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SAMPLING INSPECTION BY VARIABLE FOR MEAN OF SYMMETRICAL POPULATION WITH KNOWN COEFFICIENT OF VARIATION

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Abstract

In this paper we have examined the superiority of single sampling plan with known coefficient of variation on usual single sampling plan with known standard deviation with the help of a plan by calculating the Operating Characteristic (OC).

Key Words: - Variable Sampling Plan, OC Function, Coefficient of Variation.

1. Introduction

In variable sampling plans, an underlying process distribution form is assumed. Normally, the proportion defective in the lot can then be estimated by estimating the parameters of that distribution. The variables model thus requires more restrictive assumptions on the manufacturing process. If these assumptions can be justified however, this results a substantial savings in sample size corresponding to a given sampling risk. Because of the intractability of the some of the expressions encountered, the sample sizes derived for the finite-lot variable sampling plans are indexed on the process quality associated with the lot rather than the lot quality itself. Once the sample size is decided, the decision whether to accept or reject the lot (the acceptance region) is determined from a lot quality approach. Coefficient of variation (CV) is widely used as a measure of variability in engineering experiments although standard error of CV is well known (Kendall and Stuart (1973), Serfling (1980)). Wetherill (1960) and Wetherill and Campling (1966) used this approach to develop variable sampling plans applicable to large lots.

In this paper, an attempt has been made to study the variable sampling inspection for mean of symmetrical population with known coefficient of variation. The OC of these sampling plans is calculated, which shows good performance with the OC of Singh and Mangal (1995) and known standard case.

1.1 Model Descriptions and OC Function with Known Coefficient of Variation.

Prior knowledge of CV of study variate and information on the characteristic are used in developing some efficient estimator of mean of symmetrical population. Srivastava and Banarasi (1982) has found that $\overline{x}*$ estimators are more efficient than sample mean in every situation. where $\overline{x}*$ is defined as

$$\bar{x}^* = \bar{x} + \frac{s^2\bar{x}}{(n\bar{x}^2 + s^2)} - \frac{s^4\bar{x}}{(n\bar{x}^2 + s^2)^2} \ . \tag{2.1}$$

To evaluate the Mean Square Error (MSE) of the estimator to the $O(n^{-2})$ Srivastava and Banarasi (1982) have shown that

$$\text{MSE } (\bar{x}^*) = \frac{\sigma^2}{n}\left(1 - \frac{c}{n}\right), \tag{2.2}$$

$$= \frac{\sigma^2}{n} M^2,$$

where $M^2 = \left(1 - \frac{c}{n}\right)$.

In connection with a single sampling variable plan, when CV is known $\left(C = \frac{\sigma}{\mu}\right)$, the following symbols will be used.

L = lower specification limit
U = Upper specification limit
k = Acceptance parameter

$\bar{x}^*$ = weighted sample mean for n samples

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz,$$

where $Z \sim N(0,1)$.

We now calculate the OC function of single sampling plan. The acceptance criterion for weighted mean with known CV plan is; accept the lot if $\bar{x}^*$+k$\sigma \le U$ and reject the lot otherwise for the upper specification limit. When used with L, the acceptance criterion is; accept the lot if $\bar{x}^*$ - k$\sigma \ge L$ and reject the lot otherwise. If p is the proportion defective in the lot

$$\frac{U - \mu}{\sigma} = K_p \ ,$$

where
$$\int_{K_p}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) dz = p \ . \tag{2.3}$$

The value of n and k are determined for a given set of values of the producer's risk α, consumer's risk β, AQL (p_1) and LTPD (p_2) by the formulae

$$n = [(K_\alpha + K_\beta)/(K_{p_1} - K_{p_2})]^2 \tag{2.4}$$

$$k = [(K_\alpha K_{p_2} + K_\beta K_{p_1}) / (K_\alpha + K_\beta)] \tag{2.5}$$

Where K_α and K_β are the acceptance parameter of producer's and consumer's risk respectively, and K_{p_1} and K_{p_2} are acceptance parameter of AQL(p_1) and LTPD(p_2) respectively.

The expression for probability of acceptance i.e. OC function of the plan is

$$L(p) = \text{Prob.}[\bar{x}^* + k\sigma \le U = \mu + k_p\sigma] \tag{2.6}$$

Following Schilling (1982), the OC function with known CV works out to be as

$$L(p) = \Phi\left[\left(\frac{n}{M}\right)^{1/2} (K_p - k)\right], \tag{2.7}$$

Where

$$M = \left(1 - \frac{c}{n}\right)^{1/2}, \; c = (\sigma/\mu) \text{ and } \Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}x^2\right) dx.$$

The usual OC of single sampling plan for known standard deviation is

$$L(p) = \Phi[\sqrt{n}\;(K_p - k)]. \tag{2.8}$$

1.1.1 Numerical Illustration and Discussion of Results

For illustration we consider an example of producer's and consumer's oriented single sampling plan $p_1 = 0.05, \alpha = 0.05, p_2 = 0.30$ *and* $\beta = 0.10.$ The values of n and k determined from equation (2.4) and (2.5) are 7 and 1.015 respectively. The values of OC function for the above plan have been calculated for known CV as well as known standard deviation by using equations (2.7) and (2.8). These values of OC function for different values CV and usual known standard case are presented in Table 1 and plotted in Figure 1.

From Table 1 and Figure 1 it is evident, that the values of OC function increase as the CV increases upto p=0.02 to 0.15, while OC function decreases when CV increases for p > 0.15. For C=1,3,5 and p=0.06, the OC functions are 0.9357, 0.9699 and 0.9967 respectively. While from equation (2.8), the OC function for known standard deviation with p = 0.06 is 0.9192. The performance of this plan is better than Singh and Mangal (1995) plan.

P	σ known	C = 1	C = 2	C = 3	C = 4	C = 5
0.02	0.9968	0.9983	0.9993	0.9998	0.9999	0.9924
0.04	0.9725	0.9807	0.9886	0.9947	0.9985	0.9998
0.05	0.9494	0.9624	0.9744	0.9857	0.9946	0.9992
0.06	0.9192	0.9357	0.9525	0.9699	0.9857	0.9967
0.08	0.8437	0.8643	0.8868	0.913	0.9429	0.9755
0.1	0.7549	0.7733	0.7938	0.8238	0.8599	0.9098
0.13	0.614	0.6217	0.633	0.648	0.67	0.7122
0.15	0.5199	0.5239	0.5239	0.5279	0.5318	0.5398
0.17	0.4404	0.4326	0.4286	0.4169	0.4052	0.3821
0.2	0.3264	0.3121	0.2981	0.2743	0.242	0.1922
0.22	0.2644	0.2483	0.2267	0.2005	0.1636	0.1113
0.25	0.1895	0.1686	0.1469	0.1191	0.0838	0.0428
0.28	0.1314	0.1113	0.0902	0.0669	0.0401	0.0147
0.3	0.1003	0.0838	0.0643	0.0437	0.0233	0.0066
0.33	0.0669	0.0527	0.0376	0.0228	0.01	0.0019
0.37	0.0376	0.0269	0.0171	0.0087	0.0029	0.0003
0.4	0.0239	0.0158	0.0092	0.0041	0.0011	1e-04

Table 1: Values of OC function for known CV and known σ

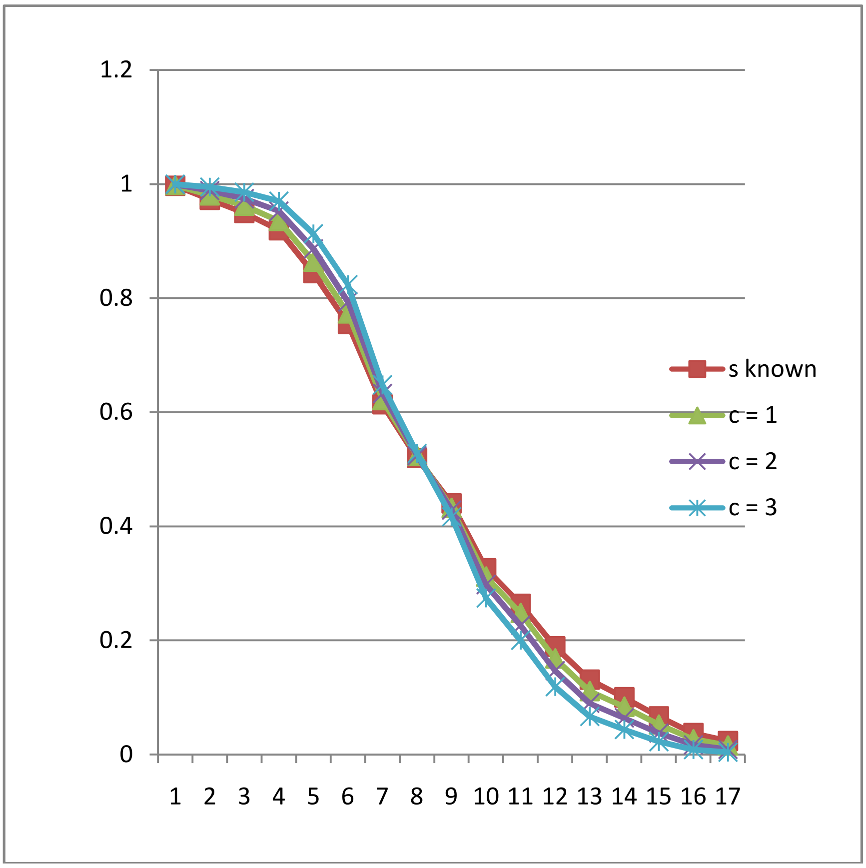

Figure 1: OC Curve for known coefficient of variation (for different values of CV)

References

1. Kendall, M.G. and Stuart, A. (1973). The Advance Theory of Statistics, Vol. 2, London : Griffin.
2. Schilling, E.G. (1982). Acceptance Sampling in Quality Control, Marcel Dekker, Inc., New York.
3. Serfling, R.J. (1980). Approximation Theorems of Mathematical Statistics, Wiley, New York.
4. Singh, J.R. and Mangal, I.K. (1995). Control charts for mean with known coefficient of variation, International Journal of Management Systems, 11(1), p. 141-146.
5. Srivastava, S.R. and Banarasi (1982). A Note on the estimation of mean of a symmetrical population, Jour. Ind. Soc. Agri. Stat., 34(3), p. 79-81.

6. Wetherill, G.B. (1960). Some remarks on the Bayesian solution of the single sample inspection scheme, Technometrics, 2, No. 3, p. 341-52.
7. Wetherill, G.B. and Campling, G.E.G. (1966). The decision theory approach to sampling inspection, Journal of the Royal Statistical Society, Ser. B, 28, No. 3, p. 381-407.