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A NEW FORM OF MULTIVARIATE GENERALIZED DOUBLE EXPONENTIAL FAMILY OF DISTRIBUTIONS OF KIND-2

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Abstract

This paper proposes a generalization of double exponential family of Sarmanov type continuous multivariate symmetric probability distribution with reference to a new form of Sam-Solai's multivariate generalized double exponential family of distribution of kind-2 from univariate case. Further, we have found it's Marginal, multivariate conditional distributions, generating functions in a closed series expression form and also discussed its special cases. The location, scale and shape parameters have played a significance role and it has determined the family of some existing known and familiar bivariate, mixtures of bivariate distributions, generalized and multivariate distributions as subclass of the proposed multivariate generalized double exponential family of distributions of kind-2.

Key Words: Sarmanov Type, Sam-Solai's Multivariate Generalized Exponential Family of Distributions, Closed Series, Shape Parameter, Mixtures of Bivariate Distributions.

1. Introduction

The generalization of generalized probability distributions and their application is versatile in all fields especially in finance, insurance and risk management etc. For the 5 past decades, statisticians gave more concentration to the generalization of exponential distribution and later they extended their investigation to the double exponential distribution in which the univariate generalization of generalized double exponential distribution diverted the path of distribution theory to new way. The authors Bain and Engelhard (1973) estimated confidence interval using maximum likelihood estimators and scale parameter for the double exponential distribution. On the other hand Kappenman (1977) developed the tolerance intervals for the double exponential distribution. Similarly, Mir et al., (1981) estimated quantiles based on two order statistics for exponential and double exponential distribution. In a different manner, Ulrich and Chen (1987) discussed a bivariate double exponential distribution and its generalization. Later, Govindarajulu (2001) characterized double exponential distribution using the expected values of the spacing associated with certain extreme order statistics and the relation between the differences of two product moments of certain extreme order statistics induced by random samples of arbitrary sizes. Based on the past reviews, many authors studied the univariate, bi-variate double exponential distribution and they applied in many areas. In this paper, the authors made an attempt to bring out a family of double exponential distribution in a generalized multivariate form which leads to explore more sub-class of Bivariate, mixtures of Bivariate, generalized and multivariate double exponential distributions. The structure of the proposed distribution and its special cases are discussed in the subsequent sections.

2. Sam-Solai's Multivariate Generalized Double exponential family of Distributions of Kind-2

Definition 2.1: Let $X_1, X_2, X_3 \ldots, X_p$ are the random variables following continuous univariate Generalized double exponential family of distributions (or) Generalized Double Gamma distribution having the location (μ_i), scale (λ_i, σ_i) and shape parameters (α_i, β_i) with mean (μ_i) and variance $\sigma_i^2\left(\Gamma\left((\alpha_i+2)/\beta_i\right)/(1/\lambda_i)^{2/\beta_i}\Gamma(\alpha_i/\beta_i)\right)$ for all i *(i=1 to p).* Then the Multivariate Sam-Solai's generalized double exponential family of distributions of kind-2 and its density is defined as

$$f(x_1,x_2,x_3\ldots,x_p)=\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right) \tag{1}$$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $-\infty \le \mu_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\lambda_i, \sigma_i, \alpha_i, \beta_i > 0$, $-1 \le \rho_{ij} \le +1$ and ρ_{ij} is the correlation co-efficient between i^{th} and j^{th} random variables.

Definition 2.2: From (1) and if $z_i = (x_i - \mu_i)/\sigma_i$, then using multi-dimensional Jacobian of transformation, the Sam-Solai's multivariate Generalized standard double exponential family of distribution of kind-2 and its density is defined as

$$f(z_1,z_2,z_3\ldots,z_p)=\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}z_i^{2\gamma_i-1}z_j^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\Gamma(\alpha_i/\beta_i)}\left|z_i\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}|z_i|^{\beta_i}}\right) \tag{2}$$

where $i \neq j$, $-\infty \le z_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\lambda_i, \alpha_i, \beta_i > 0$, $-1 \le \rho_{ij} \le +1$

Theorem 2.3: The cumulative distribution function of the Sam-solai's Multivariate Generalized double exponential family of distributions of kind-2 can also be represented in terms of its standardized form and it is given as

$$\begin{aligned}F(z_1,z_2,z_3\ldots,z_p)=&\left(-\frac{1}{2}\right)^p\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\Gamma((\alpha_i+2\gamma_i-1)/\beta_i)}{(1/\lambda_i)^{(2\gamma_i-1)/\beta_i}\Gamma(\alpha_i/\beta_i)}\frac{\Gamma((\alpha_j+2\gamma_j-1)/\beta_j)}{(1/\lambda_j)^{(2\gamma_j-1)/\beta_j}\Gamma(\alpha_j/\beta_j)}\right)\\&+\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\phi_i(z_i;\lambda_i,\alpha_i,\beta_i)\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\varphi_i(z_i;\lambda_i,\gamma_i,\alpha_i,\beta_i)\varphi_j(z_j;\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\phi_i(z_i;\lambda_i,\alpha_i,\beta_i)}\right)\end{aligned} \tag{3}$$

where $i \neq j$, $-\infty \le z_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\lambda_i, \alpha_i, \beta_i > 0$, $-1 \le \rho_{ij} \le +1$

Proof: Let the Multivariate Cumulative distribution function is given as

$$F(z_1,z_2,z_3\ldots,z_p)=$$

$$\int_{-\infty}^{z_1}\int_{-\infty}^{z_2}\int_{-\infty}^{z_3}\cdots\int_{-\infty}^{z_p}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}u_i^{2\gamma_i-1}u_j^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\Gamma(\alpha_i/\beta_i)}|u_i|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}|u_i|^{\beta_i}}\right)\prod_{i=1}^{p}du_i$$

$$F(z_1,z_2,z_3\ldots,z_p)=$$

$$\int_{-\infty}^{0}\int_{-\infty}^{0}\int_{-\infty}^{0}\cdots\int_{-\infty}^{0}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}u_i^{2\gamma_i-1}u_j^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\Gamma(\alpha_i/\beta_i)}|u_i|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}|u_i|^{\beta_i}}\right)\prod_{i=1}^{p}du_i$$

$$+\int_{0}^{z_1}\int_{0}^{z_2}\int_{0}^{z_3}\cdots\int_{0}^{z_p}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}u_i^{2\gamma_i-1}u_j^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\Gamma(\alpha_i/\beta_i)}|u_i|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}|u_i|^{\beta_i}}\right)\prod_{i=1}^{p}du_i$$

$$F(z_1,z_2,z_3\ldots,z_p)=$$

$$\left(-\frac{1}{2}\right)^p\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\Gamma((\alpha_i+2\gamma_i-1)/\beta_i)}{(1/\lambda_i)^{(2\gamma_i-1)/\beta_i}\Gamma(\alpha_i/\beta_i)}\frac{\Gamma((\alpha_j+2\gamma_j-1)/\beta_j)}{(1/\lambda_j)^{(2\gamma_j-1)/\beta_j}\Gamma(\alpha_j/\beta_j)}\right)$$

$$+(\frac{1}{2})^p\prod_{i=1}^{p}\phi_i(z_i;\lambda_i,\alpha_i,\beta_i)\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\varphi_i(z_i;\lambda_i,\gamma_i,\alpha_i,\beta_i)\varphi_j(z_j;\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\phi_i(z_i;\lambda_i,\alpha_i,\beta_i)}\right)$$

where $\phi_i(z_i;\lambda_i,\alpha_i,\beta_i)=\int_0^{z_i^{\beta_i}/\lambda_i}\frac{1}{\Gamma(\alpha_i/\beta_i)}s_i^{(\alpha_i/\beta_i)-1}e^{-s_i}ds_i$,

$\varphi_i(z_i;\lambda_i,\gamma_i,\alpha_i,\beta_i)=\int_0^{z_i^{\beta_i}/\lambda_i}\frac{1}{\Gamma((\alpha_i+2\gamma_i-1)/\beta_i)}s_i^{((\alpha_i+2\gamma_i-1)/\beta_i)-1}e^{-s_i}ds_i$ and

$\varphi_j(z_j;\lambda_j,\gamma_j,\alpha_j,\beta_j)=\int_0^{z_j^{\beta_j}/\lambda_j}\frac{1}{\Gamma((\alpha_j+2\gamma_j-1)/\beta_j)}s_j^{((\alpha_j+2\gamma_j-1)/\beta_j)-1}e^{-s_j}ds_j$

are the lower Incomplete Gamma Integrals.

Theorem 2.4: The Probability density function of the Sam-solai's Multivariate Generalized conditional double exponential family of distributions of kind-2 of X_1 on $X_2,X_3,\ldots X_p$ is

$$f(x_1\mid x_2,x_3\ldots,x_p)=\frac{\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{\beta_1(1/\lambda_1)^{\alpha_1/\beta_1}}{2\sigma_1\Gamma(\alpha_1/\beta_1)}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\alpha_1-1}e^{-\frac{1}{\lambda_1}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\beta_1}}\right)}{\left(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)} \quad (4)$$

where $i\neq j$, $-\infty\le x_1\le+\infty$, $-\infty\le\mu_1\le+\infty$, $\gamma_i,\gamma_j\ge1$ and real, $\lambda_1,\sigma_1,\alpha_1,\beta_1>0$, $-1\le\rho_{ij}\le+1$

Proof: Let the Multivariate Conditional distribution of X_1 on $X_2, X_3, \dots X_p$ is given as

$$f(x_1 \mid x_2, x_3 \dots, x_p) = \frac{f(x_1, x_2, x_3 \dots, x_p)}{f(x_2, x_3 \dots, x_p)}$$

$$f(x_1 \mid x_2, x_3 \dots, x_p) = \frac{\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)}{\int_{-\infty}^{+\infty}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)dx_1}$$

$$f(x_1 \mid x_2, x_3 \dots, x_p) = \frac{\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{\beta_1(1/\lambda_1)^{\alpha_1/\beta_1}}{2\sigma_1\Gamma(\alpha_1/\beta_1)}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\alpha_1-1}e^{-\frac{1}{\lambda_1}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\beta_1}}\right)}{\left(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)}$$

where $i \neq j$, $-\infty \le x_1 \le +\infty$, $-\infty \le \mu_1 \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\lambda_1, \sigma_1, \alpha_1, \beta_1 > 0$, $-1 \le \rho_{ij} \le +1$

Theorem 2.5: Mean and Variance of the Sam-solai's Multivariate Generalized conditional double exponential family of distributions of kind-2 are

$$E(X_1 \mid X_2, X_3 \dots, X_p) = \mu_1 + \left(\frac{\Gamma((\alpha_1+2\gamma_1))/\beta_1)}{(1/\lambda_1)^{2\gamma_1/\beta_1}\Gamma(\alpha_1/\beta_1)}\right)\left(\frac{\sigma_1\sum_{j=2}^{p}\rho_{1j}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}{1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}\right) \quad (5)$$

$$V(X_1 \mid X_2, X_3 \dots, X_p) = \sigma_1^2\left(\frac{\Gamma((\alpha_1+2\gamma_1))/\beta_1)}{(1/\lambda_1)^{2\gamma_1/\beta_1}\Gamma(\alpha_1/\beta_1)}\right)\left(1-\left(\frac{\sum_{j=2}^{p}\rho_{1j}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}{1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}\right)^2\right) \quad (6)$$

Proof: The Conditional expectation and Conditional variance of a multivariate distribution are given as

$$E(X_1 \mid X_2, X_3 \dots, X_p) = \int_{-\infty}^{+\infty} x_1 f(x_1 / x_2, x_3 \dots, x_p)\, dx_1$$

$$E(X_1 \mid X_2, X_3 \dots, X_p) =$$

$$\int_{-\infty}^{+\infty} x_1 \frac{\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{\beta_1(1/\lambda_1)^{\alpha_1/\beta_1}}{2\sigma_1\Gamma(\alpha_1/\beta_1)}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\alpha_1-1} e^{-\frac{1}{\lambda_1}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\beta_1}}\right)}{\left(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)} dx_1$$

$$E(X_1 \mid X_2, X_3 \dots, X_p) = \mu_1 + \left(\frac{\Gamma((\alpha_1+2\gamma_1))/\beta_1)}{(1/\lambda_1)^{2\gamma_1/\beta_1}\Gamma(\alpha_1/\beta_1)}\right)\left(\frac{\sigma_1\sum_{j=2}^{p}\rho_{1j}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}{1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}\right)$$

$$V(X_1 \mid X_2, X_3 \dots, X_p) = \int_{-\infty}^{+\infty}\left(x_1 - E(X_1 \mid X_2, X_3 \dots, X_p)\right)^2 f(x_1 \mid x_2, x_3 \dots, x_p)\,dx_1$$

$$V(X_1 \mid X_2, X_3 \dots, X_p) =$$

$$\int_{-\infty}^{+\infty}\left(x_1 - E(X_1 \mid X_2, X_3 \dots, X_p)\right)^2 \frac{\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{\beta_1(1/\lambda_1)^{\alpha_1/\beta_1}}{2\sigma_1\Gamma(\alpha_1/\beta_1)}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\alpha_1-1} e^{-\frac{1}{\lambda_1}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\beta_1}}\right)}{\left(1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)} dx_1$$

$$V(X_1 \mid X_2, X_3 \dots, X_p) = \sigma_1^2\left(\frac{\Gamma((\alpha_1+2\gamma_1))/\beta_1)}{(1/\lambda_1)^{2\gamma_1/\beta_1}\Gamma(\alpha_1/\beta_1)}\right)\left(1-\left(\frac{\sum_{j=2}^{p}\rho_{1j}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}{1+\sum_{i=2}^{p}\sum_{j=2}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}}\right)^2\right)$$

Theorem 2.6: If there are ***p=q+k*** random variables, such that ***q*** random variables $X_1, X_2, X_3, \dots X_q$ has a conditional dependence on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \dots X_{q+k}$,then the density function of the Sam-solai's Multivariate Generalized conditional double exponential family of distributions of kind-2 is

$$f(x_1, x_2, x_3, \dots x_q \mid x_{q+1}, x_{q+2}, x_{q+3}, \dots x_{q+k}) =$$

$$\frac{\left(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^q\prod_{i=1}^{q}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1} e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)}{\left(1+\sum_{i=q+1}^{q+k}\sum_{j=q+1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)} \quad (7)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $-\infty \leq \mu_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\lambda_i, \sigma_i, \alpha_i, \beta_i > 0$, $-1 \leq \rho_{ij} \leq +1$

Proof: Let the multivariate conditional law for ***q*** random variables $X_1, X_2, X_3, \ldots X_q$ on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$ is given as

$$f(x_1, x_2, x_3, \ldots x_q \mid x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{f(x_1, x_2, x_3, \ldots x_q, x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}{f(x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}$$

$$f(x_1, x_2, x_3, \ldots x_q \mid x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) =$$

$$\frac{\left(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^{q+k}\prod_{i=1}^{q+k}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)}{\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}\left(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^{q+k}\prod_{i=1}^{q+k}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)\prod_{i=1}^{q}dx_i}$$

$$f(x_1, x_2, x_3, \ldots x_q \mid x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) =$$

$$\frac{\left(1+\sum_{i=1}^{q+k}\sum_{j=1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^{q}\prod_{i=1}^{q}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)}{\left(1+\sum_{i=q+1}^{q+k}\sum_{j=q+1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)}$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $-\infty \leq \mu_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\lambda_i, \sigma_i, \alpha_i, \beta_i > 0$, $-1 \leq \rho_{ij} \leq +1$

Remarks: The result of (7) can be deduced to the bivariate case and it is given as

$$f(x_1, x_2 \mid x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) =$$

$$\frac{\left(1+\rho_{12}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^{2\gamma_1-1}\left(\frac{x_2-\mu_2}{\sigma_2}\right)^{2\gamma_2-1}\right)(\frac{1}{2})^{2}\prod_{i=1}^{2}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)}{\left(1+\sum_{i=q+1}^{q+k}\sum_{j=q+1}^{q+k}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)}$$

3. Constants of Sam-Solai's Multivariate Generalized Double exponential family of Distributions of kind-2

Theorem 3.1: The Marginal Co-variance between the generalized double exponential variables X_1 and X_2 is given as

$$COV(X_1,X_2)=\rho_{12}\sigma_1\sigma_2\left(\frac{\Gamma((\alpha_1+2\gamma_1)/\beta_1)\Gamma((\alpha_2+2\gamma_2)/\beta_2)}{(1/\lambda_1)^{2\gamma_1/\beta_1}(1/\lambda_2)^{2\gamma_2/\beta_2}\Gamma(\alpha_1/\beta_1)\Gamma(\alpha_2/\beta_2)}\right) \quad (8)$$

Proof: Let the product moment of the Sam-Solai's Multivariate Generalized double exponential family of distributions of kind-2 is given as in terms of co-variance as

$$COV(X_1,X_2)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(x_1-\mu_1)(x_2-\mu_2)f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^{p}dx_i$$

$COV(X_1,X_2)=$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}(x_1-\mu_1)(x_2-\mu_2)\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^{p}\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)\prod_{i=1}^{p}dx_i$$

$$COV(X_1,X_2)=\rho_{12}\sigma_1\sigma_2\left(\frac{\Gamma((\alpha_1+2\gamma_1)/\beta_1)\Gamma((\alpha_2+2\gamma_2)/\beta_2)}{(1/\lambda_1)^{2\gamma_1/\beta_1}(1/\lambda_2)^{2\gamma_2/\beta_2}\Gamma(\alpha_1/\beta_1)\Gamma(\alpha_2/\beta_2)}\right)$$

Remark: The result can be generalized to the co-variance between the i^{th} and j^{th} random variable are given as

$$COV(X_i,X_j)=\rho_{ij}\sigma_i\sigma_j\left(\frac{\Gamma((\alpha_i+2\gamma_i)/\beta_i)\Gamma((\alpha_j+2\gamma_j)/\beta_j)}{(1/\lambda_i)^{2\gamma_i/\beta_i}(1/\lambda_j)^{2\gamma_j/\beta_j}\Gamma(\alpha_i/\beta_i)\Gamma(\alpha_j/\beta_j)}\right) \quad (9)$$

$$\rho_{ij}=\frac{COV(X_i,X_j)(1/\lambda_i)^{2\gamma_i/\beta_i}(1/\lambda_j)^{2\gamma_j/\beta_j}\Gamma(\alpha_i/\beta_i)\Gamma(\alpha_j/\beta_j)}{\sigma_i\sigma_j\Gamma((\alpha_i+2\gamma_i)/\beta_i)\Gamma((\alpha_j+2\gamma_j)/\beta_j)}$$

where $i\neq j$, $-1\leq\rho_{ij}\leq+1$

Theorem 3.2: The Moment generating function of the Sam-solai's Multivariate Generalized double exponential family of distributions of kind-2 is

$$M_{x_1,x_2\ldots x_p}(t_1,t_2\ldots t_p)=\left(\prod_{i=1}^{p}\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\right)$$
$$\times\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\phi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\phi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}{\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\varphi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}\right) \quad (10)$$

Proof: Let the moment generating function of a Multivariate distribution is given as

$$M_{x_1,x_2,x_{3,\ldots}x_p}(t_1,t_2,t_{3,\ldots}t_p)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}e^{\sum_{i=1}^{p}t_ix_i}f(x_1,x_2,x_3,\ldots x_p)\prod_{i=1}^{p}dx_i$$

$M_{x_1,x_2,x_{3,\ldots}x_p}(t_1,t_2,t_{3,\ldots}t_p)=$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\cdots\int_{-\infty}^{+\infty}e^{\sum_{i=1}^{p}t_ix_i}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^{p}\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)\prod_{i=1}^{p}dx_i$$

(10a)

From (10a), it is observed that

$$\frac{\beta_i (1/\lambda_i)^{\alpha_i/\beta_i}}{2\sigma_i \Gamma(\alpha_i/\beta_i)} \int_{-\infty}^{+\infty} e^{t_i x_i} \left| \frac{x_i - \mu_i}{\sigma_i} \right|^{\alpha_i - 1} e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}} dx_i =$$

$$e^{\mu_i t_i}\left(1 + \frac{1}{\Gamma(\alpha_i/\beta_i)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_i t_i)^{2q} \Gamma((\alpha_i + 2q)/\beta_i)}{(2q)!(1/\lambda_i)^{2q/\beta_i}} \right)\right)$$

$$\frac{\beta_i (1/\lambda_i)^{\alpha_i/\beta_i}}{2\sigma_i \Gamma(\alpha_i/\beta_i)} \int_{-\infty}^{+\infty} e^{t_i x_i} \left(\frac{x_i - \mu_i}{\sigma_i} \right)^{2\gamma_i - 1} \left| \frac{x_i - \mu_i}{\sigma_i} \right|^{\alpha_i - 1} e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}} dx_i =$$

$$\frac{e^{\mu_i t_i}}{\Gamma(\alpha_i/\beta_i)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_i t_i)^{2q-1} \Gamma(\alpha_i + 2(\gamma_i + q - 1))/\beta_i)}{(2q-1)!(1/\lambda_i)^{2(\gamma_i+q-1)/\beta_i}} \right)$$

Thus, by Integration

$$M_{x_1,x_2..x_p}(t_1,t_2..t_p) =$$

$$\left(\prod_{i=1}^{p} \varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\right)\left(1 + \sum_{i=1}^{p}\sum_{j=1}^{p} \rho_{ij} \frac{\phi(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i)\phi(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\varphi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}\right)$$

where

$$\phi(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i) = \frac{e^{\mu_i t_i}}{\Gamma(\alpha_i/\beta_i)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_i t_i)^{2q-1} \Gamma(\alpha_i + 2(\gamma_i + q - 1))/\beta_i)}{(2q-1)!(1/\lambda_i)^{2(\gamma_i+q-1)/\beta_i}} \right)$$

$$\phi(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j) = \frac{e^{\mu_j t_j}}{\Gamma(\alpha_j/\beta_j)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_j t_j)^{2q-1} \Gamma(\alpha_j + 2(\gamma_j + q - 1))/\beta_j)}{(2q-1)!(1/\lambda_j)^{2(\gamma_j+q-1)/\beta_j}} \right)$$

$$\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i) = e^{\mu_i t_i}\left(1 + \frac{1}{\Gamma(\alpha_i/\beta_i)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_i t_i)^{2q} \Gamma((\alpha_i + 2q)/\beta_i)}{(2q)!(1/\lambda_i)^{2q/\beta_i}} \right)\right)$$

$$\varphi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j) = e^{\mu_j t_j}\left(1 + \frac{1}{\Gamma(\alpha_j/\beta_j)} \sum_{q=1}^{\infty} \left(\frac{(\sigma_j t_j)^{2q} \Gamma((\alpha_j + 2q)/\beta_j)}{(2q)!(1/\lambda_j)^{2q/\beta_j}} \right)\right)$$

and Γ is the Gamma function respectively.

Theorem 3.3: The Cumulant of Sam-solai's Multivariate Generalized double exponential family of distributions of kind-2 is

$$C_{x_1,x_2..x_p}(t_1,t_2...t_p) =$$

$$\sum_{i=1}^{p} \log\left(\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\right) + \log\left(1 + \sum_{i=1}^{p}\sum_{j=1}^{p} \rho_{ij} \frac{\phi(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i)\phi(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\varphi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\varphi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}\right)$$

(11)

Proof: It is found from $C_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p) = \log\left(M_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)\right)$

Theorem 3.4: The Characteristic function of Sam-solai's Multivariate Generalized double exponential family of distributions of kind-2 is

$$\phi_{x_1,x_2\dots x_p}(t_1,t_2\dots t_p) = \left(\prod_{i=1}^{p}\psi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\right)\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\omega(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i)\omega(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\psi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\psi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}\right) \tag{12}$$

where

$$\omega(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i) = \frac{e^{i\mu_i t_i}}{\Gamma(\alpha_i/\beta_i)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_i t_i)^{2q-1}\Gamma(\alpha_i+2(\gamma_i+q-1))/\beta_i)}{(2q-1)!(1/\lambda_i)^{2(\gamma_i+q-1)/\beta_i}}\right)$$

$$\omega(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j) = \frac{e^{i\mu_j t_j}}{\Gamma(\alpha_j/\beta_j)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_j t_j)^{2q-1}\Gamma(\alpha_j+2(\gamma_j+q-1))/\beta_j)}{(2q-1)!(1/\lambda_j)^{2(\gamma_j+q-1))/\beta_j}}\right)$$

$$\psi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i) = e^{i\mu_i t_i}\left(1+\frac{1}{\Gamma(\alpha_i/\beta_i)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_i t_i)^{2q}\Gamma((\alpha_i+2q)/\beta_i)}{(2q)!(1/\lambda_i)^{2q/\beta_i}}\right)\right)$$

$$\psi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j) = e^{i\mu_j t_j}\left(1+\frac{1}{\Gamma(\alpha_j/\beta_j)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_j t_j)^{2q}\Gamma((\alpha_j+2q)/\beta_j)}{(2q)!(1/\lambda_j)^{2q/\beta_j}}\right)\right)$$

and Γ is the Gamma function respectively.

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} e^{i\sum_{i=1}^{p}t_i x_i} f(x_1,x_2,x_3,\dots x_p)\prod_{i=1}^{p}dx_i$$

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty} e^{i\sum_{i=1}^{p}t_i x_i}\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)\left(\frac{1}{2}\right)^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)\prod_{i=1}^{p}dx_i \tag{12a}$$

From (12a), it is observed that

$$\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{2\sigma_i\Gamma(\alpha_i/\beta_i)}\int_{-\infty}^{+\infty}e^{it_ix_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}dx_i = e^{i\mu_i t_i}\left(1+\frac{1}{\Gamma(\alpha_i/\beta_i)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_i t_i)^{2q}\Gamma((\alpha_i+2q)/\beta_i)}{(2q)!(1/\lambda_i)^{2q/\beta_i}}\right)\right)$$

$$\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{2\sigma_i\Gamma(\alpha_i/\beta_i)}\int_{-\infty}^{+\infty}e^{it_ix_i}\left(\frac{x_i-\mu_i}{\sigma_i}\right)\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}dx_i = \frac{e^{i\mu_i t_i}}{\Gamma(\alpha_i/\beta_i)}\sum_{q=1}^{\infty}\left(\frac{(i\sigma_i t_i)^{2q-1}\Gamma(\alpha_i+2(\gamma_i+q-1))/\beta_i)}{(2q-1)!(1/\lambda_i)^{2(\gamma_i+q-1)/\beta_i}}\right)$$

Thus, by Integration

$$\phi_{x_1,x_2..x_p}(t_1,t_2\ldots t_p)=$$

$$\left(\prod_{i=1}^{p}\psi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\right)\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\frac{\omega(t_i;\mu_i,\sigma_i,\lambda_i,\gamma_i,\alpha_i,\beta_i)\omega(t_j;\mu_j,\sigma_j,\lambda_j,\gamma_j,\alpha_j,\beta_j)}{\psi(t_i;\mu_i,\sigma_i,\lambda_i,\alpha_i,\beta_i)\psi(t_j;\mu_j,\sigma_j,\lambda_j,\alpha_j,\beta_j)}\right)$$

4. Some Special cases

Result 4.1: If ρ_{ij} = 0,where *i,j=1,2,...,p* then there is no correlation between the random variables and the Sam-Solai's multivariate Generalized double exponential density function is reduced to product of uni-variate Generalized double exponential distribution.

Result 4.:2 From (1) and if *p*=2, then the density of Sam-solai's Multivariate Generalized double exponential family of distributions of kind-2 reduces to

$$f(x_1,x_2)=\left(1+\rho_{12}\left(\frac{x_1-\mu_1}{\sigma_1}\right)^{2\gamma_1-1}\left(\frac{x_2-\mu_2}{\sigma_2}\right)^{2\gamma_2-1}\right)\times$$
$$\left(\frac{\beta_1\beta_2(1/\lambda_1)^{\alpha_1/\beta_1}(1/\lambda_2)^{\alpha_2/\beta_2}}{4\sigma_1\sigma_2\Gamma(\alpha_1/\beta_1)\Gamma(\alpha_2/\beta_2)}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\alpha_1-1}\left|\frac{x_2-\mu_2}{\sigma_2}\right|^{\alpha_2-1}e^{-\left(\frac{1}{\lambda_1}\left|\frac{x_1-\mu_1}{\sigma_1}\right|^{\beta_1}+\frac{1}{\lambda_2}\left|\frac{x_2-\mu_2}{\sigma_2}\right|^{\beta_2}\right)}\right) \quad (13)$$

where $-\infty\le x_1,x_2\le+\infty$, $-\infty\le\mu_1,\mu_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real, $\sigma_1,\sigma_2,\lambda_1,\lambda_2,\alpha_1,\alpha_2,\beta_1,\beta_2>0$, $-1\le\rho_{ij}\le+1$

This is called Sam-Solai's Bi-variate generalized double exponential family of distributions of kind-2.

Result 4.3: From (10),if $z_1=(x_1-\mu_1)/\sigma_1$ and $z_2=(x_2-\mu_2)/\sigma_2$, then using two-dimensional Jacobian of transformation, the Sam-solai's Bi-variate Generalized standard double exponential family of distributions of kind-2 and its density is given as

$$f(z_1,z_2)=\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\beta_1\beta_2(1/\lambda_1)^{\alpha_1/\beta_1}(1/\lambda_2)^{\alpha_2/\beta_2}}{4\Gamma(\alpha_1/\beta_1)\Gamma(\alpha_2/\beta_2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{\lambda_1}|z_1|^{\beta_1}+\frac{1}{\lambda_2}|z_2|^{\beta_2}\right)}\right) \quad (14)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real, $\sigma_1,\sigma_2,\lambda_1,\lambda_2,\alpha_1,\alpha_2,\beta_1,\beta_2>0$, $-1\le\rho_{ij}\le+1$

This is called Sam-Solai's Bi-variate generalized double exponential family of distributions of kind-2.

Result 4.4: shown below some Bivariate distributions as special cases of Sam-solai's Bivariate generalized standard double exponential family of distributions of kind-2 from (14) for different settings of scale and shape parameters given

- Standard Laplace distribution, With Scale parameters (1,1) and Shape parameters (γ_1,γ_2,1,1,1,1), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{4}e^{-(|z_1|+|z_2|)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $-1\le\rho_{12}\le+1$
- Standard normal distribution, With Scale parameters (2,2) and Shape parameters (γ_1,γ_2,1,1,2,2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\pi}e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $-1\le\rho_{12}\le+1$
- Double Gamma distribution, With Scale parameters ($\frac{1}{\nu_1},\frac{1}{\nu_2}$) and Shape parameters ($\gamma_1,\gamma_2,\alpha_1,\alpha_2$,1,1), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\nu_1^{\alpha_1}\nu_2^{\alpha_2}}{4\Gamma(\alpha_1)\Gamma(\alpha_2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1}e^{-(\nu_1|z_1|+\nu_2|z_2|)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real, $\nu_1,\nu_2,\alpha_1,\alpha_2>0$, $-1\le\rho_{12}\le+1$
- Standard Double Rayleigh distribution, With Scale parameters (2,2) and Shape parameters (γ_1,γ_2,2, 2, 2, 2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{4}|z_1||z_2|e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $-1\le\rho_{12}\le+1$
- Double chi-square distribution, With Scale parameters (2,2) and Shape parameters ($\gamma_1,\gamma_2,n_1/2,n_2/2$,1,1),
  $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}(1/2)^{n_2/2}}{4\Gamma(n_1/2)\Gamma(n_2/2)}|z_1|^{(n_1/2)-1}|z_2|^{(n_2/2)-1}e^{-\frac{1}{2}(|z_1|+|z_2|)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $n_1,n_2>0$, $-1\le\rho_{12}\le+1$
- Double Maxwell-Boltzmann distribution, With Scale parameters ($2\sigma_1^2$, $2\sigma_2^2$) and Shape parameters (γ_1,γ_2,3,3,2,2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\pi\sigma_1\sigma_2}\left|\frac{z_1}{\sigma_1}\right|^2\left|\frac{z_2}{\sigma_2}\right|^2e^{-\frac{1}{2}\left(\left|\frac{z_1}{\sigma_1}\right|^2+\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real, $\sigma_1,\sigma_2>0$, $-1\le\rho_{12}\le+1$
- Double Nagakami distribution, With Scale parameters (Ω_1/m_1 , Ω_2/m_2) and Shape parameters ($\gamma_1,\gamma_2,2m_1,2m_2$,2,2),
  $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_1/\Omega_1)^{m_1}(m_2/\Omega_2)^{m_2}}{\Gamma(m_1)\Gamma(m_2)}|z_1|^{2m_1-1}|z_2|^{2m_2-1}e^{-\left(\frac{m_1}{\Omega_1}|z_1|^2+\frac{m_2}{\Omega_2}|z_2|^2\right)}\right)$
  where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $m_1,m_2\ge1/2$, $\Omega_1,\Omega_2>0$ $-1\le\rho_{12}\le+1$

- Double chi distribution, With Scale parameters (2,2) and Shape parameters $(\gamma_1, \gamma_2, \alpha_1, \alpha_2, 2, 2)$,

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_1/2}(1/2)^{\alpha_2/2}}{\Gamma(\alpha_1/2)\Gamma(\alpha_2/2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real, $\alpha_1, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Erlang-k distribution, With Scale parameters $(1/k_1\alpha_1,\ 1/k_2\alpha_2)$ and Shape parameters $(\gamma_1, \gamma_2, \alpha_1, \alpha_2, 1, 1)$,

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(k_1\alpha_1)^{\alpha_1}(k_1\alpha_1)^{\alpha_2}}{4\Gamma(\alpha_1)\Gamma(\alpha_2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-\left(k_1\alpha_1|z_1|+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real, $\alpha_1, \alpha_2, k_1, k_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double-Weibull distribution, With Scale parameters $(k_1^{\alpha_1}, k_2^{\alpha_2})$ and Shape parameters $(\gamma_1, \gamma_2, \alpha_1, \alpha_2, \alpha_1, \alpha_2)$,

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{\alpha_1\alpha_2(1/k_1^{\alpha_1})(1/k_2^{\alpha_2})}{4}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-\left(\left|\frac{z_1}{k_1}\right|^{\alpha_1}+\left|\frac{z_1}{k_1}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real, $\alpha_1, \alpha_2, k_1, k_2 > 0$, $-1 \le \rho_{12} \le +1$

Result 4.5: shown below some selected Bi-variate mixture of distribution as special cases of Sam-solai's Bi-variate generalized standard double exponential family of distributions of kind-2 from (14) for different settings of scale and shape parameters given

- Laplace-normal distribution, With Scale parameters (1, 2) and shape parameters $(\gamma_1, \gamma_2, 1, 1, 1, 2)$, $\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\sqrt{2\pi}} e^{-\left(|z_1|+\frac{1}{2}|z_2|^2\right)}\right)$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real, $-1 \le \rho_{12} \le +1$

- Laplace-Double Gamma distribution, With Scale parameters $(1, {}^{1}/\nu_2)$ and shape parameters $(\gamma_1, \gamma_2, 1, \alpha_2, 1, 1)$,

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{\nu_2^{\alpha_2/\beta_2}}{2\Gamma(\alpha_2)}|z_2|^{\alpha_2-1} e^{-\left(|z_1|+\nu_2|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real, $\nu_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace-Double Rayleigh distribution, With Scale parameters (1, 2) and shape parameters $(\gamma_1, \gamma_2, 1, 2, 1, 2)$,

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\sqrt{2\pi}}|z_2| e^{-\left(|z_1|+\frac{1}{2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $-1 \le \rho_{12} \le +1$

- Laplace-Double chi-square distribution, With Scale parameters (1, 2) and shape parameters (γ_1, γ_2, 1, n$_2$/2, , 1, 1),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_2/2}}{4\Gamma(n_2/2)}\left|z_2\right|^{(n_2/2)-1} e^{-\left(|z_1|+\frac{1}{2}|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $n_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace-Double Maxwell Boltzmann distribution, With Scale parameters $(1, 2\sigma_2^2)$ and shape parameters (γ_1, γ_2, 1, 3, 1, 2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\sigma_2\sqrt{2\pi}}\left|\frac{z_2}{\sigma_2}\right|^2 e^{-\left(|z_1|+\frac{1}{2}\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\sigma_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace-Double Nagakami distribution, With Scale parameters $(1, \Omega_2/m_2)$) and shape parameters (γ_1, γ_2, 1, 2m$_2$, , 1, 2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(m_2/\Omega_2)^{m_2}}{2\Gamma(m_2)}\left|z_2\right|^{2m_2-1} e^{-\left(|z_1|+\frac{m_2}{\Omega_2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $m_2 \ge 1/2$, $\Omega_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace-Double chi distribution, With Scale parameters (1, 2) and shape parameters (γ_1, γ_2, 1, α$_2$, 1, 2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_2/2}}{2\Gamma(\alpha_2/2)}\left|z_2\right|^{\alpha_2-1} e^{-\left(|z_1|+\frac{1}{2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace- Double Erlang-k distribution, With Scale parameters $(1, 1/k_2\alpha_2)$) and shape parameters (γ_1, γ_2, 1, α$_2$, 1, 1),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(k_2\alpha_2)^{\alpha_2}}{4\Gamma(\alpha_2)}\left|z_2\right|^{\alpha_2-1} e^{-\left(|z_1|+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $k_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Laplace- Double-Weibull distribution, With Scale parameters $(1, k_2^{\alpha_2})$) and shape parameters (γ_1, γ_2, 1, α$_2$, 1, α$_2$),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\alpha_2(1/k_2^{\alpha_2})}{4}|z_2|^{\alpha_2-1}e^{-\left(|z_1|+\left|\frac{z_2}{\alpha_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $k_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal-Double Gamma distribution, With Scale parameters $(2, 1/\nu_2)$) and shape parameters (γ_1 , γ_2 , 1, α_2, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\nu_2^{\alpha_2}}{\sqrt{2\pi}(2\Gamma(\alpha_2))}|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}|z_1|^2+\nu_2|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\nu_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal-Double Rayleigh distribution, With Scale parameters (2, 2) and shape parameters (γ_1 , γ_2 , 1, 2, 2, 2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\sqrt{2\pi}}|z_2|e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $-1 \le \rho_{12} \le +1$

- Normal-Double chi-square distribution, With Scale parameters (2, 2) and shape parameters (γ_1 , γ_2 , 1, $n_2/2$, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_2/2}}{2\sqrt{2\pi}\Gamma(n_2/2)}|z_2|^{(n_2/2)-1}e^{-\frac{1}{2}\left(|z_1|^2+|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $n_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal-Double Maxwell distribution, With Scale parameters $(2, 2\sigma_2^2)$) and shape parameters (γ_1 , γ_2 , 1, 3, 2, 2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\pi\sigma_2}\left|\frac{z_2}{\sigma_2}\right|^2 e^{-\frac{1}{2}\left(|z_1|^2+\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\sigma_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal –Double Nagakami distribution, With Scale parameters $(2, \Omega_2/m_2)$) and shape parameters (γ_1 , γ_2 , 1,2m_2, 2, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_2/\Omega_2)^{m_2}}{\sqrt{2\pi}\Gamma(m_2)}|z_2|^{2m_2-1}e^{-\left(\frac{1}{2}|z_1|^2+\frac{m_2}{\Omega_2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $m_2 \ge 1/2, \Omega_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal-Double chi distribution, With Scale parameters (2, 2) and shape parameters (γ_1 , γ_2 , 1, α_2, 2, 2), $\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_2/2}}{\sqrt{2\pi}\Gamma(\alpha_2/2)}|z_2|^{\alpha_2-1}e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Normal- Double Erlang-k distribution, With Scale parameters $(2,1/k_2\alpha_2))$ and shape parameters (γ_1 , γ_2 , 1,α_2, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(k_2\alpha_2)^{\alpha_2}}{\sqrt{2\pi}\,2\Gamma(\alpha_2)}\left|z_2\right|^{\alpha_2-1}e^{-\left(\frac{1}{2}\left|z_1\right|^2+k_2\alpha_2\left|z_2\right|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $k_2,\alpha_2>0$, $-1\le\rho_{12}\le+1$

- Normal- Double-Weibull distribution, With Scale parameters $(2,k_2^{\alpha_2}))$ and shape parameters (γ_1 , γ_2 , 1, α_2, , 2, α_2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\alpha_2(1/k_2^{\alpha_2})}{2\sqrt{2\pi}}\left|z_2\right|^{\alpha_2-1}e^{-\left(\frac{1}{2}\left|z_1\right|^2+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $k_2,\alpha_2>0$, $-1\le\rho_{12}\le+1$

- Double Gamma-Double Rayleigh distribution, With Scale parameters $(1/\nu_1,2)$ and shape parameters (γ_1 , γ_2 , α_1, 2, 1, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(\nu_1)^{\alpha_1}}{4\Gamma(\alpha_1)}\left|z_1\right|^{\alpha_1-1}\left|z_2\right|e^{-\left(\nu_1\left|z_1\right|+\frac{1}{2}\left|z_2\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\nu_1,\alpha_1>0$, $-1\le\rho_{12}\le+1$

- Double Gamma-Double chisquare distribution, With Scale parameters $(1/\nu_1,2)$ and shape parameters (γ_1 , γ_2 , α_1, n_2/2, 1, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\nu_1^{\alpha_1}(1/2)^{n_2/2}}{4\Gamma(\alpha_1)\Gamma(n_2/2)}\left|z_1\right|^{\alpha_1-1}\left|z_2\right|^{(n_2/2)-1}e^{-\left(\nu_1\left|z_1\right|+\frac{1}{2}\left|z_2\right|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\nu_1,\alpha_1,n_2>0$, $-1\le\rho_{12}\le+1$

- Double Gamma-Double Maxwell Boltzmann distribution, With Scale parameters ($(1/\nu_1,2\sigma_2^2)$), and shape parameters (γ_1 , γ_2 ,α_1, 3, 1, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(\nu_1)^{\alpha_1}}{2\Gamma(\alpha_1)\sigma_2\sqrt{2\pi}}\left|z_1\right|^{\alpha_1-1}\left|\frac{z_2}{\sigma_2}\right|^2e^{-\left(\nu_1\left|z_1\right|+\frac{1}{2}\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\nu_1,\alpha_1,\sigma_2>0$, $-1\le\rho_{12}\le+1$

- Double Gamma-Double Nagakami distribution, With Scale parameters $(1/\nu_1,\Omega_2/m_2))$ and shape parameters (γ_1 , γ_2 , α_1, 2m_2, 1, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\nu_1^{\alpha_1}(m_2/\Omega_2)^{m_2}}{2\Gamma(\alpha_1)\Gamma(m_2)}\left|z_1\right|^{\alpha_1-1}\left|z_2\right|^{2m_2-1}e^{-\left(\nu_1\left|z_1\right|+\frac{m_2}{\Omega_2}\left|z_2\right|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $m_2 \ge 1/2$, $v_1, \alpha_1, \Omega_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Gamma-Double Chi distribution, With Scale parameters ($(1/v_1, 2)$ and shape parameters (γ_1, γ_2, α_1, α_2, 1, 2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{v_1^{\alpha_1}(1/2)^{\alpha_2/2}}{2\Gamma(\alpha_1)\Gamma(\alpha_2/2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-\left(v_1|z_1|+\frac{1}{2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $v_1, \alpha_1, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Gamma-Double Erlang-k distribution, With Scale parameters $(1/v_1, 1/k_2\alpha_2)$) and shape parameters (γ_1, γ_2, α_1, α_2, 1, 1),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{v_1^{\alpha_1}(k_2\alpha_2)^{\alpha_2}}{4\Gamma(\alpha_1)\Gamma(\alpha_2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-(v_1|z_1|+k_2\alpha_2|z_2|)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $v_1, \alpha_1, \alpha_2, k_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Gamma-Double Weibull distribution, With Scale parameters $(1/v_1, k_2^{\alpha_2})$ and shape parameters (γ_1, γ_2, α_1, α_2, 1, α_2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{v_1^{\alpha_1}\alpha_2 k_2^{\alpha_2}}{4\Gamma(\alpha_1)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1} e^{-\left(v_1|z_1|+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $v_1, \alpha_1, \alpha_2, k_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double chi-square distribution, With Scale parameters (2, 2) and shape parameters (γ_1, γ_2, 2, $n_2/2$, 2, 1),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_2/2}}{4\Gamma(n_2/2)}|z_1||z_2|^{(n_2/2)-1} e^{-\frac{1}{2}\left(|z_1|^2+|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $n_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double Maxwell Boltzmann distribution, With Scale parameters $(2, 2\sigma_2^2)$) and shape parameters (γ_1, γ_2, 2, 3, 2, 2),

$$\left(1+\rho_{12} z_1^{2\gamma_1-1} z_2^{2\gamma_2-1}\right)\left(\frac{1}{2\sigma_2\sqrt{2\pi}}|z_1|\left|\frac{z_2}{\sigma_2}\right|^2 e^{-\left(\frac{1}{2}|z_1|^2+\frac{1}{2}\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\sigma_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double Nagakami distribution, With Scale parameters $(2, \Omega_2/m_2)$) and shape parameters (γ_1, γ_2, 2, $2m_2$, 2, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_2/\Omega_2)^{m_2}}{2\Gamma(m_2)}|z_1||z_2|^{2m_2-1}e^{-\left(\frac{1}{2}|z_1|^2+\frac{m_2}{\Omega_2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $m_2 \ge 1/2$ $\Omega_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double Chi distribution, With Scale parameters (2, 2) and shape parameters (γ_1 , γ_2 , 2, α_2, , 2, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_2/2}}{2\Gamma(\alpha_2/2)}|z_1||z_2|^{\alpha_2-1}e^{-\frac{1}{2}\left(|z_1|^2+|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double Erlang-k distribution, With Scale parameters $(2, 1/k_2\alpha_2)$ and shape parameters (γ_1 , γ_2 , 2, α_2 , 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(k_2\alpha_2)^{\alpha_2}}{4\Gamma(\alpha_2)}|z_1||z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}|z_1|^2+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $k_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Rayleigh-Double Weibull distribution, With Scale parameters $(2, k_2^{\alpha_2})$ and shape parameters (γ_1 , γ_2 , 2, α_2, 2, α_2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\alpha_2 k_2^{\alpha_2}}{4}|z_1||z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}|z_1|^2+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $k_2, \alpha_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double chi-square-Double Maxwell Boltzmann distribution, With Scale parameters $(2, 2\sigma_2^2)$ and shape parameters (γ_1 , γ_2 , $n_1/2$, 3, 1, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}}{\Gamma(n_1/2)\sigma_2\sqrt{2\pi}}|z_1|^{(n_1/2)-1}\left|\frac{z_2}{\sigma_2}\right|^2 e^{-\frac{1}{2}\left(|z_1|+\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $n_1, \sigma_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double chi-square-Double Nagakami distribution, With Scale parameters $(2, \Omega_2/m_2)$) and shape parameters (γ_1 , γ_2 , $n_{1/2,}$ $2m_2$, 1, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}(m_2/\Omega_2)^{m_2}}{2\Gamma(n_1/2)\Gamma(m_2)}|z_1|^{(n_1/2)-1}|z_2|^{2m_2-1}e^{-\left(\frac{1}{2}|z_1|+\frac{m_2}{\Omega_2}|z_2|^2\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $m_2 \ge 1/2$, $n_1, \sigma_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double chi-square-Double Chi distribution, With Scale parameters (2, 2) and shape parameters (γ_1, γ_2, $n_1/2$, α_2, 1, 2),

$$\left(1+\rho_{12}{z_1}^{2\gamma_1-1}{z_2}^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}(1/2)^{\alpha_2/2}}{2\Gamma(n_1/2)\Gamma(\alpha_2/2)}\left|z_1\right|^{(n_1/2)-1}\left|z_2\right|^{\alpha_2-1}e^{-\frac{1}{2}\left(\left|z_1\right|+\left|z_2\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $n_1,\alpha_2>0$, $-1\le\rho_{12}\le+1$

- Double chi-square-Double Erlang-k distribution, With Scale parameters $(2,1/k_2\alpha_2)$ and shape parameters (γ_1, γ_2, $n_1/2$, α_2, 1, 1),

$$\left(1+\rho_{12}{z_1}^{2\gamma_1-1}{z_2}^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}(k_2\alpha_2)^{\alpha_2}}{4\Gamma(n_1/2)\Gamma(\alpha_2)}\left|z_1\right|^{(n_1/2)-1}\left|z_2\right|^{\alpha_2-1}e^{-\left(\frac{1}{2}\left|z_1\right|+k_2\alpha_2\left|z_2\right|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $n_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double chi-square-Double Weibull distribution, With Scale parameters $(2,{k_2}^{\alpha_2})$ and shape parameters (γ_1, γ_2, $n_1/2$, α_2, 1, α_2),

$$\left(1+\rho_{12}{z_1}^{2\gamma_1-1}{z_2}^{2\gamma_2-1}\right)\left(\frac{(1/2)^{n_1/2}}{2\Gamma(n_1/2)\sigma_2\sqrt{2\pi}}\left|z_1\right|^{(n_1/2)-1}\left|\frac{z_2}{\sigma_2}\right|^2e^{-\frac{1}{2}\left(\left|z_1\right|+\left|\frac{z_2}{\sigma_2}\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $n_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double Maxwell Boltzmann- Double Nagakami distribution, With Scale parameters $(2\sigma_1^2,\Omega_2/m_2)$ and shape parameters (γ_1, γ_2, 3, $2m_2$, 2, 2),

$$\left(1+\rho_{12}{z_1}^{2\gamma_1-1}{z_2}^{2\gamma_2-1}\right)\left(\frac{(m_2/\Omega_2)^{m_2}}{2\sigma_1\sqrt{2\pi}\Gamma(m_2)}\left|\frac{z_1}{\sigma_1}\right|^2\left|z_2\right|^{2m_2-1}e^{-\left(\frac{1}{2}\left|\frac{z_1}{\sigma_1}\right|^2+\frac{m_2}{\Omega_2}\left|z_2\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $m_2\ge1/2$, $\sigma_1,\Omega_2>0$, $-1\le\rho_{12}\le+1$

- Double Maxwell Boltzmann- Double Chi distribution, With Scale parameters $(2\sigma_1^2,2)$ and shape parameters (γ_1, γ_2, 3, α_2, 2, 2),

$$\left(1+\rho_{12}{z_1}^{2\gamma_1-1}{z_2}^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_2/2}}{2\sigma_1\sqrt{2\pi}\Gamma(\alpha_2/2)}\left|\frac{z_1}{\sigma_1}\right|^2\left|z_2\right|^{\alpha_2-1}e^{-\frac{1}{2}\left(\left|\frac{z_1}{\sigma_1}\right|^2+\left|z_2\right|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\sigma_1,\alpha_2>0$, $-1\le\rho_{12}\le+1$

- Double Maxwell Boltzmann- Double Erlang-k distribution, With Scale parameters $(2\sigma_1^2,1/k_2\alpha_2)$ and shape parameters (γ_1, γ_2, 3, α_2, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(k_2\alpha_2)^{\alpha_2}}{2\sigma_1\sqrt{2\pi}\Gamma(\alpha_2)}\left|\frac{z_1}{\sigma_1}\right|^2|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}\left|\frac{z_1}{\sigma_1}\right|^2+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\sigma_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double Maxwell Boltzmann- Double Weibull distribution, With Scale parameters $(2\sigma_1^2,k_2^{\alpha_2})$ and shape parameters (γ_1, γ_2, 3, α₂, 2, α₂),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{\alpha_2(1/k_2^{\alpha_2})}{2\sigma_1\sqrt{2\pi}}\left|\frac{z_1}{\sigma_1}\right|^2|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}\left|\frac{z_1}{\sigma_1}\right|^2+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\sigma_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double Nagakami-Double chi distribution, With Scale parameters $(\Omega_1/m_1,2)$ and shape parameters (γ_1, γ_2, $2m_1$, α_2, 2, 2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_1/\Omega_1)^{m_1}(1/2)^{\alpha_2/2}}{\Gamma(m_1)\Gamma(\alpha_2/2)}|z_1|^{2m_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{m_1}{\Omega_1}|z_1|^2+\frac{1}{2}|z_2|^2\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $m_1\ge1/2$, $\Omega_1,\alpha_2>0$, $-1\le\rho_{12}\le+1$

- Double Nagakami-Double Erlang-k distribution, With Scale parameters $(\Omega_1/m_1,1/k_2\alpha_2)$ and shape parameters (γ_1, γ_2, $2m_1$, α_2, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_1/\Omega_1)^{m_1}(k_2\alpha_2)^{\alpha_2}}{2\Gamma(m_1)\Gamma(\alpha_2)}|z_1|^{2m_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{m_1}{\Omega_1}|z_1|^2+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $m_1\ge1/2$, $\Omega_1,\alpha_2,k_2>0$ $-1\le\rho_{12}\le+1$

- Double Nagakami-Double Weibull distribution, With Scale parameters $(\Omega_1/m_1,k_2^{\alpha_2})$ and shape parameters (γ_1, γ_2, $2m_1$, α_2, 2, α_2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(m_1/\Omega_1)^{m_1}\alpha_2(1/k_2^{\alpha_2})}{2\Gamma(m_1)}|z_1|^{2m_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{m_1}{\Omega_1}|z_1|^2+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $m_1\ge1/2$, $\Omega_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double chi- Double Erlang-k distribution, With Scale parameters $(2,1/k_2\alpha_2)$ and shape parameters (γ_1, γ_2, α_1, α_2, 2, 1),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_1/2}(k_2\alpha_2)^{\alpha_2}}{2\Gamma(\alpha_1/2)\Gamma(\alpha_2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}|z_1|^2+k_2\alpha_2|z_2|\right)}\right)$$

where $-\infty\le z_1,z_2\le+\infty$, $\gamma_1,\gamma_2\ge1$ and real , $\alpha_1,\alpha_2,k_2>0$, $-1\le\rho_{12}\le+1$

- Double chi- Double Weibull distribution, With Scale parameters $(2, k_2^{\alpha_2})$ and shape parameters (γ_1 , γ_2 , α_1, α_2, 2, α_2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(1/2)^{\alpha_1/2}\alpha_2(1/k_2^{\alpha_2})}{2\Gamma(\alpha_1/2)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1}e^{-\left(\frac{1}{2}|z_1|^2+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\alpha_1, \alpha_2, k_2 > 0$, $-1 \le \rho_{12} \le +1$

- Double Erlang-k- Double Weibull distribution, With Scale parameters $(1/k_1\alpha_1, k_2^{\alpha_2})$ and shape parameters (γ_1 , γ_2 , α_1, α_2, 1, α_2),

$$\left(1+\rho_{12}z_1^{2\gamma_1-1}z_2^{2\gamma_2-1}\right)\left(\frac{(k_1\alpha_1)^{\alpha_1}\alpha_2(1/k_2^{\alpha_2})}{4\Gamma(\alpha_1)}|z_1|^{\alpha_1-1}|z_2|^{\alpha_2-1}e^{-\left(k_1\alpha_1|z_1|+\left|\frac{z_2}{k_2}\right|^{\alpha_2}\right)}\right)$$

where $-\infty \le z_1, z_2 \le +\infty$, $\gamma_1, \gamma_2 \ge 1$ and real , $\alpha_1, \alpha_2, k_2 > 0$, $-1 \le \rho_{12} \le +1$

Result 4.6: shown below some selected Multivariate distributions as special cases of Sam-Solai's Multivariate generalized double exponential family of distribution of kind-2 from (1) for different settings of Location, scale and shape parameters given

- Generalized Laplace, distribution with location parameters (μ_i) , scale parameters (σ_i ,1,), shape parameters (γ_i ,1, β_i ,),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{\beta_i}{\sigma_i\Gamma(1/\beta_i)}e^{-\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)$$

where $i \ne j$, $-\infty \le x_i \le +\infty$, $-\infty \le \mu_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\sigma_i, \beta_i > 0$, $-1 \le \rho_{ij} \le +1$

- Laplace, distribution with location parameters (μ_i) scale parameters(σ_i , 1) , shape parameters (γ_i , 1, 1) ,

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{1}{\sigma_i}e^{-\left|\frac{x_i-\mu_i}{\sigma_i}\right|}\right)$$

where $i \ne j$, $-\infty \le x_i \le +\infty$, $-\infty \le \mu_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\sigma_i > 0$, $-1 \le \rho_{ij} \le +1$

- Generalized normal, distribution with location parameters (μ_i) scale parameters (σ_i , β_i) , shape parameters (γ_i , 1, β_i) ,

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{(1/\beta_i)^{(1/\beta_i)-1}}{\sigma_i\Gamma(1/\beta_i)}e^{-\frac{1}{\beta_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $-\infty \leq \mu_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\lambda_i, \sigma_i, \beta_i > 0$, $-1 \leq \rho_{ij} \leq +1$

- Normal, distribution with location parameters (μ_i) scale parameters (σ_i, 2), shape parameters (γ_i, 1, 2),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j-\mu_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2\pi})^{p/2}\prod_{i=1}^{p}\left(\frac{1}{\sigma_i}e^{-\frac{1}{2}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^2}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $-\infty \leq \mu_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $-1 \leq \rho_{ij} \leq +1$

- Generalized Double Gamma, distribution with location parameters (0), scale parameters(1, $1/\nu_i$), shape parameters (γ_i, α_i, β_i),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)(\frac{1}{2})^{p}\prod_{i=1}^{p}\left(\frac{\beta_i \nu_i^{\alpha_i/\beta_i}}{\Gamma(\alpha_i/\beta_i)}|x_i|^{\alpha_i-1}e^{-\nu_i|x_i|^{\beta_i}}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$ $\gamma_i, \gamma_j \geq 1$ and real,, $\nu_i, \alpha_i, \beta_i > 0$, $-1 \leq \rho_{ij} \leq +1$

- Double Gamma, distribution with location parameters (0), scale parameters (1, $1/\nu_i$), shape parameters (γ_i, α_i, 1),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)(\frac{1}{2})^{p}\prod_{i=1}^{p}\left(\frac{\nu_i^{\alpha_i}}{\Gamma(\alpha_i)}|x_i|^{\alpha_i-1}e^{-\nu_i|x_i|}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\nu_i, \alpha_i > 0$, $-1 \leq \rho_{ij} \leq +1$

- Double Rayleigh, distribution with location parameters (0), scale parameters (σ_i,2), shape parameters (γ_i, 2, 2),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^{p}\prod_{i=1}^{p}\left(\frac{1}{\sigma_i}\left|\frac{x_i}{\sigma_i}\right|e^{-\frac{1}{2}\left|\frac{x_i}{\sigma_i}\right|^2}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\sigma_i > 0$, $-1 \leq \rho_{ij} \leq +1$

- Double chi-square, distribution with location parameters (0), scale parameters (1, 2), shape parameters (γ_i, $n_i/2$, 1),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)(\frac{1}{2})^{p}\prod_{i=1}^{p}\left(\frac{(1/2)^{n_i/2}}{\Gamma(n_i/2)}|x_i|^{(n_i/2)-1}e^{-\frac{1}{2}|x_i|}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $n_i > 0$, $-1 \leq \rho_{ij} \leq +1$

- Double Maxwell-Boltzmann, distribution with location parameters (0), scale parameters (σ_i, 2), shape parameters (γ_i, 3, 2),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2\pi})^p\prod_{i=1}^{p}\left(\frac{1}{\sigma_i}\left|\frac{x_i}{\sigma_i}\right|^2 e^{-\frac{1}{2}\left|\frac{x_i}{\sigma_i}\right|^2}\right)$$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\sigma_i > 0$, $-1 \le \rho_{ij} \le +1$

- Double Nagakami, distribution with location parameters (0), scale parameters $(1, \Omega_i / m_i)$, shape parameters (γ_i, $2m_i$, 2),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)\prod_{i=1}^{p}\left(\frac{(m_i/\Omega_i)^{\alpha_i/\beta_i}}{\Gamma(m_i)}|x_i|^{2m_i-1}e^{-\frac{m_i}{\Omega_i}|x_i|^2}\right)$$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $m_i \ge 1/2$, $\Omega_i > 0$, $-1 \le \rho_{ij} \le +1$

- Double chi, distribution with location parameters (0), Scale parameters (1, 2), shape parameters (γ_i, α_i, 2), $\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)\prod_{i=1}^{p}\left(\frac{(1/2)^{\alpha_i/2}}{\Gamma(\alpha_i/2)}|x_i|^{\alpha_i-1}e^{-\frac{1}{2}|x_i|^2}\right)$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $\alpha_i > 0$, $-1 \le \rho_{ij} \le +1$

- Double Erlang-k, distribution with location parameters (0), scale parameters $(1, 1/k_i\alpha_i)$, shape parameters (γ_i, α_i, 1),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{(k_i\alpha_i)^{\alpha_i}}{\Gamma(\alpha_i)}|x_i|^{\alpha_i-1}e^{-k_i\alpha_i|x_i|}\right)$$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $k_i, \alpha_i > 0$, $-1 \le \rho_{ij} \le +1$

- Double-Weibull, distribution with location parameters (0), scale parameters (σ_i, 1), shape parameters (γ_i, α_i, α_i),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i}{\sigma_i}\right)^{2\gamma_i-1}\left(\frac{x_j}{\sigma_j}\right)^{2\gamma_j-1}\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{\alpha_i}{\sigma_i}\left|\frac{x_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\left|\frac{x_i}{\sigma_i}\right|^{\alpha_i}}\right)$$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\sigma_i, \alpha_i > 0$, $-1 \le \rho_{ij} \le +1$

- Double error, distribution with location parameters (0), scale parameters (1, 1), shape parameters (γ_i, 1, 2), $\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}x_i^{2\gamma_i-1}x_j^{2\gamma_j-1}\right)(\frac{1}{\pi})^{p/2}\prod_{i=1}^{p}\left(e^{-|x_i|^2}\right)$

where $i \neq j$, $-\infty \le x_i \le +\infty$, $\gamma_i, \gamma_j \ge 1$ and real, $-1 \le \rho_{ij} \le +1$

- Generalized Double exponential of kind-1, distribution with location parameters (μ_i) scale parameters (σ_i, λ_i), shape parameters (1, α_i, β_i),

$$\left(1+\sum_{i=1}^{p}\sum_{j=1}^{p}\rho_{ij}\left(\frac{x_i-\mu_i}{\sigma_i}\right)\left(\frac{x_j-\mu_j}{\sigma_j}\right)\right)(\frac{1}{2})^p\prod_{i=1}^{p}\left(\frac{\beta_i(1/\lambda_i)^{\alpha_i/\beta_i}}{\sigma_i\Gamma(\alpha_i/\beta_i)}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\alpha_i-1}e^{-\frac{1}{\lambda_i}\left|\frac{x_i-\mu_i}{\sigma_i}\right|^{\beta_i}}\right)$$

where $i \neq j$, $-\infty \leq x_i \leq +\infty$, $-\infty \leq \mu_i \leq +\infty$, $\gamma_i, \gamma_j \geq 1$ and real, $\lambda_i, \sigma_i, \alpha_i, \beta_i > 0$, $-1 \leq \rho_{ij} \leq +1$

Conclusion

The multivariate generalization of generalized double exponential family of distributions in a natural form is impossible and the authors adopted the Sarmanov system of multivariate generalization of distributions. At first, the marginal univariate distributions of the Sam-Solai's Multivariate Generalized double exponential family of distributions of kind-2 are univariate Generalized double exponential distributions. Secondly, the co-variance and correlation co-efficient of any two generalized double exponential variables will change based on the shape parameters of the distribution. Similarly, based on the parameter settings of the multivariate generalized double exponential family of distributions of kind-2, the authors further derived the multivariate distributions for the existing univariate continuous distributions. Thus the authors recommend that the generalization of Sarmanov type family of symmetric multivariate distribution opens the way for logical extension of the generalization of symmetric family of all univariate continuous probability distributions in the statistical literature.

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