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RELIABILITY ANALYSIS OF MULTI-STATE COMPLEX SYSTEM HAVING TWO MULTI-STATE SUBSYSTEMS UNDER UNCERTAINTY

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Abstract

In this paper a non-repairable multi-state complex system with two subsystems A and B is taken for study. The subsystems A and B are multi-state consecutive *r*-out-of-*k*-from-*n*: G systems connected in parallel configuration. Analysis of the system reliability is carried out incorporating the uncertainty in the probabilities and degradation rates of the subsystem elements. The uncertainty representation in probabilities and degradation rates is done by their interval values. The probability intervals are evaluated by computing bound of interval valued ordinary differential equation of the system. Interval universal generating function is used to obtain reliability and mean time to failure of the proposed system. Finally, the considered model is demonstrated with the help of a numerical example.

Key Words: Multi-State System, Consecutive *r*-out of-*k*-from-*n* System, Interval Universal Generating Function, Reliability, MTTF.

1. Introduction

Most of the literatures in reliability theory deal with the binary theory where a system and its components can have two possible states namely perfect functioning or complete failure. However, a system and its components can have more than two states characterized by different levels of performance. Such systems are referred to as Multi-State Systems (MSSs). Reliability analysis of binary state system (BSS) is a foundation for mathematical treatment of reliability theory. But BSS approach fails to describe the condition when system has more than two states. MSS models clearly describe the system state distribution and its gradual development (Meenakshi and Singh, 2016). It is more than obvious to reliability engineers that MSS reliability models provide tremendous opportunity to researchers because of its flexibility. This is one of the reasons due to which now a days it is used more popularly in different industries such as transportation system, power generating system, computer generation system etc. Barlow and Wu (1978) and El-Neveihi et al. (1978) have given the first main contributions in development of the theory of MSS. In MSS, the system has many performance levels and reliability is considered as a measure of the ability of the system to meet the demand performance. Practical methods of MSS reliability assessment are based on different approaches: the structure function approach (Pourret et al. 1999), the stochastic process (mainly Markov) approach (Marquez and Coit, 2005, Zio et al., 2007), Monte-Carlo simulation (Xue and Yang, 1995, Lisnianski, 2007) and universal generating function (UGF) approach. UGF approach is introduced by Ushakov in 1986 and applied by Lisnianski & Levitin (2003) and Levitin (2005) for different systems. UGF approach is one of the efficient and best known methods to deal with MSS which is based on simple recursive procedures and provides systematic way to evaluate system reliability and other reliability indices of the complex systems. Different authors have applied UGF to analyse the systems reliability indies such as, Levitin and Lisnianski (1999) proposed a method for the evaluation of element reliability importance in a MSS using UGF approach. In the study, Levitin and Xing (2010) suggested UGF based algorithm for reliability and performance evaluation of MSS. (Ding and Lisnianski, 2008) extended UGF as fuzzy (FUGF) for reliability assessment of the system.

It is well known that the MSSs are more complex system structures than the binary state systems. The Classical theory of MSS reliability requires two major assumptions (Ding and Lisnianski, 2008): (i) the state probabilities of MSS components can be characterized by probability measures and (ii) the performance rates of a MSS component can be precisely determined. But there may exist different type of uncertainties about the state probabilities and performance rates of elements of MSS. Two of the major uncertainties incentered in real life are: aleatory and epistemic uncertainties. Various authors proposed different techniques to modeled uncertainties. Li et al. (2011) proposed an approach based on the use of interval arithmetic with interval-valued probability masses for modeling the probability distributions. Xiao et al. (2012) considered intervals and p-boxes to model ill-known probability distributions of elements states. Destercke and Sallak (2013) developed an approach based on extension of UGF to model epistemic uncertainties in MSS. Simon and Weber (2009) considered MSS by modeling evidential networks.

The consecutive-r-out-of-k-from-n: F system was first introduced by Tong (1985). In the last few years many researchers are dealing with this type of systems. Sfakianakis et al. (1992) and Papastavridis and Koutras (1993) developed a procedure to derive bounds for reliability of the consecutive-r-out-of-k-from-n: F system. Papastavridis and Sfakianakis (1991) provided method for optimal arrangements and importance analysis for the considered system. Levitin (2005) introduced this type of system as sliding window system (SWS) and proposed a method to study reliability analysis of the SWS system in MSS case. Such systems are used in oil pipeline systems, telecommunication systems, and mobile communication systems. Some other applications of SWSs can be seen in the radar detection, quality control, inspection procedures and in a series of microwave towers. Habib et al. (2007) generalized the consecutive-k-out-of-r-from-n: G system to multi-state case, where the generalized system consists of n linearly ordered multi-state elements, the system works if k elements out of r consecutive elements work and sum of their performances are greater than demand performance. Study on reliability and mean time to failure (MTTF) of the multi-state complex system having two non-reparable multi-state consecutive-r-out-of-k-from-n: G subsystems with imprecise probability and degradation rate has not been carried out earlier with the help of interval universal generating function (IUGF) and probability interval analysis (Ramdani et al., 2010) by ordinary differential equations with the application of well-defined stochastic process.

In the present study, the non-reparable multi-state consecutive *r*-out-of-*k*-from-*n*: G system are considered as subsystems (A and B) of multistate complex system. These subsystems consist of different elements with imprecise probabilities and degradation rates corresponding to their states. The imprecise probabilities and degradation rates are represented by interval values. The reliability and MTTF of considered system under uncertainty evaluated with the application of IUGF method. The probability intervals are obtained with help of ordinary differential equations with the application of well-defined stochastic process. Finally, a numerical example is illustrated to demonstrate the presented model.

2. Notation

E_j^A	j^{th} elements of subsystem A
E_j^B	j^{th} elements of subsystem B
$\underline{p}_{ji}(t)$	lower bound of time dependent probability of j^{th} element at state i
$\overline{p}_{ji}(t)$	upper bound of time dependent probability of j^{th} element at state i
$[\underline{\lambda}_j^{ik}, \overline{\lambda}_j^{ik}]$	interval of failure rate of element j from i to k state
$\eta_{ji}(p, \lambda, t)$	function of probability (p), failure rate (λ), time (t)
$[\underline{P}_i, \overline{P}_i]^A$	probability interval of i^{th} element of subsystem A
$[\underline{P}_l, \overline{P}_l]^B$	probability interval of l^{th} element of subsystem B
$U_A(z)$	interval universal generating function of A
$U_B(z)$	interval universal generating function of B
$U_{stm}(z)$	interval universal generating function of the system
MT_{stm}	MTTF of the system
$R_L(t)$	lower bound of the system reliability interval
$R_U(t)$	upper bound of the system reliability interval
$R_{stm}(t)$	Reliability of the system

3. Interval Universal Generating Function

Consider a discrete random variable X having k possible values $x_1, x_2, ..., x_N$ and let $[\underline{p}_1, \overline{p}_1], [\underline{p}_2, \overline{p}_2] ..., [\underline{p}_N, \overline{p}_N]$ be corresponding probability intervals. These vectors $x_j, [\underline{p}_j, \overline{p}_j],\ j = 1, 2, 3, ..., N$ are probability mass function representation of X variable. The Z-transform of X variables represents its probability mass functions in a polynomial which is called interval universal generating function (IUGF) of discrete random variable (X) and defined as follows

$$U_X(z) = \sum_{j=1}^{N} [\underline{p}_j, \overline{p}_j] z^{x_j}, \text{ where } z \text{ is a variable} \quad (1)$$

4. Analysis of Probability Intervals for the Multi-state Complex System's Elements

Suppose E_j^A is the element of subsystem A of a non-repairable complex MSS and P_{ji} is probability corresponding to the state g_{ij}, where j=1, 2... n denote number of components of the system and i =1, 2… k_j denote the system states. Let λ_j^{ik} be degradation rate of subsystem's element E_j^A from one state i to another state k (where k =1, 2… k_j) expressed as interval $[\underline{\lambda}_j^{ik}, \overline{\lambda}_j^{ik}]$. The characteristics of element E_j^A of non-repairable subsystem are described by ordinary differential equation with the help of stochastic process bearing probability as a variables and degradation rate as parameter. Derivatives of the system element's probabilities are evaluated as

$$\eta_{ji}(p,\lambda,t) = \frac{dp_{ji}(t)}{dt} = \sum_{k=i+1}^{k_j} \lambda_j^{ki} p_{jk}(t) - p_{ji}(t)\sum_{k=1}^{i-1}\lambda_j^{ik} \qquad (2)$$

where $\sum_{k=i^*+1}^{i^*} = 0$ when $i^* = 0,1,2,\ldots n$.

Lower and upper probability bounds can be evaluated by substituting value of transition rates λ_j^{ik} and λ_j^{ki} in differential equation (2). The same can be expressed as follows

If $\frac{\partial \eta_{ji}(p,\lambda,t)}{\partial p_{ji}(t)} \geq 0,\ \forall j,i,k,\ \ \forall \lambda_j^{ik} \in [\underline{\lambda}_j^{ik}, \overline{\lambda}_j^{ik}], \lambda_j^{ki} \in [\underline{\lambda}_j^{ik}, \overline{\lambda}_j^{ik}]\, k \neq i,\ t \geq t_0$

Then, lower bound of probability can be obtained as follows

(i) If $\frac{\partial \eta_{ji}(p,\lambda,t)}{\partial \lambda_j^{ki}} \geq 0$ for all $t \geq 0$ then substitute $\lambda_j^{ki} = \underline{\lambda}_j^{ki}$ in equation (2)

(ii) If $\frac{\partial \eta_{ji}(p,\lambda,t)}{\partial \lambda_j^{ik}} \leq 0$ for all $t \geq 0$ then substitute $\lambda_j^{ik} = \overline{\lambda}_j^{ik}$ in equation (2).

With the help of equations (i) and (ii), we get

$$\frac{d\underline{p}_{ji}(t)}{dt} = \sum_{k=i+1}^{k_j} \underline{\lambda}_j^{ki} \underline{p}_{jk}(t) - \underline{p}_{ji}(t)\sum_{k=1}^{i-1}\overline{\lambda}_j^{ik} \qquad (3)$$

Similarly, upper bound of probability can be obtained as

(iii) If $\frac{\partial \eta_{ji}(p,\lambda,t)}{\partial \lambda_j^{ki}} \geq 0$ for all $t \geq 0$ then substitute $\lambda_j^{ki} = \overline{\lambda}_j^{ki}$

(iv) if $\frac{\partial \eta_{ji}(p,\lambda,t)}{\partial \lambda_j^{ik}} \leq 0$ for all $t \geq 0$ then substitute $\lambda_j^{ik} = \underline{\lambda}_j^{ik}$

Applying (iii) and (iv) in differential equation (2), we have

$$\frac{d\overline{p}_{ji}(t)}{dt} = \sum_{k=i+1}^{k_j} \overline{\lambda}_j^{ki} \overline{p}_{jk}(t) - \overline{p}_{ji}(t)\sum_{k=1}^{i-1}\underline{\lambda}_j^{ik} \qquad (4)$$

The probability bounds can be obtained after solving equations (3) and (4) for lower and upper bound respectively by using Laplace-Stieltjes transform. Similarly, probability interval can be evaluated for elements of subsystem B.

5. UGF, Reliability and MTTF of the System

Preposition 5.1. IUGF of a multi-state consecutive *r*-out-of-*k*-from-*n*: G system with the multi-state elements is obtained as

$$U(z)=\sum_{i=1}^{s}[\underline{P}_i,\overline{P}_i]z^{g_i} \tag{5}$$

Proof. Consider a multi-state consecutive *r*-out-of-*k*-from-*n*: G system consisting of *n* elements. Let $[\underline{p}_{jh_j},\overline{p}_{jh_j}]$ be probability of $j=1, 2,\ldots,n$ component at $h_j=1,2,...,M_j$ state corresponding to g_{jh_j} performance then IUGFs of *n* components of multistate system is obtained from equation (1) and IUGF of system is given by

$$U(z)=\sum_{h_1=1}^{M_1}[\underline{p}_{1h_1},\overline{p}_{1h_1}]z^{g_{1h_1}}\otimes\sum_{h_2=1}^{M_2}[\underline{p}_{2h_2},\overline{p}_{2h_2}]z^{g_{2h_2}}\otimes....\otimes\sum_{h_n=1}^{M_n}[\underline{p}_{nh_n},\overline{p}_{nh_n}]z^{g_{nh_n}}$$

$$=\sum_{h_1=1}^{M_1}\sum_{h_2}^{M_2}\cdots\sum_{h_n=1}^{M_n}[\prod_{j=1}^{n}\underline{p}_{jh_j},\prod_{j=1}^{n}\overline{p}_{jh_j}]z^{\phi(g_{1h_1},g_{2h_2},...,g_{nj_n})})$$

where $\phi=\begin{cases}g_i, & if\ g_i\ge D\\ 0, & if\ g_i<D\end{cases},\quad i=1,2,...,s$

and W is sum of performances of *r* consecutive elements and *D* is demand performances.
Finally, IUGF of system can be expressed as

$$U(z)=\sum_{i=1}^{s}[\underline{P}_i,\overline{P}_i]z^{g_i}$$

where $\underline{P}_i=\prod_{j=1}^{n}\underline{p}_{jh_j},\ \overline{P}_i=\prod_{j=1}^{n}\overline{p}_{jh_j}$.

Preposition 5.2. The reliability of the multistate system consisting of two subsystems A and B connected in parallel is given by

$$R_{stm}(t)=\sum_{i=1}^{n}\sum_{l=1}^{m}[\underline{P}_i\underline{P}_l,\overline{P}_i\overline{P}_l]^{\mathrm{AB}}f(z^H),$$

$$\text{where } f=\begin{cases}1, & H\ge D_1\\ 0, & H<D_1\end{cases} \tag{6}$$

Proof. Let $[\underline{P}_i,\overline{P}_i]^{\mathrm{A}}$ and $[\underline{P}_l,\overline{P}_l]^{\mathrm{B}}$ be probabilities corresponding to the performances g_i, and g_l of subsystems A and B respectively. Then, IUGF of the subsystems A and B are obtained from equation (5) as follows

$$U_{\mathrm{A}}(z)=\sum_{i=1}^{s}[\underline{P}_i,\overline{P}_i]^{\mathrm{A}}z^{g_i}$$

$$U_{\mathrm{B}}(z) = \sum_{l=1}^{m} [\underline{P}_l, \overline{P}_l]^{\mathrm{B}} z^{g_l}$$

IUGF of the whole system is given by

$$U_{stm}(z) = \sum_{i=1}^{s} [\underline{P}_i, \overline{P}_i]^{\mathrm{A}} z^{g_i} \underset{par}{\otimes} \sum_{l=1}^{m} [\underline{P}_l, \overline{P}_l]^{\mathrm{B}} z^{g_l}$$

$$= \sum_{i=1}^{s} \sum_{l=1}^{m} [\underline{P}\underline{P}_i, \overline{P}_i \overline{P}_l]^{\mathrm{AB}} z^{+(g_i, g_l)}$$

Now reliability of the system is obtained as

$$R_{stm}(t) = \sum_{i=1}^{n} \sum_{l=1}^{m} [\underline{P}_i \underline{P}_l, \overline{P}_i \overline{P}_l]^{\mathrm{AB}} f(z^H)$$

where $f = \begin{cases} 1, & H \geq D \\ 0, & H < D \end{cases}$

Finally, reliability of the considered system is re-expressed as

$$R_{stm}(t) = [R_L(t), R_U(t)] \tag{7}$$

where $H = \oplus(g_i, g_l)$, lower bound of reliability $R_L(t) = \sum_{i=1}^{n} \sum_{l=1}^{m} \underline{P}_i \underline{P}_l$ and upper bound of reliability $R_U(t) = \sum_{i=1}^{n} \sum_{l=1}^{m} \overline{P}_i \overline{P}_l$.

Preposition 5.3. If $R_{stm}(t)$ is the reliability of the system and MT_{stm} is MTTF of the system, then MTTF of the considered system is obtained as

$$MT_{stm} = [MT_L, MT_U] \tag{8}$$

Proof. Let $R_{stm}(t)$ be reliability of the multistate system consisting of two subsystems A and B, the MTTF of the system is given as

$$MT_{\mathrm{stm}} = \int_0^{\infty} R_{stm}(t)dt \tag{9}$$

then MTTF of the considered system is obtained as

$$MT_{\mathrm{stm}} = \int_{t=0}^{\infty} [R_L(t), R_U(t)]\, dt \qquad \text{(from equations (7) and (9))}$$

$$= [\int_{t=0}^{\infty} \underline{R}_L(t)\, dt, \int_{t=0}^{\infty} \overline{R}_U(t)\, dt]$$

$$= [MT_L, MT_U].$$

6. Illustrative Example

Let us consider a non-repairable multistate system having two subsystems A and B connected in parallel where subsystem A (multi-state consecutive 2-out-of-3-from-4: G system) has four elements $\mathrm{E}_1^{\mathrm{A}}, \mathrm{E}_2^{\mathrm{A}}, \mathrm{E}_3^{\mathrm{A}}, \mathrm{E}_4^{\mathrm{A}}$ and the subsystem B (multi-state consecutive 2-out-of-3-out-of-4: G system) also consists of four elements

$E_1^B, E_2^B, E_3^B, E_4^B$. Let the elements E_1^A, E_1^B, E_2^B have three states having interval valued degradation rates $[\underline{\lambda}_1^{32}, \bar{\lambda}_1^{32}]^A = [.0009, .004]$, $[\underline{\lambda}_1^{21}, \bar{\lambda}_1^{21}]^A = [.0002, .0007]$, $[\underline{\lambda}_1^{31}, \bar{\lambda}_1^{31}]^A =$ $[.00006, .00099]$, $[\underline{\lambda}_1^{32}, \bar{\lambda}_1^{32}]^B = [.0003, .005]$, $[\underline{\lambda}_1^{21}, \bar{\lambda}_1^{21}]^B = [.00001, .00029]$, $[\underline{\lambda}_1^{31}, \bar{\lambda}_1^{31}]^B = [.00001, .0001]$, $[\underline{\lambda}_2^{32}, \bar{\lambda}_2^{32}]^B = [.0008, .004]$, $[\underline{\lambda}_2^{21}, \bar{\lambda}_2^{21}]^B = [.0001, .0003]$, $[\underline{\lambda}_2^{31}, \bar{\lambda}_2^{31}]^B = [.00003, .0001]$ respectively. Similarly, other elements E_2^A, E_3^A, E_4^A, E_3^B, E_4^B of the subsystems have two states and $[\underline{\lambda}_2^{21}, \bar{\lambda}_2^{21}]^A = [.002, .006]$, $[\underline{\lambda}_3^{21}, \bar{\lambda}_3^{21}]^A = [.00007, .0002]$, $[\underline{\lambda}_4^{21}, \bar{\lambda}_4^{21}]^A = [.02, .1]$, $[\underline{\lambda}_3^{21}, \bar{\lambda}_3^{21}]^B = [.00008, .0008]$, $[\underline{\lambda}_4^{21}, \bar{\lambda}_4^{21}]^B = [.0099, .099]$ are their corresponding interval valued degradation rates. Let demand performance of the system is $D \geq 32.5$. For the considered system interval valued probabilities listed in Table 1 are obtained by using equations (3) and (4). Interval universal generating function of every element of the subsystems A and B can be evaluated with the application of equation (1) as

$$u_j^A(z) = \sum_{k=1}^{s} [\underline{p}_{jk}, \bar{p}_{jk}]^A z^{g_j^k} \tag{10}$$

$$u_j^B(z) = \sum_{k=1}^{f} [\underline{p}_{jk}, \bar{p}_{jk}]^B z^{q_j^k} \tag{11}$$

where j=1,2,3,4 number of elements, k possible number of states corresponding to every element of the subsystems A and B.

If g_j^k and q_j^k are performances of every element j at state k corresponding to the subsystems A and B respectively then the performances of elements of the subsystems are taken as

$$g_1^1 = 5, g_1^2 = 3, g_1^3 = 0, g_2^1 = 4, g_2^2 = 0, g_3^1 = 2, g_3^2 = 0, g_4^1 = 6, g_4^2 = 0,$$
$$q_1^1 = 2, q_1^2 = 1, q_1^3 = 0, q_2^1 = 4.5, q_2^2 = 2.5, q_2^3 = 0, q_3^1 = 7, q_3^2 = 0, q_4^1 = 8, q_4^2 = 0,$$

The interval universal generating function U_A (z) of components of subsystem A with demand performance $G_A(z) \geq 15$ is evaluated with the help of equation (5)

$$\begin{aligned}
U_A(z) = {} & [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^A z^{(17)} + ([\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^A + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^A) z^{(15)} \\
& + ([\underline{p}_{12}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^A + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^A + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^A \\
& + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^A + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^A + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^A \\
& + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^A + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^A + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^A \\
& + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^A + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^A + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^A \\
& + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{32}\bar{p}_{42}]^A + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^A + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^A \\
& + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^A + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^A + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{32}\bar{p}_{42}]^A \\
& + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^A + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^A + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^A) z^{(0)}
\end{aligned} \tag{12}$$

The interval universal generating function U_{B} (z) of components of subsystem B with demand performance $\mathrm{G}_{\mathrm{B}}(z) \geq 17.5$ is obtained with the help of equation (5) as

$$
\begin{aligned}
U_{\mathrm{B}}(z) = {} & [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} z^{(21.5)} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} z^{(20.5)} + ([\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}}) z^{(19.5)} + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} z^{(18.5)} + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} z^{(17.5)} \\
& + ([\underline{p}_{11}\underline{p}_{23}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{23}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{23}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{23}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{23}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{23}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{23}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{23}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} \\
& + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{23}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{23}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{23}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{23}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{13}\underline{p}_{23}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{23}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{23}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{23}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{23}\underline{p}_{31}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{23}\bar{p}_{31}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{21}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{22}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{23}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{11}\bar{p}_{23}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{23}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{12}\bar{p}_{23}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}} + [\underline{p}_{13}\underline{p}_{23}\underline{p}_{32}\underline{p}_{42}, \bar{p}_{13}\bar{p}_{23}\bar{p}_{32}\bar{p}_{42}]^{\mathrm{B}}) z^{(0)}
\end{aligned}
\tag{13}
$$

After composing IUGFs of subsystems A and B we can obtain IUGF of the illustrated system. With the help of considered complex system's IUGF reliability can be evaluated at demand performance ($D \geq 32.5$) as

$$
\begin{aligned}
R_{stm}(t) = {} & [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} \\
& + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \\
& \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} \\
& + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + ([\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \\
& \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{13}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}}) + [\underline{p}_{11}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{11}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{A}} \\
& [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}} + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{\mathrm{A}} [\underline{p}_{12}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{B} \\
& + [\underline{p}_{12}\underline{p}_{21}\underline{p}_{31}\underline{p}_{41} + \underline{p}_{11}\underline{p}_{21}\underline{p}_{32}\underline{p}_{41}, \bar{p}_{12}\bar{p}_{21}\bar{p}_{31}\bar{p}_{41} + \bar{p}_{11}\bar{p}_{21}\bar{p}_{32}\bar{p}_{41}]^{A} [\underline{p}_{13}\underline{p}_{22}\underline{p}_{31}\underline{p}_{41}, \bar{p}_{13}\bar{p}_{22}\bar{p}_{31}\bar{p}_{41}]^{\mathrm{B}}
\end{aligned}
\tag{14}
$$

t	0	1	2	3	4	5
$[\underline{p}_{11},\bar{p}_{11}]^A$	[1,1]	[0.99502, 0.99904]	[0.99007, 0.99808]	[0.98514, 0.99712]	[0.98024, 0.99617]	[0.97536, 0.99521]
$[\underline{p}_{12},\bar{p}_{12}]^A$	[0,0]	[0.0009, 0.004]	[0.00179, 0.00799]	0.00267, 0.01198]	[0.00356, 0.01596]	[0.00444, 0.01994]
$[\underline{p}_{13},\bar{p}_{13}]^A$	[0,0]	[.00008, 0.00737]	[0.00017, 0.01474]	[0.00025, 0.0221]	[0.00033, 0.02946]	[0.00042, 0.03680]
$[\underline{p}_{21},\bar{p}_{21}]^A$	[1,1]	[0.99402, 0.998]	[0.98807, 0.99601]	[0.98216, 0.99402]	[0.97628, 0.99203]	[0.97045, 0.99005]
$[\underline{p}_{22},\bar{p}_{22}]^A$	[0,0]	[0.00199, 0.0059]	[0.00398, 0.01198]	[0.00595, 0.01795]	[0.0079, 0.0239]	[0.00985, 0.02985]
$[\underline{p}_{31},\bar{p}_{31}]^A$	[1,1]	[0.9998, 0.9999]	[0.9996, 0.99986]	[0.9994, 0.99979]	[0.9992, 0.99972]	[0.999, 0.99965]
$[\underline{p}_{32},\bar{p}_{32}]^A$	[0,0]	[.00007, 0.0002]	[0.00014, 0.0004]	[0.00021, 0.0006]	[0.00028, 0.0008]	[0.00035, 0.001]
$[\underline{p}_{41},\bar{p}_{41}]^A$	[1,1]	[0.90484, 0.9802]	[0.81873, 0.96079]	[0.74082, 0.94176]	[0.67032, 0.92312]	[0.60653, 0.90484]
$[\underline{p}_{42},\bar{p}_{42}]^A$	[0,0]	[0.01903, 0.0990]	[0.03625, 0.19605]	[0.05184, 0.29118]	[0.06594, 0.38442]	[0.07869, 0.47581]
$[\underline{p}_{11},\bar{p}_{11}]^B$	[1,1]	[0.9949, 0.9997]	[0.9899, 0.9994]	[0.9848, 0.9991]	[0.9798, 0.9988]	[0.9748, 0.9985]
$[\underline{p}_{12},\bar{p}_{12}]^B$	[0,0]	[0.0003, 0.005]	[0.0006, 0.01]	[0.0009, 0.015]	[0.0012, 0.02]	[0.0015, 0.025]
$[\underline{p}_{13},\bar{p}_{13}]^B$	[0,0]	[0.0000, 0.0097]	[0.00000, 0.01933	[0.00000, 0.02899]	[0.00000, 0.03866]	[0.00000, 0.04832]
$[\underline{p}_{21},\bar{p}_{21}]^B$	[1,1]	[0.9959, 0.9992]	[0.99834, 0.00159]	[0.997513, 0.00238]	[0.983734, 0.99669]	[0.97971, 0.99586]
$[\underline{p}_{22},\bar{p}_{22}]^B$	[0,0]	[0.00079, 0.00399]	[0.00159, 0.0079]	[0.00238, 0.01198]	[0.00317, 0.01597]	[0.00395, 0.01995]
$[\underline{p}_{23},\bar{p}_{23}]^B$	[0,0]	[0.00000, 0.00329]	[0.00000, 0.00659]	[0.00013, 0.00986]	[0.00017, 0.01314]	[0.00021, 0.01642]
$[\underline{p}_{31},\bar{p}_{31}]^B$	[1,1]	[0.99920, 0.99992]	[0.99840, 0.99984]	[0.99760, 0.99976]	[0.99681, 0.99968]	[0.99601, 0.99960]
$[\underline{p}_{32},\bar{p}_{32}]^B$	[0,0]	[0.00000, 0.0008]	[0.00016, 0.0016]	[0.00024, 0.0024]	[0.00032, 0.0032]	[0.0004, 0.004]
$[\underline{p}_{41},\bar{p}_{41}]^B$	[1,1]	[0.90574, 0.99014]	[0.82037, 0.98039]	[0.74304, 0.97073]	[0.67301, 0.96117]	[0.60957, 0.95171]
$[\underline{p}_{42},\bar{p}_{42}]^B$	[0,0]	[0.00943, 0.09851]	[0.01796, 0.19605]	[0.02569, 0.29263]	[0.0327, 0.38826]	[0.03904, 0.48295]

t	6	7	8	9	10
$[\underline{p}_{11},\bar{p}_{11}]^A$	[0.9705, 0.99426]	[0.96567, 0.9933]	[0.96087, 0.99235]	[0.95608, 0.9914]	[0.95132, 0.99045]
$[\underline{p}_{12},\bar{p}_{12}]^A$	[0.00531, 0.02392]	[0.00618, 0.02789]	[0.00704, 0.03185]	[0.00789, 0.03581]	[0.00875, 0.03977]
$[\underline{p}_{13},\bar{p}_{13}]^A$	[0.0005, 0.04416]	[0.00058, 0.0515]	[0.00066, 0.05884]	[0.00075, 0.06618]	[0.00083, 0.07351]
$[\underline{p}_{21},\bar{p}_{21}]^A$	[0.96464, 0.98807]	[0.95887, 0.9861]	[0.95313, 0.98413]	[0.94743, 0.98216]	[0.94176, 0.9802]
$[\underline{p}_{22},\bar{p}_{22}]^A$	[0.01179, 0.03578]	[0.01371, 0.04171]	[0.01562, 0.04762]	[0.01752, 0.05352]	[0.01941, 0.05940]
$[\underline{p}_{31},\bar{p}_{31}]^A$	[0.9988, 0.99958]	[0.9984, 0.99944]	[0.9986, 0.9995]	[0.9982, 0.99937]	[0.998, 0.9993]
$[\underline{p}_{32},\bar{p}_{32}]^A$	[0.00042, 0.0012]	[0.00049, 0.00114]	[0.00056, 0.0016]	[0.00063, 0.0018]	[0.0007, 0.002]
$[\underline{p}_{41},\bar{p}_{41}]^A$	[0.54881, 0.88692]	[0.49658, 0.86936]	[0.44933, 0.85214]	[0.40657, 0.83527]	[0.36788, 0.81873]
$[\underline{p}_{42},\bar{p}_{42}]^A$	[0.09024, 0.5654]	[0.10068, 0.65321]	[0.11013, 0.73928]	0.11869, 0.82365]	[0.12642, 0.90635]
$[\underline{p}_{11},\bar{p}_{11}]^B$	[0.9699, 0.9981]	[0.9649, 0.9978]	[0.96, 0.9975]	[0.9551, 0.9972]	[0.9503, 0.9969]
$[\underline{p}_{12},\bar{p}_{12}]^B$	[0.0018, 0.03]	[0.0021, 0.035]	[0.0023, 0.0399]	[0.00026, 0.0445]	[0.00029, 0.0449]
$[\underline{p}_{13},\bar{p}_{13}]^B$	[0.00000, 0.05798]	[0.00000, 0.06763]	[0.00001, 0.07729]	[0.00001, 0.08694]	[0.00011, 0.09659]
$[\underline{p}_{21},\bar{p}_{21}]^B$	[0.9757, 0.99503]	[0.97171, 0.99421]	[0.96773, 0.99338]	[0.96377, 0.99256]	[0.95983, 0.99173]
$[\underline{p}_{22},\bar{p}_{22}]^B$	[0.00474, 0.02393]	[0.00551, 0.02791]	[0.00629, 0.03188]	[0.00706, 0.03585]	[0.00783, 0.03981]
$[\underline{p}_{23},\bar{p}_{23}]^B$	[0.00025, 0.01969]	[0.00029, 0.02298]	[0.00033, 0.02625]	[0.00037, 0.02952]	[0.00042, 0.03281]
$[\underline{p}_{31},\bar{p}_{31}]^B$	[0.99521, 0.99952]	[0.99442, 0.99944]	[0.99362, 0.99936]	[0.99283, 0.99928]	[0.99203, 0.9992]
$[\underline{p}_{32},\bar{p}_{32}]^B$	[0.00048, 0.0048]	[0.00056, 0.0056]	[0.00064, 0.00639]	[0.00072, 0.0072]	[0.0008, 0.008]
$[\underline{p}_{41},\bar{p}_{41}]^B$	[0.55211, 0.94233]	[0.50007, 0.93305]	[0.45294, 0.92386]	[0.41025, 0.91474]	[0.37157, 0.90574]
$[\underline{p}_{42},\bar{p}_{42}]^B$	[0.04478, 0.57670]	[0.04999, 0.66953]	[0.05471, 0.76145]	[0.05898, 0.8525]	[0.06284, 0.94257]

Table 1: Probability intervals of system components w.r.t. time

Reliability of the considered multi-state complex system in interval form can be evaluated with the help of equation (14) and Table 1 as

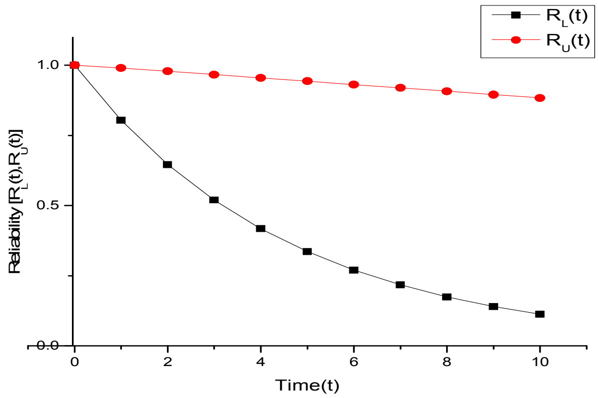

Fig. 2: Interval valued reliability of the multi-state complex system w.r.t. time

Mean Time to Failure

MTTF (MT_{stm}) of proposed multi-state complex system with two subsystems A and B can be obtained by using equations (8) and (10). The effect with respect to lower and upper bounds of failure rates for the considered systems is shown in Table 2.

λ_1^{32A}	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5683, 59.7060]	[4.5704, 59.4433]	[4.5725, 59.1827]	[4.5747, 58.9244]	[4.5768, 58.6682]
λ_1^{21A}	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.7317]	[4.5854, 57.7279]	[4.5854, 57.7245]	[4.5854, 57.7209]	[4.5854, 57.7172]
λ_1^{31A}	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.7873]	[4.5854, 57.7624]	[4.5854, 57.7376]	[4.5854, 57.7130]	[4.5854, 57.6881]
λ_2^{21A}	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5854, 63.5335]	[4.5854, 63.1978]	[4.5854, 62.8651]	[4.5854, 62.5356]	[4.5854, 62.2095]
λ_3^{21A}	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5840, 57.8326]	[4.5842, 57.8043]	[4.5845, 57.7761]	[4.5847, 57.7479]	[4.5849, 57.7196]
λ_4^{21A}	0.01	0.02	0.03	0.04	0.05
MT_{stm}	[4.5854, 107.5489]	[4.5854, 57.6635]	[4.5854, 38.2624]	[4.5854, 28.3103]	[4.5854, 22.3510]

$\underline{\lambda}_1^{32B}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5810, 84.9373]	[4.5831, 64.5622]	[4.5854, 57.6635]	[4.5875, 54.1345]	[4.5896, 51.9540]
$\underline{\lambda}_1^{21B}$	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.6635]	[4.5854, 58.1206]	[4.5854, 58.6107]	[4.5854, 59.1370]	[4.5854, 59.7043]
$\underline{\lambda}_1^{31B}$	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.6635]	[4.5854, 57.2127]	[4.5854, 56.7891]	[4.5854, 56.3903]	[4.5854, 56.0139]
$\underline{\lambda}_2^{32B}$	0.0001	0.0002	0.0003	00.0004	0.0005
MT_{stm}	[4.5705, 59.4398]	[4.5726, 59.1799]	[4.5747, 58.9220]	[4.5769, 58.6663]	[4.5790, 58.4124]
$\underline{\lambda}_2^{21B}$	0.0001	0.00011	0.00012	0.00013	0.00014
MT_{stm}	[4.5854, 57.6635]	[4.5854, 57.6599]	[4.5854, 57.6562]	[4.5854, 57.6527]	[4.5854, 57.6491]
$\underline{\lambda}_2^{31B}$	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.7129]	[4.5854, 57.6881]	[4.5854, 57.6635]	[4.5854, 57.6388]	[4.5854, 57.6142]
$\underline{\lambda}_3^{21B}$	0.00001	0.00002	0.00003	0.00004	0.00005
MT_{stm}	[4.5854, 57.8621]	[4.5854, 57.8337]	[4.5854, 57.8051]	[4.5854, 57.7767]	[4.5854, 57.7485]
$\underline{\lambda}_4^{21B}$	0.001	0.002	0.003	0.004	0.005
MT_{stm}	[4.5854, 98.7482]	[4.5854, 91.7879]	[4.5854, 85.6503]	[4.5854, 80.2059]	[4.5854, 75.3502]
$\overline{\lambda}_1^{32A}$	0.001	0.002	0.003	0.004	0.005
MT_{stm}	[4.6490, 51.1911]	[4.6276, 53.3485]	[4.6065, 55.5061]	[4.5854, 57.6635]	[4.5439, 59.8210]
$\overline{\lambda}_1^{21A}$	0.0002	0.0003	0.0004	0.0005	0.0006
MT_{stm}	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]
$\overline{\lambda}_1^{31A}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.6041, 57.6635]	[4.6020, 57.6635]	[4.5998, 57.6635]	[4.5977, 57.6635]	[4.5956, 57.6635]
$\overline{\lambda}_2^{21A}$	0.002	0.003	0.004	0.005	0.006
MT_{stm}	[4.6710, 57.6635]	[4.6493, 57.6635]	[4.6278, 57.6635]	[4.6065, 57.6635]	[4.5854, 57.6635]
$\overline{\lambda}_3^{21A}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5875, 57.4538]	[4.5854, 57.6635]	[4.5833, 57.8733]	[4.5812, 58.0830]	[4.5791, 58.2927]
$\overline{\lambda}_4^{21A}$	0.1	0.2	0.3	0.4	0.5
MT_{stm}	[4.5854, 57.6635]	[3.1437, 57.6635]	[2.3917, 57.6635]	[1.9301, 57.6635]	[1.6178, 57.6635]
$\overline{\lambda}_1^{32B}$	0.001	0.002	0.003	0.004	0.005
MT_{stm}	[4.6710, 41.4269]	[4.6493, 45.4860]	[4.6277, 49.5452]	[4.6065, 53.6043]	[4.5854, 57.6635]
$\overline{\lambda}_1^{21B}$	0.0001	0.0002	0.0003	0.0004	0.0005

MT_{stm}	[4.5854, 48.8825]	[4.5854, 53.5041]	[4.5854, 58.1255]	[4.5854, 62.7472]	[4.5854, 67.3688]
$\bar{\lambda}_1^{31B}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5854, 57.6635]	[4.5833, 57.6641]	[4.5812, 57.6650]	[4.5791, 57.6656]	[4.5770, 57.6662]
$\bar{\lambda}_2^{32B}$	0.001	0.002	0.003	0.004	0.005
MT_{stm}	[4.6491, 51.1236]	[4.6276, 53.3035]	[4.6064, 55.4836]	[4.5854, 57.6635]	[4.5645, 59.8434]
$\bar{\lambda}_2^{21B}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]
$\bar{\lambda}_2^{31B}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.5854, 57.6635]	[4.5833, 57.6635]	[4.5791, 57.6635]	[4.5790, 57.6635]	[4.5770, 57.6635]
$\bar{\lambda}_3^{21B}$	0.0001	0.0002	0.0003	0.0004	0.0005
MT_{stm}	[4.6001, 57.6635]	[4.5980, 57.6635]	[4.5959, 57.6635]	[4.5938, 57.6635]	[4.5917, 57.6635]
$\bar{\lambda}_4^{21B}$	0.01	0.02	0.03	0.04	0.05
MT_{stm}	[7.7480, 57.6635]	[7.1906, 57.6635]	[6.7081, 57.6635]	[6.2863, 57.6635]	[5.9144, 57.6635]

$\underline{\lambda}_1^{32A}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5789, 58.4139]	[4.5810, 58.1617]	[4.5831, 57.9115]	[4.5854, 57.6635]	[4.5875, 57.4173]
$\underline{\lambda}_1^{21A}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5854, 57.7137]	[4.5854, 57.7100]	[4.5854, 57.7065]	[4.5854, 57.7028]	[4.5854, 57.6992]
$\underline{\lambda}_1^{31A}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5854, 57.6635]	[4.5854, 57.6386]	[4.5854, 57.6142]	[4.5854, 57.5893]	[4.5854, 57.5648]
$\underline{\lambda}_2^{21A}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5854, 61.8863]	[4.5854, 61.5661]	[4.5854, 61.2489]	[4.5854, 60.9348]	[4.5854, 60.6234]
$\underline{\lambda}_3^{21A}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5851, 57.6916]	[4.5854, 57.6635]	[4.5856, 57.6354]	[4.5858, 57.6072]	[4.5860, 57.5793]
$\underline{\lambda}_4^{21A}$	0.06	0.07	0.08	0.09	0.1
MT_{stm}	[4.5854, 18.4149]	[4.5854, 15.6336]	[4.5854, 13.5697]	[4.5854, 11.9801]	[4.5854, 10.7197]
$\underline{\lambda}_1^{32B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5918, 50.4487]	[4.5939, 49.3293]	[4.5960, 48.4518]	[4.5983, 47.7678]	[4.6004, 47.1329]
$\underline{\lambda}_1^{21B}$	0.00006	0.00007	0.00008	0.00009	0.0001

MT_{stm}	[4.5854, 60.3170]	[4.5854, 60.9809]	[4.5854, 61.7028]	[4.5854, 62.4907]	[4.5854, 63.3538]
$\underline{\lambda}_1^{31B}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5854, 55.6582]	[4.5854, 55.3213]	[4.5854, 55.0019]	[4.5854, 54.6983]	[4.5854, 54.4097]
$\underline{\lambda}_2^{32B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5811, 58.1608]	[4.5832, 57.9111]	[4.5854, 57.6635]	[4.5875, 57.4179]	[4.5895, 57.1741]
$\underline{\lambda}_2^{21B}$	0.00015	0.00016	0.00017	0.00018	0.00019
MT_{stm}	[4.5854, 57.6455]	[4.5854, 57.6419]	[4.5854, 57.6383]	[4.5854, 57.6346]	[4.5854, 57.6311]
$\underline{\lambda}_2^{31B}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5854, 57.5894]	[4.5854, 57.5650]	[4.5854, 57.5404]	[4.5854, 57.5158]	[4.5854, 57.4912]
$\underline{\lambda}_3^{21B}$	0.00006	0.00007	0.00008	0.00009	0.0001
MT_{stm}	[4.5854, 57.7201]	[4.5854, 57.6917]	[4.5854, 57.6635]	[4.5854, 57.6352]	[4.5854, 57.6070]
$\underline{\lambda}_4^{21B}$	0.006	0.007	0.008	0.009	0.01
MT_{stm}	[4.5854, 70.9976]	[4.5854, 67.0782]	[4.5854, 63.5335]	[4.5854, 60.3150]	[4.5854, 57.3820]
$\overline{\lambda}_1^{32A}$	0.006	0.007	0.008	0.009	0.01
MT_{stm}	[4.5444, 61.9784]	[4.5234, 64.1359]	[4.5031, 67.4068]	[4.4829, 68.4507]	[4.4630, 70.6082]
$\overline{\lambda}_1^{21A}$	0.0007	0.0008	0.0009	0.001	0.002
MT_{stm}	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]	[4.5854, 57.6635]
$\overline{\lambda}_1^{31A}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5935, 57.6635]	[4.5914, 57.6635]	[4.5893, 57.6635]	[4.5872, 57.6635]	[4.5851, 57.6635]
$\overline{\lambda}_2^{21A}$	0.007	0.008	0.009	0.01	0.02
MT_{stm}	[4.5644, 57.6635]	[4.5437, 57.6635]	[4.5230, 57.6635]	[4.5027, 57.6635]	[4.3087, 57.6635]
$\overline{\lambda}_3^{21A}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5770, 58.5025]	[4.5749, 58.7123]	[4.5728, 58.9220]	[4.5707, 59.1319]	[4.5686, 59.3416]
$\overline{\lambda}_4^{21A}$	0.6	0.7	0.8	0.9	1.0
MT_{stm}	[1.3925, 57.6635]	[1.2223, 57.6635]	[1.0892, 57.6635]	[0.9822, 57.6635]	[0.8944, 57.6635]
$\overline{\lambda}_1^{32B}$	0.006	0.007	0.008	0.009	0.01
MT_{stm}	[4.5644, 61.7226]	[4.5437, 65.7817]	[4.5232, 69.8409]	[4.5028, 73.7467]	[4.4826, 77.9592]
$\overline{\lambda}_1^{21B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5854, 71.9904]	[4.5854, 76.6120]	[4.5854, 81.2336]	[4.5854, 85.8552]	[4.5854, 90.4768]
$\overline{\lambda}_1^{31B}$	0.0006	0.0007	0.0008	0.0009	0.001

MT_{stm}	[4.5749, 57.6670]	[4.5728, 57.6676]	[4.6469, 4.5707]	[4.5686, 57.6690]	[4.5665, 57.6697]
$\overline{\lambda}_2^{32B}$	0.006	0.007	0.008	0.009	0.01
MT_{stm}	[4.5438, 62.0233]	[4.5234, 64.2032]	[4.5031, 66.3831]	[4.4829, 68.5631]	[4.4629, 70.7430]
$\overline{\lambda}_2^{21B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]	[4.6276, 53.3035]
$\overline{\lambda}_2^{31B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5749, 57.6635]	[4.5728, 57.6635]	[4.5707, 57.6635]	[4.5686, 57.6635]	[4.5665, 57.6635]
$\overline{\lambda}_3^{21B}$	0.0006	0.0007	0.0008	0.0009	0.001
MT_{stm}	[4.5896, 57.6635]	[4.5875, 57.6635]	[4.5854, 57.6635]	[4.5833, 57.6635]	[4.5812, 57.6635]
$\overline{\lambda}_4^{21B}$	0.06	0.07	0.08	0.09	0.1
MT_{stm}	[5.5841, 57.6635]	[5.2887, 57.6635]	[5.0229, 57.6635]	[4.7827, 57.6635]	[4.5644, 57.6635]

Table 2: variation on MTTF of the multi-state complex system w.r.t. lower and upper bounds of failure rate

7. Conclusion

In this paper, a multi-state complex system having two subsystems A and B (non-repairable multi-state consecutive *r*-out-of-*k*-from-*n*: G systems) under imprecise probability and degradation rate has been studied. Our main concern is to analyse interval valued reliability and MTTF of the considered system by using interval universal generating function and probability interval analysis with stochastic process approach. Numerical example has been taken to show the efficiency of the applied method. It is seen from numerical example that the lower and upper reliability bonds of the considered systems are decreasing simultaneously but not with the same rate. Further, the example also reveals the variation in MTTF of the system with respect to lower and upper bound of failure rate. The uncertainty in MTTF decreases with respect to increment in lower bounds $\underline{\lambda}_1^{32A}$, $\underline{\lambda}_1^{21A}$, $\underline{\lambda}_1^{31A}$, $\underline{\lambda}_2^{21A}$, $\underline{\lambda}_3^{21A}$, $\underline{\lambda}_4^{21A}$, $\underline{\lambda}_1^{32B}$, $\underline{\lambda}_1^{31B}$, $\underline{\lambda}_2^{32B}$, $\underline{\lambda}_2^{31B}$, $\underline{\lambda}_2^{21B}$, $\underline{\lambda}_3^{21B}$, $\underline{\lambda}_4^{21B}$ of failure rates, MTTF uncertainty increases with increment of $\underline{\lambda}_1^{21B}$. Further, MTTF found to be increasing with increment in upper bounds $\overline{\lambda}_1^{32A}$, $\overline{\lambda}_1^{31A}$, $\overline{\lambda}_2^{21A}$, $\overline{\lambda}_4^{21A}$ $\overline{\lambda}_3^{21A}$, $\overline{\lambda}_1^{32B}$, $\overline{\lambda}_1^{21B}$, $\overline{\lambda}_1^{31B}$, $\overline{\lambda}_2^{32B}$, $\overline{\lambda}_2^{31B}$, $\overline{\lambda}_3^{21B}$, $\overline{\lambda}_4^{21B}$ of failure rates corresponding to components states of subsystems A and B while the value of MTTF fixed with increment of upper bounds $\overline{\lambda}_1^{21A}$, $\overline{\lambda}_2^{21B}$ of subsystems A's and B's components failures rates.

References

1. Barlow, R. and Wu, A. (1978). Coherent systems with multi-state elements, Mathematics of Operations Research, 3, p. 275–281.
2. Desterche, S. and Sallak, M. (2013). An extension of universal generating function in multi-state system considering epistemic uncertainties, IEEE Transactions on Reliability, 62(2), p. 504-514.
3. Ding, Y. and Lisnianski, A. (2008). Fuzzy universal generating functions for multi-state system reliability assessment, Fuzzy Sets and Systems, 159(3), p. 307-324.
4. El-Neveihi, E., Prochan, F. and Setharaman (1978). Multi-state coherent systems, Journal of Applied Probability, 15, p. 675-688.
5. Habib, A., Al-Seedy, R. O. and Radwan, T. (2007). Reliability evaluation of multi-state consecutive *k*-out-of-*r*-from-*n*: G system, Applied Mathematical Modelling, 31(11), p. 2412-2423.
6. Meenakshi and Singh, S.B. (2016). Availability assessment of multi-state system by hybrid universal generating function and probability intervals, International Journal of Performability Engineering, 4, p. 321-339.
7. Levitin, G. (2005). Reliability of linear multistate multiple sliding window systems, Nav. Res. Log., 52 (3), p. 212-223.
8. Levitin, G. and Lisnianski, A. (1999). Importance and sensitivity analysis of multi state systems using the universal generating function method, Reliability Engineering & System Safety, 65(3), p. 271-282.
9. Levitin, G. and Xing, L. (2010). Reliability and performance of multi state systems with propagated failures having selective effect, Reliability Engineering & System Safety, 95(6), p. 655–661.
10. Levitin, G. (2005). The Universal Generating Function in Reliability Analysis and Optimization, London: Springer-Verlag.
11. Li, C.-Y., Chen, X., Yi, X.-S. and Tao, J.-Y. (2011). Interval-valued reliability analysis of multi-state systems, IEEE Transactions on Reliability, 60, p. 323 - 330.
12. Lisnianski, A. (2007). Extended block diagram method for a multi-state system reliability assessment, Reliability Engineering & System Safety, 92(12), p. 1601-1607.
13. Lisnianski, A. and Levitin, G. (2003). Multi-State System Reliability: Assessment, Optimization and Applications, World Scientific Publishing Co. Pte. Ltd.
14. Marquez, J. R. and Coit, D. (2005). A Monte-Carlo simulation approach for approximating multi-state two-terminal reliability, Reliability Engineering & System Safety, 87(2), p. 253–264.
15. Papastavridis, S. and Koutras, M. V. (1993). Bounds for reliability of consecutive-*k*-within-*m*-out-of-*n* systems, IEEE Trans. Reliability, R-42, p. 156–160.
16. Papastavridis, S. and Sfakianak, M. E. (1991). Optimal arrangement and importance of the components in a consecutive-*k*-out-of-*r*-from-*n*: F system, IEEE Trans. Reliability, R-40, p. 277–279.
17. Pourret, O., Collet, J. and Bon, J. L. (1999). Evaluation of the unavailability of a multi-state component system using a binary model, Reliability Engineering & System Safety, 64(1), p. 13-17.

18. Ramdani, N., Meslem, N. and Candau, Y. (2010). Computing reachable sets for uncertain nonlinear monotone systems, Nonlinear Analysis: Hybrid System, 4, p. 263-278.
19. Sfakianakis, M., Kounias, S. and Hillaris, A. (1992). Reliability of a consecutive-*k*-out-of-*r*-from-*n*: F system, IEEE Trans. Reliability, R-41, p. 442-447.
20. Simon, C. and Weber, P. (2009). Evidential networks for reliability analysis and performance evaluation of systems with imprecise knowledge. IEEE Transactions on Reliability, 58(1), p. 69–87.
21. Tong, Y.L. (1985). Rearrangement inequality for the longest run with an application to network reliability, J. Appl. Prob., 22, p. 286-393.
22. Xiao, N.-C., Huang, H.-Z., Liu, Y., Li, Y. and Wang, Z. (2012). Unified uncertainty analysis by the extension universal generating functions. in International Conference on Quality, Reliability, Risk, Maintenance, and Safety Engineering (ICQR2MSE), p. 1160 -1166.
23. Xue, J. and Yang, K. (1995). Dynamic reliability analysis of coherent multistate systems, IEEE Transactions on Reliability, 44, p. 253–264.
24. Zio, E., Marella, M. and Podofillini, L. (2007). A Monte Carlo simulation approach to the availability assessment of multi-state systems with operational dependencies, Reliability Engineering & System Safety, 92(7), p. 871-882.