Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 8, Issue 1 (2015): 63-75

RELIABILITY MODELING OF A MAINTAINED SYSTEM WITH WARRANTY AND DEGRADATION

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Received January 22, 2015
Modified May 09, 2015
Accepted May 23, 2015

Abstract

This paper discusses a maintained system with the concept of warranty and degradation. Repair and preventive maintenance (PM) costs are carried by the manufacturer during warranty while the cost of repair will be charged to the user beyond warranty. There is single repairman who is always available with the system for PM, repair and replacement of the unit. During warranty, unit goes under PM and works as new after PM. The unit beyond warranty works with some reduced capacity after its repair and so is called a degraded unit. Degraded unit is replaced by new unit after its failure. The time to failure of the system follows negative exponential distribution while repair, replacement and PM time distributions are taken as arbitrary. The expressions for reliability, mean time to system failure (MTSF), availability and profit function have been determined by using supplementary variable technique. Using Abel's lemma, steady state behaviour of the system has been examined. Numerical results for reliability and profit function are also evaluated for particular values of various parameters and repair cost.

Key Words: Reliability Modeling, Degradation, Warranty, Profit Analysis.

1. Introduction

Warranty is a contractual agreement under which the manufacturer agrees to repair or replacement of a product with own cost if it fails to meet customer's requirement within warranty period. Various researchers including Goel et al. [2], Gupta and Tyagi [3], Kadyan et al. [5], Malik [6], Nailwal and Singh [8], Singh et al. [9], Tuteja and Malik [10] and Yuan and Meng [11] analyzed reliability models under various set of assumptions on failure and repair policies without considering any warranty to the system. But, warranty plays a vital role in marketing the product and also provides assurance to users against early failures of a product at least the length of the warranty period with each purchase. Consequently, the concept of warranty is important to both manufacturer and the users. Also, performing PM in cost effective manner is one of the requirement of the management. So, during warranty, well performing PM will reduce the cost of repairing deteriorated product, extend life of the component and also may provide consumer a better product service in beyond warranty. Mokaddis et al. [7] have analyzed a two-unit system with a warm standby subject to preventive maintenance without considering degradation of the unit after its repair.

However, the failed unit does not always work as new after its repair. Due to continuous usage and ageing effect, failure rate of a unit may have increased after its repair. In such a situation unit becomes degraded after its repair. Kadyan et al. [4] discussed a two-unit parallel system with the concept of degradation without any warranty.

Keeping the above facts in view, authors have analysed a single-unit system with PM under warranty and degradation. During warranty, repair and PM costs are carried by the manufacturer. The cost of repair will be charged to the user beyond warranty. There is single repairman, who is always available with the system for PM, repair and replacement of the unit. During warranty, unit goes under PM and works as new after PM. The unit beyond warranty works with some reduced capacity after its repair and so is called a degraded unit. Degraded unit is replaced by new unit after its failure. The time to failure of the system follows negative exponential distribution while repair, replacement and PM time distributions are taken as arbitrary. The expressions for reliability, MTSF, availability and profit function have been determined by using supplementary variable technique. Using Abel's lemma, steady state behavior of the system has been examined. Numerical results for reliability and profit function are also evaluated for particular values of various parameters and repair cost.

2. Assumptions

(1) The system has a single unit.
(2) There is single repairman, who is always available with the system.
(3) The cost of repair during warranty is borne by the manufacturer, provided failures are not due to the negligence of users such as cracked screen, accident, misuse, physical damage, damage due to liquid and unauthorized modifications etc.
(4) Unit goes under PM during warranty.
(5) Beyond warranty, unit works with reduced capacity and so is called a degraded unit.
(6) The degraded unit, after its failure is replaced by a new one.
(7) The distribution of failure time is taken as negative exponential while the PM, replacement and repair time distributions are considered as arbitrary.

3. State-specification

S_0 / S_1	The new unit is operative under warranty/ beyond warranty.
S_2 / S_4	The new unit is in failed state under warranty/ beyond warranty.
S_3	The new unit is under PM within warranty.
S_5	The degraded unit is operative beyond warranty.
S_6	The failed degraded unit is under replacement beyond warranty.

4. Notations

λ / λ_1	Constant failure rate of the new unit within warranty/beyond warranty.
λ_2	Constant failure rate of the degraded unit beyond warranty.
λ_m	The transition rate by which unit goes for PM.
α	Transition rate of completion of warranty.
$\mu(x), s(x) / \mu_1(x), s_1(x)$	Repair rate of the unit and probability density function, for the elapsed repair time x within warranty/ beyond warranty.

$\mu_2(y), s_2(y)$	PM rate of the unit and probability density function, for the elapsed PM time *y*.
$\mu_3(z), s_3(z)$	Replacement rate of the failed degraded unit and probability density function, for the elapsed replacement time *z*.
$p_0(t)/p_1(t)$	The Probability that at time *t*, the system is in good state within warranty/ beyond warranty.
$p_2(x,t)\Delta / p_4(x,t)\Delta$	The Probability that at time *t*, the new unit is in failed state within warranty/ beyond warranty, the elapsed repair time lies in the interval [x, x+Δ).
$p_3(y,t)\Delta$	The Probability that at time *t*, the system is under PM, the elapsed PM time lies in the interval [y, y+Δ).
$p_5(t)$	The Probability that at time *t*, the system is operable and in degraded state.
$p_6(z,t)\Delta$	The Probability that at time *t*, the failed degraded unit is under replacement, the elapsed replacement time lies in the interval [z, z+Δ).
$p(s)$	Laplace transform of function $p(t)$

$$S(x) = \mu(x)\exp[-\int_0^x \mu(x)dx]$$

$$S_1(x) = \mu_1(x)\exp[-\int_0^x \mu_1(x)dx]$$

$$S_2(y) = \mu_2(y)\exp[-\int_0^y \mu_2(y)dy]$$

$$S_3(z) = \mu_3(z)\exp[-\int_0^z \mu_3(z)dz]$$

5. Formulation of mathematical model

Using the probabilistic arguments and limiting transitions, we have the following difference-differential equations (Cox, D.R. [1]):

$$\left[\frac{d}{dt} + \lambda + \alpha + \lambda_m\right] p_0(t) = \int_0^\infty \mu(x) p_2(x,t)dx + \int_0^\infty \mu_2(y) p_3(y,t)dy \quad (1)$$

$$\left[\frac{d}{dt} + \lambda_1\right] p_1(t) = \alpha p_0(t) + \int_0^\infty \mu_3(z) p_6(z,t)dz \quad (2)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \mu(x)\right] p_2(x,t) = 0 \quad (3)$$

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial y} + \mu_2(y)\right] p_3(y,t) = 0 \quad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_1(x)\right]p_4(x,t)=0 \quad (5)$$

$$\left[\frac{d}{dt}+\lambda_2\right]p_5(t)=\int_0^\infty \mu_1(x)p_4(x,t)dx \quad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_3(z)\right]p_6(z,t)=0 \quad (7)$$

Boundary Conditions

$$p_2(0,t)=\lambda p_0(t) \quad (8)$$

$$p_3(0,t)=\lambda_m p_0(t) \quad (9)$$

$$p_4(0,t)=\lambda_1 p_1(t) \quad (10)$$

$$p_6(0,t)=\lambda_2 p_5(t) \quad (11)$$

Initial Conditions

$$p_i(0)=1; \quad \text{when} \quad i=0$$

$$p_i(0)=0; \quad \text{when} \quad i\neq 0 \quad (12)$$

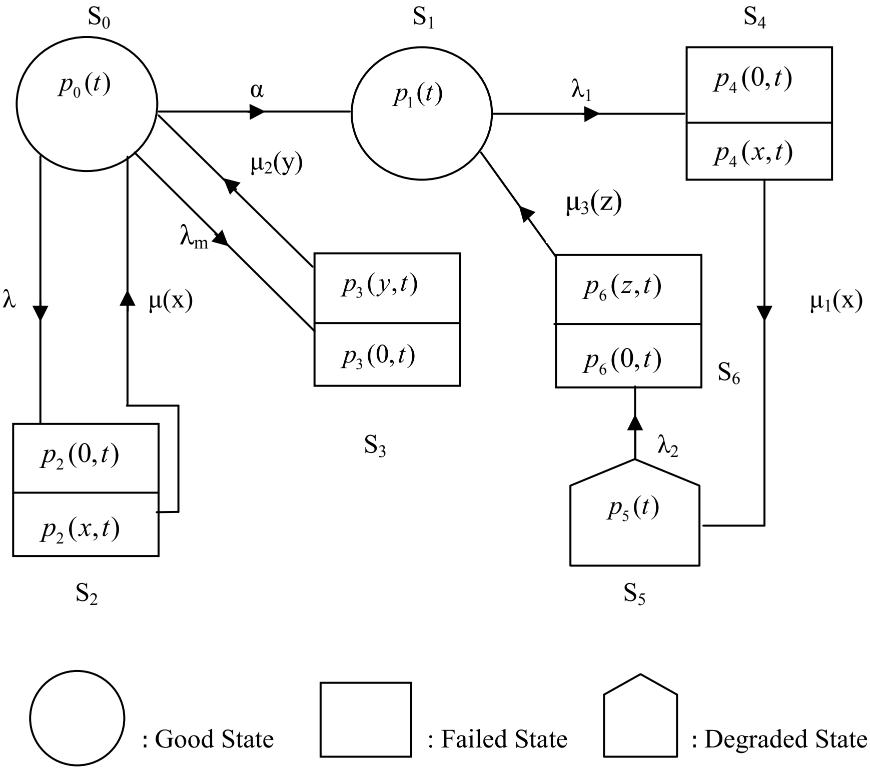

Figure 1: State Transition diagram of the model

6. Model analysis

6.1. Solution of the equations

Taking Laplace transforms of equations (1)-(11) and using (12), we obtain

$$[s+\lambda+\alpha+\lambda_m]p_0(s) = 1+\int_0^\infty \mu(x)p_2(x,s)dx+\int_0^\infty \mu_2(y)p_3(y,s)dy \tag{13}$$

$$[s+\lambda_1]p_1(s) = \alpha p_0(s)+\int_0^\infty \mu_3(z)p_6(z,s)dz \tag{14}$$

$$\left[\frac{\partial}{\partial x}+s+\mu(x)\right]p_2(x,s)=0 \tag{15}$$

$$\left[\frac{\partial}{\partial y}+s+\mu_2(y)\right]p_3(y,s)=0 \tag{16}$$

$$\left[\frac{\partial}{\partial x}+s+\mu_1(x)\right]p_4(x,s)=0 \tag{17}$$

$$[s+\lambda_2]p_5(s)=\int_0^\infty \mu_1(x)p_4(x,s)dx \tag{18}$$

$$\left[\frac{\partial}{\partial z}+s+\mu_3(z)\right]p_6(z,s)=0 \tag{19}$$

$$p_2(0,s)=\lambda p_0(s) \tag{20}$$

$$p_3(0,s)=\lambda_m p_0(s) \tag{21}$$

$$p_4(0,s)=\lambda_1 p_1(s) \tag{22}$$

$$p_6(0,s)=\lambda_2 p_5(s) \tag{23}$$

Taking integration of equations (15), (16), (17) and (19), we get the following equations

$$p_2(x,s)=p_2(0,t)\exp(-(sx+\int_0^x \mu(x)dx)) \tag{24}$$

$$p_3(y,s)=p_3(0,t)\exp(-(sy+\int_0^y \mu_2(y)dy)) \tag{25}$$

$$p_4(x,s)=p_4(0,s)\exp\left(-sx-\int_0^x \mu_1(x)dx\right) \tag{26}$$

and

$$p_6(z,s)=p_6(0,t)\exp(-(sz+\int_0^z \mu_3(z)dz)) \tag{27}$$

Using equations (20), (21), (24) and (25), equation (13) yields

$$\left[s+\lambda+\alpha+\lambda_m\right]p_0(s)=1+p_2(0,s)\int_0^{\infty}\mu(x)\exp(sx-(\int_0^x\mu(x)dx))dx$$
$$+p_3(0,s)\int_0^{\infty}\mu_2(y)\exp(sy-(\int_0^y\mu_2(y)dy))dy$$
$$=1+\lambda p_0(s)S(s)+\lambda_m p_0(s)S_2(s)$$

$$p_0(s)=\frac{1}{T(s)} \tag{28}$$

where $T(s)=s+\lambda+\alpha-\lambda S(s)+\lambda_m(1-S_2(s)$ (29)

Using equation (22) and (26), equation (18) yields

$$\left[s+\lambda_2\right]p_5(s)=\lambda_1 p_1(s)\int_0^{\infty}\mu_1(x)\exp(sx-(\int_0^x\mu_1(x)dx))dx$$

$$p_5(s)=\frac{\lambda_1 S_1 p_1(s)}{\left(s+\lambda_2\right)} \tag{30}$$

Using equations (23), (27) and (30) in equation (14), we get

$$\left[s+\lambda_1\right]p_1(s)=\alpha p_0(s)+\left(\frac{\lambda_1\lambda_2 p_1(s)S_1S_2}{\left(s+\lambda_2\right)}\right)$$

$$p_1(s)=\frac{A(s)}{T(s)} \tag{31}$$

where, $A(s)=\dfrac{\alpha\left(s+\lambda_2\right)}{\left(s+\lambda_1\right)\left(s+\lambda_2\right)-\lambda_1\lambda_2S_1S_3}$ (32)

Using equation (31) in equation (30), we get

$$p_5(s)=\frac{\left(\lambda_1 A(s)S_1(s)\right)}{T(s)\left(s+\lambda_2\right)} \tag{33}$$

Now, the Laplace transform of the probability that the system is in the failed state is given by

$$p_2(s)=\int_0^{\infty}p_2(x,s)dx=\lambda p_0(s)\left(\frac{1-S(s)}{s}\right)$$

$$p_2(s)=\frac{\lambda B(s)}{T(s)} \tag{34}$$

where $B(s)=\left(\dfrac{1-S(s)}{s}\right)$ (35)

Similarly, $p_3(s)=\int_0^{\infty}p_3(y,s)dy=\lambda_m p_0(s)\left(\dfrac{1-S_2(s)}{s}\right)$

$$p_3(s) = \frac{\lambda_m C(s)}{T(s)} \tag{36}$$

where $C(s) = \left(\frac{1 - S_2(s)}{s}\right)$ (37)

Similarly, $p_4(s) = \int_0^\infty p_4(x,s)dx = \lambda_1 p_1(s)\left(\frac{1 - S_1(s)}{s}\right)$

$$p_4(s) = \frac{(\lambda_1 A(s)D(s))}{T(s)} \tag{38}$$

where $D(s) = \left(\frac{1 - S_1(s)}{s}\right)$ (39)

Now, $p_6(s) = \int_0^\infty p_6(z,s)dz = \lambda_2 p_5(s)\left(\frac{1 - S_3(s)}{s}\right)$

$$p_6(s) = \frac{(\lambda_1 \lambda_2 A(s)S_1(s)E(s))}{T(s)(s + \lambda_2)} \tag{40}$$

where, $E(s) = \left(\frac{1 - S_3(s)}{s}\right)$ (41)

It is worth noticing that

$$p_0(s) + p_1(s) + p_2(s) + p_4(s) + p_5(s) + p_6(s) = \frac{1}{s} \tag{42}$$

6.2. Evaluation of Laplace transforms of up and down state probabilities

Let Av(t) is the probability that the system is operating satisfactorily at time '*t*'. The Laplace transforms of Av(t) or probabilities that the system is in up state (P_{up}(t)) (i.e. Good and Degraded State) and down state (P_{down}(t)) (i.e. Failed State) at time '*t*' are as follows

$A_v(s)$ *or* $p_{up}(s) = p_0(s) + p_1(s) + p_5(s)$

$$A_v(s) = \frac{(1 + A(s) + \lambda_1 S_1(s)A(s)B(s))}{T(s)(s + \lambda_2)}$$

(43)

$p_{down}(s) = p_2(s) + p_3(s) + p_4(s) + p_6(s)$

$$p_{down}(s) = \frac{\left(\lambda B(s) + \lambda_m C(s) + \lambda_1 A(s)D(s) + \left(\frac{\lambda_1 \lambda_2 A(s)E(s)S_1(s)}{(s + \lambda_2)}\right)\right)}{T(s)} \tag{44}$$

6.3. Steady-state behaviour of the system

Using Abel's Lemma in Laplace transforms, viz.

$\lim_{s\to 0} s\left(A_v(s)\right) = \lim_{t\to\infty} A_v(t) = A_v(say),$ Provided the limit on the right hand side exists, the following time independent probabilities have been obtained.

$$A_v = \frac{(\lambda_1 + \lambda_2)}{\left(\lambda_1 + \lambda_2 - \lambda_1\lambda_2 S_1^{'}(0) - \lambda_1\lambda_2 S_3^{'}(0)\right)} \tag{45}$$

$$P_{down} = \frac{-\lambda_1\lambda_2 S_1^{'}(0) - \lambda_1\lambda_2 S_3^{'}(0)}{\left(\lambda_1 + \lambda_2 - \lambda_1\lambda_2 S_1^{'}(0) - \lambda_1\lambda_2 S_3^{'}(0)\right)} \tag{46}$$

6.4. Reliability of the system (R(t))

In order to obtain system reliability, the differential–difference equations are:

$$\left[\frac{d}{dt} + \lambda + \alpha + \lambda_m\right] p_0(t) = 0 \tag{47}$$

$$\left[\frac{d}{dt} + \lambda_1\right] p_1(t) = \alpha p_0(t) \tag{48}$$

Taking Laplace transforms of equations (47) and (48), using (12), we get

$$\left[s + \lambda + \alpha + \lambda_m\right] p_0(s) = 1 \tag{49}$$

$$\left[s + \lambda_1\right] p_0(s) = \alpha p_0(s) \tag{50}$$

The solution can be written as

$$p_0(s) = \frac{1}{(s + \lambda + \alpha + \lambda_m)} \tag{51}$$

$$p_1(s) = \frac{\alpha}{(s + \alpha + \lambda + \lambda_m)(s + \lambda_1)} \tag{52}$$

$$R(s) = p_0(s) + p_1(s) = \frac{1}{(s + \lambda + \alpha + \lambda_m)} + \frac{\alpha}{(s + \lambda + \alpha + \lambda_m)(s + \lambda_1)} \tag{53}$$

Taking inverse Laplace transform, we get

$$R(t) = \exp(-(\lambda + \alpha + \lambda_m)t)\left[\frac{\lambda - \lambda_1 + \lambda_m}{\lambda - \lambda_1 + \lambda_m + \alpha}\right] + \exp(-\lambda_1 t)\left[\frac{\alpha}{\lambda - \lambda_1 + \lambda_m + \alpha}\right] \tag{54}$$

6.5. Mean time to system failure (MTSF)

$$MTSF = \int_0^\infty R(t)dt$$

$$= \int_0^\infty \left(\exp(-(\lambda + \alpha + \lambda_m)t)\left[\frac{\lambda - \lambda_1 + \lambda_m}{\lambda - \lambda_1 + \lambda_m + \alpha}\right] + \exp(-\lambda_1 t)\left[\frac{\alpha}{\lambda - \lambda_1 + \lambda_m + \alpha}\right]\right) dt$$

$$MTSF = \left[\frac{\lambda - \lambda_1 + \lambda_m}{(\lambda - \lambda_1 + \lambda_m + \alpha)(\lambda + \lambda_m + \alpha)} \right] + \left[\frac{\alpha}{(\lambda - \lambda_1 + \lambda_m + \alpha)(\lambda_1)} \right] \tag{55}$$

7. Particular cases

7.1. Availability of the system

When repair, PM and replacement times follow exponential distribution i.e. $S(s) = \frac{\mu}{(s+\mu)}$, $S_1(s) = \frac{\mu_1}{(s+\mu_1)}$,$S_2(s) = \frac{\mu_2}{(s+\mu_2)}$ *and* $S_3(s) = \frac{\mu_3}{(s+\mu_3)}$

Where μ and μ_1 are constant repair rates, μ_2 is constant PM rate and μ_3 is constant replacement rate. Putting these values in equations (28)-(33), we get

$$p_0(s) = \frac{1}{I(s)} \tag{56}$$

Where $$I(s) = \frac{\left((s+\lambda+\alpha+\lambda_m)(s+\mu)(s+\mu_2) - \lambda(s+\mu_2) - \lambda_m(s+\mu)\right)}{(s+\mu)(s+\mu_2)} \tag{57}$$

$$p_1(s) = \frac{J(s)}{I(s)} \tag{58}$$

Where $$J(s) = \left[\frac{\alpha(s+\mu_1)(s+\mu_3)(s+\lambda_2)}{(s+\mu_1)(s+\mu_3)(s+\lambda_1)(s+\lambda_2) - \lambda_1\lambda_2\mu_1\mu_3} \right] \tag{59}$$

$$p_5(s) = \frac{J(s)K(s)}{I(s)} \tag{60}$$

Where $$K(s) = \left[\frac{\lambda_1\mu_1}{(s+\mu_1)(s+\lambda_2)} \right] \tag{61}$$

$$A_v(s) \quad or \quad p_{up}(s) = p_0(s) + p_1(s) + p_5(s)$$

$$= \left[\frac{\left(s^4 + b_3 s^3 + b_2 s^2 + b_1 s + b_0\right)\left(s^2 + s(\mu+\mu_2) + \mu\mu_2\right)}{s\left(s^3 + a_2 s^2 + a_1 s + a_0\right)\left(s^3 + c_2 s^2 + c_1 s + c_0\right)} \right] \tag{62}$$

Where

$b_3 = (\lambda_1 + \lambda_2 + \alpha + \mu_1 + \mu_3)$, $b_2 = (\lambda_1\mu_1 + \lambda_1\mu_3 + \mu_3\mu_1 + \lambda_2\alpha + \lambda_2\mu_1 + \lambda_1\lambda_2 + \lambda_2\mu_3 + \mu_1\alpha + \mu_3\alpha)$,

$b_1 = (\lambda_1\mu_3\mu_1 + \lambda_1\lambda_2\mu_1 + \lambda_1\lambda_2\mu_3 + \lambda_2\mu_1\mu_3 + \lambda_2\alpha\mu_3 + \lambda_2\alpha\mu_1 + \lambda_1\alpha\mu_1)$, $b_0 = (\alpha\mu_3\mu_1\lambda_2 + \alpha\mu_3\mu_1\lambda_1)$

$a_2 = (\mu_1 + \lambda_2 + \lambda_1 + \mu_3)$, $a_1 = (\lambda_1\mu_1 + \lambda_1\lambda_2 + \mu_1\lambda_2 + \lambda_1\mu_3 + \mu_1\mu_3 + \mu_3\lambda_2)$ *and* $a_0 = (\lambda_1\mu_1\mu_3 + \mu_1\lambda_1\lambda_2 + \lambda_1\lambda_2\mu_3 + \lambda_2\mu_1\mu_3)$

$c_2 = (\mu + \lambda_m + \lambda + \alpha + \mu_2)$, $c_1 = (\lambda\mu_2 + \alpha\mu + \mu\lambda_m + \alpha\mu_2 + \mu\mu_2)$ *and* $c_0 = (\alpha\mu\mu_2)$

$$a_2 = (\mu_1 + \lambda_1 + \mu_3 + \lambda_2), a_1 = (\lambda_1\mu_1 + \lambda_1\mu_3 + \mu_1\mu_3 + \lambda_1\lambda_2 + \mu_1\lambda_2 + \mu_3\lambda_2)$$

$$a_0 = (\lambda_1\mu_1\mu_3 + \mu_1\lambda_1\lambda_2 + \lambda_2\lambda_1\mu_3 + \lambda_2\mu_1\mu_3)$$

$c_2 = (\mu + \mu_2 + \lambda + \alpha + \lambda_m)$, $c_1 = (\lambda\mu_2 + \mu\mu_2 + \mu\lambda_m + \alpha\mu + \alpha\mu_2)$ and $c_0 = \alpha\mu\mu_2$

Taking inverse Laplace transforms of equation (62), we get

$$A_v(t) = \frac{b_0\mu\mu_2}{z_1z_2z_3z_4z_5z_6} + \left\{\frac{\left(z_1^4 + b_3z_1^3 + b_2z_1^2 + b_1z_1 + b_0\right)\left(z_1^2 + (\mu+\mu_2)z_1 + \mu\mu_2\right)}{z_1(z_1-z_2)(z_1-z_3)(z_1-z_4)(z_1-z_5)(z_1-z_6)}\right\}\exp(z_1t)$$

$$+\left\{\frac{\left(z_2^4 + b_3z_2^3 + b_2z_2^2 + b_1z_2 + b_0\right)\left(z_2^2 + (\mu+\mu_2)z_2 + \mu\mu_2\right)}{z_2(z_2-z_1)(z_2-z_3)(z_2-z_4)(z_2-z_5)(z_2-z_6)}\right\}\exp(z_2t)$$

$$+\left\{\frac{\left(z_3^4 + b_3z_3^3 + b_2z_3^2 + b_1z_3 + b_0\right)\left(z_3^2 + (\mu+\mu_2)z_3 + \mu\mu_2\right)}{z_3(z_3-z_1)(z_3-z_2)(z_3-z_4)(z_3-z_5)(z_3-z_6)}\right\}\exp(z_3t)$$

$$+\left\{\frac{\left(z_4^4 + b_3z_4^3 + b_2z_4^2 + b_1z_4 + b_0\right)\left(z_4^2 + (\mu+\mu_2)z_4 + \mu\mu_2\right)}{z_4(z_4-z_1)(z_4-z_2)(z_4-z_3)(z_4-z_5)(z_4-z_6)}\right\}\exp(z_4t)$$

$$+\left\{\frac{\left(z_5^4 + b_3z_5^3 + b_2z_5^2 + b_1z_5 + b_0\right)\left(z_5^2 + (\mu+\mu_2)z_5 + \mu\mu_2\right)}{z_5(z_5-z_1)(z_5-z_2)(z_5-z_3)(z_5-z_4)(z_5-z_6)}\right\}\exp(z_5t)$$

$$+\left\{\frac{\left(z_6^4 + b_3z_6^3 + b_2z_6^2 + b_1z_6 + b_0\right)\left(z_6^2 + (\mu+\mu_2)z_6 + \mu\mu_2\right)}{z_6(z_6-z_1)(z_6-z_2)(z_6-z_3)(z_6-z_4)(z_6-z_5)}\right\}\exp(z_6t) \quad (63)$$

z_1, z_2 *and* z_3 are roots of the equation $\left(s^3 + a_2s^2 + sa_1 + a_0\right) = 0$ and z_4, z_5 *and* z_6 are roots of the equation $\left(s^3 + c_2s^2 + sc_1 + c_0\right) = 0$

7.2. Profit analysis of the user

Suppose that the warranty period of the system is (0, w]. Since the repairman is always available with the system, therefore beyond warranty period, it remains busy during the interval (w, t]. Let K_1 be the revenue per unit time and K_2 be the repair cost per unit time, then the expected profit H(t) during the interval (0, t] is given by

$$H(t) = K_1\int_0^t A_v(t)dt - K_2(t - w)$$

$$H(t)=\left\{\begin{array}{l}\dfrac{b_0\mu\mu_2 t}{z_1z_2z_3z_4z_5z_6}+\left\{\dfrac{\left(z_1^4+b_3z_1^3+b_2z_1^2+b_1z_1+b_0\right)\left(z_1^2+\left(\mu+\mu_2\right)z_1+\mu\mu_2\right)}{z_1^2\left(z_1-z_2\right)\left(z_1-z_3\right)\left(z_1-z_4\right)\left(z_1-z_5\right)\left(z_1-z_6\right)}\right\}\exp\left(z_1t-1\right)\\
+\left\{\dfrac{\left(z_2^4+b_3z_2^3+b_2z_2^2+b_1z_2+b_0\right)\left(z_2^2+\left(\mu+\mu_2\right)z_2+\mu\mu_2\right)}{z_2^2\left(z_2-z_1\right)\left(z_2-z_3\right)\left(z_2-z_4\right)\left(z_2-z_5\right)\left(z_2-z_6\right)}\right\}\exp\left(z_2t-1\right)\\
+\left\{\dfrac{\left(z_3^4+b_3z_3^3+b_2z_3^2+b_1z_3+b_0\right)\left(z_3^2+\left(\mu+\mu_2\right)z_3+\mu\mu_2\right)}{z_3^2\left(z_3-z_1\right)\left(z_3-z_2\right)\left(z_3-z_4\right)\left(z_3-z_5\right)\left(z_3-z_6\right)}\right\}\exp\left(z_3t-1\right)\\
+\left\{\dfrac{\left(z_4^4+b_3z_4^3+b_2z_4^2+b_1z_4+b_0\right)\left(z_4^2+\left(\mu+\mu_2\right)z_4+\mu\mu_2\right)}{z_4^2\left(z_4-z_1\right)\left(z_4-z_2\right)\left(z_4-z_3\right)\left(z_4-z_5\right)\left(z_4-z_6\right)}\right\}\exp\left(z_4t-1\right)\\
+\left\{\dfrac{\left(z_5^4+b_3z_5^3+b_2z_5^2+b_1z_5+b_0\right)\left(z_5^2+\left(\mu+\mu_2\right)z_5+\mu\mu_2\right)}{z_5^2\left(z_5-z_1\right)\left(z_5-z_2\right)\left(z_5-z_3\right)\left(z_5-z_4\right)\left(z_5-z_6\right)}\right\}\exp\left(z_5t-1\right)\\
+\left\{\dfrac{\left(z_6^4+b_3z_6^3+b_2z_6^2+b_1z_6+b_0\right)\left(z_6^2+\left(\mu+\mu_2\right)z_6+\mu\mu_2\right)}{z_6^2\left(z_6-z_1\right)\left(z_6-z_2\right)\left(z_6-z_3\right)\left(z_6-z_4\right)\left(z_6-z_5\right)}\right\}\exp\left(z_6t-1\right)\end{array}\right\}-K_2\left(t-w\right) \quad (64)$$

8. Numerical analysis

Time (t)	λ_1=0.02, α=0.003, λ_m=0.04	λ_1=0.02, α=0.003, λ_m=0.04	λ=0.01, α=0.003, λ_m=0.04	λ=0.01, λ_1=0.02, λ_m=0.04	λ=0.01, λ_1=0.02, α=0.003
	R(t) (for λ=0.01)	R(t) (for λ=0.02)	R(t) (for λ_1=0.03)	R(t) (for α=0.005)	R(t) (for λ_m=0.05)
10	0.609525	0.552555	0.608459	0.61149	0.552555
11	0.58043	0.521174	0.579182	0.582709	0.521174
12	0.55279	0.491663	0.551354	0.555391	0.491663
13	0.52653	0.463908	0.524901	0.529458	0.463908
14	0.501581	0.437803	0.499754	0.504837	0.437803
15	0.477876	0.413247	0.475848	0.481461	0.413247
16	0.45535	0.390148	0.45312	0.459264	0.390148
17	0.433946	0.368417	0.431511	0.438184	0.368417

Table 1: Effect of failure rates (λ and λ_1), transition rate by which unit goes for PM (λ_m) and transition rate of completion of warranty (α) on Reliability (R(t))

Time (t)	λ =0.01, λ_1=0.02, λ_2=0.03, λ_m=0.04, α =0.003, μ =0.2, μ_1 =0.1, μ_2 =0.3, μ_3=0.4, W=3, K_1=500	λ =0.01, λ_1=0.02, λ_2=0.03, λ_m=0.04, α =0.003, μ =0.2, μ_1 =0.1, μ_2 =0.3, μ_3=0.4, W=3, K_1=500	λ =0.01, λ_1=0.02, λ_2=0.03, λ_m=0.04, α =0.003 , μ =0.2, μ_1 =0.1, μ_3=0.4, W=3, K_1=500, K_2=150	λ =0.01, λ_1=0.02, λ_m=0.04, α =0.003, μ =0.2, μ_1 =0.1, μ_2 =0.3, μ_3=0.4, W=3, K_1=500, K_2=150	λ =0.01, λ_1=0.02, λ_2=0.03, α =0.003, μ =0.2, μ_1 =0.1, μ_2 =0.3, μ_3=0.4, W=3, K_1=500, K_2=150	λ =0.01, λ_1=0.02, λ_2=0.03, λ_m=0.04, α =0.003, μ =0.2, μ_1 =0.1, μ_2 =0.3, W=3, K_1=500, K_2=150
	H(t) (For K_2=150)	H(t) (For K_2=100)	H(t) (For μ_2=0.4)	H(t) (For λ_2=0.02)	H(t) (For λ_m=0.03)	H(t) (For μ_3=0.5)
10	3413.133	3763.133	3476.806	3413.142	3506.667	3413.715
11	3689.55	4089.55	3764.296	3689.565	3795.059	3690.202
12	3965.278	4415.278	4051.325	3965.303	4082.771	3966.003
13	4240.517	4740.517	4338.018	4240.554	4369.983	4241.315
14	4515.411	5065.411	4624.467	4515.464	4656.829	4516.283
15	4790.067	5390.067	4910.739	4790.139	4943.408	4791.015
16	5064.564	5714.564	5196.884	5064.659	5229.797	5065.589
17	5338.959	6038.959	5482.941	5339.081	5516.051	5340.065

Table 2: Effect of repair cost (K_2), PM rate (μ_2), failure rate of degraded unit (λ_2), transition rate by which unit goes for PM (λ_m) and replacement rate of failed degraded unit (μ_3) on expected profit (H(t))

9. Interpretation and conclusion

The reliability of the system model is shown in table 1. It can be observed that reliability of the system decreases with the increase of failure rates (λ and λ_1) and transition rate by which the unit goes under PM (λ_m) while it increases with the increase of transition rate of completion of warranty (α) with respect to time and for fixed values of other parameters. It means that the system will become more reliable as management/users pay more attention towards decreasing the failure rates within/beyond warranty. Table 2 depicts the behaviour of expected profit function (H(t)) and it is analyzed that the value of H(t) increases with the increase of PM rate

(μ_2) and replacement rate of failed degraded unit (μ_3). Also, H(t) increases with the decrease of failure rate (λ_2), transition rate by which unit goes for PM (λ_m) and repair cost (K_2) with respect to time. It means that providing PM during warranty will be economically beneficial because it extends the life of the component and maximize the expected profit. We also observed that the system will become more profitable to use after replacing the failed degrade unit by new one. Consequently, the concept of performing PM during warranty is profitable to both user and manufacturer because during warranty all type repair charges are carried by the manufacturer and well performing PM will reduce the cost of repairing deteriorated product, extend life of the component and may provide consumer a better product service beyond warranty.

Hence, on the basis of the above results, here we conclude that after getting PM under warranty, a system in which replacement of failed degraded unit by new one beyond warranty will be economically beneficial to use by increasing the PM rate, replacement rate of the failed degraded unit and decreasing the repair cost.

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