

PROBABILISTIC DROUGHT ANALYSIS OF WEEKLY RAINFALL DATA USING MARKOV CHAIN MODEL

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Abstract

This paper makes an attempt to investigate the pattern of occurrence of wet and dry weeks in three different locations of drought prone areas of India, i.e. Datia in Madhya Pradesh, Bellary in Karnataka and Solapur in Maharashtra. An index of drought proneness to evaluate its extent of degree has been worked out. Besides the probability of getting wet weeks consecutively for more than six, eight and ten weeks have been worked out. Also, the probability of sequence of more than three dry weeks is computed. The results of application of the Markov models are presented and discussed, exhibiting in particular the usefulness of transition probability matrix to agricultural planners and policy makers to understand the climatology of drought in rain fed areas so as to plan long term drought mitigation strategy.

Key Words: Drought Proneness Index, Dry Spell, Markov Chain Probability Model, Stationary Distribution, Steady State Probability.

1. Introduction

Drought is a naturally occurring event, causing temporary imbalance of water availability and vegetation damage. Drought has negative effects on surface and ground water resources, and soil moisture and agricultural productivity. When thinking of natural hazards, droughts are often perceived by society to play a less dominant role compared to floods. It affects natural environment of an area when it persists for a longer period. So, drought analysis plays an important role in the planning and management of natural resources and water resource system of an area. Unlike the effects of a flood which can be immediately seen and felt, droughts build up rather slowly, creeping and steadily growing (Lehner *et al.*, 2001). When drought occurs, it also leads to diminished power generation, disturbed riparian habits and suspended recreation activities (Mishra and Singh, 2010). Drought is considered by many researchers to be the most complex but least understood of all natural hazards, affecting more people than any other hazard (Siva Kumar *et al.*, 2005). To reduce the devastating effect of drought and minimize the losses, preparedness and early warning system can help decision and policy makers to implement policies timely. Once the drought started,

the probability of occurrence of drought severity category changing from one severity level to another severity level (i.e. probability that the drought will move to more severity or less severity state) should be known for taking mitigation measures against adverse drought effect. In some recent studies Alam *et al.* (2014a) used seasonal and non-seasonal ARIMA model to forecast drought for Bundelkhand region in Central India. Alam *et al.* (2014b) study aims to analyzed temporal variation and frequency analysis of meteorological droughts using SPI in Bellary region and to undertake frequency analysis of drought using the generalized extreme value (GEV) distribution.

Markov chain modelling approach is useful in understanding the stochastic characteristics of drought through the analysis of probabilities for each severity category changing from any drought severity category (Paulo and Pereire, 2007). Paulo and Pereire (2007) used Markov chain approach to characterize the stochasticity of droughts. They found that the approach can be satisfactorily used as a predictive tool for forecasting transitions among drought severity classes ahead of time. Alam *et al.* (2012) analysed the severity and magnitude of drought using SPI and developed a futuristic precipitation scenario using Markov model for Datia district in Madhya Pradesh, a representative area of Bundelkhand region.

Banik *et al.* (2002) used Markov Chain model to evaluate the probabilities of getting wet and dry weeks during monsoon period. Ochola and Kerkides (2003) applied a Markov chain model to predict dry spells. An evaluation of some drought indices with a multistate Markov model has been performed by Steinemann (2003), who concluded that there were differences among the performance of the drought indicators and their trigger thresholds, which can influence drought responsive measures.

This paper aims at characterizing weekly droughts in Datia in Madhya Pradesh, Bellary in Karnataka and Solapur in Maharashtra of India. The attempt to compute absolute probability of a week being wet during the monsoon period is presented and discussed. An index of drought proneness has been worked out to measure extent of drought proneness.

2. Materials and methods

2.1 Study area and data

The weekly rainfall data of Bellary (15.15° N, 76.93° E), Datia (25.67° N, 78.46° E) and Solapur (17.68° N, 75.92° E) were collected for the period from 1968-2011. Bellary region falls in the southern state of Karnataka. The state is the 9th largest state in India, covering an area of 191976 sq.km, but has the 2nd largest arid zone after Rajasthan. Drought analysis indicates that 5 droughts of varying intensities occur in a decade. Low rainfall and a short growing season (8-14 weeks) restrict the choice of crops, limit ground water recharge and often lead to high soil erosion rates due to the nature of the soils, which are highly dispersible clays (Vertisols). Datia is located in Bundelkhand region which represents the southern part of Uttar Pradesh (spread over 7 districts) and northern part of Madhya Pradesh (spread over 6 districts) occupying an area of 7 million hectares. The area has a flat to rolling topography (300-450 msl) with hard rock formations, poor aquifer recharge and high run off potential. Unexpected breaks, delayed onset or early withdrawal are fairly common in the region and rainfall deficit was 25%, 33%, 45% and 56% during 2004-05, 2005-06, 2006-07 and 2007-08,

respectively. Solapur District is one of the worst drought-hit districts in the state of Maharashtra in India. Presently, Solapur covers more than 141 villages which are entirely dependent on tankers for drinking water.

It has been established by several authors (Gabriel and Neumann, 1962, Caskey, 1963, Katz, 1974 and Rahman 1999 etc.) that rainfall sequence can be described by Markov chain model. A Markov chain (Çinlar, 1975) is a stochastic process X , such as at any time t , X_{t+1} is conditionally independent from $X_0, X_1, X_2, \dots, X_{t-1}$, given X_t ; the probability that X_{t+1} takes a particular value j depends on the past preceding only through its most recent value X_t . $P\{X_{t+1} = j | X_0, X_1, \dots, X_t\} = P\{X_{t+1} = j | X_t = i\} \forall i, j, i \in S, j \in T$.

2.2 Method 1

Let $X_0, X_1, X_2, \dots, X_t$, be random variables distributed identically and taking only two values, namely 0 and 1, with probability one, i.e., $X_t=0$ if n^{th} week is dry and 1 if n^{th} week is wet. The week with rainfall less than the threshold value (a minimum amount, say 2.5 mm) is considered to be a dry week.

$$P\{X_{t+1} = j | X_0, X_1, \dots, X_t\} = P\{X_{t+1} = j | X_t = i\} \forall i, j, i \in S, j \in T$$

i.e. probability of wetness of any week depends only on whether the previous week was wet or dry. For the transition matrix

$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$$

where $P_{ij} = P(X_{t+1}=j | X_t=i); i, j=0, 1$.

Let p is the absolute probability of a week being wet during the monsoon period i.e. $p=P(X_0=1)$. Therefore, $P(X_0=0)=1-p$. For a stationary distribution

$$[1 - p \quad p] \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} = [1 - p \quad p]$$

which gives

$$p = \frac{P_{01}}{1 - (P_{11} - P_{01})} \tag{1}$$

It is further assumed that the P_{ij} 's remaining constant over the year. The maximum likelihood estimate of P_{01} and P_{11} are appropriate relative functions (Woolhiser and Pegram, 1979).

A wet spell of length k is defined as sequences of k wet weeks preceded and followed by weeks. Dry spells are defined correspondingly. By "probability of wet spell of length k " we mean the probability of a wet spell of length k given that this week is wet, i.e.

$$P(W = k) = (1 - P_{11})P_{11}^{k-1} \tag{2}$$

and probability of wet sequences with length greater than k is

$$P(W > k) = P_{11}^k \tag{3}$$

Similarly, probability of dry sequences with length greater than m is:

$$P(W > m) = (1 - P_{01})^m \tag{4}$$

Transition probabilities can be analysed for short term and long term planning, and the probabilistic characterization of the procession and recession of droughts.

2.2.1 Index of drought-proneness

P_{11} gives the probability of a week to be wet given that previous week was wet also. When P_{11} is large, the chance of wet weeks is also large. But only a small of P_{11} may not indicate high drought-proneness. In this case, large value of P_{01} implies a large number of short wet spells which can prevent occurrence of drought. Hence an index of drought-proneness may be defined as:

$$DI = P_{11} \times P_{01} \quad (5)$$

Zero and one bound this index of droughts. Higher the value of DI, lower will be the degree of drought-proneness (Table 1).

Criteria	Degree of drought proneness
$0.0 < DI \leq 0.125$	Chronic
$0.125 < DI \leq 0.180$	Severe
$0.180 < DI \leq 0.235$	Moderate
$0.235 < DI \leq 0.310$	Mild
$0.310 < DI \leq 1.000$	Occasional

Table 1: Index of drought proneness

Sequence of wet and dry week probability was calculated using Markov chain model for annual as well as for the four seasons *viz.* monsoon (June to September), pre-monsoon (March to May), post monsoon (October to December) and winter (January and February). Transition probability, probability of expected number of wet weeks, probability of getting wet weeks of different length and also probability of getting continuous 3 dry weeks and also the index of drought proneness for each season been calculated.

2.3 Method 2

Under the same set up, let us assume that probability of wetness of any week depends only on whether the two preceding weeks were wet or dry. In other words, given the event on previous two weeks, the probability of wetness is independent of further preceding weeks i.e.

$$P(X_{n+1} = x_{n+1} | X_n = x_n, \dots, X_0 = x_0) = P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1})$$

where $x_0, x_1, \dots, x_{n+1} \in \{0, 1\}$.

Now define

$$Y_0 = (X_0, X_1), Y_1 = (X_2, X_3), Y_2 = (X_4, X_5), \dots, Y_n = (X_{2n}, X_{2n+1})$$

Now,

$$P(Y_{n+1} = y_{n+1} | Y_n = y_n, Y_{n-1} = y_{n-1}, \dots, Y_0 = y_0) = P(Y_{n+1} = y_{n+1} | Y_n = y_n)$$

where $y_0, y_1, \dots, y_{n+1} \in \{(0,1), (0,1), (1,0), (1,1)\}$. Now the stochastic process $\{Y_n, n=0, 1, 2, \dots\}$ is a Markov chain (Bhat, 1972; Ochi, 1990 and Chung, 1974).

Consider the transition matrix

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix}$$

where $a_{ij} = P(Y_1 = j | Y_0 = i)$

$$i \text{ or } j = \begin{cases} 0 & \text{stands for the state } (0, 0) \\ 1 & \text{stands for the state } (0, 1) \\ 2 & \text{stands for the state } (1, 0) \\ 3 & \text{stands for the state } (1, 1) \end{cases}$$

Let, $P(0,0) \rightarrow P_1, P(0,1) \rightarrow P_2, P(1,0) \rightarrow P_3, P(1,1) \rightarrow P_4$.

Note that $\sum_{i=1}^4 P_i = 1; \sum_{j=0}^3 a_{ij} = 1, i = 0, 1, 2, 3$.

For a stationary distribution

$$[P_1 \ P_2 \ P_3 \ P_4] \begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix} = [P_1 \ P_2 \ P_3 \ P_4]$$

Here, when we find the probability of a wet spell of length k , we actually mean it is the probability of a wet spell of length k given that this week is wet and the previous week was a dry one. Similarly we account for dry spells also.

$P(W = k) = P(\text{Wet spell of length } k \mid \text{this week is wet and previous week was dry})$

For odd k and $k \geq 3$

$$\begin{aligned} P(W = k) &= P(W = 2m - 1), \quad m \geq 2 \\ &= a_{13} a_{33}^{m-2} (a_{30} + a_{31}) \end{aligned} \tag{6}$$

For even k and $k \geq 4$

$$\begin{aligned} P(W = k) &= P(W = 2n), \quad n \geq 2 \\ &= a_{13} a_{33}^{n-2} a_{32} \end{aligned} \tag{7}$$

$P(W=2) = P((1,0) \mid (0,1)) = a_{12}$

$$\begin{aligned} P(W \geq 2t) &= \sum_{k=2t}^{\infty} P(W = k), \quad t \geq 2 \\ &= (a_{32} + a_{33}(a_0 + a_{31})) \frac{a_{13} a_{33}^{t-2}}{1 - a_{33}} \end{aligned} \tag{8}$$

$$\begin{aligned} P(W \geq 2t - 1) &= P(W \geq 2t) + P(W = 2t - 1), \quad t \geq 2 \\ &= a_{13} a_{33}^{t-2} \end{aligned} \tag{9}$$

Similarly, $P(D=k) = P(\text{Dry spell of length } k \mid \text{this week is dry and the previous week was wet})$

For odd k and $k \geq 3$

$$\begin{aligned} P(W = k) &= P(W = 2m - 1), \quad m \geq 2 \\ &= a_{20} a_{00}^{m-2} (a_{02} + a_{03}) \end{aligned} \tag{10}$$

For even k and $k \geq 4$

$$\begin{aligned} P(W = k) &= P(W = 2n), \quad n \geq 2 \\ &= a_{20} a_{00}^{n-2} a_{01} \end{aligned} \tag{11}$$

Also,

$$P(D \geq 2t) = (a_{01} + a_{00}(a_{02} + a_{03})) \frac{a_{02} a_{00}^{t-2}}{1 - a_{00}}, \quad t \geq 2 \tag{12}$$

$$P(W \geq 2t - 1) = a_{20} a_{00}^{t-2}, \quad t \geq 2 \tag{13}$$

Let U be the random variable such that $U = \text{number of wet weeks among } 2n \text{ week period}$. Therefore, $U = F(Y_0) + F(Y_1) + \dots + F(Y_{n-1})$ where

$f(0,0) = 0; f(0,1) = 1; f(1,0) = 1$ and $f(1,1) = 2$

For large n , $U \sim (n\mu, n\sigma^2)$ [for large n , $2n \sim 2n-1$] (Medhi, 1981)

where

$$\mu = f(0,0) \times P_1 + f(0,1) \times P_2 + f(1,0) \times P_3 + f(1,1) \times P_4 \tag{14}$$

$$\sigma^2 = \mathbf{F}'\mathbf{C}\mathbf{F} \tag{15}$$

where, $\mathbf{F} = \begin{bmatrix} f(0,0) \\ f(0,1) \\ f(1,0) \\ f(1,1) \end{bmatrix}$

and $\mathbf{C} = (c_{ij})$ where $c_{ij} = \mathbf{P}_i z_{ij} \mathbf{P}_j z_{ji} \mathbf{P}_i \delta_{ij} \mathbf{P}_i \mathbf{P}_j$

where $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$

$Z = ((z_{ij}))$

$$= \begin{bmatrix} 1 + P_1 - a_{00} & P_2 - a_{01} & P_3 - a_{02} & P_4 - a_{03} \\ P_1 - a_{10} & 1 + P_2 - a_{11} & P_3 - a_{12} & P_4 - a_{13} \\ P_1 - a_{20} & P_2 - a_{21} & 1 + P_3 - a_{22} & P_4 - a_{23} \\ P_1 - a_{30} & P_2 - a_{31} & P_3 - a_{32} & 1 + P_4 - a_{33} \end{bmatrix}^{-1}$$

3. Results and discussion

At the first step towards the fitting of a 2-state Markov chain model, the raw data on the weekly rainfall are classified into four classes according to the conditional events E_{00}, E_{01}, E_{10} and E_{11} . From the actual frequencies of these classes, the corresponding relative frequencies are computed in order to obtain the maximum likelihood estimates of the transition conditional probabilities P_{00}, P_{01}, P_{10} and P_{11} alongwith absolute probability p for the three centres.

3.1. Method 1

Transition probability matrix developed for the three region using Markov chain model produces the answer of this question: once the drought starts (1) what is the probability that the drought will move to more severity or less severity state: (2) what is the probability that the drought continues in the same severity. Once these questions are understood it will be helpful in taking mitigation measures against adverse drought effect.

3.1.1 Annual

Drought Index is less (<0.150) in all the three areas indicating that they are severe drought prone zones. Annually, Datia receives almost 17 rainfall weeks which is lesser than Bellary (19) and Solapur (21) but probability of getting two consecutive wet weeks is lesser in Bellary (32%) than Datia and Solapur (46%) (Table 2).

Station	p	P ₀₁	P ₁₁	E(Y)	DI	Probability of getting wet weeks					P(D>3)
						2	4	6	8	10	
Bellary	0.37	0.25	0.57	19.05	0.142	0.32	0.10	0.03	0.01	0.003	0.42
Datia	0.32	0.15	0.68	16.87	0.105	0.46	0.21	0.10	0.05	0.021	0.60
Solapur	0.40	0.21	0.67	20.58	0.144	0.46	0.217	0.09	0.04	0.020	0.49

Table 2: Annual probabilistic drought characteristics using Method-1

3.1.2 Monsoon season

Higher DI value (>0.310) in all the three places indicates that during monsoon season there is less dry week in the monsoon season. The expected number of wet

weeks is also greater than 10 for the three places. Among the three Solapur is the wettest during monsoon season (Table 3).

Station	p	P ₀₁	P ₁₁	E(Y)	DI	Probability of getting wet weeks					P(D>3)
						2	4	6	8	10	
Bellary	0.59	0.54	0.63	10.63	0.348	0.39	0.15	0.06	0.02	0.01	0.10
Datia	0.71	0.47	0.81	12.78	0.397	0.65	0.43	0.28	0.18	0.12	0.15
Solapur	0.77	0.64	0.80	13.78	0.515	0.65	0.42	0.27	0.17	0.11	0.05

Table 3: Probabilistic drought characteristics for Monsoon season (22-39 weeks) using Method-1

3.1.3 Pre-monsoon season

Probability of getting wet weeks during pre-monsoon season is very less in all the three places. Expected number of wet week during this period is <2 days for Datia, <3 days for Solapur and <4 days for Bellary. Probability of getting 3 continuous dry weeks is also very high and maximum has been recorded for Datia followed by Solapur (Table 4).

Station	p	P ₀₁	P ₁₁	E(Y)	DI	Probability of getting wet weeks			P(D>3)
						2	4	6	
Bellary	0.28	0.22	0.43	3.69	0.097	0.190	0.030	0.010	0.47
Datia	0.13	0.12	0.18	1.65	0.022	0.034	0.001	0.000	0.68
Solapur	0.22	0.21	0.26	2.86	0.054	0.067	0.004	0.000	0.49

Table 4: Probabilistic drought characteristics for Pre-Monsoon season (9-21 week) using Method-1

3.1.4 Post-monsoon season

Though expected number of wet weeks is increases as compared to pre monsoon season for Bellary (<5) and Solapur (<3), Datia become more dry in post monsoon season, where probability of getting three consecutive dry week is as high as 75% (Table 5).

Station	P	P ₀₁	P ₁₁	E(Y)	DI	Probability of getting wet weeks			P(D>3)
						2	4	6	
Bellary	0.34	0.26	0.49	4.40	0.128	0.245	0.060	0.015	0.41
Datia	0.11	0.09	0.28	1.49	0.026	0.079	0.006	0.000	0.75
Solapur	0.26	0.16	0.52	3.33	0.086	0.275	0.076	0.021	0.59

Table 5: Probabilistic drought characteristics for Post-Monsoon season (40-52 week) using Method-1

3.1.5 Winter season

Winter is very dry where expected number of wet weeks is less than 1 in all the three places. Probability of getting more than three dry weeks 0.91 for Bellary, 0.83 for Solapur and 0.74 for Datia. Though Datia is drier than other places during other three seasons, situation become more severe in Bellary and Solapur during winter season (Table 6).

Station	P	P ₀₁	P ₁₁	E(Y)	DI	Probability of getting wet weeks			P(D>3)
						at least			
						2	4	6	
Bellary	0.04	0.03	0.21	0.32	0.007	0.046	0.002	0.000	0.91
Datia	0.12	0.10	0.29	0.96	0.028	0.086	0.007	0.001	0.74
Solapur	0.07	0.06	0.21	0.57	0.013	0.043	0.002	0.000	0.83

Table 6: Probabilistic drought characteristics for Winter season (1-8 week) using Method-1

3.2 Method 2

Tables 7-11 give the values of P₁, P₂, P₃ and P₄ as per Method-2. The probabilities of getting at least 4,6,8 and 10 wet weeks were worked out using equations (4) and (5) under the assumptions of normality. But, the probabilities worked out using Method-1 as given in Tables 2-6 are more reliable because of the conditions for assumption of normality are more favourable for method1 (Rahmann 1999).

Station	P ₁	P ₂	P ₃	P ₄	E(U)	Probability of getting wet weeks at least				P(D>3)
						4	6	8	10	
						Bellary	0.468	0.157	0.016	
Datia	0.002	0.095	0.176	0.727	44.84	0.195	0.123	0.078	0.049	0.554
Solapur	0.116	0.126	0.189	0.569	37.76	0.289	0.152	0.079	0.042	0.475

Table 7: Annual probabilistic drought characteristics using Method-2

Station	P ₁	P ₂	P ₃	P ₄	E(U)	Probability of getting wet weeks at least				P(D>3)
						4	6	8	10	
						Bellary	0.189	0.223	0.222	
Datia	0.103	0.049	0.136	0.762	15.38	0.495	0.341	0.235	0.162	0.314
Solapur	0.079	0.154	0.183	0.584	13.54	0.519	0.327	0.206	0.130	0.106

Table 8: Probabilistic drought characteristics for Monsoon season (22-39 weeks) using Method-2

Station	P ₁	P ₂	P ₃	P ₄	E(U)	Probability of getting wet weeks at least			P(D>3)
						4	6	8	
						Bellary	0.450	0.239	
Datia	0.027	0.065	0.079	0.830	11.72	0.013	0.004	0.001	0.702
Solapur	0.079	0.154	0.183	0.584	13.54	0.519	0.327	0.206	0.106

Table 9: Probabilistic drought characteristics for Pre-Monsoon season (9-21 week) using Method-2

Station	P ₁	P ₂	P ₃	P ₄	E(U)	Probability of getting wet weeks at least			P(D>3)
						4	6	8	
						Bellary	0.490	0.171	
Datia	0.409	0.059	0.459	0.071	4.30	0.042	0.005	0.0005	0.869
Solapur	0.397	0.127	0.174	0.303	5.88	0.047	0.006	0.0003	0.379

Table 10: Probabilistic drought characteristics for Post-Monsoon season (40-52 week) using Method-2

Station	P ₁	P ₂	P ₃	P ₄	E(U)	Probability of getting wet weeks at least			P(D>3)
						4	6	8	
						Bellary	0.433	0.045	
Datia	0.101	0.117	0.189	0.594	5.97	0.057	0.023	0.009	0.724
Solapur	0.456	0.329	0.080	0.134	2.71	0.000	0.000	0.000	0.850

Table 11: Probabilistic drought characteristics for Winterseason (1-8 week) using Method-2

4. Conclusion

To obtain sequences of dry and wet spells during different seasons, Markov Chain model has been fitted to weekly precipitation data. These sequences of wet and dry spells can be an aid to understand drought-proneness which has been identified with the help of a simple index. In chronic drought-prone areas the crop failure is very frequent. A good crop may be raised in about 35% of the years in severe drought prone areas. In moderately and mild drought-prone areas a good crop may be harvested in about 40-50% and 50-55% of the years respectively and crop prospect is high in occasionally drought-prone areas. The results of this paper will be useful to agricultural planners and irrigation engineers to identifying the areas where agricultural development should be focused as a short term and long term drought mitigation strategy. The models developed in this study seem promising in the analysis of rainfall

in arid and semi-arid zones. A rigorous data collection scheme is needed to get reliable data for such analysis.

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