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ECONOMIC DESIGN OF $\overline{X}$ CONTROL CHART UNDER DEWMA MODEL

Manzoor A. Khanday* and J. R. Singh
School of Studies in Statistics, Vikram University, Ujjain, India
E Mail:Corresponding Author: *manzoorstat@gmail.com

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Abstract

In this paper, mathematical investigation has been made to study the effect of double exponentially weighted moving average (DEWMA) model on economic design of $\overline{X}$ control chart. Formulae are derived for calculating the value of n and h when the characteristics of an item possess DEWMA model. A numerical example is derived to verify the performance of DEWMA model in presence of normality. The DEWMA charts working together with normality affects the control chart scheme when small to moderate shifts in the mean of the controlled parameter are expected. It is found that when shifts are uncertain the optimal design for DEWMA chart should be more conservative.

Key Words: Economic Design, Control Chart, DEWMA.

1. Introduction

A popular control chart used to detect and identify small shifts in a process mean is the EWMA (Roberts, 1959). The attempt to increase the sensitivity of EWMA control chart to detect small shifts and drift in a process, a double EWMA (DEWMA) control chart was developed by Shamma and Shamma (1992). Zhang (2002) has conducted extensive studies on DEWMA control charts for the mean. Like most commonly used control charts, the traditional EWMA and DEWMA control charts for monitoring process means were developed under the assumption of normality. Simulation studies on the robustness of an EWMA control chart for process mean monitoring have been conducted by Borror, et al. (1999).As quality has become a crucial factor in global market competition, statistical process control (SPC) techniques are becoming significant in both manufacturing and service industries that aim at 6σ excellence. With modern measurement and inspection technologies, It is common to collect large volumes of data from individual units usually on very short time intervals. Such nearly continuous measurement unavoidably results in data that tend to be non-normally distributed. However, most existing SPC techniques were not designed for such environments. It is known that conventional SPC techniques are affected by skewed data. Specifically, false alarm rates are so high that true alarms are often ignored. Since the primary purpose of SPC is to detect quickly unusual sources of variability so that their root cause can be properly addressed, data skewness has severe adverse impacts on the economic benefits of implementing SPC. If the time series model adequately represents the process behaviour the residuals will be uncorrelated. Thus, conventional SPC methods, such as Shewhart charts and exponentially weighted average (EWMA) charts, which were developed for uncorrelated data, and can be

applied directly to the residuals. For detecting process changes, Early applications on EWMA appear in economics, inventory control and forecasting (See, Cox (1961) , Hunter (1986), Mac Gregor (1988), most special attention from the semiconductor fabrication process (See, Ingolfsson and Sachs (1993), Del Castilo and Hurwitz (1997).
Economic design of control charts is used to determine various design parameters that minimize total economic costs. The effect of production lot size on the quality of the product may also be significant. If the production process shifts to an out-of- control state at the beginning of the production run, the entire lot will contain more defective items. Hence it is better to reduce the production cycle to decrease the fraction of defective items and, thus improve output quality. On the other hand, reduction of the production cycle may result in an increase in cost due to frequent setups. A balance must be maintained so that the total cost is minimized. The operating condition of the machine tools; however, the performance of machine tools depends up on the maintenance policy. It is assumed that that the cost of maintaining the equipment increases with the age; therefore, an age replacement strategy is needed to minimize the total cost of the system, which will simultaneously improve quality control and maintenance policy. The behavior of the DEWMA control chart performance for non-normal populations has been investigated.Singh et al. (2013) Studies the problem on Variables sampling plan for correlated data, Khanday and Singh (2015) study the effect of Markof's model on Economic design of $\overline{X}$ control charts under independent observations. Zhang (2002) has conducted extensive studies on DEWMA control charts for the mean. Recently, many researchers have contributed to a wide variety of control charts to improve process monitoring, such as Saghaeiet al. (2014), Amiriet al. (2015) and Lee et al. (2014).

2. Duncan's model for the cost function

Duncan (1956) obtained an approximate function for the average net income per hour of using the control chart for mean of normal variables as:

$$I = V_0 - \frac{\eta MB + (\alpha T / h) + \eta W}{1 + \eta B} - \frac{b + cn}{h} \tag{2.1}$$

Duncan's cost model indicates
(i) the cost of an out-of –control conditions,
(ii) the cost of false alarms,
(iii) the cost of finding an assignable cause and
(iv) the cost of sampling inspection, evolution, and plotting.

Notations

V_0 = the average per hour when process is in control and process average is μ ,

V_1 = the average income per hour when process is not in control and process average is $\mu' = \mu + \delta\sigma$,

$M = V_0 - V_1$

η = the average number of times the assignable cause occur within an interval of time,

$B = a\,h + Cn + D$

$a = \frac{1}{P} - \frac{1}{2} + \frac{\eta h}{12}$,

$h =$ Sampling interval in hours

$Cn =$ the time required to take and inspect a sample of size n .

$D =$ average time taken to find the assignable cause after a point plotted on the chart falls outside the control limits,

$P =$ Probability of detecting an assignable cause when it exists,

$$= \int_{-\infty}^{\mu - k\sigma/\sqrt{n}} g(\bar{x}/\mu')d\,\bar{x} + \int_{\mu + k\sigma/\sqrt{n}}^{\infty} g(\bar{x}/\mu')d\,\bar{x}$$

$\cong 1 - \Phi(k - \delta\sqrt{n})$ for $\delta > 0$

Where $g(\bar{x}/\mu')$ is the density function of $\bar{x}$ when the true mean μ and $\Phi(x)$ is the normal probability

$\alpha =$ probability of wrongly indicating the presence of assignable cause.

$$= \int_{\mu - \frac{k\sigma}{\sqrt{n}}}^{\mu + \frac{k\sigma}{\sqrt{n}}} g(\bar{x}/\mu)\, d\,\bar{x} = 2\Phi(-k) \tag{2.2}$$

$T =$ The cost per occasions of looking for an assignable cause when no assignable cause exists,

$W =$ the average cost per occasion of finding the assignable cause when it exist,

$b =$ per sample cost of sampling and plotting, that is independent of sample size,

c= the cost per unit of measuring an item in a sample.

The average cost per hour involved for maintaining the control chart is $\frac{(b+cn)}{h}$. The average net income per hour of the process under the surveillance of the control chart for mean can be rewritten as, $I = V_0 - L$

Where

$$L = \frac{\eta MB + (\alpha T/h) + \eta W}{1 + \eta B} + \frac{b + cn}{h} \tag{2.3}$$

L Can now be treated as the per hour cost due to the surveillance of the process under the control chart.

3. Derivation for optimum value of sample size n and sampling interval h

One can determine the optimum value of sample size n and sampling interval h either by maximizing the gain function I or by minimizing the cost function L with respect to n and h , and we get,

$$\frac{\partial L}{\partial n} = \frac{(1+\eta B)\left(\eta M \frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha'}{\partial n}\right) - \left(\eta MB + \frac{\alpha' T}{h} + \eta W\right)\eta \frac{\partial B}{\partial n}}{(1+\eta B)^2} + \frac{c}{h} = 0 \tag{3.1}$$

$$\frac{\partial L}{\partial h} = \frac{(1+\eta B)\left(\eta M \frac{\partial B}{\partial h} - \frac{\alpha' T}{h^2}\right) - \left(\eta MB + \frac{\alpha' T}{h} + \eta W\right)\eta \frac{\partial B}{\partial n}}{(1+\eta B)^2} - \left(\frac{b+cn}{h^2}\right) = 0 \tag{3.2}$$

Where,

$$\frac{\partial B}{\partial n} = \frac{-h}{p'^2}\frac{\partial p'}{\partial n} + c\,, \quad \frac{\partial L}{\partial h} = \frac{1}{p'} - \frac{1}{2} + \frac{\eta h}{\sigma} \quad \text{and} \quad \frac{\partial \alpha'}{\partial n} = 0 - \frac{\partial \alpha_c}{\partial n} = 0 \tag{3.3}$$

$$\frac{\partial P'}{\partial n} = \frac{\delta}{2\sqrt{n}}\phi(\xi) \text{ ,where } \xi = (k - \delta\sqrt{n}) \tag{3.4}$$

The solutions of the equations (2.1) and (2.2) for n and h yield the required optimum values. The equations (2.1) and (2.2) can be rewritten as follows:

$$\eta h\left(M - \eta MB - \frac{\alpha' T}{h} - \eta W\right)\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha'}{\partial n} + \eta B\left(\eta M\frac{\partial B}{\partial n} + \frac{T}{h}\frac{\partial \alpha'}{\partial n}\right) + c(1+\eta B)^2 = 0 \tag{3.5}$$

$$\eta h^2\left(M - \eta MB - \frac{\alpha' T}{h} - \eta W\right)\frac{\partial B}{\partial h} - \alpha' T(1+\eta B) + \eta^2 h^2 MB\frac{\partial B}{\partial n} - (b+cn)(1+\eta B)^2 = 0 \tag{3.6}$$

By assuming η to be small and noting that the optimum h is roughly of order of $\frac{1}{\sqrt{\eta}}$, we neglect terms with ηB containing ηWc, $\frac{\alpha' T}{h}$ and the terms equating higher powers of η . The equations (3.5) and (3.6) are simplified and put in the following form

$$\frac{-\eta h^2 M}{p'^2}\frac{\partial p'}{\partial n} - \eta\alpha' T + c = 0 \tag{3.7}$$

$$\eta M h^2\left(\frac{1}{P'} - \frac{1}{2}\right) - (\alpha' T + b + cn) = 0 \tag{3.8}$$

From the equation (3.8) we get

$$h = \left\{\frac{\alpha' T + b + cn}{\eta M\left(\frac{1}{P'} - \frac{1}{2}\right)}\right\}^{\frac{1}{2}} \tag{3.9}$$

By eliminating h from the equation (2.7), we get,

$$-\frac{\alpha' T + b + cn}{P'^2\left(\frac{1}{P'} - \frac{1}{2}\right)}\cdot\frac{\partial p'}{\partial n} - \eta\alpha' T + c = 0 \quad (3.10)$$

The values of n for which the equation (3.10) satisfy yield us the required optimum value of sample size n. Substituting this value of n in equation (3.9), we find the optimum value of the sampling interval h.

4. Derivation of the optimum values of sample size n and sampling interval h under DEWMA

Suppose that X_t , (*t=1, 2, 3, ...*) is a sequence of random variables taken from a normal distribution with mean μ_0 and variance σ^2. Note that Y_t is the usual EWMA control statistic, and the DEWMA control statistics Z_t is defined as the system of equations (4.1) and (4.2)

$$Y_t = \psi X_t + (1-\psi)Y_{t-1} , \tag{4.1}$$

$$Z_t = \psi Y_t + (1-\psi)Z_{t-1} , \tag{4.2}$$

such that $0 < \psi < 1$ (smoothing parameter) and $Y_0 = Z_0 = \mu_0$, repeated substitutions are applied to equations (4.1) and (4.2) and rewritten as:

$$Y_t = \psi \sum_{j=0}^{t-l}(1-\psi)^j X_{t-j} + (1-\psi)^t Y_0 , \tag{4.3}$$

$$Z_t = \psi \sum_{j=0}^{t-l}(1-\psi)^j Y_{t-j} + (1-\psi)^t Z_0 . \tag{4.4}$$

Using values of (4.3) in equation (4.4) we get:

$$Z_t = \psi \sum_{j=0}^{t-l}(1-\psi)^j \left[\psi \sum_{k=0}^{t-j-l}(1-\psi)^k X_{t-j-k} + (1-\psi)^{t-j} Y_0 \right] + (1-\psi)^t Z_0 ,$$

$$Z_t = \psi^2 \sum_{j=0}^{t-l}(1-\psi)^j \sum_{k=0}^{t-j-l}(1-\psi)^k X_{t-j-k} + t\psi(1-\psi)^t Y_0 + (1-\psi)^t Z_0 ,$$

$$Z_t = \psi^2 \sum_{j=0}^{t-l}(1-\psi)^j \sum_{l=0}^{t-j}(1-\psi)^{t-j-1} X_l + t\psi(1-\psi)^t Y_0 + (1-\psi)^t Z_0 ,$$

$$Z_t = \psi^2 \sum_{l=1}^{t}(t-l+1)(1-\psi)^{t-l} X_l + t\psi(1-\psi)^t Y_0 + (1-\psi)^t Z_0 . \tag{4.5}$$

Replacing *l* with *j* in equation (4.5) we get

$$Z_t = \psi^2 \sum_{j=1}^{t}(t-j+1)(1-\psi)^{t-j} X_j + t\psi(1-\psi)^t Y_0 + (1-\psi)^t Z_0 . \tag{4.6}$$

It is assumed, without loss of generality, that $Y_0 = Z_0 = \mu_0$.

Here we mentioned some quantities below for evaluating mean and variance, then, for $a \neq 0$

$$\sum_{k=1}^{n} k a^k = \frac{a(1-a^n)}{(1-a)^2} - \frac{n\, a^{n+1}}{1-a} \text{ and} \tag{4.7}$$

$$\sum_{k=1}^{n} k^2 a^k = \frac{a + a^2 - (n+1)^2 a^{n+1} + (2n^2+2n-1)a^{n+2} - n^2 a^{n+3}}{(1-a)^3} . \tag{4.8}$$

On taking the expectation of equation (4.6), we will have:

$$\begin{aligned}
\mu_{Z_t} &= E(Z_t) \\
&= E\left[\psi^2 \sum_{j=1}^{t}(t-j+1)(1-\psi)^{t-j} X_j + t\psi(1-\psi)^t Y_0 + (1-\psi)^t Z_0 \right], \\
&= \psi^2 \sum_{j=1}^{t}(t-j+1)(1-\psi)^{t-j} E(X_j) + t\psi(1-\psi)^t E(Y_0) + (1-\psi)^t E(Z_0), \\
&= \frac{\psi^2}{1-\psi} \sum_{j=1}^{t}(t-j+1)(1-\psi)^{t-j+1} \mu_0 + t\psi(1-\psi)^t \mu_0 + (1-\psi)^t \mu_0 ,
\end{aligned}$$

Put $k = t-j+1$ in the above equation, we get

$$\mu_{Z_t} = \frac{\psi^2}{1-\psi} \sum_{j=1}^{t}(t-j+1)(1-\psi)^k \mu_0 + t\psi(1-\psi)^t \mu_0 + (1-\psi)^t \mu_0 .$$

Now using equation (4.7) in the first term with $a=(1-\psi)$ and $n=t$ we get

$$\mu_{Z_t}=\frac{\psi^2}{1-\psi}\left[\frac{(1-\psi)\left(1-(1-\psi)^t\right)}{\left[1-(1-\psi)\right]^2}-\frac{t(1-\psi)^{t+1}}{1-(1-\psi)}\right]\mu_0$$

$$+t\psi(1-\psi)^t\mu_0+(1-\psi)^t\mu_0\,,$$

$$=\psi^2\left[\frac{1-(1-\psi)^t}{\psi^2}-\frac{t(1-\psi)^t}{\psi}\right]\mu_0+t\psi(1-\psi)^t\mu_0+(1-\psi)^t\mu_0\,,$$

$$=\left(1-(1-\psi)^t-t\psi(1-\psi)^t+(1-\psi)^t+(1-\psi)^t\right)\mu_0\,,$$

$$\mu_{Z_t}=E(Z_t)=\mu_0\,. \tag{4.9}$$

Now, taking variance of equation (4.6), we will have:

$$\sigma^2_{Zt}=Var(Z_t),$$

$$\sigma^2_{Zt}=Var\left[\psi^2\sum_{j=1}^{t}(t-j+1)(1-\psi)^{t-j}X_j+t\psi(1-\psi)^tY_0+(1-\psi)^tZ_0\right],$$

$$=\psi^4\sum_{j=1}^{t}(t-j+1)^2\left[(1-\psi)^2\right]^{t-j}V(X_j)+0\,,$$

$$=\frac{\psi^4}{(1-\psi)^2}\sum_{j=1}^{t}(t-j+1)^2\left[(1-\psi)^2\right]^{t-j+1}\sigma^2\,.$$

Now using equation (4.8) with $a=(1-\psi)^2, k=t-j+1$ and $n=t$ we get:

$$\sigma^2_{Zt}=\frac{\psi^4}{(1-\psi)^2}\left[\frac{\begin{array}{c}(1-\psi)^2+(1-\psi)^4-(t+1)^2\,(1-\psi)^{2t+2}\\+(2t^2+2t-1)(1-\psi)^{2t+4}-t^2(1-\psi)^{2t+6}\end{array}}{\left(1-(1-\psi)^2\right)^3}\right]\sigma_0^2$$

$$\sigma^2_{Zt}=\psi^4\left[\frac{\begin{array}{c}1+(1-\psi)^2-(t+1)^2\,(1-\psi)^{2t}\\+(2t^2+2t-1)(1-\psi)^{2t+2}-t^2(1-\psi)^{2t+4}\end{array}}{\left(1-(1-\psi)^2\right)^3}\right]\sigma_0^2\,. \tag{4.10}$$

The control limits for DEWMA control chart are:

$$UCL=\mu_0+L'\sigma\sqrt{\sigma^2_{Zt}}\,,$$

$$CL=\mu_0\,,$$

$$LCL=\mu_0-L'\sigma\sqrt{\sigma^2_{Zt}}\,, \tag{4.11}$$

where L' is the distance between the control limits and the central line (CL) measured in σ units. For large values of t the control limit becomes For large values of L', the control limits become :

$$UCL=\mu_0+L'\sigma\sqrt{\frac{\psi(2-2\psi+\psi^2)}{(2-\psi)^3}}\,,$$

$$CL = \mu_0 ,$$

$$LCL = \mu_0 - L'\sigma\sqrt{\frac{\psi(2-2\psi+\psi^2)}{(2-\psi)^3}} . \tag{4.12}$$

Assuming that X_t is drawn independently from a normal distribution with variance σ^2 so that 't' is sufficiently large. One of the disturbing thing here is that ψ is quite arbitrary and lies between 0 and 1. Suppose that a machine whose performance can be effectively represented by a single unknown quality μ is inspected regularly to see whether the quality of performance is deteriorated. The successive performance level μ_1 , μ_2 , μ_3 ,…, μ_t are tracked by the observations $x_1, x_2, x_3, \ldots, x_t$. The operation continues until a decision is made to overhaul it in which case the level is set to zero instantaneously and the whole sequence begins again. This resetting after overhaul may be subject to error and so it is assumed that μ_0 is $N(0,\frac{\sigma^2}{n})$ and each subsequent state of repair is drawn independently from this distribution. Thus we get,

$$E(Z_t) = \mu_0 , \; Var(Z_t) = \frac{\sigma^2}{n}\frac{\lambda(2-2\psi+\psi^2)}{(2-\psi)^3} = \frac{\sigma^2}{n}g^2 ,$$

where $g^2 = \frac{\psi(2-2\psi+\psi^2)}{(2-\psi)^3}$. (4.13)

So for the DEWMA model, the probability density function for independent case is represented by

$$P_e^{'} = 1-\Phi(\xi_e)$$

$$\alpha_e^{'} = \alpha_{Ne} , \text{ where } \xi_e = \frac{(k-\delta\sqrt{n})}{g} \tag{4.14}$$

$$\alpha_{Ne} = 2\Phi\left(\frac{-k}{g}\right)$$

For DEWMA model, the equation (3.1) and (3.2) will reduce in following form

$$\frac{\partial L}{\partial n} = \frac{(1+\eta B)\left(\eta M\frac{\partial B}{\partial n}+\frac{T}{h}\frac{\partial\alpha_e^{'}}{\partial n}\right)-\left(\eta MB+\frac{\alpha_e^{'}T}{h}+\eta W\right)\eta\frac{\partial B}{\partial n}}{(1+\eta B)^2}+\frac{c}{h}=0 \tag{4.15}$$

$$\frac{\partial L}{\partial h} = \frac{(1+\eta B)\left(\eta M\frac{\partial B}{\partial h}-\frac{\alpha_e^{'}T}{h^2}\right)-\left(\eta MB+\frac{\alpha_e^{'}T}{h}+\eta W\right)\eta\frac{\partial B}{\partial h}}{(1+\eta B)^2}-\left(\frac{b+cn}{h^2}\right)=0 \tag{4.16}$$

Where,

$$\frac{\partial B}{\partial n} = \frac{-h}{p_e^{'2}}\frac{\partial p_e^{'}}{\partial n}+c , \; \frac{\partial B}{\partial h} = \frac{1}{p_e^{'}}-\frac{1}{2}+\frac{\eta h}{6} \; \text{ and } \; \frac{\partial\alpha_e^{'}}{\partial n}=0$$

$$\frac{\partial P_e^{'}}{\partial n} = \frac{\delta}{2\sqrt{ng}}\phi(\xi_e) \tag{4.17}$$

By solving the equation (4.15) and (4.16) we get

$$h_{e0} = h = \left\{ \frac{\alpha_e^{'}T + b + cn}{\eta M\left(\frac{1}{P_e^{'}} - \frac{1}{2}\right)} \right\}^{\frac{1}{2}} \tag{4.18}$$

and $$-\frac{\alpha_e' T + b + cn}{P_e'^2\left(\frac{1}{P_e'} - \frac{1}{2}\right)} \cdot \frac{\partial p_e'}{\partial n} - \eta\alpha_e' T + c = 0 \tag{4.19}$$

The values of n for which the equation (4.19) is satisfied, yield us the required optimum value of sample size *n*. Substituting this value *n* in equation (4.18), we find the optimum value of the sampling interval h_{oe} under DEWMA model for $\overline{X}$ chart.

ψ		k = 3		k = 2.5		k = 2		k = 1.5		k = 1	
	δ	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**	**n**	**h**
1	**1**	23	2.3371	20	2.4134	19	3.0258	35	4.6216	29	6.2076
	1.5	11	1.8026	9	1.9963	9	2.7064	16	4.1978	14	5.9551
	2	6	1.5648	6	1.8157	6	2.574	9	4.0314	8	5.8593
0.8	**1**	18	2.0317	13	1.8385	10	1.7802	9	2.3943	16	4.3788
	1.5	8	1.5707	6	1.4686	5	1.508	5	2.2144	8	4.1802
	2	5	1.367	4	1.3065	3	1.3901	3	2.1396	4	4.1032
0.6	**1**	15	1.8999	11	1.7054	8	1.526	6	1.4797	5	2.5773
	1.5	7	1.4896	5	1.3818	4	1.2868	3	1.3229	3	2.4855
	2	4	1.3104	3	1.2419	2	1.1837	2	1.2552	2	2.4483
0.2	**1**	11	1.7286	8	1.5475	6	1.382	4	1.2379	2	1.1242
	1.5	5	1.3842	4	1.2856	3	1.1984	2	1.1248	1	1.0683
	2	3	1.2377	2	1.176	2	1.1223	1	1.0776	1	1.0438

Table 1: Optimum sample size *n* and sampling interval *h* under DEWMA for $\overline{X}$ control chart.

5. Numerical illustration

In order to illustrate the results, we take *k*= 1, 1.5, 2.0, 2.5, 3.0, δ=1.0, 1.5, 2.0, λ =0.01, *M*=100, *W*=25, *T*=50, *C*=0.05, *D*=2, *b*=0.5, *c*=0.1 and ψ =1, 0.8, 0.6 and 0.2

to determine the optimum values of sample size and sampling interval. The values of n and h are presented in the above Table, which shows, as would be expected, that small values of ψ are better for detecting small shifts and large values of ψ are better for detecting large shifts as the value of ψ increases, the values of n and h increase. From the Table it is seen that for a given k and δ the value of n and h increase with increase in the value of ψ . This shows the degree of robustness for economic design of $\overline{X}$ control chart for DEWMA model to smaller values of smoothing parameter. The DEWMA for economic design of $\overline{X}$ control chart working together with normality affects the control chart scheme when small moderate shifts in the mean of the controlled parameter are expected. It is found that when shifts are uncertain the optimal design for DEWMA economic design for $\overline{X}$ control chart should be more conservative, i.e., the optimal design for random shifts are comparable to traditional designs for smaller deterministic shifts. For naive practitioners, the DEWMA chart design for ψ =1, and independence case is suggested a very good control chart to start with.

6. Conclusion

It may be inferred that when the rate of occurrence of assignable cause is fixed, the value of sample size and sampling interval are different for different value of ψ . The effect of non-normality is more serious for DEWMA model for different parameters. Since the variability in n and h of DEWMA generally smaller, therefore, due to these properties, we should motivate the use of DEWMA in industrial process. We also find that the DEWMA chart performs better only when shifts are more certain and large. From economic point of view, under some contaminated normal distribution, the DEWMA $\overline{X}$ control chart out performs the other control chart available in the literature. Therefore, we recommend the economic design of $\overline{X}$ control chart for DEWMA model be employed when there is concern about the non-normality assumption.

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