fulfil the requirements in the form of durability and cost effectiveness. It is a common practice that the parallel structure of the components is being preferred over the others in order to reduce the working stress of the system. Besides, several research papers have been written by the scholars on the theoretical evaluation of reliability measures of a system under different configurations of the components. EI-Damcese (2009) evaluated reliability and mean time to system failure of Series-Parallel system using Weibull distribution. Mustafa et al. (2012) discussed reliability equivalence factors of a general parallel system with mixture of lifetimes. Elsayed (2012) developed reliability and mean time of system failure (MTSF) of some system configurations using Exponential, Rayleigh and Weibull distributions. Nandal et al. (2015) proved that a parallel system is more reliable to use then that of a series system having constant failure rate of the components. Chauhan et al. (2016) determine reliability measures of a series system with Weibull failure laws.

Keeping in view, the wide applications of the parallel structure of the components in operating systems, here we determine reliability and mean time to system failure (MTSF) of a system by considering Weibull failure laws of the identical and non-identical components. The behaviour of these measures has been observed for arbitrary values of the failure rates of the components, operating time of the components, shape parameter and number of components used in the parallel structure. The results of these measures have also been obtained for a particular case of Weibull distribution i.e. Rayleigh distribution in order to examine the effect of various parameters.

2. Notations

R(t) = Reliability of the system,
$R_i(t)$ = Reliability of the i^{th} component
h(t)= Instantaneous failure rate of the system,
$h_i(t)$ = Instantaneous failure rate of i^{th} component
λ = Constant failure rate
T = Life time of the system,
T_i= Life time of the i^{th} component

3. System Description

Here, a parallel system of 'n' components is considered which can fail at the failure of all components. The state transition diagram is shown in Fig.: 1

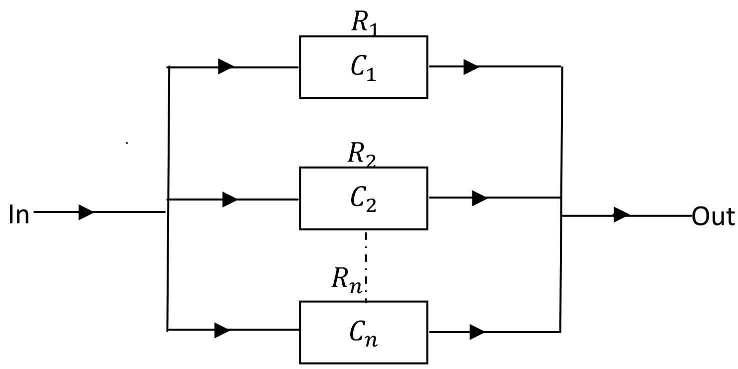

Fig. 1: A Parallel System of 'n' Components

The reliability of the System is given by
R(t) = Pr[T>t] = 1- $\prod_{i=1}^{n}[1 - R_i(t)]$ (1)
The mean time to system failure is given by
MTSF= $\int_0^\infty$[1- $\prod_{i=1}^{n}[1 - R_i(t)]$]dt (2)

4. Reliability Measures

Suppose failure rate of all the components are governed by Weibull failure law i.e. $h_i(t) = \lambda_i t^{\beta_i}$
Then, the i[th] component's reliability is given by

$$R_i(t) = e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}$$

Therefore, the system reliability is given by

$$R_s(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t)) = 1 - \prod_{i=1}^{n}\left(1 - e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)$$

$$= 1 - \left(1 - e^{-\lambda_1 \frac{t^{\beta_1+1}}{\beta_1+1}}\right)\left(1 - e^{-\lambda_2 \frac{t^{\beta_2+1}}{\beta_2+1}}\right) \dots \dots \left(1 - e^{-\lambda_n \frac{t^{\beta_n+1}}{\beta_n+1}}\right)$$

$$R_s(t)=\sum_{i=1}^{n} e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}} \sum_{i=1}^{n}\sum_{j=i+1}^{n} e^{-\left[\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1} + \lambda_j \frac{t^{\beta_j+1}}{\beta_j+1}\right]} + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n} e^{-\left[\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1} + \lambda_j \frac{t^{\beta_j+1}}{\beta_j+1} + \lambda_k \frac{t^{\beta_k+1}}{\beta_k+1}\right]} + \dots\dots + (-1)^{n+1} e^{-\sum_{i=1}^{n} \lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}} \quad (3)$$

and,

$$\text{MTSF}= \sum_{i=1}^{n} \frac{\Gamma^{1}/_{\beta_i+1}}{\left[\lambda_i(\beta_i+1)^{\beta_i}\right]^{\frac{1}{\beta_i+1}}} - \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\frac{\Gamma^{1}/_{\beta_i+1}}{\left[\lambda_i(\beta_i+1)^{\beta_i}\right]^{\frac{1}{\beta_i+1}}} \frac{\Gamma^{1}/_{\beta_j+1}}{\left[\lambda_j(\beta_j+1)^{\beta_j}\right]^{\frac{1}{\beta_j+1}}}\right) + \dots\dots + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n}\left(\frac{\Gamma^{1}/_{\beta_i+1}}{\left[\lambda_i(\beta_i+1)^{\beta_i}\right]^{\frac{1}{\beta_i+1}}} \frac{\Gamma^{1}/_{\beta_j+1}}{\left[\lambda_j(\beta_j+1)^{\beta_j}\right]^{\frac{1}{\beta_j+1}}} \frac{\Gamma^{1}/_{\beta_k+1}}{\left[\lambda_k(\beta_k+1)^{\beta_k}\right]^{\frac{1}{\beta_k+1}}}\right) + \dots + (-1)^{n+1}\sum_{i=1}^{n} \frac{\Gamma^{1}/_{\beta_i+1}}{\left[\lambda_i(\beta_i+1)^{\beta_i}\right]^{\frac{1}{\beta_i+1}}} \quad (4)$$

For identical components, we have $h_i(t) = \lambda t^{\beta}$
Then the system reliability is given by

$$R_s(t) = 1 - \left(1 - e^{-\lambda \frac{t^{\beta+1}}{\beta+1}}\right)^n$$

$$= \binom{n}{1} e^{-\lambda \frac{t^{\beta+1}}{\beta+1}} - \binom{n}{2} e^{-2\lambda \frac{t^{\beta+1}}{\beta+1}} + \binom{n}{3} e^{-3\lambda \frac{t^{\beta+1}}{\beta+1}} + \cdots \dots + (-1)^{n+1} e^{-n\lambda \frac{t^{\beta+1}}{\beta+1}}$$

$$R_s(t) = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i} e^{-i\lambda \frac{t^{\beta+1}}{\beta+1}} \quad (5)$$

$$\text{MTSF}= \int_0^{\infty} R(t)dt = \int_0^{\infty}\left[\sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i} e^{-i\lambda \frac{t^{\beta+1}}{\beta+1}}\right]dt$$

$$\text{MTSF} = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\int_0^{\infty} e^{-i\lambda \frac{t^{\beta+1}}{\beta+1}}dt = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i} \frac{\left(\Gamma\frac{1}{\beta}+1\right)}{\left[n\lambda(\beta+1)^{\beta}\right]^{\frac{1}{\beta+1}}} \quad (6)$$

Illustration

Suppose system has two components, then the system reliability is given by

$R_s(t) = 1 - \prod_{i=1}^{2}(1 - R_i(t)) = 1 - \prod_{i=1}^{2}\left(1 - e^{-\lambda_i \frac{t^{\beta_i+1}}{\beta_i+1}}\right)$

$$R_s(t) = e^{-\lambda_1 \frac{t^{\beta_1+1}}{\beta_1+1}} + e^{-\lambda_2 \frac{t^{\beta_2+1}}{\beta_2+1}} - e^{-\left[\lambda_1 \frac{t^{\beta_1+1}}{\beta_1+1} + \lambda_2 \frac{t^{\beta_2+1}}{\beta_2+1}\right]} \quad (7)$$

$$\text{MTSF}=\frac{\Gamma^{1}/_{\beta_1+1}}{[\lambda_1(\beta_1+1)^{\beta_1}]^{\frac{1}{\beta_1+1}}} + \frac{\Gamma^{1}/_{\beta_2+1}}{[\lambda_2(\beta_2+1)^{\beta_2}]^{\frac{1}{\beta_2+1}}} - \frac{\Gamma^{1}/_{\beta_1+1}}{[\lambda_1(\beta_1+1)^{\beta_1}]^{\frac{1}{\beta_1+1}}} \frac{\Gamma^{1}/_{\beta_2+1}}{[\lambda_2(\beta_2+1)^{\beta_2}]^{\frac{1}{\beta_2+1}}} \quad (8)$$

For identical components, we have $h_i(t) = \lambda t^{\beta}$

$$R_s(t) = 1 - \left(1 - e^{-\lambda \frac{t^{\beta+1}}{\beta+1}}\right)^2 = 2e^{-\lambda \frac{t^{\beta+1}}{\beta+1}} - e^{-2\lambda \frac{t^{\beta+1}}{\beta+1}} \quad (9)$$

$$\text{MTSF}= \frac{\left(\Gamma \frac{1}{\beta}+1\right)}{[\lambda(\beta+1)^{\beta}]^{\frac{1}{\beta+1}}} \left[2 - \frac{1}{2^{\frac{1}{\beta+1}}}\right] \quad (10)$$

In a similar way can obtain reliability and MTSF of a system having three or more component connected in parallel.

5. Special Case for Parameters following Weibull Distribution

Reliability and mean time to system failure (MTSF) of the system have been obtained for arbitrary values of the parameters associated with number of components (n), failure rate (λ), operating time of the component (t) and shape parameter (β). The results are shown numerically and graphically as:

n	Reliability				
	λ=0.01, t=10,β=0.1	λ=0.02, t=10,β=0.1	λ=0.03, t=10,β=0.1	λ=0.04, t=10,β=0.1	λ=0.05, t=10,β=0.1
1	0.891858524	0.795411626	0.7093946	0.6326796	0.564261
2	0.988305421	0.958143597	0.9155485	0.8650757	0.810131
3	0.998735331	0.991436667	0.9754579	0.9504395	0.917267
4	0.999863237	0.998248042	0.9928679	0.9817954	0.963949
5	0.99998521	0.99964157	0.9979274	0.9933131	0.984291
6	0.999998401	0.999926669	0.9993977	0.9975438	0.993155
7	0.999999827	0.999984997	0.9998249	0.9990978	0.997017
8	0.999999981	0.999996931	0.9999491	0.9996686	0.998700
9	0.999999998	0.999999372	0.9999852	0.9998783	0.999434
10	0.999999999	0.999999872	0.9999957	0.9999553	0.999753

Table 1: Reliability Vs No. of Components (n)

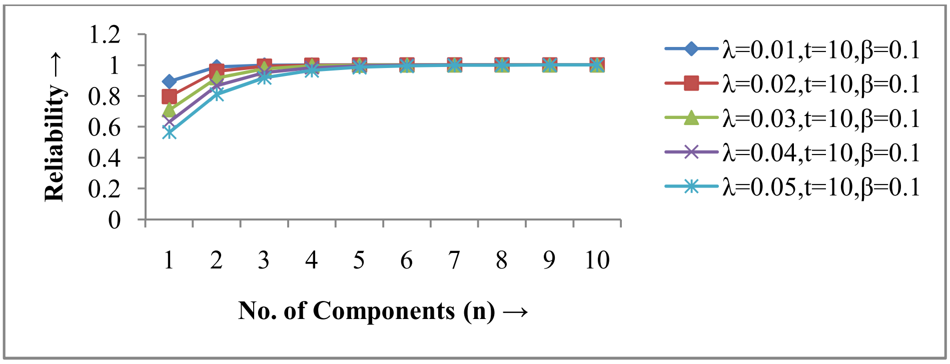

Fig. 2: Reliability Vs No. of Components (n)

n	MTSF				
	λ=0.01, t=10,β=0.1	λ=0.02, t=10,β=0.1	λ=0.03, t=10,β=0.1	λ=0.04, t=10,β=0.1	λ=0.05, t=10,β=0.1
1	69.23057	36.8667	25.50065	19.63228	16.02768
2	101.5944	54.10113	37.42169	28.80996	23.52029
3	122.5923	65.2829	45.15611	34.76449	28.38153
4	138.0924	73.53705	50.86549	39.15999	31.96999
5	150.3587	80.06909	55.3837	42.63844	34.80978
6	160.4983	85.46864	59.11856	45.51381	37.15722
7	169.1344	90.06752	62.2996	47.9628	39.15656
8	176.6518	94.0707	65.06859	50.09458	40.89693
9	183.3049	97.61361	67.51922	51.98125	42.4372
10	189.2703	100.7903	69.71656	53.67293	43.81827

Table 2: MTSF Vs No. of Components (n)

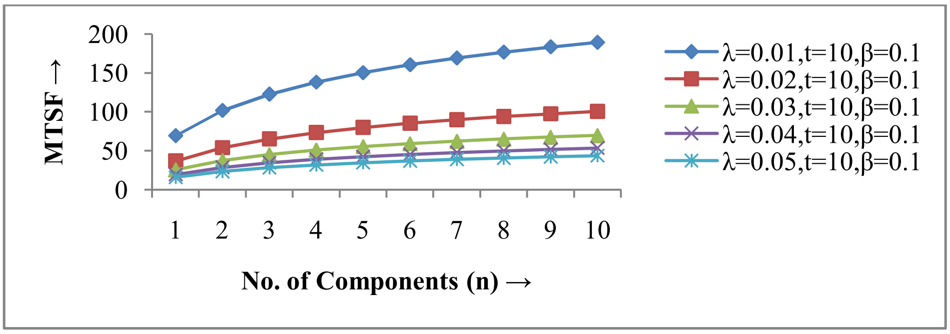

Fig. 3: MTSF Vs No. of Components (n)

n	Reliability				
	λ=0.01, t=10,β=0.1	λ=0.01, t=10,β=0.2	λ=0.01, t=10,β=0.3	λ=0.01, t=10,β=0.4	λ=0.01, t=10,β=0.5
1	0.891858524	0.876275769	0.85771644	0.83575442	0.809921
2	0.988305421	0.984692315	0.97975539	0.97302339	0.96387
3	0.998735331	0.998106068	0.99711952	0.99556921	0.9931324
4	0.999863237	0.999765675	0.99959015	0.99927226	0.9986946
5	0.99998521	0.999971008	0.99994168	0.99988047	0.9997519
6	0.999998401	0.999996413	0.99999170	0.99998037	0.9999528
7	0.999999827	0.999999556	0.99999881	0.99999678	0.999991
8	0.999999981	0.999999945	0.99999983	0.99999947	0.9999983
9	0.999999998	0.999999993	0.99999997	0.99999991	0.9999997
10	0.9999999997	0.999999999	0.99999999	0.99999999	0.9999999

Table 3: Reliability Vs No. of Components (n)

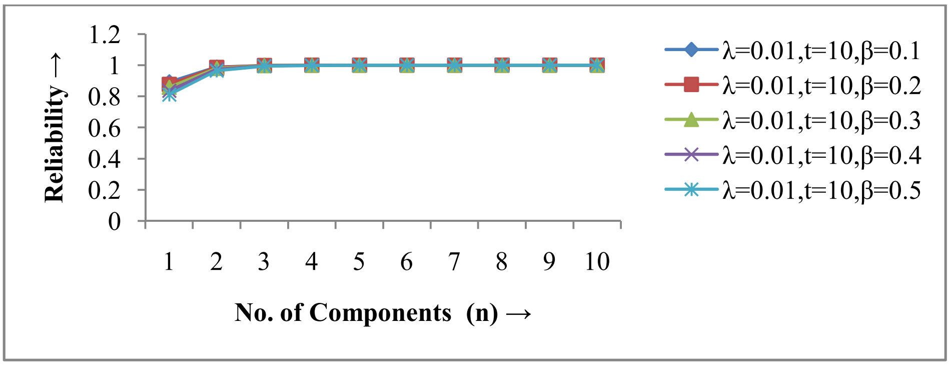

Fig. 4: Reliability Vs No. of Components (n)

n	MTSF				
	β=0.1, λ=0.01, t=10	β=0.2, λ=0.01,t=10	β=0.3, λ=0.01,t=10	β=0.4, λ=0.01,t=10	β=0.5, λ=0.0, t=10
1	69.23057	50.82565	39.04661	31.09362	25.48514
2	101.59444	73.12637	55.18339	43.23546	34.91564
3	122.59227	87.24829	65.18166	50.61187	40.5435
4	138.09242	97.52847	72.37083	55.85796	44.50689
5	150.3587	105.5874	77.96021	59.90692	47.54602
6	160.4983	112.2028	82.5205	63.19274	50.00064
7	169.13435	117.8065	86.36503	65.9513	52.05379

8	176.65178	122.6626	89.68385	68.32461	53.81494
9	183.30488	126.9444	92.6007	70.40458	55.35458
10	189.27034	130.7714	95.20055	72.25404	56.72068

Table 4: MTSF Vs No. of Components (n)

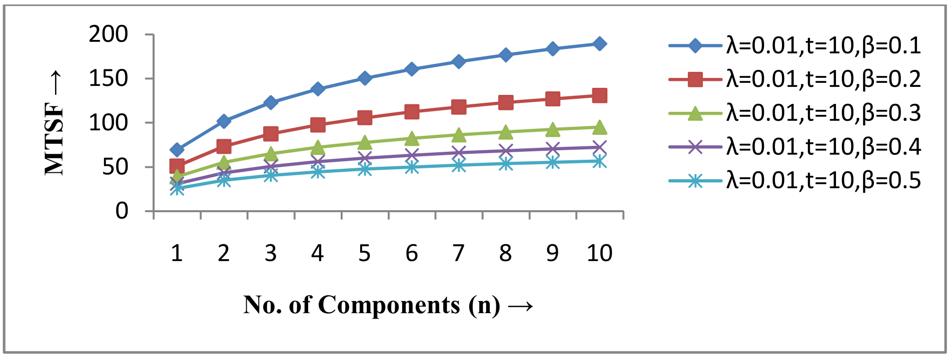

Fig. 5: MTSF Vs No. of Components (n)

n	Reliability				
	t=5, λ=0.01, β=0.1	t=10, λ=0.01, β=0.1	t=15, λ=0.01, β=0.1	t=20, λ=0.01, β=0.1	t=25, λ=0.01, β=0.1
1	0.948008538	0.89185852	0.83629373	0.78245086	0.73082962
2	0.997296888	0.98830542	0.97320026	0.95267237	0.92754731
3	0.999859461	0.99873533	0.99561271	0.98970392	0.98049788
4	0.999992693	0.99986324	0.99928177	0.9977601	0.99475061
5	0.99999962	0.99998521	0.99988242	0.99951271	0.99858702
6	0.99999998	0.99999840	0.99998075	0.99989399	0.99961967
7	0.999999999	0.99999983	0.99999685	0.99997694	0.99989763
8	0.99999999994	0.99999998	0.99999948	0.99999498	0.99997244
9	0.99999999999	0.99999999	0.99999992	0.99999891	0.99999258
10	0.999999999999	0.999999999	0.99999999	0.99999976	0.999998

Table 5: Reliability Vs No. of Components (n)

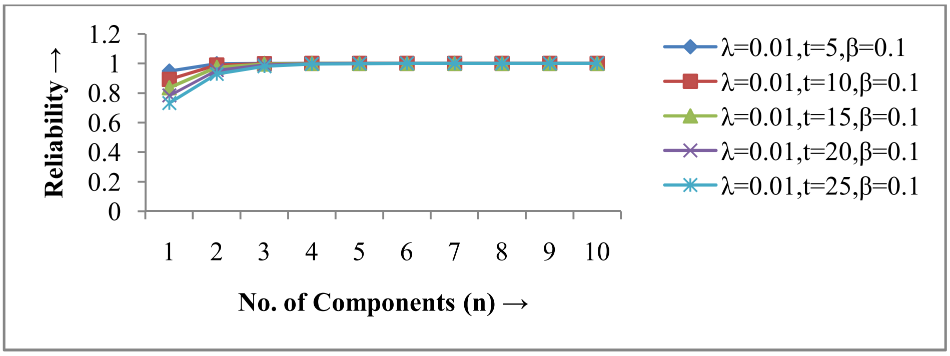

Fig. 6: Reliability Vs No. of Components (n)

6. Reliability Measures for a Special Case of Weibull Distribution: Rayleigh Distribution

The Rayleigh distribution is a special case of Weibull distribution with the shape parameter β=1.The i[th] component's reliability in this case is given by

$R_i(t) = e^{\frac{-\lambda_i t^2}{2}}$, where $h_i(t) = \lambda_i t$

and, the system reliability is given by

$$R_s(t) = 1 - \prod_{i=1}^{n}(1 - R_i(t)) = 1 - \prod_{i=1}^{n}\left(1 - e^{-\frac{\lambda_i t^2}{2}}\right)$$

$$= 1\text{-}\left(1 - e^{-\frac{\lambda_1 t^2}{2}}\right)\left(1 - e^{-\frac{\lambda_2 t^2}{2}}\right)\dots\dots\dots.\left(1 - e^{-\frac{\lambda_n t^2}{2}}\right)$$

$$R_s(t) = \sum_{i=1}^{n} e^{\frac{-\lambda_i t^2}{2}} - \sum_{i=1}^{n}\sum_{j=i+1}^{n} e^{\frac{-(\lambda_i+\lambda_j)t^2}{2}} + \dots + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n} e^{\frac{-(\lambda_i+\lambda_j+\lambda_k)t^2}{2}} + \dots + (-1)^{n+1} e^{-\sum_{i=1}^{n}\frac{\lambda_i t^2}{2}} \quad (11)$$

Also,

$$\text{MTSF}=\sqrt{\frac{\Pi}{2\sum_{i=1}^{n}\lambda_i}} - \sqrt{\frac{\Pi}{2\sum_{i=1}^{n}\sum_{j=i+1}^{n}(\lambda_i+\lambda_j)}} + \sqrt{\frac{\Pi}{2\sum_{i=1}^{n}\sum_{j=i+1}^{n}\sum_{k=j+1}^{n}(\lambda_i+\lambda_j+\lambda_k)}}\dots\dots + (-1)^{n+1}\sqrt{\frac{\Pi}{2\sum_{i=1}^{n}\lambda_i}} \quad (12)$$

For identical components, suppose $\lambda_i t = \lambda t$. The system reliability is given by

$$R_s(t) = 1\text{-}\left(1 - e^{\frac{-\lambda t^2}{2}}\right)^n = \binom{n}{1}e^{\frac{-\lambda t^2}{2}} - \binom{n}{2}e^{\frac{-2\lambda t^2}{2}} + \binom{n}{3}e^{\frac{-3\lambda t^2}{2}} + \cdots\dots.+(-1)^{n+1}e^{\frac{-n\lambda t^2}{2}}$$

$$R_s(t) = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}e^{\frac{-i\lambda t^2}{2}} \quad (13)$$

$$\text{MTSF}= \int_0^{\infty} R(t)dt = \int_0^{\infty}\left[\sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}e^{\frac{-i\lambda t^2}{2}}\right]dt$$

$$=\sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\int_0^{\infty} e^{\frac{-i\lambda t^2}{2}}dt = \sum_{i=1}^{n}(-1)^{i+1}\binom{n}{i}\sqrt{\frac{\Pi}{2i\lambda}} \quad (14)$$

Illustration

The system reliability with two components is given by

$R_s(t) = 1 - \prod_{i=1}^{2}(1 - R_i(t)) = 1 - \prod_{i=1}^{2}\left(1 - e^{\frac{-\lambda_i t^2}{2}}\right)$

$$R_s(t) = 1 - \left(1 - e^{\frac{-\lambda_1 t^2}{2}}\right)\left(1 - e^{\frac{-\lambda_2 t^2}{2}}\right) = e^{\frac{-\lambda_1 t^2}{2}} + e^{\frac{-\lambda_2 t^2}{2}} - e^{\frac{-(\lambda_1+\lambda_2) t^2}{2}} \quad (15)$$

MTSF=$\int_0^\infty R_s(t)dt = \int_0^\infty [e^{\frac{-\lambda_1 t^2}{2}} + e^{\frac{-\lambda_2 t^2}{2}} - e^{\frac{-(\lambda_1+\lambda_2) t^2}{2}}]dt$

$$\text{MTSF} = \sqrt{\frac{\Pi}{2\lambda_1}} + \sqrt{\frac{\Pi}{2\lambda_2}} - \sqrt{\frac{\Pi}{2(\lambda_1+\lambda_2)}} \quad (16)$$

For identical case, we can assume $\lambda_i t = \lambda t$. The system reliability is given by

$$R_s(t) = 1 - \left(1 - e^{\frac{-\lambda t^2}{2}}\right)^2 = 1 - \left[1 + e^{\frac{-2\lambda t^2}{2}} - 2e^{\frac{-\lambda t^2}{2}}\right] = 2e^{\frac{-\lambda t^2}{2}} - e^{-\lambda t^2} \quad (17)$$

And, MTSF= $\int_0^\infty \left(2e^{\frac{-\lambda t^2}{2}} - e^{-\lambda t^2}\right) dt = \sqrt{\frac{\Pi}{2\lambda}} - \frac{1}{2}\sqrt{\frac{\Pi}{\lambda}} = \sqrt{\frac{\Pi}{2\lambda}}\left[1 - \frac{1}{\sqrt{2}}\right]$ (18)

The reliability and MTSF of a system having three or more components connected in parallel can be obtained in a similar way.

7. Special Case for Parameters following Rayleigh Distribution

For arbitrary values of the parameters, reliability and mean time to system failure (MTSF) of the system have been evaluated which are shown numerically and graphically as:

n	Reliability				
	λ=0.01,t=10	λ=0.02,t=10	λ=0.03,t=10	λ=0.04,t=10	λ=0.05,t=10
1	0.60653066	0.367879441	0.22313016	0.13533528	0.082085
2	0.845181878	0.600423599	0.39647325	0.25235493	0.1574321
3	0.939083816	0.747419542	0.53113827	0.35353769	0.2265942
4	0.976031349	0.8403387	0.63575546	0.44102685	0.2900793
5	0.990569071	0.89907481	0.71702941	0.51667564	0.3483531
6	0.996289218	0.936203112	0.78016868	0.58208648	0.4018435
7	0.998539921	0.959672676	0.82921968	0.63864492	0.4509432
8	0.999425504	0.974508269	0.86732592	0.68754901	0.4960125
9	0.999773953	0.983886153	0.89692951	0.72983466	0.5373823
10	0.999911058	0.989814106	0.91992764	0.76639756	0.5753563

Table 6: Reliability Vs No. of Components (n)

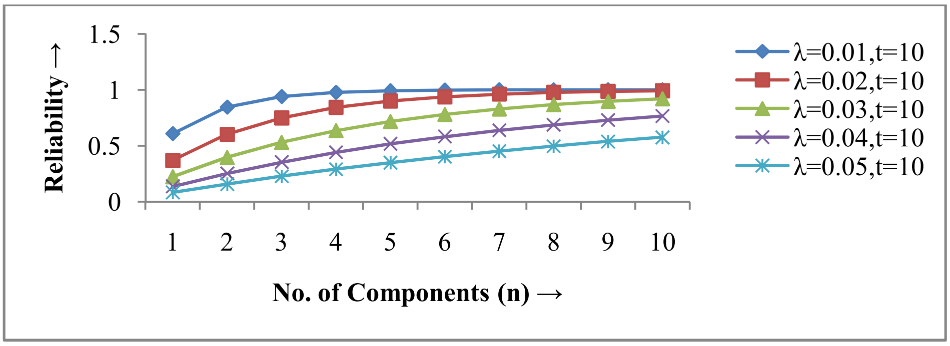

Fig.7: Reliability Vs No. of Components (n)

n	MTSF				
	λ=0.01, t=10	λ=0.02, t=10	λ=0.03, t=10	λ=0.04, t=10	λ=0.05, t=10
1	12.53314	8.86227	7.23601	6.26657	5.60499
2	16.20401	11.45797	9.35539	8.10201	7.24666
3	18.24863	12.90373	10.53585	9.12431	8.16103
4	19.63643	13.88505	11.3371	9.81821	8.78168
5	20.67528	14.61963	11.93688	10.33764	9.24627
6	21.49981	15.20266	12.41292	10.74991	9.61501
7	22.18026	15.68381	12.80578	11.09013	9.91932
8	22.75764	16.09208	13.13913	11.37882	10.17753
9	23.25786	16.44579	13.42793	11.62893	10.40123
10	23.69832	16.75724	13.68223	11.84916	10.59821

Table 7: MTSF Vs No. of Components (n)

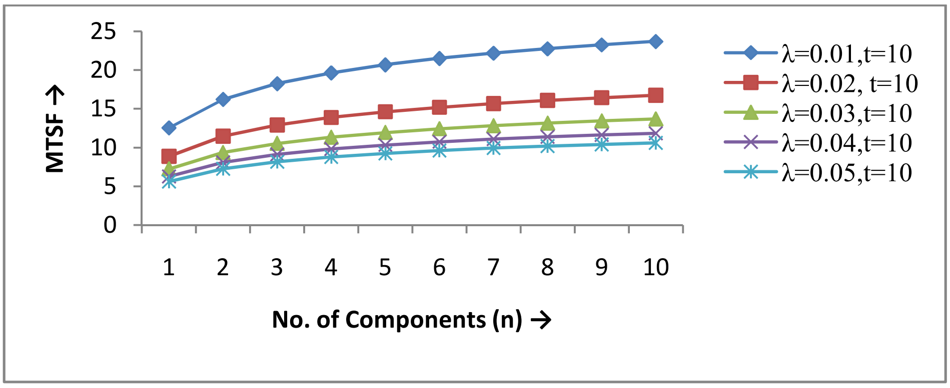

Fig. 8: MTSF Vs No. of Components (n)

n	Reliability				
	λ=0.01,t=5	λ=0.01,t=10	λ=0.01,t=15	λ=0.01,t=20	λ=0.01,t=25
1	0.882496903	0.6065306	0.32465246	0.135335	0.0439369
2	0.986193022	0.84518187	0.5439057	0.252354	0.0859434
3	0.998377637	0.93908381	0.69197785	0.353537	0.1261043
4	0.999809367	0.97603135	0.79197799	0.441026	0.1645006
5	0.9999776	0.99056907	0.85951285	0.516675	0.2012098
6	0.999997368	0.99628922	0.90512235	0.582086	0.2363062
7	0.999999691	0.99853992	0.93592461	0.638644	0.2698606
8	0.999999964	0.9994255	0.95672684	0.687549	0.3019407
9	0.999999996	0.99977395	0.97077558	0.729834	0.3326113
10	0.999999999	0.99991106	0.98026336	0.766397	0.3619343

Table 8: Reliability Vs No. of Components (n)

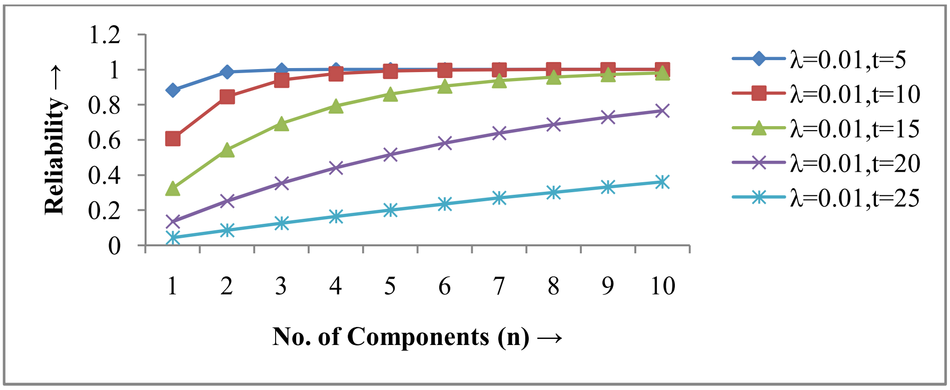

Fig. 9: Reliability Vs No. of Components (n)

8. Discussion of the Research Findings

A parallel system of ten identical components with Weibull failure laws has been analyzed to see the effect of number of components, failure rates of the components, shape parameter and operating time of the components on reliability and mean time to system failure. It is found that these measures keep on increasing with the increase of number of components while they decline with the increase of failure rates of the components. The values of reliability and MTSF have also been obtained for a special case of Weibull distribution i.e. Rayleigh distribution. The system has more values of reliability and MTSF when components follow Weibull failure laws. However, the effect of number of components on reliability is much more when components follow Rayleigh failure laws than that of Weibull failure laws. Further, reliability of a parallel system goes on decreasing with the increase of operating time irrespective of distributions related to failure time of the components. But, comparatevily the system has more reliability when its components follow Weibull distribution.

The reliability and MTSF of this system decrese with the increase of the value of the shape parameter β. The results are shown graphically and numerically in respective tables and figures.

9. Conclusion

The reliability of a parallel system can be increased by increasing the number of components irrespective of the distributions related to failure time of the components. However, reliability of such a system becomes less with the increase of operating time of the components.

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