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STOCHASTIC MODEL FOR ESTIMATING THE EXPECTED TIME TO RECRUITMENT BASED ON LINDLY DISTRIBUTION

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Abstract

The recruitment of persons in every organization is very important because the survival of the organization very much depends upon the availability of the manpower. The depletion of manpower in any organization may be due to the policy decisions taken by the management. To make up the loss of manpower, recruitments cannot be done after every decision making epoch. It is due to the fact that recruitment involves cost, time and manpower. So when the cumulative depletion of manpower due to successive decisions exceeds the threshold level, recruitment is necessary. The threshhold level of manpower depletion which can be managed is assumed to be a random variable. In this paper, a stochastic model is developed to find the expected time recruitment under the assumption that the threshold level follows Lindly Distribution. Numerical illustrations are also provided.

Key Words: Expected Time, Variance, Threshold, Lindley Distribution.

1. Introduction

There are many organizations which are engaged in productions, marketing and other business activities. All these organizations are very much dependent on the availability of enough manpower. Decisions regarding the targets, work scheduled, pay revisions, promotions and codes of conduct are taken by the management at random epoch. Hence the time interval between successive decisions which is the so called inter arrival times is of random character. After every decisions epoch, due to the decisions taken by the management, it may result in leaving of personnel due to unsatisfactory packages. Hence the depletion of manpower has to be compensated by suitable recruitments in order to keep the manpower availability at stable level.

In this paper the expected time to recruitment is derived using the concept of shock model and cumulative damage processes due to Esary et.al., (1973). It is assumed that the threshold level is a random variable which follows the so called extended Lindly distribution introduced by Bakouch et.al., (2012). Such models have been discussed by the authors Sathiyamoorthi (1980), Pandian et.al., (2010), Kannadasan et.al., (2013), Rao and Rao (2013).

The expression for mean and variance of time to recruitment are derived in this paper. Numerical illustrations are also provided.

2. Assumptions of the Model

- Exit of person from an organization takes place whenever the policy decisions regarding targets, incentives and promotions are made.
- The exit of every person from the organization results in a random amount of depletion of manpower (in man hours).
- The process of depletion is linear and cumulative.
- The inter arrival times between successive occasions of wastage are i.i.d. random variables.
- If the total depletion exceeds a threshold level Y which is itself a random variable, the breakdown of the organization occurs. In other words recruitment becomes inevitable.
- The process, which generates the exits, the sequence of depletions and the threshold are mutually independent.

Notations

X_i :a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the ith occasion of policy announcement, $i = 1,2,3, \dots k$ and $X_i^{'}s$ are i.i.d and $X_i = X$ for all i.

Y : a continuous random variable denoting the threshold level having the Lindley distribution .

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of $g(.)$; $g_k^*(.)$: Laplace transform of $g_k(.)$

$h(.)$: The probability density functions of random threshold level 'Y' which has the Lindley distribution and $H(.)$is the corresponding probability distribution function.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding c.d.f. F(.)

$V_k(t) : F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly *'k'* policies decisions in (0, t]

$S(.)$: The survivor function i.e. $P[T > t]$;

L(t) = $1 - S(t)$

3. Model Description

The Lindley distribution was originally proposed by Lindley (1958) in the context of Bayesian statistics, as a counter example of fiducial statistics. The Lindley distribution has the following probability density function (PDF)

$$f(x,\theta)=\frac{\theta^2}{1+\theta}(1+x)e^{-\theta x} \qquad x>0,\theta>0$$

The corresponding cumulative distribution function (CDF)

$$F(x,\theta)=1-\left(1+\frac{\theta}{1+\theta}x\right)e^{-\theta x}$$

The corresponding Survival Function is (SF)

$$\overline{H}(x)=1-F(x)$$

$$=\left(1+\frac{\theta}{1+\theta}x\right)e^{-\theta x}$$

The shock survival probability are given by

$$P(X_i<Y)=\int_0^\infty g_k(x)\overline{H}(x)dx$$

$$=\int_0^\infty g_k(x)\left(1+\frac{\theta}{1+\theta}x\right)e^{-\theta x}dx$$

On simplification,

$$=\left(g^*(\theta)\right)^k-\left(\frac{\theta}{1+\theta}\right)\left(g^{*'}(\theta)\right)^k$$

The survival function $S(t)$ which is the probability that the total depletion does not cross the threshold level is given as

$$S(t)=P(T>t)=\sum_{k=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]P(X_k<Y)$$

It is also known from renewal process that

$$P(T > t) = \sum_{k=0}^{\infty} \left[F_k(t) - F_{k+1}(t) \right] P(X_k < Y)$$

Now $L(t) = 1 - S(t)$

On simplification

$$= 1 - \left[1 - g^*(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^*(\theta)\right]^{k-1} - \left(\frac{\theta}{1+\theta}\right) + \left(\frac{\theta}{1+\theta}\right) \left[1 - g^{*'}(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^{*'}(\theta)\right]^{k-1}$$

Taking Laplace transformation of L(t) one can get,

$$L(t) = 1 - \left\{1 - \left[1 - g^*(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^*(\theta)\right]^{k-1} - \left(\frac{\theta}{1+\theta}\right) + \left(\frac{\theta}{1+\theta}\right) \left[1 - g^{*'}(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^{*'}(\theta)\right]^{k-1}\right\}$$

$$= \left[1 - g^*(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^*(\theta)\right]^{k-1} + \left(\frac{\theta}{1+\theta}\right) - \left(\frac{\theta}{1+\theta}\right) \left[1 - g^{*'}(\theta)\right] \sum_{k=1}^{\infty} F_k(t) \left[g^{*'}(\theta)\right]^{k-1}$$

After simplification, one can get

$$l^*(S) = \frac{\left[1 - g^*(\theta)\right] f^*(s)}{\left[1 - g^*(\theta) f^*(s)\right]} - \left(\frac{\theta}{1+\theta}\right) \frac{\left[1 - g^{*'}(\theta)\right] f^*(s)}{\left[1 - g^{*'}(\theta) f^*(s)\right]}$$

Let the random variable U denoting the inter arrival time which follows exponential distribution with parameter c. Now, $f^*(s) = \frac{c}{c+s}$, substituting in the above equation we get,

$$l^*(S) = \frac{\left[1 - g^*(\theta)\right] \frac{c}{c+s}}{\left[1 - g^*(\theta) \frac{c}{c+s}\right]} - \left(\frac{\theta}{1+\theta}\right) \frac{\left[1 - g^{*'}(\theta)\right] \frac{c}{c+s}}{\left[1 - g^{*'}(\theta) \frac{c}{c+s}\right]}$$

$$= \frac{\left[1 - g^*(\theta)\right] c}{\left[c + s - g^*(\theta) c\right]} - \left(\frac{\theta}{1+\theta}\right) \frac{\left[1 - g^{*'}(\theta)\right] c}{\left[c + s - g^{*'}(\theta) c\right]}$$

$$E(T) = -\frac{d}{ds}L^*(S) \qquad = \frac{\left[1-g^*(\theta)\right]c}{\left[c+s-g^*(\theta)c\right]^2} - \left(\frac{\theta}{1+\theta}\right)\frac{\left[1-g^{*'}(\theta)\right]c}{\left[c+s-g^{*'}(\theta)c\right]^2}$$

$$= \frac{1}{\left[1-g^*(\theta)\right]c} - \left(\frac{\theta}{1+\theta}\right)\frac{1}{\left[1-g^{*'}(\theta)\right]c}$$

$$E(\mathrm{T}^2) = \frac{d^2}{ds^2}L^*(s)$$

$$= \frac{1}{\left[1-g^*(\theta)\right]^2 c^2} - \left(\frac{\theta}{1+\theta}\right)\frac{1}{\left[1-g^{*'}(\theta)\right]^2 c^2}$$

$$g^*(\theta) \sim \left(\frac{\mu}{\mu+\theta}\right), g^{*'}(\theta) \sim -\frac{\mu}{(\mu+\theta)^2}$$

$$E(T) = \frac{1}{\left(1-\frac{\mu}{\mu+\theta}\right)c} - \left(\frac{\theta}{1+\theta}\right)\frac{1}{\left(1+\frac{\mu}{(\mu+\theta)^2}\right)c}$$

$$= \frac{\mu+\theta}{\theta c} - \frac{\theta}{(1+\theta)c}\frac{(\mu+\theta)^2}{\left[(\mu+\theta)^2+\mu\right]}$$

$$E(T^2) = \frac{(\mu+\theta)^2}{\theta^2 c^2} - \frac{\theta}{(1+\theta)c^2}\frac{(\mu+\theta)^4}{\left[(\mu+\theta)^2+\mu\right]^2}$$

From which the variance can be obtained

$$V(T) = E(T^2) - (E(T))^2$$

$$= \frac{(\mu+\theta)^2}{\theta^2 c^2} - \frac{\theta}{(1+\theta)c^2}\frac{(\mu+\theta)^4}{\left[(\mu+\theta)^2+\mu\right]^2} - \left\{\frac{\mu+\theta}{\theta c} - \frac{\theta}{(1+\theta)c}\frac{(\mu+\theta)^2}{\left[(\mu+\theta)^2+\mu\right]}\right\}^2$$

On simplification one can get

$$= \frac{2(\mu+\theta)^3}{c^2(1+\theta)\left[(\mu+\theta)^2+\mu\right]} - \frac{\theta(\mu+\theta)^4(1+2\theta)}{(1+\theta)^2 c^2\left[(\mu+\theta)^2+\mu\right]^2}$$

4. Numerical Illustrations

The mean and variance of time to recruitment is numerically illustrated by varying one parameter and keeping other parameters fixed. The effect of the parameters θ, μ and c on the performance measures is shown in the following table.

c	μ=0.5	μ=1	μ=1.5	μ=2
1	1.568	2.397	3.22	4.044
2	0.784	1.199	1.61	2.022
3	0.523	0.799	1.073	1.348
4	0.392	0.599	0.805	1.011
5	0.314	0.479	0.644	0.809
6	0.261	0.4	0.537	0.678
7	0.224	0.342	0.46	0.578
8	0.196	0.3	0.403	0.505
9	0.174	0.266	0.358	0.449
10	0.157	0.24	0.322	0.404

Table 1: Effect of θ, μ and c on the performance measures E(T)

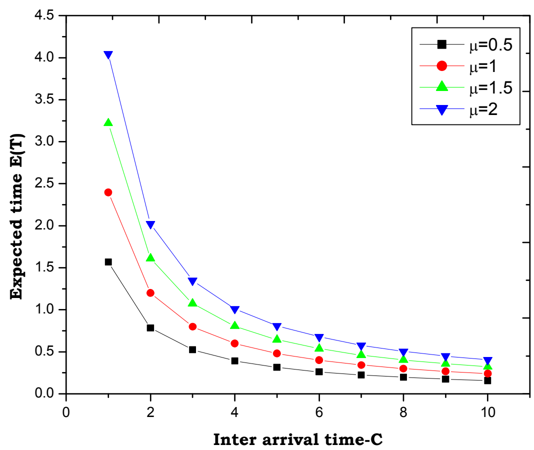

Figure 1 : The changes in E(t) due to changes in μ

c	μ=0.5	μ=1	μ=1.5	μ=2
1	0.715	1.172	1.672	2.201
2	0.179	0.293	0.418	0.55
3	0.079	0.13	0.186	0.245
4	0.045	0.073	0.104	0.138
5	0.029	0.047	0.067	0.088
6	0.02	0.033	0.046	0.061
7	0.015	0.024	0.034	0.045
8	0.011	0.018	0.026	0.034
9	0.009	0.014	0.021	0.027
10	0.007	0.012	0.017	0.022

Table 2: Effect of θ, μ and c on the performance measures V(T)

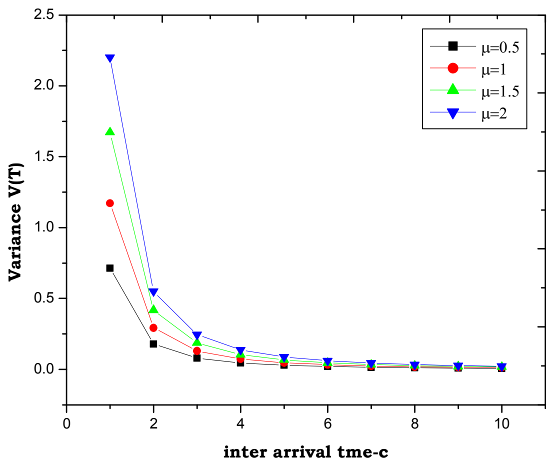

Figure 2 : Changes in V(t) due to changes in μ

c	θ=0.5	θ=1	θ=1.5	θ=2
1	1.373	0.875	0.651	0.519
2	0.687	0.438	0.325	0.26
3	0.458	0.292	0.217	0.173
4	0.343	0.219	0.163	0.13
5	0.275	0.175	0.13	0.104
6	0.229	0.146	0.108	0.087
7	0.196	0.125	0.093	0.074
8	0.172	0.109	0.081	0.065
9	0.153	0.097	0.072	0.058
10	0.137	0.088	0.065	0.052

Table 3: Effect of θ, μ and c on the performance measures E(T)

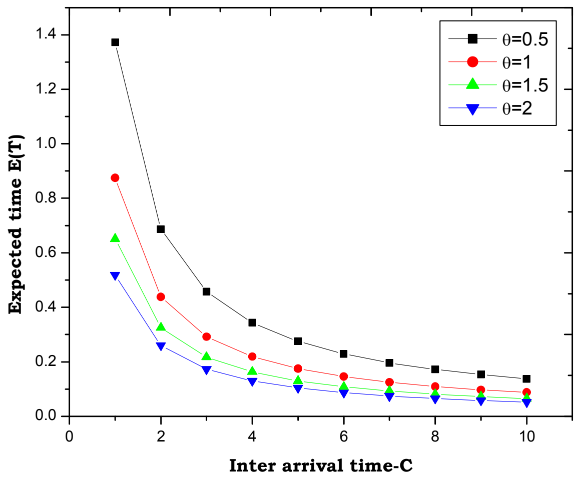

Figure 3 : Changes in E(t) due to changes in θ

c	**θ=0.5**	**θ=1**	**θ=1.5**	**θ=2**
1	0.52	0.563	0.514	0.456
2	0.13	0.141	0.128	0.114
3	0.058	0.063	0.057	0.051
4	0.033	0.035	0.032	0.028
5	0.021	0.023	0.021	0.018
6	0.014	0.016	0.014	0.013
7	0.011	0.011	0.01	0.01
8	0.008	0.009	0.008	0.007
9	0.006	0.007	0.006	0.006
10	0.005	0.006	0.005	0.005

Table 4: Effect of θ, μ and c on the performance measures V(T)

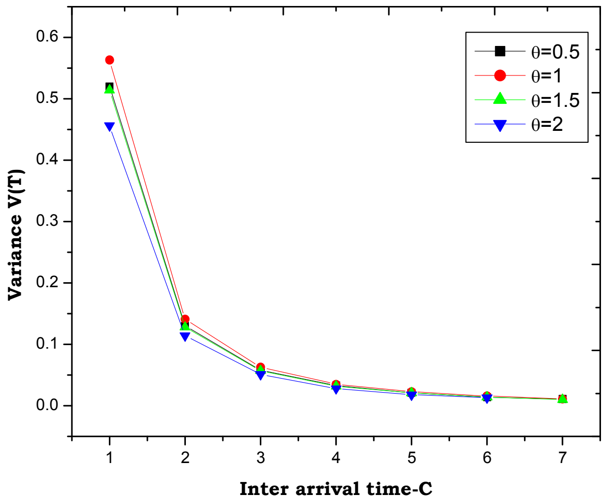

Figure 4: Changes in V(t) due to changes in θ

Conclusions

The results revealed that when μ is kept fixed the inter-arrival time $'c'$ which follows exponential distribution, is increasing in the time to recruitment, then the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $\mu = 0.5, 1, 1.5, 2$. When the value of the parameter μ increases, the expected time is also found decreasing, this is observed in Figure 1. The same case is found in Variance $V(T)$ which is observed in Figure 2.

When θ is kept fixed and the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the time to recruitment is found to be decreasing, in all the cases of the parameter value $\theta = 0.5, 1, 1.5, 2$. When the value of the parameter θ increases, the expected time is found increasing, this is indicated in Figure 3. The same case is observed in the variance $V(T)$ which is observed in Figure 4.

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