Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 8, Issue 1 (2015): 39-50

# EXACT SAMPLING DISTRIBUTION OF SAMPLE COEFFICIENT OF VARIATION

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Received July 02, 2014
Modified May 05, 2015
Accepted May 18, 2015

## Abstract

This paper proposes the sampling distribution of sample coefficient of variation from the normal population. We have derived the relationship between the sample coefficient of variation, standard normal and chi-square variate. We have derived density function of the sample coefficient of variation in terms of the confluent hyper-geometric distribution. Moreover, the first two moments of the distribution are derived and we have proved that the sample coefficient of variation (*cv*) is the biased estimator of the population coefficient of variation (*CV*). Moreover, the shape of the density function of sample co-efficient of variation is also visualized and the critical points of sample (*cv*) at 5% and 1% level of significance for different sample sizes have also been computed.

**Key Words:** Sample Coefficient of Variation, Sampling Distribution, Standard Normal Variate, Chi-Square Variate , Hyper-Geometric Distribution, Moments.

## 1. Introduction and related work

The coefficient of variation is the widely used measures of dispersion especially quantifying the consistency of random variable and its distribution. Hendricks and Robey (1936) made an attempt to extent its use in biometry by proposing a sampling distribution. Sharma and Krishna (1994) developed the asymptotic sampling distribution of the inverse of the coefficient of variation where the distribution is used for making statistical inference about the population CV or Inverse CV without making an assumption about the population distribution. Hurlimann (1995) proposed a uniform approximation to the sampling distribution of the coefficient of variation proposed by Hendricks and Robey (1936). Bedeian and Mossholder (2000) used coefficient of variation as measure of diversity to statistical measure commonly used for comparing diversity in work groups. Reef et.al (2002) derived the mathematical relationship between the coefficient of variation associated with repeated measurements from quantitative assays and the expected fraction of pairs of those measurements that differ by at least some given factor. Albrecher et.al, (2009) proposed an asymptotic of the sample coefficient of variation and the sample dispersion which are examples of widely used measures of variation. Castagliola et.al (2015) monitored the CV using a variable sample size control chart. In this paper we have extended the work of Hendricks and Robey (1936) where they proposed the distribution of sample *'cv'* from the normal population had a constraint of using odd and even sample size. The authors believed that the proposed distribution is the extension and alternative to the Henricks and Robbey's (1936) work for testing the significance of sample *cv* from the normal population irrespective of any sample size and proposed the sampling distribution of sample coefficient of variation from the normal population and discussed it's properties with numerical example in the subsequent sections.

## 2. Relationship of sample co-efficient of variation with standard normal, chi-square variates

If $X \sim (\mu, \sigma^2)$ then the population co-efficient of variation *(Cx)* is given as

$$C_x = \frac{\sigma}{\mu} \qquad \text{-- (1)}$$

The population *Cx is* used to know the consistency of the random variables and it may called as inverse signal to noise ratio. If the $C_x < 1$, then the distribution of the random variable is said to be low-variance distribution and if $C_x > 1$ then it is said to be high-variance distribution. In order to give the inference about the population *Cx*, the calculation of sample coefficient of variation is inevitable. The sample coefficient of variation *($c_x$)* of a random variable from the normal population is by

$$c_x = \frac{s}{\bar{x}} \qquad \text{-- (2)}$$

Where $s$ and $\bar{x}$ are the estimates of the population standard deviation and mean respectively. Based on the sample coefficient of variation ($c_x$) we have derived the relationship between the $c_x$, standard normal variate $(z)$ and chi-square $(\chi^2)$ variate. The inverse of the sample coefficient of variation $c_x$ can be written as

$$\frac{1}{c_x} = \frac{\bar{x}}{s} \qquad \text{-- (3)}$$

We can rewrite (3) in terms of the population mean $\mu$ and $\sigma / \sqrt{n}$

$$\frac{1}{c_x} = \frac{\bar{x} - \mu}{s} + \frac{\mu}{s} \qquad \text{-- (4)}$$

$$\frac{1}{c_x} = \frac{\sqrt{n}\left(\bar{x} - \mu\right)/\sigma}{\sqrt{ns^2 / \sigma^2}} + \frac{\sqrt{n} /(\sigma / \mu)}{\sqrt{ns^2 / \sigma^2}} \qquad \text{-- (5)}$$

Based on the central limit theorem, $\sqrt{n}\left(\bar{x} - \mu\right) / \sigma$ follows standard normal distribution (*z)* with mean 0 and variance 1, the term $ns^2 / \sigma^2$ follows chi-square distribution $\chi^2_{n-1}$ with n-1 degrees of freedom and $\sigma / \mu$ is equal to the population (*$C_x$)*.Without loss of generality we can rewrite (5) in terms of standard normal variate (z), chi-square variate $\chi^2_{n-1}$ and population (*$C_x$)* as given below

$$\frac{1}{c_x} = \frac{z + \left(\sqrt{n} / C_x\right)}{\sqrt{\chi^2_{n-1}}} \qquad \text{-- (6)}$$

$$c_x = \frac{\sqrt{\chi^2_{n-1}}}{z + \left(\sqrt{n} / C_x\right)} \quad \text{-- (7)}$$

$$c_x = \frac{C_x \sqrt{\chi^2_{n-1}}}{z(C_x) + \sqrt{n}} \quad \text{-- (8)}$$

Finally from (7) we have derived the expression for the sample coefficient of variation ($c_x$) which lies $0 \le c_x \le \infty$ and it is defined as the ratio of the square root of independent chi-square variate with $n$-1 degrees of freedom divided by the standard normal variate (z) plus the quantity $\sqrt{n} / C_x$. Based on the identified relationship from (8) we have derived the sampling distribution of the sample co-efficient of variation and it is discussed in the next section.

## 3. Sampling distribution of sample co-efficient of variation

Using the technique of two-dimensional Jacobian of transformation, the joint probability density function of the standard normal variate and the chi-square variate with $n$-1 degrees of freedom was transformed into density function of sample coefficient of variation *($c_x$)* and it is given by

$$f(c_x, u) = f\left(z, \chi^2_{n-1}\right) |J| \quad \text{-- (9)}$$

From (8) we know that z and $\chi^2_{n-1}$ are independent then (9) can be written as

$$f(c_x, u) = f(z) f(\chi^2_{n-1}) |J| \quad \text{-- (10)}$$

Using the change of variable technique, substitute $z = u$ in (8) we get $\chi^2_{n-1} = (c_x / C_x)^2 \left(u(C_x) + \sqrt{n}\right)^2$.Then partially differentiate the above substitution, Calculate the Jacobian determinant and rewrite (10) as

$$f(c_x, u) = f(z) f(\chi^2_{n-1}) \left| \frac{\partial\left(z, \chi^2_{n-1}\right)}{\partial(c_x, u)} \right| \quad \text{-- (11)}$$

$$f(c_x, u) = f(z) f(\chi^2_{n-1}) \begin{vmatrix} \dfrac{\partial z}{\partial(c_x)} & \dfrac{\partial z}{\partial u} \\ \dfrac{\partial \chi^2_{n-1}}{\partial(c_x)} & \dfrac{\partial \chi^2_{n-1}}{\partial u} \end{vmatrix} \quad \text{-- (12)}$$

From (12) we know standard normal variate ($z$) and chi-square variate $\chi^2_{n-1}$ are independent, hence the density function of the joint distribution of $z$ and $\chi^2_{n-1}$ can be written as

$$f\left(z, \chi^2_{n-1}\right) = f(z) f\left(\chi^2_{n-1}\right)$$

$$f\left(z, \chi^2_{n-1}\right) = \left(\frac{1}{\sqrt{2\pi}} e^{-Z^2/2}\right) \left(\frac{(1/2)^{(n-1)/2}}{\Gamma((n-1)/2)} \left(\chi^2_{n-1}\right)^{((n-1)/2)-1} e^{-\chi^2_{n-1}/2}\right) \quad \text{-- (13)}$$

and

$$\begin{vmatrix} \dfrac{\partial z}{\partial (c_x)} & \dfrac{\partial z}{\partial u} \\ \dfrac{\partial \chi^2{}_{n-1}}{\partial (c_x)} & \dfrac{\partial \chi^2{}_{n-1}}{\partial u} \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 2c_x\left(\dfrac{1}{C_x}\right)^2 \left(u(C_x)+\sqrt{n}\right)^2 & \dfrac{2(c_x)^2}{C_x}\left(u(C_x)+\sqrt{n}\right) \end{vmatrix}$$

-- (14)

Then replace (13) and (14) in (12) in terms of the substitution *(u)*, we get the joint distribution of sample *($c_x$)* and *u* as

$$f(c_x,u)=\left(\frac{1}{\sqrt{2\pi}}e^{-u^2/2}\right)\left(\frac{(1/2)^{(n-1)/2}}{\Gamma((n-1)/2)}\left(\frac{c_x}{C_x}\right)^2\left(\left(u(C_x)+\sqrt{n}\right)^2\right)^{((n-1)/2)-1} e^{-\left(\frac{1}{2}\left(\frac{c_x}{C_x}\right)^2\right)\left(u(C_x)+\sqrt{n}\right)^2}\right)$$

$$\times\left(2c_x\left(\frac{1}{C_x}\right)^2\left(u(C_x)+\sqrt{n}\right)^2\right) \quad \text{-- (15)}$$

where $0 \le c_x \le \infty, -\infty \le u \le +\infty$

Rearranging (15) and integrating with respect to *u,* we get the marginal distribution of *u* as given by

$$f(c_x)=\frac{2(1/2)^{(n-1)/2}(c_x)^{n-2}\, e^{-\frac{n}{2(C_x)^2}\left(\frac{(c_x)^2}{(1+(c_x)^2}\right)}}{(C_x)^{n-1}\Gamma((n-1)/2)\sqrt{2\pi}}$$

$$\times\int_{-\infty}^{+\infty}\left(u(C_x)+\sqrt{n}\right)^{n-1} e^{-\frac{1}{2}\left(u\sqrt{1+(c_x)^2}+\frac{(c_x)^2\sqrt{n}}{C_x\sqrt{1+(c_x)^2}}\right)^2} du$$

$$f(c_x)=\frac{2(n/2)^{(n-1)/2}}{(C_x)^{n-1}\Gamma((n-1)/2)}(c_x)^{n-2}\left(1+(c_x)^2\right)^{-(n-1/2)} e^{-\frac{n}{2(C_x)^2}\left(\frac{(c_x)^2}{1+(c_x)^2}\right)}$$

$$\times{}_1F_1\left(\frac{-n}{2}+1;\frac{-n+1}{2};\frac{2(C_x)^2\left(1+(c_x)^2\right)}{n}\right) \quad \text{-- (16)}$$

where $0 \le c_x \le \infty$ , $C_x > 0$ , $n > 1$

From (16) it is the density function of sample coefficient of variation ($c_x$) from the normal population and it involves $\Gamma((n-1)/2)$ and ${}_1F_1\left(\frac{-n}{2}+1;\frac{-n+1}{2};\frac{2(C_x)^2\left(1+(c_x)^2\right)}{n}\right)$ are the Gamma function and confluent hyper-geometric function respectively with two parameters ( $n, C_x$ ), where $n$ is the sample size and $C_x$ is the population co-efficient of variation. Moreover, the first two moments of the distribution of sample coefficient of variation *($c_x$)* in terms of mean and variance are given as.

$$E(c_x)=\int_0^{\infty}(c_x)f(c_x)d(c_x)$$

$$E(c_x)=\int_0^{\infty}(c_x)\frac{2(n/2)^{(n-1)/2}}{(C_x)^{n-1}\Gamma((n-1)/2)}(c_x)^{n-2}\left(1+(c_x)^2\right)^{-(n-1/2)}e^{-\frac{n}{2(C_x)^2}\left(\frac{(c_x)^2}{1+(c_x)^2}\right)}$$
$$\times{}_1F_1\left(\frac{-n}{2}+1;\frac{-n+1}{2};\frac{2(C_x)^2\left(1+(c_x)^2\right)}{n}\right)d(c_x) \quad \text{-- (20)}$$

$$E(c_x)=\frac{\sqrt{2n}\,\Gamma((n+1)/2)}{C_x\Gamma(n/2)}\left(1+\frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty}\left(C_x/\sqrt{n}\right)^{2k}2^k\,\Gamma(k+1/2)\right)$$
$$\text{-- (21)}$$

$$E(c_x)^2=\int_0^{\infty}(c_x)^2\frac{2(n/2)^{(n-1)/2}}{(C_x)^{n-1}\Gamma((n-1)/2)}(c_x)^{n-2}\left(1+(c_x)^2\right)^{-(n-1/2)}e^{-\frac{n}{2(C_x)^2}\left(\frac{(c_v)^2}{1+(c_v)^2}\right)}$$
$$\times{}_1F_1\left(\frac{-n}{2}+1;\frac{-n+1}{2};\frac{2(C_x)^2\left(1+(c_x)^2\right)}{n}\right)d(c_x)$$

$$E(c_x)^2=\left(n^2/C_x\right)\left(1+\frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty}(2k+1)\left(C_x/\sqrt{n}\right)^{2k}2^k\,\Gamma(k+1/2)\right)$$
$$\text{-- (22)}$$

We know that

$$V(c_x)=E(c_x)^2-\left(E(c_x)\right)^2 \quad \text{-- (23)}$$

Substitute (21) and (22) in (23) we get

$$V(c_x)=\left(n^2/C_x\right)\left(1+\frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty}(2k+1)\left(C_x/\sqrt{n}\right)^{2k}2^k\,\Gamma(k+1/2)\right)$$
$$-\left(\frac{\sqrt{2n}\,\Gamma((n+1)/2)}{C_x\Gamma(n/2)}\left(1+\frac{1}{\sqrt{\pi}}\sum_{k=1}^{\infty}\left(C_x/\sqrt{n}\right)^{2k}2^k\,\Gamma(k+1/2)\right)\right)^2$$

From (21) it is clear that the sample coefficient of variation (*cv*) is the biased estimator of population co-efficient of variation ($C_x$) because $E(c_x)\neq C_x$. The following simulation graphs shows the shape of the density function of the distribution of sample coefficient of variation ($c_x$) for different values of $(n, C_x)$.

$(n,C_x)=(2,1)$ $(n,C_x)=(3,1)$

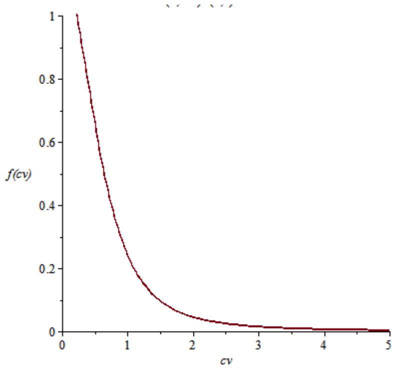

fig.1

$(n,C_x)=(4,1)$

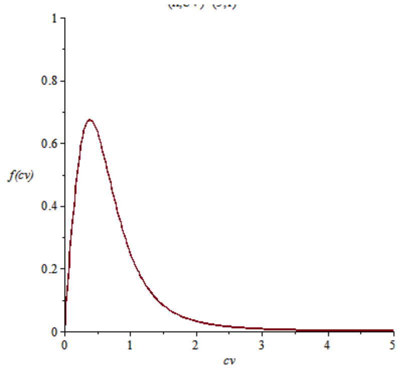

fig.2

$(n,C_x)=(5,1)$

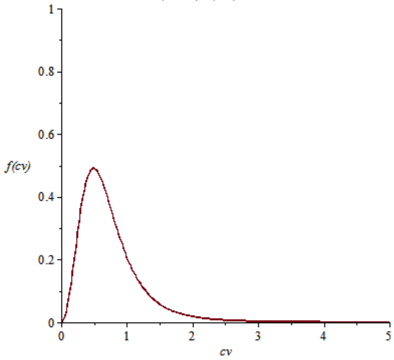

fig.3

$(n,C_x)=(6,1)$

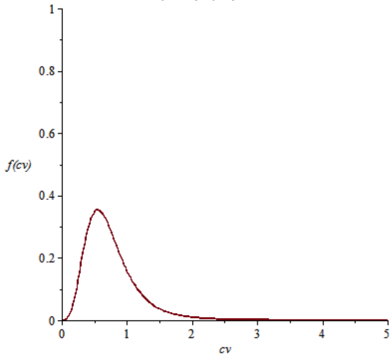

fig.4

$(n,C_x)=(7,1)$

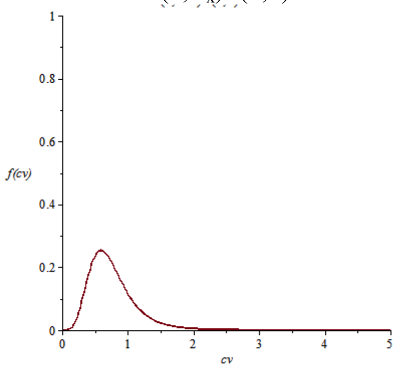

fig.5

$(n,C_x)=(8,1)$

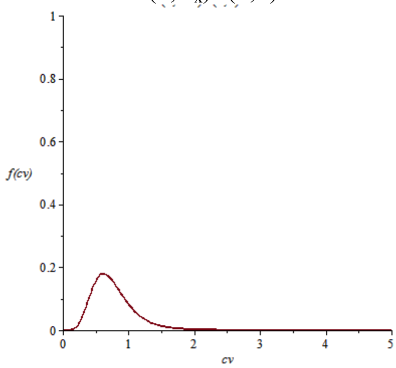

fig.6

$(n,C_x)=(9,1)$

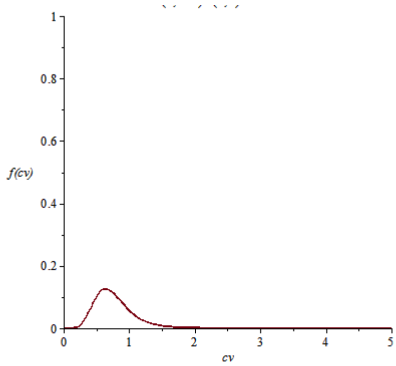

fig.7

(n,$C_x$)=(10,1)

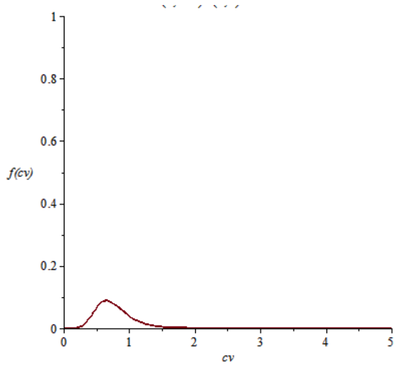

fig.8

(n,$C_x$)=(2,2)

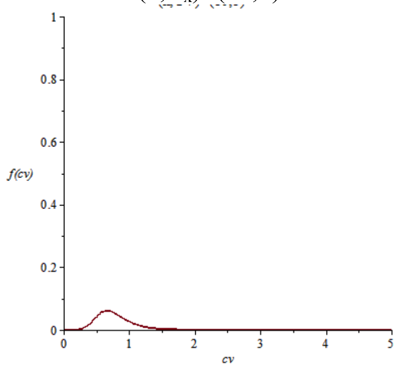

fig.9

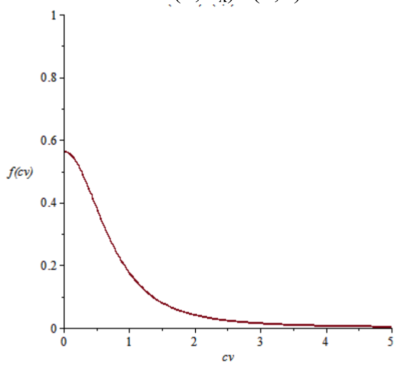

fig.10

(n,$C_x$)=(3,2)

(n,$C_x$)=(4,2)

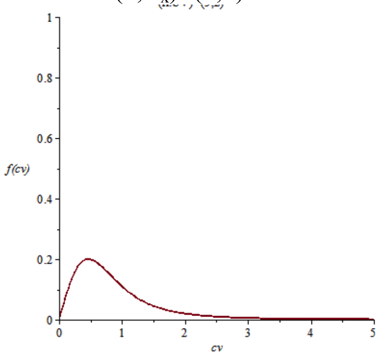

fig.11

(n,$C_x$)=(5,2)

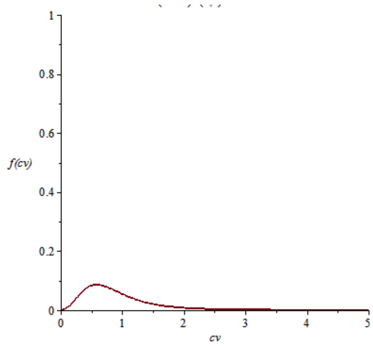

fig.12

(n,$C_x$)=(2,3)

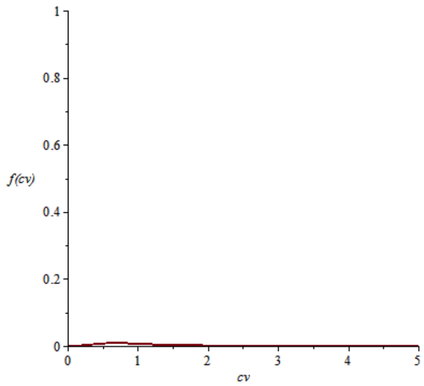


fig 13

$(n,C_x)=(3,3)$

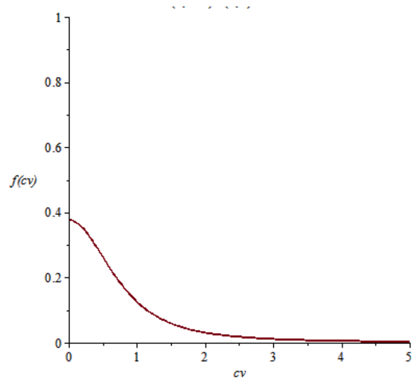


fig 14

$(n,C_x)=(4,3)$

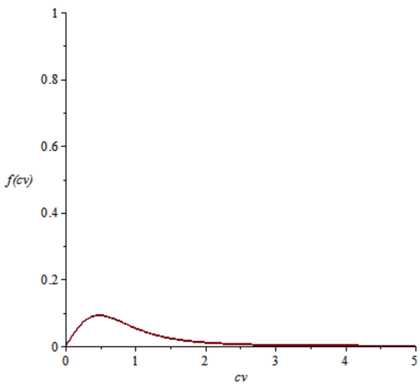


fig 15

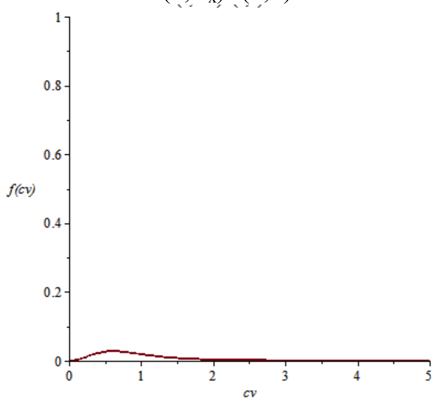


fig 16

$(n,C_x)=(5,3)$

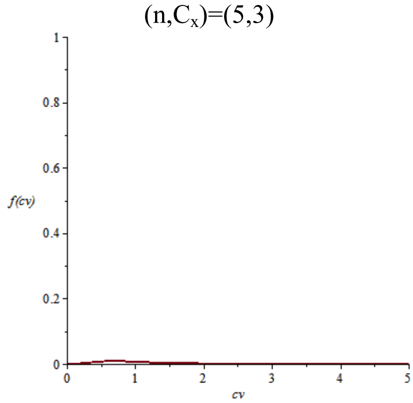


fig 17

Moreover the critical points of the sample coefficient of variation by using the relations in (8) for different values of $(n, C_x)$ for varying sample size and the significance probability is given as $p\left(c_x > c_{xn,C_x}(\alpha)\right) = \alpha$ and the critical points are visualized in Tables 1 and 2.

| ***n*** | $C_x$ | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| 2 | .5809 | .73 | .81 | .85 | .87 |
| 3 | .6630 | .87 | .96 | 1.02 | 1.06 |
| 4 | .7059 | .94 | 1.06 | 1.14 | 1.18 |
| 5 | .7341 | 1.00 | 1.14 | 1.22 | 1.28 |
| 6 | .7546 | 1.04 | 1.20 | 1.29 | 1.36 |
| 7 | .7704 | 1.08 | 1.25 | 1.35 | 1.43 |
| 8 | .7833 | 1.11 | 1.29 | 1.41 | 1.48 |
| 9 | .7939 | 1.14 | 1.33 | 1.45 | 1.54 |
| 10 | .8030 | 1.16 | 1.36 | 1.50 | 1.59 |
| 11 | .8109 | 1.18 | 1.40 | 1.53 | 1.63 |
| 12 | .8178 | 1.20 | 1.42 | 1.57 | 1.67 |
| 13 | .8239 | 1.22 | 1.45 | 1.60 | 1.71 |
| 14 | .8294 | 1.23 | 1.47 | 1.63 | 1.75 |
| 15 | .8343 | 1.25 | 1.50 | 1.66 | 1.78 |
| 16 | .8389 | 1.26 | 1.52 | 1.69 | 1.81 |
| 17 | .8430 | 1.28 | 1.54 | 1.71 | 1.84 |
| 18 | .8468 | 1.29 | 1.56 | 1.74 | 1.87 |
| 19 | .8503 | 1.30 | 1.57 | 1.76 | 1.90 |
| 20 | .8536 | 1.31 | 1.59 | 1.78 | 1.92 |
| 21 | .8566 | 1.32 | 1.61 | 1.80 | 1.95 |
| 22 | .8595 | 1.33 | 1.62 | 1.82 | 1.97 |
| 23 | .8621 | 1.34 | 1.64 | 1.84 | 2.00 |
| 24 | .8647 | 1.34 | 1.65 | 1.86 | 2.02 |
| 25 | .8670 | 1.35 | 1.66 | 1.88 | 2.04 |
| 26 | .8693 | 1.36 | 1.68 | 1.90 | 2.06 |
| 27 | .8714 | 1.37 | 1.69 | 1.91 | 2.08 |
| 28 | .8734 | 1.38 | 1.70 | 1.93 | 2.10 |
| 29 | .8753 | 1.38 | 1.71 | 1.94 | 2.12 |
| 30 | .8772 | 1.39 | 1.72 | 1.96 | 2.14 |
| 40 | .8929 | 1.45 | 1.82 | 2.10 | 2.30 |
| 50 | .9027 | 1.49 | 1.89 | 2.19 | 2.42 |
| 60 | .9102 | 1.52 | 1.95 | 2.27 | 2.52 |
| 80 | .9209 | 1.56 | 2.03 | 2.40 | 2.68 |
| 100 | .9285 | 1.60 | 2.10 | 2.49 | 2.81 |
| 120 | .9341 | 1.62 | 2.15 | 2.57 | 2.91 |
| 140 | .9386 | 1.64 | 2.19 | 2.63 | 3.00 |
| 160 | .9423 | 1.66 | 2.23 | 2.69 | 3.07 |
| 180 | .9453 | 1.68 | 2.26 | 2.74 | 3.13 |
| 200 | .9479 | 1.69 | 2.29 | 2.78 | 3.19 |

**Table 1: Significant two-tail percentage points of sample co-efficient of variation ($c_x$) at 5% significance level** $p\left(c_x > c_{xn,C_x}(0.05)\right) = 0.05$

| **n** | $C_x$ | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| 2 | .645 | .784 | .844 | .878 | .900 |
| 3 | .704 | .881 | .961 | 1.007 | 1.037 |
| 4 | .735 | .941 | 1.037 | 1.094 | 1.130 |
| 5 | .757 | .985 | 1.096 | 1.161 | 1.204 |
| 6 | .772 | 1.021 | 1.144 | 1.217 | 1.265 |
| 7 | .785 | 1.051 | 1.184 | 1.265 | 1.319 |
| 8 | .795 | 1.076 | 1.220 | 1.308 | 1.366 |
| 9 | .803 | 1.099 | 1.252 | 1.346 | 1.410 |
| 10 | .811 | 1.119 | 1.281 | 1.381 | 1.449 |
| 11 | .817 | 1.137 | 1.307 | 1.413 | 1.485 |
| 12 | .823 | 1.153 | 1.331 | 1.443 | 1.519 |
| 13 | .828 | 1.168 | 1.354 | 1.471 | 1.551 |
| 14 | .832 | 1.182 | 1.375 | 1.497 | 1.581 |
| 15 | .837 | 1.195 | 1.395 | 1.521 | 1.609 |
| 16 | .840 | 1.207 | 1.413 | 1.545 | 1.636 |
| 17 | .844 | 1.219 | 1.431 | 1.567 | 1.662 |
| 18 | .847 | 1.229 | 1.447 | 1.588 | 1.686 |
| 19 | .850 | 1.240 | 1.463 | 1.608 | 1.709 |
| 20 | .853 | 1.249 | 1.478 | 1.627 | 1.731 |
| 21 | .856 | 1.258 | 1.492 | 1.645 | 1.753 |
| 22 | .858 | 1.267 | 1.506 | 1.663 | 1.774 |
| 23 | .861 | 1.275 | 1.519 | 1.680 | 1.793 |
| 24 | .863 | 1.283 | 1.532 | 1.696 | 1.813 |
| 25 | .865 | 1.291 | 1.544 | 1.712 | 1.831 |
| 26 | .867 | 1.298 | 1.555 | 1.727 | 1.849 |
| 27 | .869 | 1.305 | 1.567 | 1.742 | 1.867 |
| 28 | .871 | 1.311 | 1.578 | 1.756 | 1.884 |
| 29 | .872 | 1.318 | 1.588 | 1.770 | 1.900 |
| 30 | .874 | 1.324 | 1.598 | 1.783 | 1.916 |
| 40 | .888 | 1.380 | 1.693 | 1.909 | 2.067 |
| 50 | .898 | 1.419 | 1.759 | 1.999 | 2.177 |
| 60 | .905 | 1.450 | 1.814 | 2.074 | 2.270 |
| 80 | .915 | 1.497 | 1.899 | 2.194 | 2.420 |
| 100 | .923 | 1.532 | 1.965 | 2.288 | 2.539 |
| 120 | .928 | 1.560 | 2.018 | 2.365 | 2.638 |
| 140 | .933 | 1.583 | 2.062 | 2.430 | 2.722 |
| 160 | .937 | 1.603 | 2.100 | 2.486 | 2.794 |
| 180 | .940 | 1.619 | 2.133 | 2.535 | 2.859 |
| 200 | .942 | 1.633 | 2.162 | 2.579 | 2.916 |

**Table 2: Significant two-tail percentage points of sample co-efficient of variation ($c_x$) at 1% significance level** $p\left(c_x > c_{xn,C_x}(0.01)\right) = 0.01$

## 4. Numerical results and discussion

In this section, authors have used a time-series study for testing the significance of the sample co-efficient of variation. The sample coefficient of variation is calculated for 12 macro-economic factors of India with different sample sizes from the period 1950-51 to 2011-12 and the results are presented in Table 3.

| Variable no. | Sample size (n) | Macroeconomic factors of India | Sample 'cv' | *critical 'cv'* | |
|---|---|---|---|---|---|
| | | | | 5% | 1% |
| 1 | 62 | **Consumption of Fixed Capital** | | .9115 | .9060 |
| 2 | 62 | **NDP at Factor Cost** | 0.890 | .9115 | .9060 |
| 3 | 62 | **Indirect Taxes less Subsidies** | 0.835 | .9115 | .9060 |
| 4 | 62 | **GDP at Market Prices** | 0.900 | .9115 | .9060 |
| 5 | 62 | **NDP at Market Prices** | 0.883 | .9115 | .9060 |
| 6 | 62 | **Net factor income from abroad** | 1.138* | .9115 | .9060 |
| 7 | 62 | **GNP at Factor Cost** | 0.906 | .9115 | .9060 |
| 8 | 62 | **NNP at Factor Cost** | 0.867 | .9115 | .9060 |
| 9 | 62 | **GNP at Market Prices** | 0.899 | .9115 | .9060 |
| 10 | 62 | **NNP at Market Prices** | 0.882 | .9115 | .9060 |
| 11 | 50 | **GDP of Public sector** | 0.828 | .9027 | .8977 |
| 12 | 50 | **NDP of public sector** | 0.862 | .9027 | .8977 |
| 13 | 61 | **Gross Domestic Capital Formation** | 1.221* | .9108 | .9054 |
| 14 | 61 | **Net domestic capital formation** | 1.320* | .9108 | .9054 |
| 15 | 62 | **Per Capita GNP at factor cost (Rs)** | 0.551 | .9115 | .9060 |
| 16 | 62 | **Per Capita NNP at factor cost (Rs)** | 0.530 | .9115 | .9060 |

**Table 3: Result of Two-tail test of significance for Sample co-efficient of variation**

**p-value <0.01 & p-value <0.05* *under* $H_O : CV = 1$

Table 3 visualizes the result of the two-tailed test of significance of the sample coefficient of variation for different sample sizes. The sample coefficient of variation for the macro economic factors such as consumption of fixed capital, net factor income from abroad, gross domestic capital and net domestic capital formation are greater than the critical 'cv' at 5%,1% level of significance, hence reject the null hypothesis $H_o : C_x = 1$ and accept the alternative hypothesis $H_o : C_x \neq 1$.This shows that the above mentioned macro-economic factors of India has high variation, less consistent and the distribution of these macro-economic factors are having a high variance for selected period of time.

## 5. Conclusion

In this paper authors have derived the sampling distribution of sample coefficient of variation *($c_x$)* and it's density function in terms of the confluent hyper-geometric distribution and the first two moments are also derived in terms of mean and variance. The simulation study is shown with a view of testing the significance of the coefficient of variation *($c_x$)* from which we got an insight to check the consistency and the variability of the distribution of the major macroeconomic factors of India. Henricks and Robbey (1936) proposed the distribution of sample *'$c_x$'* from the normal population had a constraint of using odd and even sample size. The authors believed that the proposed distribution is the extension and alternative to the Henricks and Robbey's (1936) work for testing the significance of sample $c_x$ from the normal population irrespective of any sample size. Finally, the sampling distribution of the difference and ratios of the two sample coefficient of variation can also be derived and the authors left it for future research.

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