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RELIABILITY MEASURES OF A SYSTEM OF TWO NON-IDENTICAL UNITS WITH PRIORITY SUBJECT TO WEATHER CONDITIONS

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Abstract

In this paper, reliability measures of a system of two non-identical units operating under normal and abnormal weather conditions are obtained in steady state using semi-Markov process and regenerative point technique. Initially, one original unit (called main unit) is operative while the other substandard unit (called duplicate unit) is taken as spare in cold standby. Each unit has direct complete failure from normal mode. There is a single server who visits the system immediately to carry out repair of the failed unit. However, repair of the failed unit is not allowed in abnormal weather while system remains operative. Priority is given to operation and repair of main unit over duplicate unit. The distributions for failure times of the units and time to change of weather conditions are taken as negative exponential while that of repair time of the units are arbitrary. All random variables are statistically independent. The results for some important reliability measures have been analyzed graphically for arbitrary values of various parameters and costs.

Key Words: Non-identical Units, Priority, Weather Conditions, Reliability Measures.

1. Introduction

A lot of research work on reliability modeling of maintained systems has been carried out by the researchers in the field of reliability by considering identical units and static environmental conditions. Osaki and Asakura, (1970), Goel and Sharma (1989) and Kadyan et al.(2012) investigated reliability models of such systems with different repair policies. But, sometimes it is very difficult to afford a high cost identical unit in spare. In such a situation, a substandard unit might be taken as spare in cold standby not only to protect operation of the system but also to minimize the operating cost. Kishan and Jain (2012) and Mokkadis et al. (1989) discussed standby systems of non-identical units with different sets of assumptions on failure and repair. Further, the performance of repairable systems can be improved by giving priority in repair disciplines. Kadian et al.(2004) introduced the concept of priority while analyzing a system of non-identical units. Furthermore, it is very difficult to keep the environmental conditions under control which may fluctuate due to changing climate and other natural catastrophic. Therefore, Goel and Sharma(1985), Gupta and Goel(1991), Gupta et al. (2010) have obtained reliability measures of cold standby repairable systems operating under different weather conditions.

While considering above facts and practical situations in mind, here reliability measures of a system of non-identical units operating under different weather conditions- normal and abnormal are obtained using semi-Markov process and regenerative point technique. Initially, one original unit (called main unit) is operative while the other substandard unit (called duplicate unit) is taken as spare in cold standby. Each unit has direct complete failure from normal mode. A single server is available immediately for repairing the failed unit. Priority is given to operation and repair of main unit over duplicate unit. Repair activities are not allowed in abnormal weather whereas system remains operative. The distributions of failure time of the units and time to change of weather conditions are taken as negative exponential while the distributions for repair time of the units are arbitrary with different probability density functions. The expressions for some measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure (MTSF), availability, busy period of the server and profit function are derived in steady state. The graphs for MTSF, availability and profit have been drawn with respect to normal weather rate for arbitrary values of various parameters and costs.

2. System description and assumptions

(i) There is a system of two non-identical units- one original unit (called main unit) and the other substandard unit(called duplicate unit).
(ii) The main unit is initially operative and the duplicate unit is taken as spare in cold standby.
(iii) A single repair facility is provided immediately as and when required.
(iv) There are two weather conditions-normal and abnormal.
(v) Priority is given to operation and repair of the main unit over duplicate unit.
(vi) Repair activities are not allowed in abnormal weather.
(vii) System remains operative in both abnormal and normal weather conditions.
(viii)The random variables are statistically independent.
(ix) The unit works as new after repair.
(x) Switch devices are perfect.
(xi) The distributions of failure time of the units and time to change of weather conditions are taken as negative exponential while the distributions for repair time of the units are arbitrary with different probability density functions.

3. Notations

E	: The set of regenerative states
MO/DO	: Main/Duplicate unit is good and operative
$\overline{MO}$ / $\overline{DO}$	: Main/Duplicate unit is good and operative in abnormal weather
DCs/ $\overline{DCs}$	: Duplicate unit is in cold standby in normal weather/ abnormal weather
λ/λ_1	: Constant failure rate of Main /Duplicate unit
β/β_1	: Constant rate of change of weather from normal to abnormal/abnormal to normal weather

MFur/DFur : Main/duplicate unit failed and under repair
MFUR/DFUR : Main/duplicate unit failed and under repair continuously from previous state
MFwr/DFwr : Main/duplicate unit failed and waiting for repair
MFWR/DFWR : Main/duplicate unit failed and waiting for repair continuously from previous state
$\overline{\text{MFwr}}$ / $\overline{\text{DFwr}}$: Main/Duplicate unit failed and waiting for repair due to abnormal weather
$\overline{\text{MFWR}}$ / $\overline{\text{DFWR}}$: Main/Duplicate unit failed and waiting for repair continuously from previous state due to abnormal weather
$g(t)/G(t)$: pdf/cdf of repair time of Main unit
$g_1(t)/ G_1(t)$: pdf/cdf of repair time of Duplicate unit
$q_{ij}(t)/Q_{ij}(t)$: pdf/cdf of passage time from regenerative state i to a regenerative state j or to a failed state j without visiting any other regenerative state in (0,t]
$q_{ij.kr}(t)/ Q_{ij.kr}(t)$: pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k,r once in (0,t]
$q_{ij.k,(r,s)}{}^n(t)/Q_{ij.k,(r,s)}{}^n(t)$: pdf/cdf of direct transition time from regenerative state i to a regenerative state j or to a failed state j visiting state k once and n times through states r and s.
$M_i(t)$: Probability that the system is up initially in regenerative state S_i at time t without visiting to any other egenerative state
$W_i(t)$: Probability that the server is busy in state S_i up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states
m_{ij} : The contribution to mean sojourn time in regenerative state S_i when system is to make transition in to regenerative state S_j. Mathematically, it can be written as

$$m_{ij} = E(T_{ij}) = \int_0^\infty t d[Q_{ij}(t)] = -q_{ij}^{*'}(0)\text{, where}$$

T_{ij} is the transition time from state S_i to S_j; S_i , S_j ε E.

μ_i : The mean Sojourn time in state S_i this is gven by

$$\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}\text{, where } T_i \text{ is}$$

the sojourn time in state S_i

Ⓢ/©/©^{n} : Symbol for Laplace Stieltjes convolution/Laplace convolution/ Laplace convolution n times
~ / * : Symbol for Laplace Steiltjes Transform (LST)/ Laplace Transform (LT)

The following are the possible transition states of the system

S_0 = (MO, DCs), S_1 = (MFur, DO), S_2 = ($\overline{\text{MO}}$, $\overline{\text{DCs}}$), S_3= ($\overline{\text{MFwr}}$, $\overline{\text{DO}}$),

S_4= (MFUR,DFwr),S_5=($\overline{\text{MFWR}}$, $\overline{\text{DFwr}}$), S_6=($\overline{\text{MFwr}}$, $\overline{\text{DFWR}}$), S_7=(MO,DFur),

S_8= (MFur,DFWR),S_9=($\overline{\text{MO}}$, $\overline{\text{DFwr}}$),S_{10}=(MFur,DFwr)

The statesS_0, S_1, S_2, S_3, S_7, S_9, S_{10} are regenerative while the states S_4, S_5, S_6, S_8 are non regenerative as shown in figure1

State transition diagram

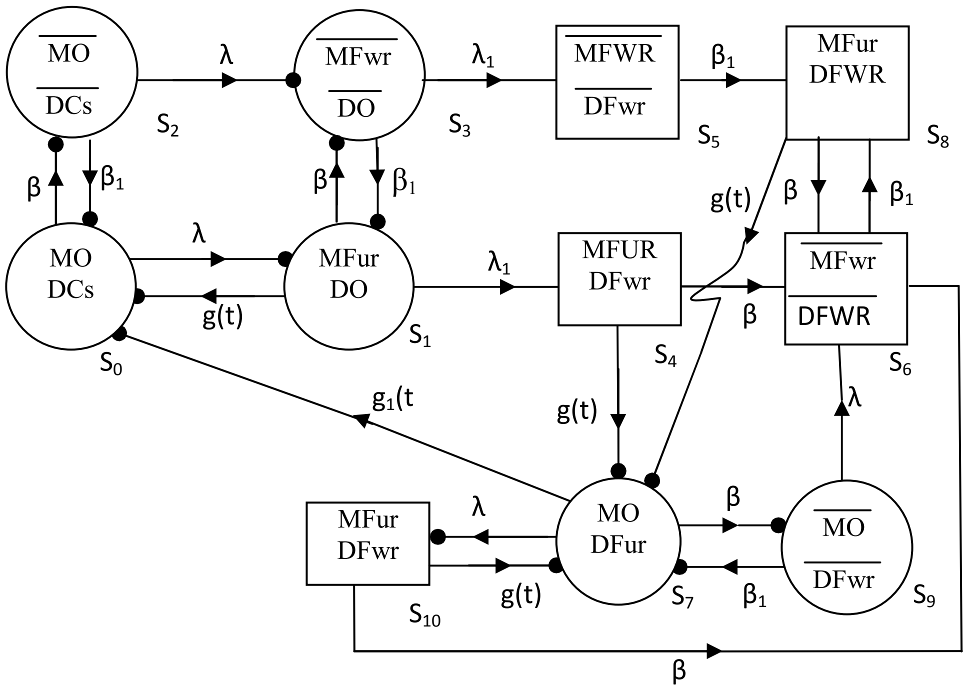

Fig.1

○ Up-state □ Failed State ● Regenerative point

4. Transition probabilities and mean sojourn times

Simple probabilistic considerations yield the following expressions for the non-zero elements: $p_{ij}= \lim_{t\to\infty} Q_{ij}(t)$ =∫q_{ij}(t)dt. The transition probabilities are as follows:

$$dQ_{01}(t) = \lambda e^{-(\beta+\lambda)t}dt,\ dQ_{02}(t) = \beta e^{-(\beta+\lambda)t}dt$$

and similarly we can obtain other differential transition probabilities. The transiton probabilities between states can be determined by using Laplace transform Technique. We have

$p_{01}=\frac{\lambda}{\beta+\lambda}$, $p_{02}=\frac{\beta}{\beta+\lambda}$, $p_{10}=g^*(\beta+\lambda_1)$, $p_{46}=1-g^*(\beta)$, $p_{86}=1-g^*(\beta)$,

$p_{13}=\frac{\beta}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{14}=\frac{\lambda_1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $p_{47}=g^*(\beta)$, $p_{58}=1$,

$p_{20}=\frac{\beta_1}{\beta_1+\lambda}$, $p_{23}=\frac{\lambda}{\beta_1+\lambda}$, $p_{31}=\frac{\beta_1}{\beta_1+\lambda_1}$, $p_{35}=\frac{\lambda_1}{\beta_1+\lambda_1}$, $p_{68}=1$, $p_{70}=g_1^*(\beta+\lambda)$, $p_{79}=$
$\frac{\beta}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{7,10}=\frac{\lambda}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $p_{87}=g^*(\beta)$, $p_{10,6}=1-g^*(\beta)$,

$p_{96}=\frac{\lambda}{\beta_1+\lambda}$, $p_{97}=\frac{\beta_1}{\beta_1+\lambda}$, $p_{10,7}=g^*(\beta)$ (1)

It can be easily verified that

$p_{01}+p_{02}=p_{10}+p_{13}+p_{14}=p_{20}+p_{23}=p_{31}+p_{35}=p_{46}+p_{47}=p_{58}=1$
$p_{68}=p_{70}+p_{79}+p_{7,10}=p_{86}+p_{87}=p_{96}+p_{97}=p_{10,1}+p_{10,5}=1$ (2)

The mean sojourn times (μ_i) in the state S_i are

$\mu_0=\frac{1}{\beta+\lambda}$, $\mu_1=\frac{1}{\beta+\lambda_1}(1-g^*(\beta+\lambda_1))$, $\mu_2=\frac{1}{\beta_1+\lambda}$, $\mu_3=\frac{1}{\beta_1+\lambda_1}$, $\mu_4=\frac{1}{\beta}(1-g^*(\beta)$,

$\mu_5=\frac{1}{\beta_1}$, $\mu_6=\frac{1}{\beta_1}$, $\mu_7=\frac{1}{\beta+\lambda}(1-g_1^*(\beta+\lambda))$, $\mu_8=\frac{1}{\beta}(1-g^*(\beta))$,

$\mu_9=\frac{1}{\beta_1+\lambda}$, $\mu_{10}=\frac{1}{\beta}(1-g^*(\beta))$ (3)

Also

$m_{01}+m_{02}=\mu_0$, $m_{10}+m_{13}+m_{14}=\mu_1$, $m_{20}+m_{23}=\mu_2$, $m_{31}+m_{35}=\mu_3$, $m_{46}+m_{47}=\mu_4$,
$m_{58}=\mu_5$, $m_{68}=\mu_6$, $m_{70}+m_{79}+m_{7,10}=\mu_7$, $m_{86}+m_{87}=\mu_8$, $m_{96}+m_{97}=\mu_9$, $m_{10,1}+m_{10,5}=\mu_{10}$ (4)

and

$\mu'_1=m_{10}+m_{13}+m_{17.4}+m_{17.4,(6,8)}{}^n$, $\mu_3'=m_{31}+m_{37.58}+m_{37.58(6,8)}{}^n$,
$\mu'_9=m_{97}+m_{97.(6,8)}{}^n$, $\mu_{10}=m_{10,7}+m_{10,7.(6,8)}{}^n$ (5)

5. Reliability and mean time to system failure (MTSF)

Let $\phi_i(t)$ be the cdf of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\phi_i(t)$:

$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t) + Q_{02}(t) Ⓢ \phi_2(t)$

$\phi_1(t) = Q_{10}(t) Ⓢ \phi_0(t) + Q_{13}(t) Ⓢ \phi_3(t) + Q_{14}(t)$

$\phi_2(t) = Q_{20}(t) Ⓢ \phi_0(t) + Q_{23}(t) Ⓢ \phi_3(t), \quad \phi_3(t) = Q_{31}(t) Ⓢ \phi_1(t) + Q_{35}(t)$ (6)

Taking LST of above relations (6) and solving for $\tilde{\phi}_0(s)$ We have

$$R^*(s) = \frac{1-\tilde{\phi}_0(s)}{s} \quad (7)$$

The reliability of the system model can be obtained by taking inverse Laplace transform of (7).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to o} \frac{1-\tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1}, \text{ where} \quad (8)$$

$N_1 = (1-p_{13}p_{31})(p_{02}\mu_2+\mu_0)+p_{01}(\mu_1+p_{13}\mu_1)+p_{02}p_{23}(\mu_1 p_{31}+\mu_3)$

$D_1 = (1-p_{13}p_{31})(1-p_{02}p_{20})-p_{10}(p_{01}+p_{02}p_{23}p_{31})$ (9)

6. Steady state availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0.The recursive relations for $A_i(t)$ are given as

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t)$

$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + q_{13}(t) © A_3(t) + (q_{17.4}(t) + q_{17.4,(6,8)}^n(t)) © A_7(t)$

$A_2(t) = M_2(t) + q_{20}(t) © A_0(t) + q_{23}(t) © A_3(t), A_3(t) = M_3(t) + q_{31}(t) © A_1(t) + (q_{37.58}(t) + q_{37.5,(8,6)}n) © A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t) © A_0(t) + q_{79}(t) © A_9(t) + q_{7,10}(t) © A_{10}(t)$

$A_9(t) = M_9(t) + (q_{97}(t) + q_{97.(6,8)}^n)(t) © A_7(t), A_{10}(t) = (q_{10,7}(t) + q_{10,7.(6,8)}^n) © A_7(t)$ (10)

where

$M_0(t) = e^{-(\beta+\lambda)t}, M_1(t) = e^{-(\beta+\lambda_1)t}\overline{G(t)}, \ M_2(t) = e^{-(\beta_1+\lambda)t}, \ M_3(t) = e^{-(\beta_1+\lambda_1)t},$

$M_7(t) = e^{-(\beta+\lambda)t}\overline{G_1(t)}, \ M_9(t) = e^{-(\beta_1+\lambda)t}$ (11)

Taking LT of above relations (10) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2} \quad (12)$$

where

$N_2 = ((\mu_0+\mu_2 p_{02})((1-p_{13}p_{31})+\mu_3(p_{02}p_{23}+p_{01}p_{13})+\mu_1(p_{01}+p_{02}p_{23}p_{31}))p_{70}$
$+(p_{14}(p_{01}+p_{02}p_{23}p_{31})+p_{35}(p_{02}p_{23}+p_{01}p_{13}))(\mu_7+p_{79}\mu_9)$

$D_2 = ((\mu_0+\mu_2 p_{02})((1-p_{13}p_{31})+\mu_3'(p_{02}p_{23}+p_{01}p_{13})+\mu_1'(p_{01}+p_{02}p_{23}p_{31}))p_{70}$
$+(p_{14}(p_{01}+p_{02}p_{23}p_{31})+p_{35}(p_{02}p_{23}+p_{01}p_{13}))(\mu_7+p_{79}\mu_9'+p_{7,10}\mu_{10}'$ (13)

7. Busy period analysis for server

Let $B_i(t)$ be the probability that the server is busy in repairing the unit at an instant't' given that the system entered regenerative state S_i at t=0.The recursive relations for $B_i(t)$ are as follows:

$B_0(t) = q_{01}(t) © B_1(t) + q_{02}(t) © B_2(t)$

$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{13}(t) © B_3(t) + (q_{17.4}(t)+q_{17.4,(6,8)}^n(t)) ©B_7(t)$
$B_2(t) = q_{20}(t)©B_0(t)+q_{23}(t)©B_3(t), B_3(t)=q_{31}(t)©B_1(t)+(q_{37.58}(t)+q_{37.5,(8,6)}n)©B_7(t)$
$B_7(t) = W_7(t)+q_{70}(t)©B_0(t)+q_{79}(t)©B_9(t)+q_{7,10}(t)©B_{10}(t)$
$B_9(t) = (q_{97}(t)+q_{97.(6,8)}^n)(t)©B_7(t), B_{10}(t)=W_{10}(t)+(q_{10,7}(t)+q_{10,7.(6,8)}^n)(t) ©B_7(t)$ (14)

where

$W_1(t)=e^{-(\beta+\lambda_1)t}\overline{G(t)} + (\lambda_1 e^{-(\beta+\lambda_1)t}©1)\overline{G(t)}$, $W_7(t) = e^{-(\beta+\lambda)t}\overline{G_1(t)}$

$W_{10}(t)=e^{-\beta t}\overline{G(t)} + (\beta e^{-\beta t}©1)\overline{G(t)}$ (15)

Taking LT of above relations (14) and solving for B_0^* (s). The time for which server is busy due to repair is given by

$$B_0^*(\infty) = \lim_{s\to 0} sB_0^*(s) = \frac{N_3}{D_2} \tag{16}$$

where

$N_3=(W_7^*(0)+p_{7,10}W_{10}^*(0))(p_{14}(p_{01}+p_{02}p_{23}p_{31})+p_{35}(p_{01}p_{13}+p_{02}p_{23}))+p_{70}W_1^*(0)(p_{01}+p_{02}p_{23}p_{31})$, $W_1^*(0) = \dfrac{\alpha+\lambda_1}{\alpha(\alpha+\beta+\lambda_1)}$, $W_7^*(0) = \dfrac{1}{(\alpha_1+\beta+\lambda)}$, $W_{10}^*(0) = \dfrac{1}{\alpha}$ and D_2 is already mentioned.

8. Expected number of visits by the server

Let $N_i(t)$ be the expected number of visits by the server in (0,t] given that the system entered the regenerative state S_i at t=0. The recursive relations for $N_i(t)$ are given as

$N_0(t) = Q_{01}(t)Ⓢ(1+N_1(t)) + Q_{02}(t) ⓈN_2(t)$
$N_1(t) = Q_{10}(t)ⓈN_0(t)+ Q_{13}(t) ⓈN_3(t) +Q_{17.4}(t) ⓈN_7(t)+Q_{17.4,(6,8)}^n(t))Ⓢ(1+N_7(t))$
$N_2(t)=Q_{20}(t)ⓈN_0(t)+Q_{23}(t)ⓈN_3(t), N_3(t)=Q_{31}(t)Ⓢ(1+N_1(t))+(Q_{37.58}(t)+Q_{37.5,(8,6)}n)Ⓢ(1+N_7(t))$
$N_7(t) = Q_{70}(t)ⓈN_0(t)+Q_{79}(t)ⓈN_9(t)+Q_{7,10}(t)ⓈN_{10}(t)$
$N_9(t)=(Q_{97}(t)+Q_{97.(6,8)}^n(t))Ⓢ(1+N_7(t)), N_{10}(t)=Q_{10,7}(t) ⓈN_7(t)+Q_{10,7.(6,8)}^n)(t)Ⓢ(1+N_7(t))$ (17)

Taking LST of relations (16) and solving for $\tilde{N}_0(s)$. The expected numbers of visits per unit time by the server are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_4}{D_2} \tag{18}$$

where

$N_4=p_{70}p_{01}(1-p_{13}p_{31})-p_{14}(p_{70}p_{47}+1-p_{77.10})(p_{01}+p_{02}p_{23}p_{31})+(p_{70}+p_{35})(p_{02}p_{23}+p_{01}p_{13})$
and D_2 is already specified.

9. Profit analysis

The profit incurred to the system model in steady state can be obtained as
$P_i=K_0A_0-K_1B_0-K_2N_0$
where
K_0=Revenue per unit up-time of the system

K_1=Cost per unit for which server is busy
K_2= Cost per unit visit by the server and A_0, B_0, N_0 are already defined.

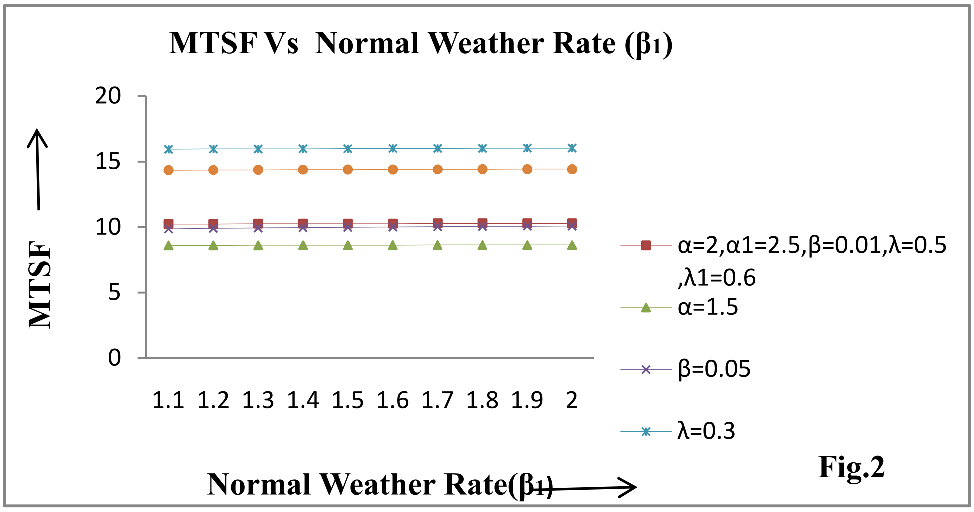

Fig. 2: MTSF vs. Normal Weather Rate (β_1)

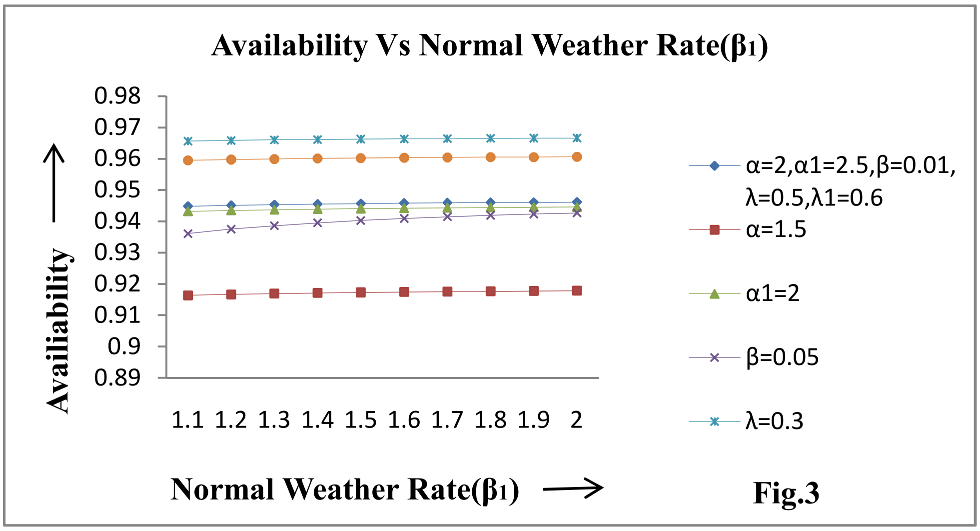

Fig. 3: Availability vs. Normal Weather Rate (β_1)

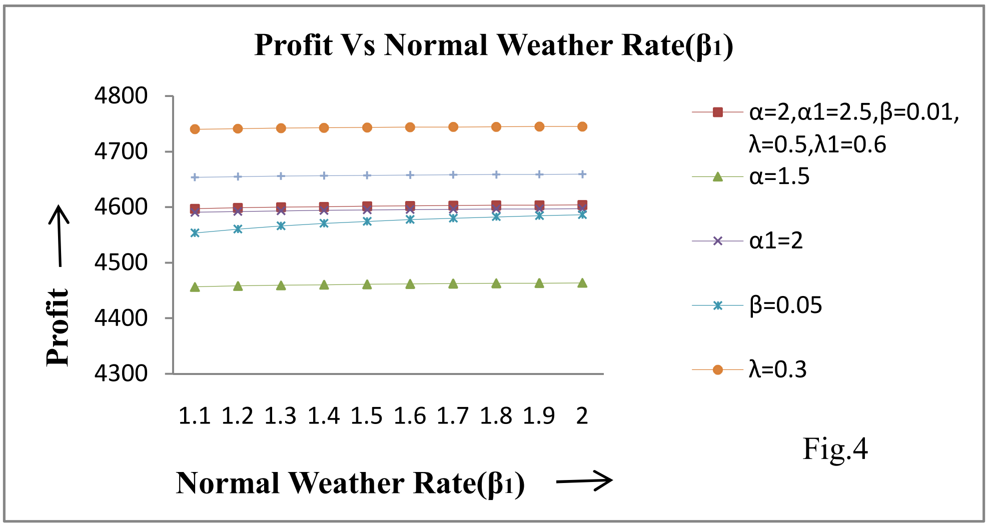

Fig. 4: Profit vs. Normal Weather Rate (β_1)

10. Conclusion

Considering $g(t) = \alpha e^{-\alpha t}$ and $g_1(t)=\alpha_1 e^{-\alpha_1 t}$, it is analyzed from figures 2, 3 and 4 that MTSF, availability and profit keep on moving up with the increase of normal weather rate (β_1) and repair rate (α) of the main unit. Also, there is an upward trend in availability and profit when repair rate (α_1) of duplicate unit increases. The values of these measures go on decline with the increase of abnormal weather rate (β) and failure rates (λ and λ_1). Thus, the study reveals that a system of two non-identical units operating in different weather conditions can be made more profitable and reliable to use by giving priority for operation and repair to main unit in case repair activities are allowed in abnormal weather.

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