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SEPARATE TYPE EXPONENTIAL ESTIMATORS OF POPULATION MEAN USING TWO AUXILIARY VARIABLES IN STRATIFIED RANDOM SAMPLING

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Abstract

In this paper separate type estimators of population mean have been suggested. The biases and mean squared errors of the suggested estimators are obtained up to the first degree of approximation. Asymptotic optimum estimator (AOE) is also obtained with its properties. Conditions under which the suggested estimators are more efficient than other considered estimators are obtained. An empirical study has been carried out to demonstrate the performance of the suggested estimators.

Key Words: Finite Population Mean, Separate Type Estimators, Mean Squared Error, Bias.

1. Introduction

Huge literature is available on simple random sampling to address the issue of the efficient estimation of the mean (or total) of a survey (study) variable when information on auxiliary variable is available, for example see Srivastava (1967),Srivastava and Jhajj (1981), Bahl and Tuteja (1991), Tracy et al. (1999), Upadhyaya and Singh (1999), Singh and Tailor (2003), Tailor and Sharma (2009),Singh et al. (2009) , Singh and Agnihotri (2008), Singh and Solanki (2011) , Tailor (2012) and Solanki et al. (2012).

When information of parameters of auxiliary variate is available in each stratum, separate ratio type estimators may be constructed easily and perform better as compared to combined estimators. In planning survey stratified random sampling has often proved needful in improving the precision of estimators over simple random sampling. Thus improving the precision of estimators by reducing heterogeneity the use of auxiliary information at the estimation stage in stratified random sampling has been made by various authors. Hansen et al. (1946), Kadilar and Cingi (2003), Singh et al. (2008), Tailor et al. (2011), Vishwakarma and Singh (2011), Solanki and Singh (2013), Tailor and Chouhan (2014) and Lone et al. (2014) have suggested some estimators/classes of estimators of population mean of the study variable in stratified random sampling.

Consider a finite population $U = U_1, U_2, ..., U_N$ of size N, which is divided into L strata of size N_h $\left(h = 1,2,3,...,L\right)$. Let y be the study variate and x and z are the two auxiliary variates taking values y_{hi}, x_{hi} and z_{hi}; $h = 1,2,3,...,L$;

$i=1,2,3,...,N_h$ respectively, on i^{th} unit of the h^{th} stratum, where x is positively correlated and z is negatively correlated with the study variate y. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n=\sum_{h=1}^{L} n_h$.

Usual separate ratio and product type estimators in stratified random sampling are defined as

$$\hat{\bar{Y}}_{RS}=\sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{X}_h}{\bar{x}_h}\right) \tag{1.1}$$

and

$$\hat{\bar{Y}}_{PS}=\sum_{h=1}^{L} W_h \bar{y}_h \left(\frac{\bar{z}_h}{\bar{Z}_h}\right). \tag{1.2}$$

Mean squared errors of the classical separate ratio and product type estimators are given as

$$MSE\left(\hat{\bar{Y}}_{RS}\right)=\sum_{h=1}^{L} W_h^2\, \gamma_h \left(S_{yh}^2+R_{1h}^2 S_{xh}^2-2R_{1h}S_{yxh}\right) \tag{1.3}$$

and

$$MSE\left(\hat{\bar{Y}}_{PS}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2+R_{2h}^2 S_{zh}^2+2R_{2h}S_{yzh}\right), \tag{1.4}$$

where

$$R_{1h}=\frac{\bar{Y}_h}{\bar{X}_h},\ R_{2h}=\frac{\bar{Y}_h}{\bar{Z}_h},\ \gamma_h=\left(\frac{1}{n_h}-\frac{1}{N_h}\right),\ \rho_{yxh}=\frac{S_{yxh}}{S_{yh}S_{xh}},\ \rho_{yzh}=\frac{S_{yzh}}{S_{yh}S_{zh}},$$

$$S_{xh}^2=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(x_{hi}-\bar{X}_h)^2,\ S_{yh}^2=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(y_{hi}-\bar{Y}_h)^2,$$

$$S_{zh}^2=\frac{1}{N_h-1}\sum_{h=1}^{N_h}(z_{hi}-\bar{Z}_h)^2,\ S_{yzh}=\frac{1}{N_h-1}\sum_{h=1}^{L}(y_{hi}-\bar{Y}_h)(z_{hi}-\bar{Z}_h) \text{ and}$$

$$S_{yxh}=\frac{1}{N_h-1}\sum_{h=1}^{L}(x_{hi}-\bar{X}_h)(y_{hi}-\bar{Y}_h).$$

Further we define usual separate ratio and product type exponential estimators in stratified random sampling as

$$\hat{\bar{Y}}_{S\mathrm{Re}}=\sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right) \tag{1.5}$$

and

$$\hat{\bar{Y}}_{SPe} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{z}_h + \bar{Z}_h}\right). \tag{1.6}$$

Upto the first degree of approximation, the mean squared errors of $\hat{\bar{Y}}_{S\mathrm{Re}}$ and $\hat{\bar{Y}}_{SPe}$ are obtained as

$$MSE\left(\hat{\bar{Y}}_{S\mathrm{Re}}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4} R_{1h}^2 S_{xh}^2 - R_{1h} S_{yxh}\right] \tag{1.7}$$

and

$$MSE\left(\hat{\bar{Y}}_{SPe}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4} R_{2h}^2 S_{zh}^2 + R_{2h} S_{yzh}\right]. \tag{1.8}$$

2. Suggested estimator

Singh et al. (2009) discussed a ratio-cum-product type exponential estimator in simple random sampling as

$$\hat{\bar{Y}}_{RPe} = \bar{y} \exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right). \tag{2.1}$$

Tailor and Chouhan (2014) suggested a generalized ratio-cum-product type exponential estimator in stratified random sampling as

$$t_{RPe} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}}\right) \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}}\right). \tag{2.2}$$

Motivated by Singh et al. (2009) and Tailor and Chouhan (2014), we suggested a separate ratio-cum-product type exponential estimator for population mean as

$$t_{RPe}^{S} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right) \exp\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{z}_h + \bar{Z}_h}\right). \tag{2.3}$$

To obtain the bias and mean squared error of the suggested estimator t_{RPe}^{S}, we write $\bar{y}_h = \bar{Y}_h(1 + e_{0h})$, $\bar{x}_h = \bar{X}_h(1 + e_{1h})$ and $\bar{z}_h = \bar{Z}_h(1 + e_{2h})$ such that

$E(e_{0h}) = E(e_{1h}) = E(e_{2h}) = 0$,

$E(e_{0h}^2) = \gamma_h \, C_{yh}^2$,

$E(e_{1h}^2) = \gamma_h C_{xh}^2$,

$E(e_{2h}^2) = \gamma_h C_{zh}^2$,

$E(e_{0h} e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh}$,

$E(e_{0h} e_{2h}) = \gamma_h \rho_{yzh} C_{yh} C_{zh}$ and

$$E(e_{1h}e_{2h}) = \gamma_h \rho_{xzh} C_{xh} C_{zh}.$$

Expressing (2.3) in terms of $e's$, we have

$$\left(t_{RPe}^{S} - \overline{Y}\right) = \sum_{h=1}^{L} W_h \overline{Y}_h \left[e_{0h} + \frac{e_{2h} - e_{1h}}{2} + \frac{e_{1h}^2 - e_{2h}^2}{4} + \frac{1}{8}\left(e_{1h}^2 + e_{2h}^2 - 2e_{1h}e_{2h}\right) + \frac{e_{0h}e_{2h} - e_{0h}e_{1h}}{2} \right] \tag{2.4}$$

Now taking expectation of both sides of (2.4), the bias of the suggested estimator t_{RPe}^{S} to the first degree of approximation is obtained as

$$B\left(t_{RPe}^{S}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \overline{Y}_h \left[\frac{1}{8}\left(3C_{xh}^2 - C_{zh}^2 - 2\rho_{xzh} C_{xh} C_{zh}\right). + \frac{1}{2}\left(\rho_{yzh} C_{yh} C_{zh} - \rho_{yxh} C_{yh} C_{xh}\right) \right] \tag{2.5}$$

Squaring and taking expectation on both sides to (2.4), we get the mean squared error of the suggested estimator t_{RPe}^{S} up to the first degree of approximation as

$$MSE\left(t_{RPe}^{S}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + \frac{1}{4}\left(R_{1h}^2 S_{xh}^2 + R_{2h}^2 S_{zh}^2 - 2R_{1h}R_{2h}S_{xzh}\right) + \left(R_{2h}S_{yzh} - R_{1h}S_{yxh}\right) \right. \tag{2.6}$$

3. Efficiency comparisons of the suggested separate ratio-cum-product type exponential estimator t_{RPe}^{S} with the estimators $\overline{y}_{st}$, $\hat{\overline{Y}}_{RS}$, $\hat{\overline{Y}}_{PS}$, $\hat{\overline{Y}}_{S\mathrm{Re}}$ and $\hat{\overline{Y}}_{SPe}$

Up to the first degree of approximation, the variance of unbiased estimator $\overline{y}_{st}$ is given as

$$V(\overline{y}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2, \tag{3.1}$$

From (1.3), (1.4), (1.7), (1.8), (2.6) and (3.1), it is observed that the suggested estimator t_{RPe}^{S} would be more efficient than

(i) $\overline{y}_{st}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[R_{1h}^2 S_{xh}^2 + R_{2h}^2 S_{zh}^2 - 2R_{1h}R_{2h}S_{xzh} + 4\left(R_{2h}S_{yzh} - R_{1h}S_{yxh}\right)\right] < 0, \tag{3.2}$$

(ii) $\hat{\overline{Y}}_{RS}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[-3R_{1h}^2 S_{xh}^2 + R_{2h}^2 S_{zh}^2 - 2R_{1h} R_{2h} S_{xzh} + 4\left(R_{2h} S_{yzh} + R_{1h} S_{yxh}\right)\right] < 0 , \quad (3.3)$$

(iii) $\hat{\bar{Y}}_{PS}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[R_{1h}^2 S_{xh}^2 - 3R_{2h}^2 S_{zh}^2 - 2R_{1h} R_{2h} S_{xzh} - 4\left(R_{2h} S_{yzh} + R_{1h} S_{yxh}\right)\right] < 0 , \quad (3.4)$$

(iv) $\hat{\bar{Y}}_{S\mathrm{Re}}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[R_{2h}^2 S_{zh}^2 - 2R_{1h} R_{2h} S_{xzh} + 4R_{2h} S_{yzh} \right] < 0 , \quad (3.5)$$

(v) $\hat{\bar{Y}}_{SPe}$ if

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[R_{1h}^2 S_{xh}^2 - 2R_{1h} R_{2h} S_{xzh} - 4R_{1h} S_{yxh} \right] < 0 \ . \quad (3.6)$$

4. Separate generalized ratio-cum-product type exponential estimator

Singh et al. (2009) suggested a generalized ratio-cum-product type exponential estimator for population mean $\bar{Y}$ in simple random sampling as

$$\hat{\bar{Y}}_{RPe}^{(a,b)} = \bar{y}_{st} \exp\left[a\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) + b\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}} \right) \right] . \quad (4.1)$$

Lone et al. (2014) suggested a generalized ratio-cum-product type exponential estimator in stratified random sampling as

$$t_{RPe}^{(a,b)} = \bar{y}_{st} \exp\left[a\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}} \right) + b\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}} \right) \right] . \quad (4.2)$$

Motivated by Singh et al. (2009) and Lone et al. (2014), we suggested a separate generalized ratio-cum-product type exponential estimator for population mean as

$$t_{RPeS}^{(a,b)} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left[a\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h} \right) + b\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{z}_h + \bar{Z}_h} \right) \right] . \quad (4.3)$$

where (a, b) are suitably chosen constants and can be determined such that $t_{RPeS}^{(a,b)}$ can be minimized.

Up to the first degree of approximation, the bias of the estimator $t_{RPeS}^{(a,b)}$ can be obtained as

$$B\left(t_{RPeS}^{(a,b)}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \bar{Y}_h \left[\frac{C_{xh}^2 a}{4}\left\{\frac{a}{2}+1\right\} + \frac{C_{zh}^2 b}{4}\left\{\frac{b}{2}-1\right\} - \frac{2ab\rho_{xzh} C_{xh} C_{zh}}{8} + \frac{1}{2} b \rho_{yzh} C_{yh} C_{zh} - \frac{1}{2} a \rho_{yxh} C_{yh} C_{xh} \right] \quad (4.4)$$

and the mean squared error of the suggested estimator $t_{RPeS}^{(a,b)}$ up to the first degree of approximation is obtained as

$$MSE\left(t_{RPeS}^{(a,b)}\right)=\sum_{h=1}^{L}W_h^2\gamma_h\left\{S_{yh}^2+\frac{1}{4}\left(R_{1h}^2\,a^2S_{xh}^2+R_{2h}^2\,b^2S_{zh}^2-2a\,b\,R_{1h}\,R_{2h}\,S_{xzh}\right)\right.$$
$$\left.+\left(bR_{2h}\,S_{yzh}-aR_{1h}S_{yxh}\right)\right\}. \tag{4.5}$$

Remarks

For $(a,b)=(0,0),(1,0),\ (0,1)$ and $(1,1)$, $t_{RPeS}^{(a,b)}$ reduces to the estimators $\bar{y}_{st}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$, $\hat{\bar{Y}}_{SPe}$ and t_{RPe}^{S} respectively. Upto the first degree of approximation , the bias and mean squared error of the estimators $\bar{y}_{st}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$, $\hat{\bar{Y}}_{SPe}$ and t_{RPe}^{S} can be obtained from (4.4) and (4.5) by putting the values of $(a\,,b)$ by $(0\,,0),(1\,,0),(0\,,1)$ and $(1\,,1)$ respectively.

The mean squared error of $t_{RPeS}^{(a,b)}$ is minimum when

$$a=\frac{2(CD-EB)}{(AD-B^2)}=a_{10}(say)\quad \text{and}\ b=\frac{2(CB-EA)}{(AD-B^2)}=b_{10}(say)$$

where

$$A=\sum_{h=1}^{L}W_h^2\gamma_h\,R_{1h}^2S_{xh}^2\ ,B=\sum_{h=1}^{L}W_h^2\gamma_h\,R_{1h}\,R_{2h}S_{xzh}\,,$$

$$C=\sum_{h=1}^{L}W_h^2\gamma_h\,R_{1h}S_{yxh}\ \ ,\ D=\sum_{h=1}^{L}W_h^2\gamma_h\,R_{2h}^2S_{zh}^2\ ,$$

$$E=\sum_{h=1}^{L}W_h^2\gamma_hR_{2h}S_{yzh}\ \text{ and }\ H=\sum_{h=1}^{L}W_h^2\gamma_hS_{yh}^2\,.$$

Substituting $a=a_{10}$ and $b=b_{10}$ in (4.3), we get the asymptotically optimum estimator (AOE) of estimator $t_{RPeS}^{(a\ ,b\ )}$ as

$$t_{RPeS}^{(a_{10},b_{10})}=\sum_{h=1}^{L}W_h\bar{y}_h\exp\left[a_{10}\left(\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\bar{x}_h}\right)+b_{10}\left(\frac{\bar{z}_h-\bar{Z}_h}{\bar{z}_h+\bar{Z}_h}\right)\right]. \tag{4.6}$$

The mean squared error of the estimator $t_{RPeS}^{(a_{10},b_{10})}$ is given by

$$Min.MSE\left(t_{RPeS}^{(a,b)}\right)=MSE\left(t_{RPeS}^{(a_{10},b_{10})}\right)=H-\frac{\left(AE^2+DC^2-2BCE\right)}{AD(1-\rho^2)}. \tag{4.7}$$

where $\rho = \dfrac{\sum_{h=1}^{L} W_h^2 \gamma_h R_{1h} R_{2h} S_{xzh}}{\sqrt{\sum_{h=1}^{L} W_h^2 \gamma_h R_{1h}^2 S_{xh}^2 \sum_{h=1}^{L} W_h^2 \gamma_h R_{2h}^2 S_{zh}^2}}$.

5. Efficiency comparison of the generalized separate ratio-cum-product type exponential estimator $t_{RPeS}^{(a,b)}$ with the estimators $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$, $\hat{\bar{Y}}_{SPe}$ and t_{RPe}^{S}

From (1.3), (1.4), (1.7), (1.8), (2.6), (3.1) and (4.5) it is observed that the suggested estimator $t_{RPeS}^{(a,b)}$ would be more efficient than

(i) $\bar{y}_{st}$ if

$$a^2 A + b^2 D - 2a\,bB + 4bE - 4aC < 0\,, \tag{5.1}$$

(ii) $\hat{\bar{Y}}_{RS}$ if

$$a^2 A - a(2Bb + 4C) + (b^2 D - 4A + 4bE + 8C) < 0\,, \tag{5.2}$$

(iii) $\hat{\bar{Y}}_{PS}$ if

$$a^2 A - a(2Bb + 4C) + (b^2 D - 4D + 4bE - 8E) < 0\,, \tag{5.3}$$

(iv) $\hat{\bar{Y}}_{S\mathrm{Re}}$ if

$$a^2 A - a(2Bb + 4C) + (b^2 D - A + 4bE + 4C) < 0\,, \tag{5.4}$$

(v) $\hat{\bar{Y}}_{SPe}$ if

$$a^2 A - a(2Bb + 4C) + (b^2 D - D + 4bE - 4E) < 0\,, \tag{5.5}$$

(vi) t_{RPe}^{S} if

$$a^2 A - a(2Bb + 4C) + (b^2 D + 4bE - A - D + 2B - 4E + 4C) < 0\,. \tag{5.6}$$

6. Estimator based on estimated optimum

If the investigator is unable to guess the values of a_{10} and b_{10}, the only alternative left him is to replace a_{10} and b_{10} by its estimates $\hat{a}_{10}$ and $\hat{b}_{10}$.Thus the estimator based on the estimated optimum is

$$t_{RPeS}^{(\hat{a}_{10},\hat{b}_{10})} = \sum_{h=1}^{L} W_h \bar{y}_h \exp\left[\hat{a}_{10}\left(\frac{\bar{X}_h - \bar{x}_h}{\bar{X}_h + \bar{x}_h}\right) + \hat{b}_{10}\left(\frac{\bar{z}_h - \bar{Z}_h}{\bar{z}_h + \bar{Z}_h}\right)\right] \tag{6.1}$$

where

$$\hat{a} = \frac{2(\hat{C}\hat{D} - \hat{E}\hat{B})}{(\hat{A}\hat{D} - \hat{B}^2)} = \hat{a}_{10}(say) \quad , \hat{b} = \frac{2(CB - EA)}{(AD - B^2)} = \hat{b}_{10}(say)\ ,$$

$$\hat{A}=\sum_{h=1}^{L}W_h^2\gamma_h\,\hat{R}_{1h}^2 s_{xh}^2\ ,\ \hat{B}=\sum_{h=1}^{L}W_h^2\gamma_h\,\hat{R}_{1h}\,\hat{R}_{2h}\,s_{xzh}\,,$$

$$\hat{C}=\sum_{h=1}^{L}W_h^2\gamma_h\,\hat{R}_{1h}s_{yxh}\ ,\ \hat{D}=\sum_{h=1}^{L}W_h^2\gamma_h\,\hat{R}_{2h}^2 s_{zh}^2\ ,$$

$$\hat{E}=\sum_{h=1}^{L}W_h^2\gamma_h\hat{R}_{2h}s_{yzh}\ \text{ and }\ \hat{H}=\sum_{h=1}^{L}W_h^2\gamma_h s_{yh}^2\,.$$

Remark

The estimator based on the estimated optimum $t_{RPeS}^{(\hat{a}_{10},\hat{b}_{10})}$ has the same mean squared error to the first degree of approximation, as that of the estimator $t_{RPeS}^{(a_{10},b_{10})}$.

7. Empirical study

To judge the performance of the suggested estimators in comparison to other considered estimators, we consider a natural population data set. Description of the population is given below.

y : Productivity (MT/Hectare) ,
x : Production in "000" Tons and
z : Area in "000" Hectare

The required values of the parameters are summarized in Table **7**.1.

Constants	Stratum I	Stratum II
n_h	4	4
N_h	10	10
$\bar{X}_h$	10.4	309.4
$\bar{Y}_h$	1.7	3.67
$\bar{Z}_h$	6.23	80.67
S_{yh}	0.54	1.41
S_{xh}	3.53	80.54
S_{zh}	1.19	10.81
S_{yxh}	1.60	83.47
S_{yzh}	-0.2	-7.06
S_{xzh}	1.75	68.57

Table 7.1: Population- I Data Statistics [Source: Chouhan, S. 2012]

Estimators	PRE (…, $\bar{y}_{st}$)
$\bar{y}_{st}$	**100.00**
$\hat{\bar{Y}}_{RS}$	**227.38**
$\hat{\bar{Y}}_{PS}$	**121.75**
$\hat{\bar{Y}}_{S\mathrm{Re}}$	**172.00**
$\hat{\bar{Y}}_{SPe}$	**114.69**
t^{S}_{RPe}	**233.83**
$t^{(a,b)}_{RPeS}$	**262.85**

Table 7.2: Percent relative efficiencies of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$, $\hat{\bar{Y}}_{SPe}$, t^{S}_{RPe} and $t^{(a,b)}_{RPeS}$ with respect to $\bar{y}_{st}$

8. Conclusion

Section 3 provides the conditions under which the suggested estimator t^{S}_{RPe} has less mean squared error as compared to the mean squared errors of $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$ and $\hat{\bar{Y}}_{SPe}$. It has been observed from the table 7.2, that the suggested estimator t^{S}_{RPe} has maximum percent relative efficiency in comparisons to $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$ and $\hat{\bar{Y}}_{SPe}$ .It has also been observed that the separate generalized ratio-cum-product type exponential estimator $t^{(a,b)}_{RPeS}$ has maximum percent relative efficiency in comparison to $\bar{y}_{st}$, $\hat{\bar{Y}}_{RS}$, $\hat{\bar{Y}}_{PS}$, $\hat{\bar{Y}}_{S\mathrm{Re}}$, $\hat{\bar{Y}}_{SPe}$ and t^{S}_{RPe} . Thus the suggested estimators t^{S}_{RPe} and $t^{(a,b)}_{RPeS}$ are recommended for use in practice if the conditions defined in the section 3 and 5 are satisfied.

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