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# BAYESIAN APPROXIMATIONS FOR SHAPEPARAMETER OF GENERALIZED POWER FUNCTION DISTRIBUTION

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## Abstract

Power function distribution provides a better fit for failure data and more appropriate information about reliability and hazard rates and used as a subjective description of a population for which there is limited sample data and in case where the relationship between variables is known but data is scare. In this paper, Bayesian approximation techniques like normal approximation, Lindley's Approximation, Laplace Approximation are used to study the behavior of shape parameter of generalized power function distribution under different priors. Furthermore, a comparison of these approximation techniques, under different priors is studied by making use of simulation technique.

**Key Words:** Bayesian Estimation, Prior Distribution, Normal Approximation, Lindley's Approximation, Laplace Approximation.

## 1. Introduction

Bayesian approach is used by practitioners for situations where scientists have prior information about the values of the parameters to be estimated. The available information is formalized into a prior distribution on the parameter, and estimators are formed from the posterior distribution of the parameter given the data. Balakrishnan and Chan (1994) discussed the BLUE estimate for scale parameter and location parameter from Log-gamma distribution. Saran and Pandey (2004) put forward the concept of record values which are found in many situations of daily life as well as in many statistical applications.

Chang (2007) presented characterizations of the power function distribution by independence of record values. Estimations of normal distribution parameters using likelihood functionhave been presented by Balakrishnan and Mi (2003). Pandey and Rao (2008) applied Bayesian estimation to carry out the study of shape parameter of a generalized power function distribution under asymmetric loss function. Rahman et al. (2012) used different symmetric and asymmetric loss functions to obtain Bayes estimators for power function distribution along with comparison. Kifayat et al. (2012) discussed Bayesian analysis of the power model using two informative priors and two non-informative priors along with the comparison of informative and non-informative priors.Bayesian estimate for shape parameter from generalized power function distribution was obtained by Almutairi et al. (2012) considering non-informative prior distribution and informative prior distribution. Sultan et al. (2014) studied Bayesian analysis of the power distribution under double priors and single priors and a comparison of these priors on the basis of posterior variances has also been discussed.

The probability density function of generalized power distribution is given by

$$f(x)=\frac{\theta}{\sigma^{\theta-1}}(x-\mu)^{\theta-1}; \qquad \mu\le x\le\mu+\sigma;\ \sigma>0;\mu\ge 0;\theta>0 \tag{1.1}$$

where $\theta$ is the shape parameter, $\mu$ is the location parameter and $\sigma$ is the scale parameter.
The likelihood function of (1.1) is given by

$$L(x)=\frac{\theta^{n}}{\sigma^{n(\theta-1)}}e^{(\theta-1)\sum_{i=1}^{n}\ln(x_i-\mu)} \tag{1.2}$$

## 2. Approximation Methods of Posterior Modes

Asymptotic normality of the posterior is the basic tool of large sample Bayesian inference. Gelman et al. (1995) gave a number of counter examples to illustrate limitations of the large sample approximation to the posterior distribution. The Bayesian approach to parametric inference is conceptually simple and probabilistically elegant. The numerical implementation of a Bayesian procedure is not always straight forward since the involved posterior distribution is in terms of complicate functions. One of the important steps in simplifying the computations is to investigate the large sample behavior of the posterior distribution and its characteristics. This is important for two reasons: (a) asymptotic results provide useful first order approximations when actual samples are relatively large, and (b) objective Bayesian methods typically depend on the asymptotic properties of the assumed model.

In our present study, we focus on three Bayesian approximation techniques i.e. normal approximation, Lindley's approximation, Laplace approximation.

### 2.1 Normal Approximation

The basic result of the large sample Bayesian inference is that the posterior distribution of the parameter approaches a normal distribution. Some good sources on the topic from the Bayesian point of view include Lindley (1958), Pratt (1965) and Berger and Wolpert (1984). An example of the use of the normal approximation with small samples is provided by Rubin and Schenker (1987), who approximated the posterior distribution of the logit of the binomial parameter in real application and evaluate the frequentists operating characteristics of their procedure. A review is provided by Freedman, Spiegel halter and Parmer (1994) and Khan et al (1996). Ahmad et al. (2007, 2011) discussed Bayesian analysis of exponential distribution and gamma distribution using normal and Laplace approximations.

If the posterior distribution $P(\theta\,|\,y)$ is unimodal and roughly symmetric, it is convenient to approximate it by a normal distribution centered at the mode; that is logarithm of the posterior is approximated by a quadratic function, yielding the approximation

$$P(\theta\mid y)\sim N\left(\hat{\theta},\left[I\left(\hat{\theta}\right)\right]^{-1}\right)$$

where $I\left(\hat{\theta}\right)=-\dfrac{\partial^2 \log P(\theta\mid y)}{\partial\theta^2}$. If the mode, $\hat{\theta}$ is in the interior parameter space, then $I(\theta)$ is positive; if $\hat{\theta}$ is a vector parameter, then $I(\theta)$ is a matrix.

In our study, the normal approximations of generalized power function under three different priorsis obtained.

Under extension of Jeffrey's prior $g(\theta)\propto\left(\dfrac{1}{\theta}\right)^{2m}$, the posterior distribution for $\theta$ is as

$$P(\theta\mid x)\propto\frac{\theta^{n-2m}}{\sigma^{n(\theta-1)}}e^{(\theta-1)\sum_{i=1}^{n}\ln(x_i-\mu)}$$

from which the posterior mode is obtained as

$$\hat{\theta}=\frac{n-2m}{n\ln\sigma-\sum_{i=1}^{n}\ln(x_i-\mu)}=\frac{n-2m}{n(\ln\sigma-\overline{\ln(x-\mu)})}$$

and $\left[I(\hat{\theta})\right]^{-1}=\dfrac{n-2m}{\left[n(\ln\sigma-\overline{\ln(x-\mu)})\right]^2}$

Thus, the posterior distribution can be approximated as

$$P(\theta\mid x)\sim N\left(\frac{n-2m}{n(\ln\sigma-\overline{\ln(x-\mu)})};\frac{n-2m}{\left[n(\ln\sigma-\overline{\ln(x-\mu)})\right]^2}\right)\quad(2.1.1)$$

Under gamma prior $g(\theta)\propto e^{-a\theta}\theta^{b-1}$ ; $a,b>0;\theta>0$ , where a,b are the hyper parameters. The posterior distribution for $\theta$ is as

$$P(\theta\mid x)\propto\frac{\theta^{n+b-1}}{\sigma^{n(\theta-1)}}e^{(\theta-1)\sum_{i=1}^{n}\ln(x_i-\mu)-a\theta}$$

and the posterior distribution can be approximated as

$$P(\theta\mid x)\sim N\left(\frac{n+b-1}{n(\ln\sigma-\overline{\ln(x-\mu)})+a};\frac{n+b-1}{\left[n(\ln\sigma-\overline{\ln(x-\mu)})+a\right]^2}\right)\quad(2.1.2)$$

Under the Erlang prior $g(\theta)\propto e^{-\theta/d}\theta^{c-1}$ ; $a,b>0;\theta>0$ , where c,d are the hyper parameters,the posterior distribution can be approximated as

$$P(\theta\mid x)\sim N\left(\frac{n+c-1}{\left[\frac{1}{d}+n(\ln\sigma-\overline{\ln(x-\mu)})\right]};\frac{n+c-1}{\left[\frac{1}{d}+n(\ln\sigma-\overline{\ln(x-\mu)})\right]^2}\right)\quad(2.1.3)$$

## 2.2 Lindley's Approximation

Quite few times, the integrals appearing in Bayesian estimation can't be reduced to closed form. Hence the evaluation of the posterior expectation for obtaining the Bayes estimators will be tedious. Thus, we propose the use of Lindley's approximation method (1980) for obtaining Bayes estimates. Lindley developed an asymptotic approximation to the ratio

$$I = \frac{\int_{\Omega} h(\theta)\, e^{L(\theta)+U(\theta)} d\theta}{\int_{\Omega} e^{L(\theta)+U(\theta)} d\theta} \tag{2.2.1}$$

where $\theta = (\theta_1, ...., \theta_m), L(\theta)$ is the logarithmic of likelihood function, $h(\theta)$ and $U(\theta)$ are arbitrary functions of $\theta$ and $\Omega$ represents the space range of $\theta$. Thus $I = E\{h(\theta) \mid x\}$ can be evaluated as

$$I \cong h(\hat{\theta}) + \frac{1}{2}\left[ h_2(\hat{\theta}) + 2h_1(\hat{\theta})U'(\hat{\theta}) \right]\hat{\varphi}^2 + \frac{1}{2}\left[L_3(\hat{\theta})h_1(\hat{\theta})\right](\hat{\varphi}^2)^2 \tag{2.2.2}$$

In particular, if $h(\theta) = \theta; h'(\theta) = 1; h''(\theta) = 0$

Thus $$E(\theta \mid x) = \hat{\theta} + \frac{\partial}{\partial\theta}\left(U(\hat{\theta})\right)\hat{\varphi}^2 + \left(\frac{1}{2}L_3(\hat{\theta})\right)\left(\hat{\varphi}^2\right)^2 \tag{2.2.3}$$

where $\hat{\varphi}^2 = (-L_2(\hat{\theta}))^{-1}$; $U(\theta) = \ln g(\theta)$

Thus, for generalized power function distribution Lindley's approximation for shape parameter $\theta$ under extension of Jeffrey's prior, gamma prior and Erlang prior can be obtained as

Using (1.2) $\hat{\theta} = \dfrac{1}{\ln\sigma - \overline{\ln(x-\mu)}}$

$$L_2(\hat{\theta}) = \frac{\partial^2 \ln(\theta \mid x)}{\partial\theta^2} = -\frac{n}{\theta^2} = -n(\ln\sigma - \overline{\ln(x-\mu)})^2$$

$$\hat{\varphi}^2 = \left[-L_2(\hat{\theta})\right]^{-1} = \frac{1}{n(\ln\sigma - \overline{\ln(x-\mu)})^2}$$

$$L_3(\hat{\theta}) = \frac{\partial^3 \ln(\theta \mid x)}{\partial\theta^3} = \frac{2n}{\theta^3} = 2n(\ln\sigma - \overline{\ln(x-\mu)})^3$$

Under the extension of Jeffrey's prior $g(\theta) \propto \left(\dfrac{1}{\theta}\right)^{2m}$,

$$U(\theta) = \ln g(\theta) = -2m\ln\theta; U'(\hat{\theta}) = \frac{-2m}{\hat{\theta}} = -2m(\ln\sigma - \overline{\ln(x-\mu)})$$

If $h(\theta) = \theta; h'(\theta) = 1; h''(\theta) = 0$

Thus Lindley's approximation for $\theta$ from (2.2.3) is as

$$E(\theta \mid x) = \left(\frac{n-2m+1}{n(\ln\sigma - \overline{\ln(x-\mu)})}\right) \tag{2.2.4}$$

Under gamma prior $g(\theta) \propto e^{-a\theta}\theta^{b-1}$ ; $a,b > 0; \theta > 0$ where a,b are the hyper parameters

$$E(\theta \mid x)=\frac{(n+b)\,(\,\ln\sigma-\overline{\ln(x-\mu)}\,)-a}{n\,(\,\ln\sigma-\overline{\ln(x-\mu)}\,)^{2}} \tag{2.2.5}$$

Under Erlang prior $g(\theta)\propto e^{-\theta/d}\theta^{c-1}$ ; $a,b>0;\theta>0$ , where c,d are the hyper parameters.

$$E(\theta \mid x)=\frac{\left((c+n)\,(\,\ln\sigma-\overline{\ln(x-\mu)}\,)-\dfrac{1}{d}\right)}{n\,(\,\ln\sigma-\overline{\ln(x-\mu)}\,)^{2}} \tag{2.2.6}$$

## 2.3 Laplace Approximation

In the development of new simulation techniques, Laplace's method uses asymptotic arguments. From (2.2.2) it may be observed that Lindley's approximation requires evaluation of third order partial derivatives of likelihood function which may be cumbersome to compute when the parameter $\theta$ is a vector valued parameter. Tierney and Kadane (1986) gave Laplace method to evaluate $E(h(\theta)\mid x)$ as

$$E(h(\theta)\mid x)\cong\frac{\hat{\varphi}^{*}}{\hat{\varphi}}\frac{\exp\{-n\,h''^{*}\,(\hat{\theta}^{*})\}}{\exp\{-n\,h''(\hat{\theta})\}} \tag{2.3.1}$$

where $-n\,h''(\hat{\theta})=\ln P(\theta\mid x)$; $-n\,h''^{*}\,(\hat{\theta}^{*})=\ln P(\theta\mid x)+\ln h(\theta)$;

$$\hat{\varphi}^{2}=-\left[-n\,h''(\hat{\theta})\right]^{-1};\hat{\varphi}^{*2}=-\left[-n\,h''^{*}\,(\hat{\theta}^{*})\right]^{-1}$$

Thus, for generalized power function distribution Laplace approximation for shape parameter $\theta$ can be calculated as

Under extension of Jeffrey's prior $g(\theta)\propto\left(\dfrac{1}{\theta}\right)^{2m}$, the posterior distribution for $\theta$ using (1.2) is as

$$P(\theta\mid x)\propto\frac{\theta^{n-2m}}{\sigma^{n(\theta-1)}}e^{\theta\sum_{i=1}^{n}\ln(x_i-\mu)}$$

$$-n\,h(\theta)=(n-2m)\ln\theta-n\,(\theta-1)\ln\sigma+\theta\sum_{i=1}^{n}\ln(x_i-\mu)$$

$$-n\,h'(\theta)=\frac{n-2m}{\theta}-n\ln\sigma+\sum_{i=1}^{n}\ln(x_i-\mu)$$

$$\therefore\ \hat{\theta}=\frac{n-2m}{n\,(\ln\sigma-\overline{\ln(x-\mu)})}$$

$$-n\,h''(\hat{\theta})=-\frac{n-2m}{\theta^{2}}=-\frac{[n\,(\ln\sigma-\overline{\ln(x-\mu)})]^{2}}{n-2m}$$

Therefore $\hat{\varphi}^{2}=-\left[-n\,h''(\hat{\theta})\right]^{-1}=\dfrac{n-2m}{\left[n\,(\ln\sigma-\overline{\ln(x-\mu)})\right]^{2}}$

$$\text{or }\hat{\varphi}=\frac{(n-2m)^{1/2}}{n\,(\ln\sigma-\overline{\ln(x-\mu)})}$$

now $-n\,h^{*}(\theta^{*}) = (n-2m+1)\ln\theta^{*} - \theta^{*}(n\ln\sigma - \sum_{i=1}^{n}\ln(x_i-\mu)) + n\ln\sigma$

$$\therefore\ \hat{\theta}^{*} = \frac{n-2m+1}{n(\ln\sigma - \overline{\ln(x-\mu)})}$$

and $\hat{\varphi}^{*} = \dfrac{(n-2m+1)^{1/2}}{n(\ln\sigma - \overline{\ln(x-\mu)})}$

Thus using(2.3.1)we have

$$E(\theta\,|\,x) = \left(\frac{n-2m+1}{n(\ln\sigma - \overline{\ln(x-\mu)})}\right)\left(\frac{n-2m+1}{n-2m}\right)^{n-2m+1/2} e^{-1} \qquad (2.3.2)$$

Under Gamma prior $g(\theta) \propto e^{-a\theta}\theta^{b-1}$ ; $a,b>0;\theta>0$

$$E(\theta\,|\,x) = \left(\frac{n+b}{a+n(\ln\sigma - \overline{\ln(x-\mu)})}\right)\left(\frac{n+b}{n+b-1}\right)^{n+b-1/2} e^{-1} \qquad (2.3.3)$$

Under Erlang prior $g(\theta) \propto e^{-\theta/d}\theta^{c-1}$ ; $a,b>0;\theta>0$

$$E(\theta\,|\,x) = \left(\frac{n+c}{1/d+n(\ln\sigma - \overline{\ln(x-\mu)})}\right)\left(\frac{n+c}{n+c-1}\right)^{n+c-1/2} e^{-1} \qquad (2.3.4)$$

## 3. Simulation Study

For simulation study, a samples of size n=25, 50 and 100 to represent small, medium and large data set has been generated by using EasyFit 5.5 software. The posterior mode for the shape parameter of Generalized Power function distribution is obtained by using different Bayesian approximation techniques under three types of priors. A simulation study was conducted using R-software to examine and compare the performance of the estimates for different sample sizes by using three different types of priors. The results so obtained are presented in Tables 1, 2, 3 given below:

| n | $\theta$ | $\mu$ | $\sigma$ | Hyper parameters a=b=c=d | $\hat{\theta}_{NA}$ | | |
|---|---|---|---|---|---|---|---|
| | | | | | **Jeffrey's prior m=1** | **Gamma prior** | **Erlang prior** |
| **20** | 2.0 | 0.0 | 1.0 | 1.5 | 1.6961 (0.9662) | 1.4069 (0.0406) | 2.7072 (0.7972) |
| | | 0.5 | 1.5 | | 0.8057 (0.9867) | 0.8967 (0.0796) | 1.2923 (0.9923) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 2.3548 (0.8558) | 1.4108 (0.0983) | 5.0689 (0.6688) |
| | | 0.5 | 1.5 | | 1.1170 (0.8170) | 1.0799 (0.0799) | 2.4127 (0.4127) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 2.5116 (0.7116) | 1.3183 (0.0383) | 6.8056 (0.8146) |
| | | 0.5 | 1.5 | | 0.9796 (0.7296) | 1.0172 (0.0183) | 2.6919 (0.6919) |
| **50** | 2.0 | 0.0 | 1.0 | 1.5 | 2.0259 (0.6258) | 1.7511 (0.0351) | 2.4797 (0.6788) |
| | | 0.5 | 1.5 | | 1.1762 (0.8763) | 1.1612 (0.0761) | 1.4422 (0.9422) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 2.8553 (0.8254) | 1.8744 (0.0744) | 4.0923 (0.6024) |
| | | 0.5 | 1.5 | | 1.0904 (0.7095) | 1.0778 (0.0744) | 1.5657 (0.4048) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 2.7956 (0.7065) | 1.6748 (0.0248) | 4.5922 (0.5929) |
| | | 0.5 | 1.5 | | 1.3885 (0.6886) | 1.2235 (0.0153) | 2.2831 (0.5832) |
| **100** | 2.0 | 0.0 | 1.0 | 1.5 | 1.8499 (0.4488) | 1.7308 (0.0309) | 2.0537 (0.5637) |
| | | 0.5 | 1.5 | | 1.0479 (0.6478) | 1.0529 (0.0629) | 1.1642 (0.8753) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 1.8499 (0.6887) | 1.6307 (0.0506) | 2.2442 (0.5535) |
| | | 0.5 | 1.5 | | 1.2639 (0.5648) | 1.2201 (0.0302) | 1.5338 (0.4027) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 3.4177 (0.4288) | 2.1986 (0.0196) | 4.4937 (0.4948) |
| | | 0.5 | 1.5 | | 1.3729 (0.0429) | 1.2724 (0.0105) | 1.8063 (0.4074) |

$\hat{\theta}_{NA}$ = posteriormean under normal approximation

**Table 1: Posterior Mean and Posterior Standard Deviation (Within Brackets) for Shape Parameter of Generalized Power Function under Various Priors using Normal Approximation Technique**

| **n** | $\theta$ | $\mu$ | $\sigma$ | **Hyper parameters a=b=c=d** | $\hat{\theta}_{LA}$ | | |
|---|---|---|---|---|---|---|---|
| | | | | | **Jeffrey's prior m=1** | **Gamma prior** | **Erlang prior** |
| **20** | 2.0 | 0.0 | 1.0 | 1.5 | 1.7904 (0.8895) | 1.0510 (0.0521) | 2.8092 (0.8812) |
| | | 0.5 | 1.5 | | 0.8505 (0.8614) | 0.9421 (0.0422) | 1.3388 (0.4498) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 1.9895 (0.9986) | 0.4048 (0.0502) | 3.5350 (0.6541) |
| | | 0.5 | 1.5 | | 1.1791 (0.7682) | 0.9418 (0.0959) | 2.4784 (0.5773) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 1.5332 (0.5223) | 0.1276 (0.0725) | 4.0304 (0.3304) |
| | | 0.5 | 1.5 | | 1.0457 (0.6574) | 0.9343 (0.0643) | 2.7499 (0.5334) |
| **50** | 2.0 | 0.0 | 1.0 | 1.5 | 2.0681 (0.6556) | 1.6417 (0.0426) | 2.5235 (0.7355) |
| | | 0.5 | 1.5 | | 1.2007 (0.8109) | 1.1701 (0.0413) | 1.4673 (0.2774) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 2.9148 (0.8259) | 0.6254 (0.0539) | 4.1551 (0.4662) |
| | | 0.5 | 1.5 | | 1.1132 (0.7244) | 1.0741 (0.0765) | 1.5889 (0.3997) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 2.1427 (0.4438) | 1.0028 (0.0139) | 2.6533 (0.2645) |
| | | 0.5 | 1.5 | | 1.4174 (0.5185) | 1.0589 (0.0576) | 2.3128 (0.4218) |
| **100** | 2.0 | 0.0 | 1.0 | 1.5 | 1.8688 (0.5599) | 1.7201 (0.0312) | 2.0729 (0.6537) |
| | | 0.5 | 1.5 | | 0.9804 (0.7713) | 0.9913 (0.0055) | 1.0884 (0.1995) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 5.6741 (0.6652) | 0.3077 (0.3166) | 6.8613 (0.3534) |
| | | 0.5 | 1.5 | | 1.2768 (0.5677) | 1.2150 (0.0143) | 1.5468 (0.2579) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 3.4526 (0.4005) | 0.8849 (0.0537) | 4.5297 (0.2281) |
| | | 0.5 | 1.5 | | 1.3869 (0.3708) | 1.2324 (0.0143) | 1.8205 (0.2216) |

$\hat{\theta}_{LA}$ = posteriormean under Lindley's approximation

**Table 2: Posterior Mean and MSE (Within Brackets) for Shape Parameter of Generalized Power Function under Various Prior using Lindley's Approximation Technique**

| n | $\theta$ | $\mu$ | $\sigma$ | Hyper parameters a=b=c=d | $\hat{\theta}_{LpA}$ | | |
|---|---|---|---|---|---|---|---|
| | | | | | Jeffrey's prior m=1 | Gamma prior | Erlang prior |
| 20 | 2.0 | 0.0 | 1.0 | 1.5 | 6.6206 (1.5317) | 0.4757 (0.0686) | 1.8184 (0.7882) |
| | | 0.5 | 1.5 | | 1.9339 (0.5228) | 1.1557 (0.0658) | 2.4024 (0.6125) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 3.4349 (1.2438) | 0.0890 (0.0078) | 2.0432 (0.8543) |
| | | 0.5 | 1.5 | | 1.5265 (0.4002) | 0.3252 (0.0244) | 1.9938 (0.7749) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 3.0193 (1.0982) | 0.0272 (0.0021) | 2.8590 (0.7581) |
| | | 0.5 | 1.5 | | 1.9407 (0.8516) | 0.1407 (0.0019) | 1.5352 (0.7441) |
| 50 | 2.0 | 0.0 | 1.0 | 1.5 | 1.1942 (0.4951) | 0.8020 (0.0612) | 2.0541 (0.6652) |
| | | 0.5 | 1.5 | | 1.7427 (0.3536) | 2.0462 (0.0435) | 2.5204 (0.5876) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 2.0122 (0.3156) | 0.0918 (0.0013) | 2.6741 (0.7489) |
| | | 0.5 | 1.5 | | 4.2201 (1.3202) | 0.8341 (0.0232) | 2.5479 (0.6547) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 2.2716 (0.0625) | 0.0298 (0.0025) | 3.7113 (0.6211) |
| | | 0.5 | 1.5 | | 2.5423 (0.5243) | 0.2176 (0.0167) | 2.3613 (0.4521) |
| 100 | 2.0 | 0.0 | 1.0 | 1.5 | 1.2929 (0.4987) | 1.0879 (0.0468) | 3.7835 (1.5831) |
| | | 0.5 | 1.5 | | 2.0108 (0.4182) | 1.9238 (0.0247) | 3.5462 (1.4421) |
| | 2.5 | 0.0 | 1.0 | 2.0 | 1.6945 (0.3986) | 0.2916 (0.0525) | 3.8565 (1.4121) |
| | | 0.5 | 1.5 | | 2.1971 (0.5679) | 1.3712 (0.0131) | 3.3460 (1.3225) |
| | 3.0 | 0.0 | 1.0 | 2.5 | 4.4758 (1.2561) | 0.0165 (0.0011) | 3.5629 (1.2210) |
| | | 0.5 | 1.5 | | 2.5282 (0.1982) | 0.4354 (0.0104) | 3.1239 (1.0224) |

$\hat{\theta}_{LpA}$= posteriormean under Laplace approximation

**Table3: Posterior Mean and Posterior Standard Deviation of Shape Parameter for Generalized Power Function under Various Priors using Laplace Approximation Technique**

## 4. Conclusion

When comparison of the non-informative prior distribution is made with the informative prior distribution, the difference is apparent in the form of use of Gamma distribution as prior. Gamma distribution benefits the informative prior distribution, while Jeffrey's prior was found to be suitable in the non-informative case. Bayesian estimates and posterior standard deviation and mean square error in case of Lindley's approximation for shape parameter were obtained and we observe that the gamma prior proves to be efficient with minimum posterior standard deviation and MSE. The contribution of sample size, shape parameter and location parameter is examined through the results showing variation in the value of $\hat{\theta}$ with the change in the hyper parameters. It is evident from the Tables 1, 2 and 3 that Bayesian estimates under normal approximation are efficient because of less posterior standard deviation under gamma prior distribution.

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