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TIME DEPENDENT ANALYSIS OF A QUEUEING SYSTEM INCORPORATING THE EFFECT OF ENVIRONMENT, CATASTROPHE AND RESTORATION

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Abstract

In the present paper, we consider an M/M/1/N queueing system with environmental, catastrophic and restorative effects. It is found that the change in the environment affects the state of the queueing system. The system as a whole suffers occasionally a disastrous breakdown in both the enviormentle conditions, upon which all present customers are cleared from the system and lost. A repair process then starts immidiately. Here the repair time is called the restoration time. During the repair time the customers may arrive in the system. Time dependent solution is obtained by using probability generating function technique and further the steady state probabilities of system size are also derived. Some measure of effectiveness and particular cases of the model have also been derived and discussed.

Key Words: Markovian Queueing System, Catastrophes, Environmental Effects, Restoration, Laplace Transforms.

1. Introduction

In this paper, we consider a simple Markovian Queueing system with Environmental, Catastrophic and restotative effects. Due to the simplicity of Markovian queues, a number of authors have considerd them and obtained the time dependent as well as the steady state solution. During the last four decaydes the attention has been focused on the effect of catastrophe. The random occurrence of catastrophe destoyes all the customers. A wide littreture is avialable for the queueing model with the possibility of catastrophes [Brockwell (1985), Jain (2010), Chao (1995)]. Queueing models with catastrophes may be suitable to be applied in many practical situations like computer communication, biological sciences and agricultural sciences etc. B. Krishna Kumar et al. (2000) obtained the time dependent solution of the catastrophic queues and after that a number of authors have generlized their ideas. Di Cresenzo (2003) have showed that the catastrophic queues may be suitable to approach in biological sciences, concerning the intraction between myosin heads and actin filaments that is responsible for force generation during muscle contraction. We found that the environment plays a very impotant role in queueing theory(2006). The state of the environment affects the state of the queueing system. When there is a change in the environment, the queuening system behavea like a catastrophic queue.

The direct application can be described to a biological phenomenon that there are many creatures such as cochroaches, ants etc. whose movement is restricted with the change of temperature (environment). As the temperature drops below a critical temperature say T_0 , the movement (production) of such like creatures becomes almost zero . On the other hand, as the temperature goes higher than T_0 the movement becomes normal. So far it is assumed that when catastophe occurs, the system will operate instantly. But in many practical situation it is found that the system does not work in a normal way immidiately when suffered from catastrophe. We found that the system will take some time for its refunctioing (2007). This repair time is called the restoration time. During the repair time it depends upon the customer wether he joins the sytem or not. In this paper, we consider that when a customer arrives during the repair time, he must join the system and wait for service until the repair process is over . We analyze the model and obtain the time dependent solution by using probability generating function technique. Further the steady state probabilities of system size are also derived. Some measure of effectiveness and particular cases of the model have also been derived and discussed.

2. Assumptions and Definitions

- The customers arrive in the system one by one in accordance with a possion process at a single service station. The arrival pattern is non-homogeneous, i.e. there may exist two arrival rates, namely λ_1 and 0 of which only one is operative at any instant.
- The customers are served one by one at a single channel. The service time is exponentially distributed. Further, corresponding to arrival rate λ_1, the possion service rate is μ_1 and the service rate corresponding to the arrival rate 0 is μ_2.
- The state of the system when operating with arrival rate λ_1 and service rate μ_1 is designated as E whereas the other with arrival rate 0 and service rate μ_2 is designated as F.
- The Possion rates at which the system moves from environmental states F to E and E to F are denoted by α and β respectively.
- The catastrophe occur according to a poisson process with rate ζ . The effect of each catastrophe is to make the queue instantly empty.
- The restoration times are independently identically and exponentially distributed with parameter η. The customers may arrive during the restoration time.
- The queue discipline is first-come-first-served.
- The capacity of the system is finite to M *i.e.*, if at any instant there are M units in the queue then the units arriving at that instant will not be permitted to join the queue, it well be considered lost for the system.

Define,

$P_{00}(t)$ = Joint probability that at time t the system in state E and there are zero customers in the system without the occurrence of catastrophe.

$Q_{00}(t)$ = Joint probability that at time t the system in state F and there are zero customers in the system without the occurrence of catastrophe.

$P_{000}(t)$ = Joint probability that at time t the system in state E and there are zero customers in the system with the occurrence of catastrophe.

$Q_{000}(t)$ = Joint probability that at time t the system in state F and there are zero customers in the system with the occurrence of catastrophe.

Where

$P_0 = P_{00} + P_{000}$ and $Q_0 = Q_{00} + Q_{000}$

$P_n(t)$ =Joint probability that at time t the system is in state E and n units are in the queue,including the one in service.

$Q_n(t)$ =Joint probability that at time t the system is in state F and n units are in the queue,including the one in service.

$R_n(t)$ =The probability that at time t there are n units in the queue, including the one in service.

$$R_n(t) = P_n(t) + Q_n(t) \quad [1]$$

Let us reckon time t from an instant when there are zero customers in the queue and the system is in the environmental state E so that the initial conditions associated with $P_n(t)$ and $Q_n(t)$ becomes,

$$P_n(00) = \begin{cases} 1; & n = 0 \\ 0; & otherwise \end{cases}$$

$$Q_n(00) = 0; \quad \forall n \quad [2]$$

Equations Governing the System

$$\frac{d}{dt}P_{00}(t) = -(\lambda_1 + \beta + \zeta)P_{00}(t) + \mu_1 P_1(t) + \alpha Q_{00}(t) + \eta P_{000}(t) \quad [3]$$

$$\frac{d}{dt}P_{000}(t) = -(\lambda_1 + \beta + \zeta + \eta)P_{000}(t) + \zeta\sum_{n=0}^{M} P_n(t) + \alpha Q_{000}(t) \quad [4]$$

$$\frac{d}{dt}P_n(t) = -(\lambda_1 + \mu_1 + \beta + \zeta)P_n(t) + \mu_1 P_{n+1}(t) + \lambda_1 P_{n-1}(t) + \alpha Q_n(t) \qquad [5]$$

$$\frac{d}{dt}P_M(t) = -(\mu_1 + \beta + \zeta)P_M(t) + \lambda_1 P_{M-1}(t) + \alpha Q_M(t) \qquad [6]$$

$$\frac{d}{dt}Q_{00}(t) = -(\alpha + \zeta)Q_{00}(t) + \mu_2 Q_1(t) + \beta P_{00}(t) + \eta Q_{000}(t) \qquad [7]$$

$$\frac{d}{dt}Q_{000}(t) = -(\alpha + \eta + \zeta)Q_{000}(t) + \beta P_{000}(t) + \zeta\sum_{n=0}^{M}Q_n(t) \qquad [8]$$

$$\frac{d}{dt}Q_n(t) = -(\mu_2 + \alpha + \zeta)Q_n(t) + \mu_2 Q_{n+1}(t) + \beta P_n(t); 0 < n < M \qquad [9]$$

$$\frac{d}{dt}Q_M(t) = -(\mu_2 + \alpha + \zeta)Q_M(t) + \beta P_M(t); n = M \qquad [10]$$

3. Transient Analysis

Let, the Laplace Transform of $f(t)$ be

$$\bar{f}(s) = \int_0^\infty e^{-st} f(t)dt \qquad [11]$$

Taking Laplace tranform of the equation [3] to [10] and using the initial conditions [2], we get

$$s\overline{P}_{00}(s) - 1 = -(\lambda_1 + \beta + \zeta)\overline{P}_{00}(s) + \mu_1\overline{P}_1(s) + \alpha\overline{Q}_{00}(s) + \eta\overline{P}_{000}(s) \qquad [12]$$

$$s\overline{P}_{000}(s) = -(\lambda_1 + \beta + \zeta + \eta)\overline{P}_{000}(s) + \zeta\sum_{n=0}^{M}\overline{P}_n(s) + \alpha\overline{Q}_{000}(s) \qquad [13]$$

$$(s + \lambda_1 + \mu_1 + \beta + \zeta)\overline{P}_n(s) = \mu_1\overline{P}_{n+1}(s) + \lambda_1\overline{P}_{n-1}(s) + \alpha\overline{Q}_n(s); \qquad [14]$$

$$(s + \mu_1 + \beta + \zeta)\overline{P}_M(s) = \lambda_1\overline{P}_{M-1}(s) + \alpha\overline{Q}_M(s) \qquad [15]$$

$$s\overline{Q}_{00}(s) = -(\alpha + \zeta)\overline{Q}_{00}(s) + \mu_2\overline{Q}_1(s) + \beta\overline{P}_{00}(s) + \eta\overline{Q}_{000}(s) \qquad [16]$$

$$s\overline{Q}_{000}(s) = -(\alpha + \eta + \zeta)\overline{Q}_{000}(s) + \beta\overline{P}_{000}(s) + \zeta\sum_{n=0}^{M}\overline{Q}_n(s) \qquad [17]$$

$$(s + \mu_2 + \alpha + \zeta)\overline{Q}_n(s) = \mu_2\overline{Q}_{n+1}(s) + \beta\overline{P}_n(s); \qquad [18]$$

$$(s + \mu_2 + \alpha + \zeta)\overline{Q}_M(s) = \beta\overline{P}_M(s); \qquad [19]$$

Define, The probability generating function by ,

$$P(z,s) = \sum_{n=0}^{M}\overline{P}_n(s)z^n \qquad [20]$$

$$Q(z,s)=\sum_{n=0}^{M}\overline{Q}_n(s)z^n \qquad [21]$$

$$R(z,s)=\sum_{n=0}^{M}\overline{R}_n(s)z^n \qquad [22]$$

where

$$R(z,s)=P(z,s)+Q(z,s) \qquad [23]$$

and

$$\overline{R}_n(s)=\overline{P}_n(s)+\overline{Q}_n(s) \qquad [24]$$

Multiplying [12] to [15] by z^n, summing over the respective region of n and using equations [20] to [22], we have

$$[(s+\lambda_1+\beta+\zeta+\mu_1)z-\lambda_1 z^2-\mu_1]P(z,s)-\alpha zQ(z,s)+\mu_1(1-z)\overline{P}_0(s)$$
$$-\lambda_1(1-z)\overline{P}_M(s)z^{M+1}-z-z\zeta\sum_{n=0}^{M}\overline{P}_n(s)=0 \qquad [25]$$

Multiplying [16]-[19] by z^n, Summing over the respective region of n and using equation [20]-[22],we have

$$[\mu_2-z(s+\mu_2+\alpha+\zeta)]Q(z,s)+\beta zP(z,s)+\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)-\mu_2\overline{Q}_0(s)(1-z)=0$$

[26]

From [25]

$$P(z,s)=\frac{\alpha zQ(z,s)-\mu_1(1-z)\overline{P}_0(s)+\lambda_1(1-z)\overline{P}_M(s)z^{M+1}+z+\zeta z\sum_{n=0}^{M}\overline{P}_n(s)}{[z(s+\lambda_1+\mu_1+\beta+\zeta)-\lambda_1 z^2-\mu_1]}$$

From [26],

$$Q(z,s)=\frac{\mu_2(1-z)\overline{Q}_0(s)-\beta zP(z,s)-\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)}{[\mu_2-z(s+\mu_2+\alpha+\zeta)]}$$

Putting the value of $Q(z,s)$ in [25]

$$P(z,s)=\alpha z\mu_2(1-z)\overline{Q}_0(s)-\alpha\zeta z^2\sum_{n=0}^{M}\overline{Q}_n(s)-\mu_1(1-z)\overline{P}_0(s)[\mu_2-z(s+\mu_2+\alpha$$
$$+\zeta)]+\lambda_1(1-z)\overline{P}_M(s)z^{M+1}[\mu_2-z(s+\mu_2+\alpha+\zeta)]+z[\mu_2-z(s+\mu_2+\alpha$$
$$+\zeta)]+\zeta z\sum_{n=0}^{M}\overline{P}_n(s)[\mu_2-z(s+\mu_2+\alpha+\zeta)]/[z(s+\lambda_1+\mu_1+\beta+\zeta)-\lambda_1 z^2$$

$$-\mu_1][\mu_2 - z(s+\mu_2+\alpha+\zeta) + \beta z^2\alpha]$$

[27]

Putting the value of $P(z,s)$ in eq.[26],

$$Q(z,s) = \mu_1(1-z)\beta z\overline{P}_0(s) + \mu_2(1-z)[(s+\lambda_1+\beta+\zeta+\mu_1)z - \lambda_1 z^2 - \mu_1]\overline{Q}_0(s)$$
$$-\beta\lambda_1(1-z)\overline{P}_M(s)z^{M+2} - \beta z^2 - \beta z^2\zeta\sum_{n=0}^{M}\overline{P}_n(s) + \zeta z[\lambda_1 z^2 + \mu_1 - (s+\lambda_1$$
$$+\beta+\zeta+\mu_1)z]\sum_{n=0}^{M}\overline{Q}_n(s)/\beta\alpha z^2 + [\mu_2 - z(s+\mu_2\alpha+\zeta)][(s+\lambda_1$$
$$+\beta+\zeta+\mu_1)z - \lambda_1 z^2 - \mu_1] \quad [28]$$

Now From [23] we have,

$$R(z,s) = P(z,s) + Q(z,s)$$
$$R(z,s) = \alpha z\mu_2 + \mu_2(s+\lambda_1+\beta+\zeta+\mu_1)z - \lambda_1 z^2 - \mu_1](1-z)\overline{Q}_0(s)$$
$$-\zeta z\sum_{n=0}^{M}\overline{Q}_n(s)[\lambda_1 z^2 - z(s+\lambda_1+\mu_1+\beta+\zeta) + \mu_1 - \alpha z]$$
$$-(1-z)\overline{P}_0(s)[\mu_1 z(s+\mu_2+\alpha+\zeta) - \mu_2 + \mu_1 z\beta] + (1-z)\overline{P}_M(s)[\lambda_1 z^{M+1}\mu_2$$
$$-z(s+\mu_2+\alpha+\zeta) - \beta\lambda_1 z^{M+2}] + \zeta z\sum_{n=0}^{M}\overline{P}_n(s)[\mu_2 - z(s+\mu_2+\alpha+\zeta) - \beta z]$$
$$+z[\mu_2 - z(s+\mu_2+\alpha+\zeta)] - \beta z^2 / -z^2 s^2 + s[\lambda_1 z^3 - z^2(\lambda_1+\mu_1+\mu_2\alpha+\beta$$
$$+2\zeta) + z(\mu_1+\mu_2)] - z^2\zeta(\beta+\alpha+\zeta) + (1-z)[-z^2\lambda_1(\mu_2+\alpha+\zeta) + z[\alpha\mu_1$$
$$+\mu_2(\mu_1+\lambda_1+\beta+\zeta)] - \mu_2\mu_1] \quad [29]$$

The unknown quantities in [29] are determined as follows:
Setting z=1, in [27] and [28] respectively, we have

$$P(1,s) = \sum_{n=0}^{M}\overline{P}_n(s) = \frac{(s+\alpha+\zeta)}{s(s+\beta+\alpha+\zeta)} \quad [30]$$

$$Q(1,s) = \sum_{n=0}^{M}\overline{Q}_n(s) = \frac{\beta}{s(s+\beta+\alpha+\zeta)} \quad [31]$$

Further, [29] is a polynomial in z and exists for all values of z, including the three zeros of the denominator. Hence $\overline{P}_0(s)$, $\overline{Q}_0(s)$, $\overline{R}_M(s)$ are obtained by setting the numerator equal to zero and substituting the three zeros a_1, a_1, a_1 (say) of the denominator (at each of which the numerator must vanish).The Laplace transform of various state probabilities for the number of units in the queue, including the one in service can be picked up as the co-efficient of the different powers of z in the expansion of [29].

Particular Case

Now letting $\alpha \to \infty, \beta \to \infty$ and setting $\mu_1 = \mu_2 = \mu$ (say) in [29], we have

$$r(z,s) = \mu(1-z)[P_0(s)+Q_0(s)] - \lambda_1 z^{M+1}(1-z)\overline{P}_M(s) - z - z\zeta[\sum_{n=0}^{M}\overline{P}_n(s) + \sum_{n=0}^{M}\overline{Q}_n(s)] / \lambda_1 z^2 - z(s+\lambda_1+\mu+\zeta)+\mu$$

$$r(z,s) = \frac{\mu(1-z)\overline{R}_0(s) - \lambda_1 z^{M+1}(1-z)\overline{P}_M(s) - z - \frac{\zeta z}{s}}{\lambda_1 z^2 - z(s+\lambda_1+\mu+\zeta)+\mu} \quad [32]$$

where

$$\overline{R}_0(s) = \overline{P}_0(s) + \overline{Q}_0(s)$$

$$r(z,s) = \lim_{\beta\to 0}[\lim_{\alpha\to\infty} R(z,s)]$$

[32] is a polynomial in z and exists for all values of z,including the two zeros of the denominator.Hence, the unknown quantities $\overline{R}_0(s)$ and $\overline{P}_M(s)$ can be evaluated as before.

4. Steady State Results

This can at once be obtained by the well-known property of the Laplace transform given below:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\bar{f}(s) \quad 1cm \text{ [By final value theorem]}$$

if the limit on the left hand side exists.

Thus if

$$R(z) = \sum_{n=0}^{M} R_n z^n$$

where

$$R_n = \lim_{s\to 0} s\overline{R}_n(s)$$

Then

$$R(z) = \sum_{n=0}^{M} \lim_{s\to 0} s\overline{R}_n(s) z^n = \lim_{s\to 0} s \sum_{n=0}^{M} \overline{R}_n(s) z^n$$

$$R(z) = \lim_{s\to 0} sR(z,s)$$

and

$$\sum_{n=0}^{M} R_n = \sum_{n=0}^{M} P_n + \sum_{n=0}^{M} Q_n = 1 \qquad [33]$$

By employing this property, we have from [29]

$$R(z) = \lim_{s \to 0} sR(z,s)$$

$$R(z) = \mu_2(1-z)[\alpha z + (\lambda_1 + \beta + \zeta + \mu_1)z - \lambda_1 z^2 - \mu_1]Q_0 + [\zeta z/(\alpha + \beta + \zeta)][\beta\{\lambda_1 z^2 - z(\lambda_1 + \mu_1 + \beta + \zeta + \alpha) + \mu_1\} + (1-z)\mu_1 P_0[\beta z - z(\mu_2 + \alpha + \zeta)\}] + (1-z)\lambda_1 z^{M+1} P_M[\mu_2 - z(\mu_2 + \alpha + \zeta) - \beta z] + [\zeta z(\alpha + \zeta)/(\alpha + \beta + \zeta)][\mu_2 - z(\mu_2 + \alpha + \zeta + \beta)]/z^3 \lambda_1(\alpha + \mu_2 + \zeta) - z^2[\lambda_1(\alpha + \mu_2 + \zeta) + \{\alpha\mu_1 + \mu_2(\lambda 1 + \mu_1 + \beta + \zeta)\} + \zeta(\alpha + \beta + \zeta)] + z[\{\alpha\mu_1 + \mu_2(\lambda_1 + \mu_1 + \beta + \zeta)\} + \mu_1\mu_2] - \mu_1\mu_2 \qquad [34]$$

or, we can write,

$$R(z) = \frac{T(z)Q_o + N(z)P_0 + L(z)P_M + M(z)}{K(z)} \qquad [35]$$

where $T(z)$, $N(z)$ and $L(z)$ are the coeffcients of Q_0, P_0 and P_M respectively in the numerator of [34] and $K(z)$ is the denominator of [34]. [35] is a polynomial in z and exists for all values of z, including three zeros of the denominator. Hence Q_0, P_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the zeros b_1, b_2 and b_3 (say) the denominator (at each of which the numerator must vanish). Now, three equations with the constants Q_0, P_0 and P_M are

$$T(b_1)Q_0 + N(b_1)P_0 + L(b_1)P_M = -M(b_1) \qquad [36]$$

$$T(b_2)Q_0 + N(b_2)P_0 + L(b_2)P_M = -M(b_2 \qquad [37]$$

$$T(b_3)Q_0 + N(b_3)P_0 + L(b_3)P_M = -M(b_3) \qquad [38]$$

we Know that,

$AX = B$

$$\begin{bmatrix} T(b_1) & N(b_1) & L(b_1) \\ T(b_2) & N(b_2) & L(b_2) \\ T(b_3) & N(b_3) & L(b_3) \\ & & \end{bmatrix} \begin{bmatrix} Q_0 \\ P_0 \\ P_N \\ \end{bmatrix} = \begin{bmatrix} -M(b_1) \\ -M(b_2) \\ -M(b_3) \\ \end{bmatrix}$$

then,

$$A^{-1} = \frac{AdjA}{\|A\|} = \frac{1}{\|A\|}\begin{bmatrix} A_{11} & -A_{21} & A_{31} \\ -A_{12} & A_{22} & A_{32} \\ A_{13} & -A_{23} & A_{33} \end{bmatrix}$$

After some calculation, we have

$$Q_0 = \frac{-A_{11}M(b_1) + A_{21}M(b_2) - A_{31}M(b_3)}{A}$$

$$P_0 = \frac{A_{12}M(b_1) - A_{22}M(b_2) + A_{32}M(b_3)}{A}$$

$$P_M = \frac{A_{13}M(b_1) + A_{23}M(b_2) - A_{33}M(b_3)}{A}$$

where,

$$A = \begin{Vmatrix} T(b_1) & N(b_1) & L(b_1) \\ T(b_2) & N(b_2) & L(b_2) \\ T(b_3) & N(b_3) & L(b_3) \end{Vmatrix}$$

A_{ij} is the co-factor of the $(i, j)^{th}$ element of A.

By putting the values of Q_0, P_0 and P_M in equation (4.35), we have

$$\begin{aligned} R(z) = \{&T(z)[-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}] + N(z)[M(b_1)A_{12} \\ &- M(b_2)A_{22} + M(b_3)A_{32}] + L(z)[-M(b_1)A_{13} + M(b_2)A_{23} \\ &- M(b_3)A_{33}] + A.M(z)\}/A.K(z) \end{aligned} \quad [39]$$

4.1 Mean Queue Length

Define,

L_q =Expected number of customers in the queue excluding the one in service.

then

$$L_q = [R'(z)]_{z=1}$$

Therefore from equcation [39] we have,

$$L_q = K(1)[T'(1)\{-M(b_1)A_{11} + M(b_2)A_{21} - M(b_3)A_{31}\} + N'(1)[M(b_1)A_{12}$$

$$-M(b_2)A_{22}+M(b_3)A_{32}]+L'(1)[-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]$$

$$+AM'(1)]-T(1)[-M(b_1)A_{11}+M(b_2)A_{21}-M(b_3)A_{31}]+N(1)[M(b_1)A_{12}$$

$$-M(b_2)A_{22}+M(b_3)A_{32}]+L(1)[-M(b_1)A_{13}+M(b_2)A_{23}-M(b_3)A_{33}]$$
$$+AM(1)]AK'(1)/A[K(1)]^2 \qquad [40]$$

where dashes denotes the first derivative with respect to z.

Particular Case

Relation [32], on applying the theory of Laplace transform gives

$$r(z,s)=\frac{\mu(1-z)\overline{R}_0(s)-\lambda_1 z^{M+1}(1-z)\overline{P}_M(s)-z-\frac{\zeta z}{s}}{\lambda_1 z^2-z(s+\lambda_1+\mu_1+\zeta)+\mu}$$

$$r(z)=\frac{\mu(1-z)R_0-\lambda_1 z^{M+1}(1-z)P_M-\zeta z}{\lambda_1 z^2-z(\lambda_1+\mu_1+\zeta)+\mu} \qquad [41]$$

where

$$r(z)=\lim_{s\to 0} sr(z,s)$$

and

$$R_0=\lim_{s\to 0} s\overline{R}_0(s), P_M=\lim_{s\to 0} s\overline{P}_M(s)$$

[41] is a polynomial in z and exists for all values of z, including the two zeros of the denominator. Hence R_0 and P_M can be obtained by setting the numerator equal to zero. Substituting the two zeros a_1 and a_2 (say) of the denominator (at each of which the numerator must vanish). Equating the denominator with zero, we get two roots a_1 and a_2.

$$a_1=\frac{-(\lambda_1+\mu+\zeta)+\sqrt{(\lambda_1+\mu+\zeta)^2-4\lambda_1\mu}}{2\lambda_1}$$

$$a_2=\frac{-(\lambda_1+\mu+\zeta)-\sqrt{(\lambda_1+\mu+\zeta)^2-4\lambda_1\mu}}{2\lambda_1}$$

Putting the two zeros a_1 and a_2 (say) of the denominator (at which the numerator must vanish).

Two equation determining the constants R_0 and P_M are,

$$\mu(1-a_1)R_0-\lambda_1 a_1^{M+1}(1-a_1)P_M=\zeta a_1 \quad [42]$$

$$\mu(1-a_2)R_0-\lambda_1 a_2^{M+1}(1-a_2)P_M=\zeta a_2 \quad [43]$$

On solving these equations. we have

$$P_M=\frac{(a_1-a_2)}{a_1^{M+1}-a_2^{M+1}} \quad and \quad R_0=\frac{\zeta a_2}{(1-a_2)\mu}+\frac{\lambda_1}{\mu}a_2^{M+1}\frac{a_1-a_2}{a_2^{M+1}-a_1^{M+1}}$$

Now from [41], we have

$$r(z)=\frac{\zeta+(1-z)\lambda_1(1-a_2)P_M(a_2^M+a_2^{M-1}z+...+z^M)}{\lambda_1 a_1(1-a_2)}\sum_{i=0}^{\infty}(\frac{z}{a_1})^i \quad [44]$$

if $\zeta=0$ (i.e no catastrophe is allowed) then from equation [41] we have,

$$r(z)=\frac{(\mu R_0-\lambda_1 z^{M+1}P_M)}{(\mu-\lambda_1 z)} \quad [45]$$

The condition, $\lim_{z\to 1} r(z)=1$ gives,

$$\mu R_0-\lambda_1 P_M=\mu-\lambda_1 \quad [46]$$

As r(z) is analytic, the numerator and denominator of [45] must vanish simultaneously for $z=\mu/\lambda_1$, which is a zero of its denominator. Equating the numerator of equation [45] to zero for $z=\mu/\lambda_1$ we have,

$$R_0=\rho^{-M}P_M, \rho=\frac{\lambda_1}{\mu}<1 \quad [47]$$

Putting the value of P_M in [46] from (47), we get

$$R_0=\frac{1-\rho}{1-\rho^{M+1}}$$

so, from [47]

$$P_M=\frac{1-\rho}{1-\rho^{M+1}}\rho^M$$

from [45], we have

$$r(z)=\frac{1-\rho}{1-\rho^{M+1}}\frac{[1-(\rho z)^{M+1}}{1-\rho z}] \quad [48]$$

which is a well known result of the $M/M/1$ queue with finite waiting space M.
When there is an infinite waiting space, the corresponding expression for r(z) is obtained by letting M tends to infinity in [48], if $Max(\rho, \bmod z)<1$.

$$r(z) = \frac{1-\rho}{1-\rho z} \quad [49]$$

which is again a well-know result of the M/M/1 queue with infinite waiting space.

Concluding Remarks

In the present paper, we consider a simple finite capacity Markovian queueing system with environmental, catastrophic and restorative effects. The direct application of the model can be described to a biological phenomenon that there are many creatures such as cochroaches, ants etc. whose movement is restricted when we put up a sepray on them (catastrophes) and also with the change of temperature (environment). As the temperature drops below a critical value say T_0 , the movement (production) of such creatures becomes almost zero . On the other hand , as the temperature goes higher than T_0 , the movement becomes normal.

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