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STOCHASTIC ANALYSIS OF A PARALLEL SYSTEM WITH FAILURE OF SERVICE FACILITY

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Abstract

The reliability measures of a parallel system are analyzed stochastically by considering failure of service facility while performing jobs. There are two identical units in the system which has two modes - operative and failure. The repair activities are carried out by a single service facility (called server) as and when required. The service facility undergoes for treatment when it fails to perform the jobs. The unit and service facility work as new after rectification with same life time as original. The random variables associated with failure time of the unit and service facility, treatment time of the service facility and repair time of the unit are statistically independent. The failure time and the time by which service facility undergoes for treatment follow negative exponential distribution while the distributions for repair and treatment time are taken as arbitrary with different probability density functions. The semi-Markov process and regenerative point technique are adopted to derive the expressions for reliability measures of the system model. The results for arbitrary values of various parameters and costs are obtained to depict the behavior of some measures of system effectiveness.

Key Words: Parallel System, Failure of Service Facility, Treatment, Reliability Measures and Stochastic Analysis.

1. Introduction

There is no doubt that cold standby redundancy is one of the best methods to enhance reliability and performance of operating systems. And, therefore, over the past few years a lot of research work has been carried out by the researchers including Said et al. (2005), Xinzhuo and Lerong (2012) and Malik (2013) on reliability modeling of cold standby systems. But in many industrial systems, the technique of cold standby redundancy has not been considered as an appropriate method for improving their efficiency and so in such cases parallel redundancy can be the best option. For example, the transformers having same polarity and voltage ratio are connected in parallel in a power supply system in order to provide continuous power supply and also to meet the load requirements. Kadyan et al. (2010) and Malik and Rathee (2014) have obtained reliability measures of parallel systems with different repair policies. However, most of these models have been discussed under a common assumption that repair facility neither fails nor deteriorate while performing jobs. In-fact, this assumption seems to be unrealistic whenever service facility (called server) meets with an accident due to the reasons like mishandling of the components, sudden electric shock, lack of expertise and carelessness.

The purpose of the present study is to analyze stochastically a parallel system of two identical units with failure of service facility while performing jobs. The unit fails from normal mode. The service facility undergoes for treatment when it fails to perform the jobs so that repair of the failed may be resumed by it. The repair activities of the unit and treatment given to the service facility are perfect. The random variable associated with failure time of the unit and server, repair time of the unit and the treatment time of the server are considered as statistically independent. The distribution for failure time of the unit and service facility assumed negative exponential while that of treatment time of the service facility and repair time of the unit are taken as arbitrary with different probability functions. The system is observed at suitable regenerative epochs to derive the expressions for some important reliability measures such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server, expected number of treatments given to service facility, expected number of repairs of the unit and profit function by using semi-Markov process (Smith, 1955) and regenerative point technique (Levi, 1954). A particular case is considered to depict behavior of MTSF, availability and profit of the function the system model for different parametric values. The possible application of the present work can be visualized in the system of electric transformers where parallel operation of the components is required to ensure continuous power supply for a considerable period of time.

2. Notations

E	:	Set of regenerative states
$\bar{E}$	:	Set of non-regenerative states
λ	:	Constant failure rate of the unit
μ	:	Constant failure rate of the server
FUr /FWr	:	The unit is failed and under repair/waiting for repair
SFUt/SFUT	:	The server is failed and under treatment/ continuously under treatment from previous state
FUR/FWR	:	The unit is failed and under repair / waiting for repair continuously from previous state
g(t)/G(t)	:	pdf/cdf of repair time of the unit
f(t)/F(t)	:	pdf/cdf of treatment time of the server
q_{ij} (t)/ Q_{ij}(t)	:	pdf / cdf of passage time from regenerative state Si to a regenerative state Sj or to a failed state Sj without visiting any other regenerative state in (0, t]
$q_{ij.kr}$ (t)/$Q_{ij.kr}$(t)	:	pdf/cdf of direct transition time from regenerative state Si to a regenerative state Sj or to a failed state Sj visiting state Sk, Sr once in (0, t]
$q_{ij.k,(r,s)}^{n}(t)/Q_{ij.k,(r,s)}^{n}(t)$	:	pdf/cdf of direct transition time from

		regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times states r and s.
$M_i(t)$	:	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	:	Probability that the server is busy in the state S_i up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.
μ_i	:	The mean sojourn time in state S_i which is given by $\mu_i = E(T) = \int_0^\infty P(T > t)\,dt = \sum_j m_{ij}$, where T denotes the time to system failure.
m_{ij}	:	Contribution to mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$
Ⓢ/©/©ⁿ	:	Symbol for Laplace-Stieltjes convolution/ Laplace convolution / Laplace convolution n times
*/**	:	Symbol for Laplace Transformation /Laplace Stieltjes Transformation

The possible transition states of the system model are shown in fig.1

3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements as

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \qquad (1)$$

$$p_{10} = g^*(\lambda+\mu), \qquad p_{12} = \frac{\lambda}{\lambda+\mu}(1-g^*(\lambda+\mu)),$$

$$p_{13} = \frac{\mu}{\lambda+\mu}(1-g^*(\lambda+\mu)), \quad p_{21} = g^*(\mu),$$

$p_{24} = 1 - g^*(\mu),\quad p_{31} = f^*(\lambda),\quad p_{35} = 1 - f^*(\lambda),\quad p_{61} = g^*(\mu),$
$p_{64} = 1 - g^*(\mu),$

$$p_{01} = p_{46} = p_{56} = 1 \tag{2}$$

It can be easily verify that

$$p_{01} = p_{10} + p_{12} + p_{13} = p_{21} + p_{24} = p_{31} + p_{35} = p_{46} = p_{56} = p_{61} + p_{64} = 1$$
$$p_{10} + p_{13} + p_{11.2} + p_{11.2(46)^n} = p_{31} + p_{31.56} + p_{31.5(64)^n} = 1$$

The mean sojourn times (μ_i) is in the state S_i are

$\mu_0 = m_{01}$, $\mu_1 = m_{10} + m_{12} + m_{13}$, $\mu_2 = m_{21} + m_{24}$, $\mu_3 = m_{31} + m_{35}$

$\mu_4 = m_{46}$, $\mu_5 = m_{56}$

$\mu_6 = m_{61} + m_{64}$, $\mu_1' = m_{10} + m_{13} + m_{11.2} + m_{11.2(46)^n}$,

$$\mu_3' = m_{31} + m_{31.56} + m_{31.5(64)^n} \tag{3}$$

4. Reliability and Mean Time to System Failure (MTSF)

Let ϕ_i(t) be the cdf of first passage time from regenerative state Si to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for ϕ_i(t):

$$\phi_0(t) = Q_{01}(t)Ⓢ\phi_1(t)$$
$$\phi_1(t) = Q_{10}(t)Ⓢ\phi_0 + Q_{13}(t)Ⓢ\phi_3 + Q_{12}(t)$$
$$\phi_3(t) = Q_{31}(t)Ⓢ\phi_1(t) + Q_{35}(t) \tag{4}$$

Taking LST of above relation (4) and solving for Φ_0^{**}(s), we have

$$R^*(s) = \frac{1 - \phi^{**}(s)}{s} \tag{5}$$

The reliability of the system model can be obtained by taking Inverse Laplace transform of (5).

The mean time to system failure (MTSF) is given by

$$MTSF = \lim_{s \to 0} \frac{1 - \phi^{**}(s)}{s} = \frac{N}{D} \tag{6}$$

Where

$$N = (1 - p_{13} + p_{13}p_{35})\mu_0 + \mu_1 + p_{13}\,\mu_3 \text{ and } D = 1 - p_{13}p_{31} - p_{10} \tag{7}$$

5. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant ‘t’ given that the system entered regenerative state S_i at t = 0.The recursive relations for $A_i(t)$ are given as:

$$A_0(t) = M_0(\mathrm{t}) + \mathrm{q}_{01}(t)©A_1(t)$$

$$A_1(t) = M_1(\mathrm{t}) + \mathrm{q}_{10}(t)©A_0(t) + \{q_{11.2}(t) + q_{11.2(46)^n}(t)\}©A_1(t) + \mathrm{q}_{13}(t)©A_3(t)$$

$$A_3(t) = M_3(\mathrm{t}) + \{\mathrm{q}_{31}(t) + q_{31.56}(t) + q_{31.5(64)^n}(t)\}©A_1(t) \tag{8}$$

where $M_0(t) = e^{-2\lambda t}$, $M_1(t) = e^{-(\mu+\lambda)t}\overline{G(t)}$, $M_3(t) = e^{-\lambda t}\overline{F(t)}$ (9)

Taking LT of above relations (8) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \tag{10}$$

Where

$$N_1 = p_{10}\mu_0 + \mu_1 + p_{13}\mu_3\} \ ; \ D_1 = \mathrm{p}_{10}\,\mu_0 + \mu_1^{'} + p_{13}\mu_3^{'} \tag{11}$$

6. Busy Period Analysis for Server Due to Repair

Let $B_i(t)$ be the probability that the server is busy in repair the unit at an instant‘t’ given that the system entered regenerative state Si at t=0.The recursive relations for $B_i(t)$ are as follows:

$$B_0(t) = \mathrm{q}_{01}(t)©B_1(t)$$

$$B_1(t) = W_1(\mathrm{t}) + \mathrm{q}_{10}(t)©B_0(t) + \{q_{11.2}(t) + q_{11.2(46)^n}(t)\}©B_1(t) + \mathrm{q}_{13}(t)©B_3(t)$$

$$B_3(t) = \{\mathrm{q}_{31}(t) + q_{31.56}(t) + q_{31.5(64)^n}(t)\}©B_1(t) \tag{12}$$

Where $W_1(t) = e^{-(\lambda+\mu)t}\overline{G(t)} + (\lambda e^{-(\lambda+\mu)t}©1)\overline{G(t)}$ (13)

Taking LT of above relations (12) and solving for $B_0^*(s)$ .The time for which server is busy due to repair is given by

$$B_0(\infty) = \lim_{s\to 0} sB_0^*(\mathrm{s}) = \frac{N_2}{D_1} \tag{14}$$

Where $N_2 = W_1^*(0)$ and D_1 is already mentioned. (15)

7. Expected Number of Repairs Done by the Server

Let $R_i(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state Si at t = 0. The recursive relations for $R_i(t)$ are given as:

$$R_0(t) = Q_{01}(t)Ⓢ R_1(t)$$

$$R_1(t) = Q_{10}(t) Ⓢ [1 + R_0(t)] + \{Q_{11.2}(t) + Q_{11.2(46)^n}(t)\} Ⓢ [1 + R_1(t)] + Q_{13}(t) Ⓢ R_3(t)$$

$$R_3(t) = Q_{31}(t) Ⓢ R_1(t) + \{q_{31.56}(t) + q_{31.5(64)^n}(t)\} Ⓢ [1 + R_1(t)] \qquad (16)$$

Taking LST of above relations (16) and solving for $R_0^{**}(s)$.The expected number of repairs per unit time by the server is giving by

$$R_0(\infty) = \lim_{s \to 0} sR_0^{**}(s) = \frac{N_3}{D_1} \qquad (17)$$

Where $N_3 = 1 - P_{24}P_{42} + P_{13}P_{20}$ and D_1 is already mentioned. (18)

8. Expected Number of Treatments Given to the Server

Let $T_i(t)$ be the expected number of treatments given to the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $T_i(t)$ are given as:

$$T_0(t) = Q_{01}(t) Ⓢ T_1(t)$$

$$T_1(t) = Q_{10}(t) Ⓢ T_0(t) + \{Q_{11.2}(t) + Q_{11.2(46)^n}(t)\} Ⓢ T_1(t) + Q_{13}(t) Ⓢ T_3(t)$$

$$T_3(t) = \{Q_{31}(t) + q_{31.56}(t) + q_{31.5(64)^n}(t)\} Ⓢ [1 + T_1(t)] \qquad (19)$$

Taking LST of above relations (19) and solving for $T_0^{**}(s)$ .The expected number of treatments given to the server per unit time is giving by

$$T_0(\infty) = \lim_{s \to 0} sT_0^{**}(s) = \frac{N_4}{D_1} \qquad (20)$$

Where $N_4 = (P_{31} + P_{31.56} + P_{31.5(64)^n})P_{01}P_{13}$

and D_1 is already mentioned. (21)

9. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0 - K_2R_0 - K_3T_0 \qquad (22)$$

Where

P = Profit of the system model

K_0 = Revenue per unit up-time of the system

K_1 = Cost per unit time for which server is busy due to repair

K_2 = Cost per repair of the failed unit per unit time

K_3 = Cost per treatment of the service facility per unit time

10. Particular Case

Suppose $f(t) = \beta e^{-\beta t}$ and $g(t) = \alpha e^{-\alpha t}$

We can obtain the following results:

$$p_{10}=\frac{\alpha}{\alpha+\lambda+\mu},\quad p_{12}=\frac{\lambda}{\alpha+\lambda+\mu},\quad p_{13}=\frac{\mu}{\alpha+\lambda+\mu},\quad p_{21}=\frac{\alpha}{\alpha+\mu},$$

$$p_{24}=\frac{\mu}{\alpha+\mu}$$

$$p_{31}=\frac{\beta}{\beta+\lambda},\quad p_{35}=\frac{\lambda}{\beta+\lambda},\quad p_{61}=\frac{\alpha}{\alpha+\mu},\quad p_{64}=\frac{\mu}{\alpha+\mu}$$

and $\mu_0=\dfrac{1}{2\lambda}$, $\mu_1=\dfrac{1}{\alpha+\lambda+\mu}$, $\mu_2=\dfrac{1}{\alpha+\mu}$, $\mu_3=\dfrac{1}{\beta+\lambda}$, $\mu_4=\mu_5=\dfrac{1}{\beta}$,

$$\mu_6=\frac{1}{\alpha+\mu}$$

$$\mu_1'=\frac{\alpha\beta+\lambda\mu+\lambda\beta}{\alpha\beta(\alpha+\lambda+\mu)},\quad \mu_3'=\frac{\alpha(\beta+\lambda)+\lambda(\beta+\mu)}{\alpha\beta(\beta+\lambda)}$$

Also

$$N=3\lambda\mu+(3\lambda+\alpha)(\lambda+\beta)$$

$$D=2\lambda^2(\beta+\lambda+\mu)$$

$$D_1=\alpha(\beta+\lambda)\{2\lambda\mu+\beta(\alpha+2\lambda)\}+2\lambda^2(\beta+\mu)(\beta+\lambda+\mu)$$

$$N_1=\alpha\beta\{2\lambda\mu+(\alpha+2\lambda)(\beta+\lambda)$$

$$N_2=2\beta\lambda(\beta+\lambda)(\lambda+\alpha)$$

$$N_3=2\lambda\beta\alpha\{\mu\lambda+(\beta+\lambda)(\lambda+\alpha)\},\quad N_4=2\lambda\mu\beta\alpha(\beta+\lambda)$$

11. Conclusion

The results for some important reliability measures including mean time to system failure (MTSF), availability and profit function are obtained for arbitrary values of various parameters and costs by assuming negative exponential distribution for the random variables associated with repair time, treatment time and arrival time of the server. The behavior of these measures is shown respectively in tables 1, 2 and 3. It observed that MTSF, availability and profit go on decreasing with the increase of failure rates of the unit and server (λ and μ). However, they keep on moving up with the increase of repair rate (α), treatment rate (β) of the server. Hence, the study reveals that a parallel system of two identical units can be made more available for use either by increasing the treatment rate of the service facility or by increasing the repair rate of the failed unit.

12. Numerical Presentation of Reliability Measures

Server Treatment Rate(β)	λ=0.1,α=2.1,μ=0.03	λ=0.11	α=2.3	μ=0.04
1.1	116.2195122	97.10411	125.9756	115.4032
1.2	116.5037594	97.35105	126.2782	115.7463
1.3	116.7482517	97.56371	126.5385	116.0417
1.4	116.9607843	97.74874	126.7647	116.2987
1.5	117.1472393	97.91121	126.9632	116.5244
1.6	117.3121387	98.05500	127.1387	116.7241
1.7	117.4590164	98.18317	127.2951	116.9022
1.8	117.5906736	98.29812	127.4352	117.0619
1.9	117.7093596	98.4018	127.5616	117.2059
2.0	117.8169014	98.49579	127.6761	117.3364

Table 1: MTSF Vs Server Treatment Rate

Server Treatment Rate (β)	λ=0.1,α=2.1,μ=0.03	λ=0.11	α=2.3	μ=0.04
1.1	0.995472787	0.994578	0.996178	0.995338
1.2	0.995519718	0.994634	0.99622	0.995400
1.3	0.99555768	0.994679	0.996254	0.995451
1.4	0.995588917	0.994716	0.996282	0.995493
1.5	0.995615003	0.994747	0.996305	0.995527
1.6	0.995637066	0.994773	0.996325	0.995557
1.7	0.995655935	0.994795	0.996341	0.995582
1.8	0.995672231	0.994815	0.996356	0.995604
1.9	0.995686428	0.994831	0.996368	0.995623
2.0	0.995698894	0.994846	0.996379	0.995639

Table 2: Availability Vs Server Treatment Rate

Server Treatment Rate(β)	λ=0.1,α=2.1,μ=0.03	λ=0.11	α=2.3	μ=0.04
1.1	14277.5538	14202.39	14309.14	14275.08
1.2	14278.1509	14203.09	14309.67	14275.88
1.3	14278.6302	14203.66	14310.10	14276.52
1.4	14279.0217	14204.12	14310.45	14277.04
1.5	14279.3464	14204.50	14310.74	14277.47
1.6	14279.6192	14204.82	14310.99	14277.84
1.7	14279.8510	14205.10	14311.19	14278.15
1.8	14280.0500	14205.33	14311.37	14278.41
1.9	14280.2223	14205.54	14311.52	14278.64
2.0	14280.3728	14205.72	14311.66	14278.84

Table 3: Profit Vs Server Treatment Rate

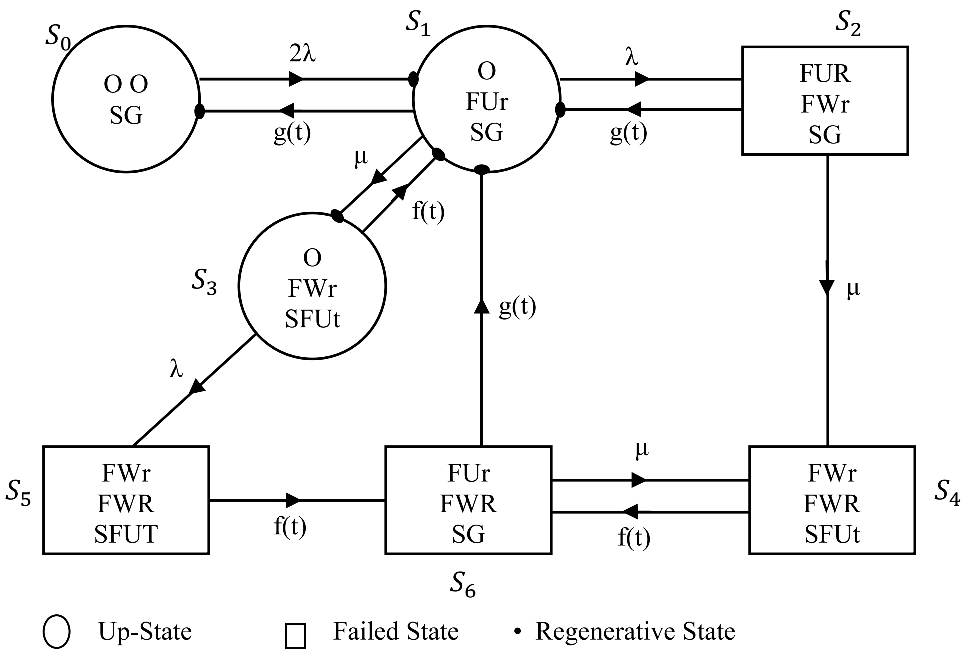

Fig. 1: State Transition Diagram

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