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ESTIMATION OF PARAMETERS IN STEP-STRESS ACCELERATED LIFE TESTS FOR THE RAYLEIGH DISTRIBUTION UNDER CENSORING SETUP

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Abstract
In this paper, step-stress accelerated life test strategy is considered in obtaining the failure time data of the highly reliable items or units or equipment in a specified period of time. It is assumed that life time data of such items follows a Rayleigh distribution with a scale parameter (θ) which is the log linear function of the stress levels. The maximum likelihood estimates (MLEs) of the scale parameters (θ_i) at both the stress levels $(s_i), i = 1, 2$ are obtained under a cumulative exposure model. A simulation study is performed to assess the precision of the MLEs on the basis of mean square error (MSE) and relative absolute bias (RABias). The coverage probabilities of approximate and bootstrap confidence intervals for the parameters involved under both the censoring setup are numerically examined. In addition to this, asymptotic variance and covariance matrix of the estimators are also presented.

Key Words: Step-stress Accelerated Life Tests, Cumulative Exposure Model, Rayleigh Distribution, Maximum Likelihood Estimation, Type-I And Type II Censoring, Fisher Information Matrix, Bootstrap Confidence Interval.

1. Introduction
In reliability theory, accelerated life tests (ALT) is one of the useful areas which is an experimental procedure used to obtain information on life distribution of products/equipments/components by testing them at higher than usual level of stress to induce early failures in a specific period of time. The ALT is achieved by subjecting the test units to conditions that are more severe than the normal ones, such as higher levels of temperature, voltage, pressure, vibration, cycling rate, load, etc. The life time data are obtained at accelerated conditions based on a several physical models, results are extrapolated to the design stress to estimate the life distribution parameters.

Initially, researchers commenced to use ALT in the 1950‘s to develop a more effective life testing technique. Chernoff (1962) and Bessler (1962) firstly introduced and studied the concept of accelerated life tests. Some of the well-known researcher’s viz., Nelson and Meeker (1978), Meeker and Hahn (1985), Nelson (1990), Meeker and Escobar (1998) and Bagdonavicius and Nikulin (2002) has attempted the life testing problems with relevant assumptions and they also discuss various features of ALT and its applications in the area of reliability engineering.

The step-stress testing is a special class of the ALT which allows the experimenter to gradually increase the stress levels at some pre-specified time points during experiment. In such life-test experiment, n identical units are placed on life test at a specified low stress, if it does not fail at a pre-specified time then stress on it is repeatedly increased until the entire test unit fails or censoring is reached. This process of applying stress in steps is known as simple step-stress tests, which means stress(s) are applied on life tests units in two steps only.

In step-stress models, usually a cumulative exposure model is assumed, in which the remaining lifetime of a unit test depends on the cumulative fraction failed and current stress, regardless how the fraction is accumulated. The cumulative exposure model defined by Nelson (1980) for simple step-stress testing with low stress S_1 and high stress S_2 is given by

$$G(t)=\begin{cases} G_1(t); & 0 \le t < \tau \\ G_2(t-\tau+\tau'); & \tau \le t < \infty \end{cases} \tag{1}$$

Where $G_i(t)$ is the cumulative distribution function of the failure time at stress levels S_i, and τ is the time to change stress and therefore τ' is the solution of $G_1(\tau)=G_2(\tau')$.

A plenty of work on step stress ALT using cumulative exposure model have focused in developing of the optimal test plans as well as attempting the inferential problems. Miller and Nelson (1983) obtained the optimal time for changing the stress level, assuming the lifetime of a unit follows exponentially distributed. Bai, Kim, and Lee (1989) extended the study of Miller and Nelson (1983) to the case of censoring. Kateri and Balakrishnan (2008) attempted simple step stress ALT problem using cumulative exposure model for Weibull parameters by using ML method. Xiong (1998) obtained the MLEs for the parameters of exponential distribution using a simple step-stress ALT with Type-II censoring. Xiong and Milliken (2002) discussed simple step stress ALT for constructing the prediction limits, where lifetime of units follows exponential distribution under Type-I and Type-II censoring. Gouno and Balakrishnan (2001) reviewed the development on step-stress accelerated life-tests. Gouno, Sen and Balakrishnan (2004) presented inference for step-stress models under the exponential distribution in the case of a progressively Type-I censored data. Balakrishnan (2009) discussed exact inferential results for exponential step-stress models and some associated optimal accelerated life-tests. Balakrishnan, Kundu, Ng and Kannan (2007) discussed the simple step-stress model under Type-II censoring under the exponential distribution and they also developed the exact distributions of the MLEs of the parameters through using conditional moment generating function. Balakrishnan and Xie (2007) discussed exact inference for a simple step-stress model with Type-II hybrid censored data from the exponential distribution and also derived the confidence intervals using exact and asymptotic distribution of the MLEs. Recently, Chandra and Khan (2012) developed a new optimum test plans for simple step-stress accelerated life testing using Rayleigh Distribution.

In this article, we concentrated on the of the parameter estimation of Rayleigh distribution in step-stress accelerated life testing using maximum likelihood method for both Type-I and Type-II censored data. We also focus in developing the coverage probabilities of approximate and bootstrap confidence intervals for the parameters of the proposed model under both the censoring.

Furthermore, the proposed model description and assumption are defined in section 2. Section 3 deals the procedure of the maximum likelihood estimation under Type-I and Type-II censoring. The approximate and bootstrap confidence intervals of model parameters are given in section 4. Simulation study of the theoretical results and its findings are given in section 5. Finally, section 6 presents the conclusion of the study.

2. Model and Assumptions

The probability density function (p.d.f.) and cumulative distribution function (c.d.f.) of the Rayleigh distribution are given respectively by:

$$f(t,\theta)=\frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad t>0,\theta>0 \tag{2}$$

and

$$F(t,\theta)=1-\exp\left(-\frac{t^2}{2\theta^2}\right), \qquad t>0,\theta>0 \tag{3}$$

where, θ is the scale parameter.
The corresponding reliability function is given by

$$R(t)=\exp\left(-\frac{t^2}{2\theta^2}\right) \tag{4}$$

And the hazard rate function of t, denoted as $h(t)=f(t)/R(t)$ is obtained as

$$h(t)=\frac{\frac{t}{\theta^2}\exp\left(-\frac{t^2}{2\theta^2}\right)}{\exp\left(-\frac{t^2}{2\theta^2}\right)}=\frac{t}{\theta^2} \tag{5}$$

2.1 Assumptions for proposed Model

The following assumptions are made

1. Under any constant stress, the failure time of a test unit follows a Rayleigh distribution with c.d.f. given in Eq.(3).
2. The scale parameter θ_i at stress level $S_i, i=1,2$ is a log linear function of stress, that is, $\log(\theta_i)=\beta_0+\beta_1 S_i$, where β_0 and β_1 are unknown parameters depending upon the nature of the product and the method of the test.
3. The life test is conducted as follows: All n units are initially put on lower stress S_1 and run until time τ. Then the stress is changed to the high stress S_2 and the test continued until all failures are reported or censoring is reached.
4. The lifetime of test units are independent and identically distributed random variables.

Thus, under the assumption of cumulative exposure model, the c.d.f. of the lifetime of a test unit under such a step-stress model is given by

$$G(t)=\begin{cases} 1-\exp\left(-\dfrac{t^2}{2\theta_1^2}\right) & ; \quad 0\le t<\tau \\ 1-\exp\left(-\dfrac{\left(t-\tau+\dfrac{\theta_2}{\theta_1}\right)^2}{2\theta_2^2}\right) & ; \quad \tau\le t<\infty \end{cases} \tag{6}$$

The corresponding p.d.f. of the lifetime of a test unit is given by

$$g(t)=\begin{cases} \dfrac{t}{\theta_1^2}\exp\left(-\dfrac{t^2}{2\theta_1^2}\right) & ; \quad 0\le t<\tau \\ \dfrac{t}{\theta_2^2}\exp\left(-\dfrac{\left(t-\tau+\dfrac{\theta_2}{\theta_1}\right)^2}{2\theta_2^2}\right) & ; \quad \tau\le t<\infty \end{cases} \tag{7}$$

The maximum likelihood estimates for both the cases of Type-I and Type-II censoring are formulated in section 3.

3. Maximum Likelihood Estimation

The maximum likelihood method is popular in statistical data analysis to estimate the model parameters given the sample data. The simple step stress ALT model considered in this study has two parameters β_0 and β_1. Given the data $\mathbf{t}=(t_1 ......t_n)$ collected from the test, the maximum likelihood estimates of the two parameters are the ones that maximize the following likelihood function

$$L(\beta_0,\beta_1\,|\,t)=\prod_{i=1}^{n} L_i(\beta_0,\beta_1\,|\,t_i)$$

where L_i is the likelihood of the i^{th} observation. In order to make the computation more convenient, the logarithm of likelihood function (called *log-likelihood function*) is maximized instead of the likelihood function. Generally, “the MLEs are obtained by taking the first partial derivatives of the logarithm of the likelihood function and setting these partials equal to zero”.

3.1 The case of Type-I censoring

In simple step-stress ALT under the case of Type-I censoring, we start with n identical units are put on a life test, and subjected to low stress S_1. The experiment continues until all units fails or censoring time η is reached. Let τ denote the fixed pre-specified time at which the stress level is changed from S_1 to S_2. Further, let n_1 denote the random number of failures that occur before τ; and n_2, denote the number of failures that occur after τ. If $n_1 = n$, then the test is terminated at first step itself. Otherwise, at time τ, the stress level is increased at second step, and the test continues until the censoring time η is reached. The ordered failure times of testing units that are observed and will be denoted by

$$\{t_{1:n} < \ldots < t_{n_1:n} < \tau \le t_{n_1+1:n} < \ldots < t_{n_1+n_2:n} < \eta\} \tag{8}$$

Considering the observed Type-I censored data given in (8), the likelihood function is then given by

$$L(t;\theta_1,\theta_2) = \begin{cases} c_1 \prod_{i=1}^{n} g_1(t_{i:n}) & ; \quad n_1 = n & (9a) \\ c_2 \prod_{i=1}^{n} g_2(t_{i:n}) \left[1 - G(t_{n_c:n})\right]^{n-n_c} & ; \quad n_1 = 0 & (9b) \\ c_3 \prod_{i=1}^{n_1} g_1(t_{i:n}) \prod_{i=1}^{n_2} g_2(t_{i:n}) \left[1 - G(t_{n_c:n})\right]^{n-n_c} & ; \; 1 \le n_1 < n_1 + n_2 - 1 & (9c) \end{cases}$$

From the likelihood function in (9a), (9b) and (9c), we observe the following:

1. When, $n_1 = n$ the MLE of θ_2 does not exist.
2. When, $n_1 = 0$ the MLE of θ_1 does not exist.
3. The MLEs of θ_1 and θ_2 exist only when, $n_1 \ge 1$ and $n_2 \ge 1$ and may be obtained by maximizing the corresponding likelihood function in (9c). Therefore, from (7) and from (9c), the log-likelihood function is given by

$$\begin{aligned} \log L(t;\theta_1,\theta_2) &= \log(c_3) - 2n_1 \log(\theta_1) + \sum_{i=1}^{n_1} \log(t_{1i}) - \sum_{i=1}^{n_1} \frac{t_{1i}^2}{2\theta_1^2} - 2n_2 \log(\theta_2) \\ &+ \sum_{i=1}^{n_2} \log\left\{t_{2i} - \tau\left(1 - \frac{\theta_2}{\theta_1}\right)\right\} + \sum_{i=1}^{n_2} \log\left\{t_{2i} - \tau\left(1 - \frac{\theta_2}{\theta_1}\right)\right\} \\ &- \sum_{i=1}^{n_2} \frac{\left\{t_{2i} - \tau\left(1 - \frac{\theta_3}{\theta_1}\right)\right\}^2}{2\theta_2^2} - (n - n_c) \frac{\left\{\eta - \tau\left(1 - \frac{\theta_2}{\theta_1}\right)\right\}^2}{2\theta_2^2} \end{aligned} \tag{10}$$

According to assumption (2), the log-likelihood function (10) becomes

$$\log L(t;\beta_0,\beta_1) = \log(c_3) - 2n_1(\beta_0+\beta_1 S_1) - 2n_2(\beta_0+\beta_1 S_2) + \sum_{i=1}^{n_1}\log(t_{1i})$$

$$-\sum_{i=1}^{n_1}\frac{t_{1i}^2}{2e^{2(\beta_0+\beta_1 S_1)}} + \sum_{i=1}^{n_2}\log\left\{t_{i2} - \tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\} - \sum_{i=1}^{n_2}\frac{\left\{t_{2i}-\tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\}^2}{2e^{2(\beta_0+\beta_2 S_2)}}$$

$$-(n-n_c)\frac{\left\{\eta-\tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\}^2}{2e^{2(\beta_0+\beta_2 S_2)}} \qquad (11)$$

3.2 The case of Type–II censoring

We suppose that n identical units are put on a life test, and subjected to low stress S_1. The experiment continues until a pre-specified number of failures r are observed. Let τ denote the fixed pre-specified time at which the stress level is changed from S_1 to S_2. Further, let n_1 denote the random number of failures that occur before τ; and $n_2 = r - n_1$, denote the number of failures that occur after τ. If $n_1 = r$, then the test is terminated at first step itself. Otherwise, at time τ, the stress level is increased at second step, and the test continues until the required r failures are reported. The ordered failure times that are observed will be denoted by

$$t_{1:n} < \ldots\ldots < t_{n_1:n} < \tau \le t_{n_1+1:n} < \ldots\ldots < t_{r:n} \qquad (12)$$

The likelihood function of observed Type-II censored observations are derived as given in (12), is

$$L(t;\theta_1,\theta_2) = \begin{cases} c_1\prod_{i=1}^{n_1} g_1(t_{i:n})\left[1-G(t_{r:n})\right]^{n-r} ; & n_1 = r \qquad (13a) \\ c_2\prod_{i=1}^{n_1} g_2(t_{i:n})\left[1-G(t_{r:n})\right]^{n-r} ; & n_1 = 0 \qquad (13b) \\ c_3\prod_{i=1}^{n_1} g_1(t_{i:n})\prod_{i=1}^{r} g_2(t_{i:n})\left[1-G(t_{r:n})\right]^{n-r} ; & 1 \le n_1 < r-1 \qquad (13c) \end{cases}$$

From the likelihood function in (13a), (13b) and (13c), we observe the following:

1. When, $n_1 = r$ the MLE of θ_2 does not exist.
2. When, $n_1 = 0$ the MLE of θ_1 does not exist.
3. The MLEs of θ_1 and θ_2 exist only when, $1 \le n_1 < r-1$ and may be obtained by maximizing the corresponding likelihood function in (13c). Therefore, from (7) and from (13c), the log-likelihood function is given by

$$\log L(t;\theta_1,\theta_2)=\log(c_3)-2n_1\log(\theta_1)+\sum_{i=1}^{n_1}\log(t_{1i})-\sum_{i=1}^{n_1}\frac{t_{1i}^2}{2\theta_1^2}-2n_2\log(\theta_2)$$

$$+\sum_{i=1}^{n_2}\log\left\{t_{2i}-\tau\left(1-\frac{\theta_2}{\theta_1}\right)\right\}-\sum_{i=1}^{n_2}\frac{\left\{t_{2i}-\tau\left(1-\frac{\theta_2}{\theta_1}\right)\right\}}{2\theta_2^2}-(n-r)\frac{\left\{t_r-\tau\left(1-\frac{\theta_2}{\theta_1}\right)\right\}^2}{2\theta_2^2} \quad (14)$$

According to assumption (2), the log-likelihood function (14) becomes

$$\log L(t;\beta_0,\beta_1)=\log(c_3)-2n_1(\beta_0+\beta_1 S_1)-2n_2(\beta_0+\beta_1 S_2)+\sum_{i=1}^{n_1}\log(t_{1i})$$

$$-\sum_{i=1}^{n_1}\frac{t_{1i}^2}{2e^{2(\beta_0+\beta_1 S_1)}}+\sum_{i=1}^{n_2}\log\left\{t_{2i}-\tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\}$$

$$-\sum_{i=1}^{n_2}\frac{\left\{t_{2i}-\tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\}^2}{2e^{2(\beta_0+\beta_2 S_2)}}-(n-r)\frac{\left\{t_r-\tau\left(1-e^{\beta_1(S_2-S_1)}\right)\right\}^2}{2e^{2(\beta_0+\beta_2 S_2)}} \quad (15)$$

Our objective now is to determine the MLE of the parameters β_0 and β_1, based on the observed Type-I and Type-II censored data given in (8) and (12), respectively. Here, numerical likelihood maximization was carried out on the log-likelihood using R software. In the R software, the function *optim()* is used to maximize this log-likelihood function.

We used the following algorithm to find MLEs:

1. Simulate n order statistics from the uniform (0,1) distribution, $U_1, U_2, \ldots\ldots\ldots U_n$.
2. Find n_1 such that $U_{n_1} \le G_1(\tau) \le U_{n_1+1}$.
3. For $i \le n_1, T_i=\theta_1 G^{-1}(U_i)$, where $G(t)=1-\exp\left(-\frac{t^2}{2\theta^2}\right)$, same for the both cases.
4. (i) For $n_1+1 \le i \le n-1,\ T_i=\theta_2 G^{-1}(U_i)+\tau\left(1-\frac{\theta_2}{\theta_1}\right)$, for Type-I censoring case.

(ii) For $n_1+1 \le i \le r,\ T_i=\theta_2^{-1}(U_i)+\tau\left(1-\frac{\theta_2}{\theta_1}\right)$, for Type-II censoring case.

5. Compute the MLEs of β_0 and β_1 based on observed failure time data in step 3 and 4, say $\hat\beta_0$ and $\hat\beta_1$ for both the cases. The maximum likelihood estimates $\hat\beta_0$ and $\hat\beta_1$ for the model parameters β_0 and β_1 can be obtained by solving numerically the following two equations:

$$\frac{\partial \log L(t;\beta_0,\beta_1)}{\partial\beta_0}=0 \quad (16)$$

$$\frac{\partial \log L(t;\beta_0,\beta_1)}{\partial \beta 1}=0 \tag{17}$$

The asymptotic variance-covariance matrix of the estimates of β_0 and β_1 are obtained by inverting the asymptotic Fisher-information matrix, where its elements are obtained from the negative of the second and mixed derivatives of the log-likelihood equation for both the Type-I and Type-II censored cases defined in (11) and (15) respectively.

4. Confidence Interval for Type-I and Type-II Censoring Data

4.1 Approximate Confidence Intervals

To construct a confidence interval for a population parameter α; we assume that $L_\alpha = L_a(t_1......t_n)$ and $U_\alpha = U_\alpha(t_1.......t_n)$ are the functions of the sample data $t_1, t_2,........t_n$ such that

$$P_\alpha\left(L_\alpha \le t \le U_\alpha\right)=\gamma \tag{18}$$

where, L_α and U_α are indicating the lower and upper confidence limits which enclose α with probability γ. The interval [L_α, Uα] is called a two sided 100 γ % confidence interval for α. It is known that the MLEs, for large sample size under appropriate regularity conditions, are consistent and normally distributed. Therefore, the two-sided approximate 100 γ% confidence limits for a population parameter can be constructed as follows:

$$P\left[-z \le \frac{\hat{\alpha}-\alpha}{\sigma(\hat{\alpha})} \le z\right] \cong \gamma \tag{19}$$

Where, z is the [100(1-γ/2)]th percentile of the standard normal. Thus the confidence limits for β_0 and β_1 are given as:

$$\left.\begin{array}{ll} L_{\beta_0}=\hat{\beta}_0 - z\sqrt{Var(\hat{\beta}_0)}, & U_{\beta_0}=\hat{\beta}_0 + z\sqrt{Var(\hat{\beta}_0)} \\ L_{\beta_1}=\hat{\beta}_1 - z\sqrt{Var(\hat{\beta}_1)}, & U_{\beta_1}=\hat{\beta}_1 + z\sqrt{Var(\hat{\beta}_1)} \end{array}\right\} \tag{20}$$

4.2 Bootstrap Confidence Intervals

Confidence intervals based on the parametric bootstrap sampling can be constructed. The following steps are used to generate the bootstrap confidence intervals:

1. Based on the original Type-I and Type-II censored sample, obtained by using the algorithm in section 3.2, compute the MLEs of β_0 and β_1 .
2. Generate a random sample of size n^* from Uniform (0, 1) distribution, and obtain the order statistic $\left(U^*_{1:n^*},............,U^*_{n^*:n^*}\right)$.
3. For a given value of the stress change time τ , find n_1^* such that $U^*_{n_1^*:n^*} \le F_1^*(\tau_1) \le U^*_{n_1^*+1:n^*}$, where $F_1^*(\tau_1)=\int_0^{\frac{\tau_1}{\hat{\theta}_1}} \frac{x}{\hat{\theta}_1^2}\exp\left(-\frac{x^2}{2\hat{\theta}_1^2}\right)dx$.

4. For $1 \le i \le n_1^*$, $T_1^* = \hat{\theta}_1 F^{*-1}(U_i)$, where $F^*(t) = 1 - \exp\left(-\frac{t^2}{2\hat{\theta}^2}\right)$, same for the both cases.
5. (i). For $n_1^* + 1 \le i \le n_1^* + n_2^* - 1$, $T_i^* = \hat{\theta}_2 F^{*-1}(U_i) + \tau\left(1 - \frac{\hat{\theta}_2}{\hat{\theta}_1}\right)$, for Type-I censoring case.

   (ii). For $n_1^* + 1 \le i \le r^*$, $T_i^* = \hat{\theta}_2 F^{*-1}(U_i) + \tau\left(1 - \frac{\hat{\theta}_2}{\hat{\theta}_1}\right)$, for Type-II censoring case, where $F^*(t) = \int_0^t \frac{x}{\hat{\theta}_1^2} \exp\left(-\frac{x^2}{2\hat{\theta}_1^2}\right) dx$ and $r^* = n_1^* + n_2^*$.
6. Based on $n^*, n_1^*, n_2^*, \tau, r^*$ and the ordered bootstrap sample given in step 4-5, we can get the bootstrap estimates $\hat{\beta}_0^*$ and $\hat{\beta}_1^*$. The value of n^* and r^* has been taken to be equal to n and r.
7. Repeat above steps 2-5 B times to obtain B sets of MLEs of β_0 and β_1.

A two-sided 100(1-α) % bootstrap confidence interval of β_0 and β_1 for both cases are then given by

$$CI_{\beta_0} = \left[\hat{\beta}_0 - z\sqrt{MSE(\hat{\beta}_0)},\ \hat{\beta}_0 + z\sqrt{MSE(\hat{\beta}_0)}\right] \tag{21}$$

$$CI_{\beta_1} = \left[\hat{\beta}_1 - z\sqrt{MSE(\hat{\beta}_1)},\ \hat{\beta}_1 + z\sqrt{MSE(\hat{\beta}_1)}\right] \tag{22}$$

5. Simulation Study

A simulation study is performed using R software for illustrating the theoretical results developed for estimation in this article. The numerical investigations are performed to analyze the performance and precision of the maximum likelihood estimates of the parameters on the basis of the $MSE(\hat{\theta}) = E(\hat{\theta} - \theta)^2$ and $RABias(\hat{\theta}) = \left(\left|\hat{\theta} - \theta\right| / \theta\right)$. Apart from this, the asymptotic variance and covariance matrix and confidence intervals under approximate and bootstrap for coverage probabilities of the maximum likelihood estimates are obtained.

We assumed the stress values for low and high stress levels: S_1 : low stress level=0.1, 0.2 and S_2 : high stress level=0.7, 0.9 and $\beta_0 = 2.7$ and $\beta_1 = -2.8$.

The following steps are carried out for the simulation study:

Step 1: 1000 random samples of sizes 60,100, 150 for Type-I censoring and 25, 50, 100 for Type-II, were generated from Rayleigh distribution.

Step 2: (i) Assuming the stress changing time τ = 2.4, 2.6, 2.8 and different values of censoring times η = 7.0, 7.2, 7.4, for the case of Type-I censoring.

(ii) Assuming the stress changing time $\tau = 2.0, 2.2, 2.4$ with different values of stress levels $S_1 = 0.1, 0.2$ and $S_2 = 0.7, 0.9$ and the total number of failure observations in the test of a step stress ALT to be $r = 0.75n$ under case of Type-II censoring.

Step 3: The parameters of the model is estimated for assumed values in Step 2(i) as well as for step 2(ii) for each sample sizes considered in type-I and type-II censoring case.

Step 4: Newton- Raphson method was used for solving the two simultaneous nonlinear likelihood equations given in (16) and (17) for β_0 and β_1, respectively.

Step 5: The MSE, RABias and confidence intervals at 95% and 99% confidence level of the estimators for all sample sizes are tabulated for both censoring cases.

Step 6: The asymptotic variance and covariance matrix of the estimators for different sample sizes are also obtained.

Simulation results are summarized in the Table 1-12, in which Table 1-6 represents the findings for Type-I censoring, where Table 1-3 shows the MLEs, MSE and RABias of the estimators and Table 4 and Table 5 indicates the confidence intervals for β_0 and β_1, respectively, at 95% and 99% confidence coefficients. Table 6 display the asymptotic variance and covariance matrix. Similarly, Table 7-12 representing the finding for Type-II censoring, in which Table 7-9 shows the MLEs, MSE and RABias of the estimators and Table 10 and Table 11 indicates the confidence intervals for β_0 and β_1, respectively, with 95% and 99% confidence coefficients. Table 12 represents the asymptotic variance and covariance matrix of the M. L. estimates.

τ	η	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.4	7.0	2.8014 -2.9551	0.7798 1.7212	0.0376 -0.0554	9.1193 2.0810	0.0064 0.0000	0.0729 0.0043
	7.2	2.7909 -2.9022	0.8311 1.8198	0.0337 -0.0365	9.1197 2.1369	0.0064 0.0000	0.0729 0.0224
	7.4	2.7430 -2.7916	0.7268 1.6076	0.0159 -0.0030	8.8877 2.2009	0.0025 0.0002	0.0456 0.0531
2.6	7.0	2.7404 -2.9005	0.3944 0.9109	0.0150 -0.0359	8.6742 2.0342	0.0005 0.0001	0.0205 0.0267
	7.2	2.7080 -2.8138	0.3144 0.7343	0.0030 -0.0049	8.5444 2.0926	0.0000 0.0000	0.0052 0.0012
	7.4	2.7478 -2.8335	0.4412 1.0008	0.0177 -0.0119	8.8560 2.1476	0.0021 0.0001	0.0419 0.0276
2.8	7.0	2.7244 -2.9151	0.1921 0.4853	0.0090 -0.0411	8.5109 1.9814	0.0000 0.0002	0.0013 0.0520
	7.2	2.7561 -2.9213	0.3502 0.8123	0.0208 -0.0433	8.7742 2.0364	0.0013 0.0000	0.0323 0.0257
	7.4	2.7051 -2.8001	0.1985 0.4764	0.0019 0.0000	8.5429 2.1066	0.0000 0.0000	0.0050 0.0079

Table 1: MLEs of (β_0, β_1), θ_1 and θ_2 with their MSEs and RABias for different stress changing times and for different censoring times. Initial parameter values: $\beta_0 = 2.7$ and $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1, 2$ **with n=60 under Type-I censoring.**

τ	η	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.4	7.0	2.7110 -2.8206	0.1592 0.3806	0.0041 -0.0073	8.5580 2.0888	0.0000 0.0000	0.0068 0.0006
	7.2	2.7110 -2.7837	0.1172 0.2862	0.0041 -0.0058	8.6215 2.1434	0.0001 0.0000	0.0143 0.0256
	7.4	2.6980 -2.7310	0.1156 0.2846	0.0008 -0.0246	8.5999 2.1952	0.0001 0.0001	0.0118 0.0503
2.6	7.0	2.7042 -2.8483	0.0815 0.2015	0.0016 -0.0172	8.4532 2.0348	0.0000 0.0000	0.0055 0.0264
	7.2	2.7062 -2.8106	0.0787 0.1960	0.0023 -0.0038	8.5343 2.0934	0.0000 0.0000	0.0040 0.0016
	7.4	2.7101 -2.7793	0.0904 0.2251	0.0037 -0.0074	8.6213 2.1481	0.0001 0.0000	0.0143 0.0278
2.8	7.0	2.7212 -2.9138	0.0592 0.1638	0.0078 -0.0406	8.4861 1.9769	0.0000 0.0001	0.0016 0.0541
	7.2	2.7323 -2.8910	0.1347 0.3165	0.0119 -0.0325	8.6198 2.0311	0.0001 0.0000	0.0141 0.0282
	7.4	2.7056 -2.8068	0.0588 0.1511	0.0021 -0.0024	8.5350 2.0975	0.0000 0.0000	0.0041 0.0036

Table 2: MLEs of (β_0, β_1), θ_1 and θ_2 **with their MSEs and RABias for different stress changing times & censoring times. Initial parameter values:** $\beta_0 = 2.7$ & $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1, 2$ **with n=100 under Type-I censoring.**

τ	η	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.4	7.0	2.7268 -2.8490	0.0617 0.1520	0.0099 -0.0175	8.6451 2.0802	0.0001 0.0000	0.0171 0.0047
	7.2	2.7007 -2.7680	0.0547 0.1371	0.0003 -0.0114	8.5602 2.1450	0.0000 0.0000	0.0071 0.0263
	7.4	2.6906 -2.7205	0.0563 0.1457	0.0035 -0.0284	8.5548 2.1951	0.0000 0.0001	0.0064 0.0503
2.6	7.0	2.7220 -2.8774	0.0388 0.1024	0.0081 -0.0276	8.5548 2.0295	0.0000 0.0000	0.0064 0.0289
	7.2	2.7348 -2.8571	0.0436 0.1109	0.0129 -0.0204	8.7010 2.0853	0.0003 0.0000	0.0236 0.0023
	7.4	2.7137 -2.7883	0.0372 0.0950	0.0051 -0.0042	8.6368 2.1423	0.0001 0.0000	0.0161 0.0250
2.8	7.0	2.7609 -2.9784	0.0322 0.1048	0.0225 -0.0637	8.7163 1.9660	0.0003 0.0001	0.0254 0.0593
	7.2	2.7752 -2.9595	0.0399 0.1139	0.0279 -0.0569	8.8760 2.0211	0.0009 0.0000	0.0442 0.0330
	7.4	2.7588 -2.8898	0.0379 0.0973	0.0218 -0.0321	8.8541 2.0875	0.0008 0.0000	0.0417 0.0012

Table 3: MLEs of (β_0, β_1), θ_1 and θ_2 **with their MSEs and RABias for different stress changing times & different censoring times. Initial parameter values:** $\beta_0 = 2.7$ & $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1, 2$ **with n=150 under Type-I censoring.**

τ	η	Approximate C.I.			Bootstrap C.I.		
		n=60	n=100	n=150	n=60	n=100	n=150
2.4	7.0	-0.18, 5.79 -1.10, 6.70	0.22, 5.20 -0.54, 5.96	0.75, 4.70 0.15, 5.32	0.76, 5.82 -0.01, 6.6	1.86, 4.18 1.48, 4.48	2.06, 3.95 1.78, 4.25
	7.2	-0.56, 6.14 -1.58, 7.16	-0.07, 5.49 -0.93, 6.35	0.39, 5.02 -0.31, 5.73	0.85,5.65 0.12,6.38	1.96, 3.97 1.66, 4.28	2.19, 3.68 1.96, 3.90
	7.4	-1.19, 6.67 -2.39, 7.88	-0.45, 5.86 -1.41, 6.81	0.09, 5.28 -0.69, 6.08	0.59, 5.89 -0.21, 6.7	1.97, 3.95 1.67, 4.26	2.18, 3.66 1.96, 3.89
2.6	7.0	-0.15, 5.65 -1.04, 6.52	0.38, 5.02 -0.32, 5.73	0.88, 4.56 0.32, 5.15	1.67,4.43 1.25,4.85	2.08, 3.84 1.82, 4.13	2.28, 3.65 2.08, 3.83
	7.2	-0.68, 6.09 -1.71, 7.13	0.07, 5.39 -0.73, 6.15	0.68, 4.78 0.05, 5.41	1.17,4.96 0.59,5.54	1.98, 3.87 1.69, 4.16	2.26, 3.70 2.04, 3.92
	7.4	-0.91, 6.41 -2.03, 7.53	-0.25, 5.67 -1.15, 6.58	0.34, 5.09 -0.39 5.87	1.16, 5.03 0.57, 5.63	2.09, 3.83 1.84, 4.07	2.28, 3.59 2.09, 3.79
2.8	7.0	0.02, 5.43 -0.81, 6.26	0.64, 4.80 0.01, 5.44	1.15, 4.36 0.66, 4.85	1.57,4.61 1.04,5.08	2.05, 3.92 1.72, 4.20	2.37, 3.71 2.15, 3.92
	7.2	-0.21, 5.73 -1.12, 6.65	0.41, 5.06 -0.31, 5.77	0.98, 4.56 0.44, 5.10	1.64,4.48 1.20, 4.90	2.17, 3.79 1.93, 4.03	2.37, 3.63 2.18, 3.88
	7.4	-1.02, 6.44 -2.16, 7.57	-0.12, 5.53 -0.98, 6.39	0.65, 4.86 0.05, 5.53	1.32, 4.70 0.80, 5.22	2.20, 3.67 1.98, 3.86	2.34, 3.66 2.15, 3.85

Table 4: Confidence Intervals (C.I.) for the parameter β_0 under Type-I censoring. The 1st and 2nd lines of each rows indicates the confidence interval at 95% and 99% confidence levels, respectively.

τ	η	Approximate C.I.			Bootstrap C.I.		
		n=60	n=100	n=150	n=60	n=100	n=150
2.4	7.0	-6.84, 0.94 -8.21, 2.11	-6.59, 0.88 -7.68, 1.99	-5.69, -0.02 -6.56, 0.89	-7.69, 0.05 -8.88, 1.19	-5.26,-1.49 -5.84,-0.92	-5.09, -1.75 -5.58, -1.25
	7.2	-7.28, 1.48 -8.63, 2.88	-6.98, 1.38 -8.23, 2.65	-6.28, 0.78 -7.36, 1.84	-7.38,-0.07 -8.53, 1.09	-5.01,-1.67 -5.53,-1.09	-4.60, -1.96 -5.01, -1.58
	7.4	-8.38, 2.79 -10.09,4.50	-7.63, 2.16 -9.13, 3.66	-6.84, 1.38 -8.08, 2.69	-7.67, 0.38 -8.89, 1.56	-4.93,-1.59 -5.44,-1.08	-4.52, -1.89 -4.96, -1.49
2.6	7.0	-7.17, 1.37 -8.48, 2.68	-6.48, 0.79 -7.60, 1.93	-5.67, -0.09 -6.53, 0.77	-5.79,-1.29 -6.48,-0.57	-4.99,-1.84 -5.46,-1.36	-4.66, -2.13 -5.05, -1.74
	7.2	-8.19, 2.56 -9.85, 4.27	-7.07, 1.38 -8.22, 2.67	-5.94, 0.23 -6.89, 1.17	-6.45,-0.56 -7.35, 0.38	-4.89,-1.69 -5.38,-1.20	-4.77, -2.05 -5.13, -1.65
	7.4	-8.29, 2.63 -9.97, 4.34	-7.57, 2.01 -9.07, 3.48	-6.59, 1.05 -7.75, 2.19	-6.49,-0.54 -7.41, 0.40	-4.78,-1.79 -5.25,-1.34	-4.48, -2.05 -4.85, -1.68
2.8	7.0	-7.18, 1.38 -8.49, 2.66	-6.24, 0.38 -7.24, 1.39	-5.36, -0.59 -6.09, 0.13	-6.10,-1.17 -6.86,-0.34	-5.12,-1.89 -5.62,-1.36	-4.87, -2.21 -5.27, -1.80
	7.2	-7.49, 1.59 -8.82, 2.98	-6.58, 0.79 -7.71, 1.98	-5.58, -0.36 -6.38, 0.47	-5.89,-1.24 -6.54,-0.54	-4.89,-1.97 -5.34,-1.56	-4.66, -2.25 -5.07, -1.84
	7.4	-9.37, 3.72 -11.32,5.72	-7.70, 2.09 -9.21, 3.59	-6.19, 0.39 -7.18, 1.38	-6.06,-0.76 -6.87, 0.04	-4.59,-1.97 -4.99,-1.57	-4.63, -2.16 -5.07, -1.79

Table 5: Confidence Intervals (C.I.) for the parameter β_1 under Type-I censoring. The 1st and 2nd lines of each rows indicates the confidence interval at 95% and 99% confidence levels, respectively.

τ	η	n=60	n=100	n=150
2.4	7.0	2.3205 -1.5575 -1.5575 3.9165	1.616 -1.3580 -1.358 3.5414	1.0146 -0.8167 -0.8167 2.1097
	7.2	2.9159 -1.9391 -1.9391 5.0099	2.0219 -1.6914 -1.6914 4.5139	1.3845 -1.2000 -1.2000 3.2171
	7.4	4.0228 -3.0098 -3.0098 8.1281	2.5803 -2.2798 -2.2798 6.2477	1.7511 -1.5947 -1.5947 4.3823
2.6	7.0	2.1803 -1.8121 -1.8121 4.7593	1.3987 -1.2937 -1.2937 3.4449	0.8815 -0.7724 -0.7724 2.0383
	7.2	2.9830 -2.7581 -2.7581 7.5216	1.8039 -1.6794 -1.6794 4.5847	1.0942 -0.9220 -0.9220 2.4812
	7.4	3.4880 -2.8259 -2.8259 7.7727	2.2833 -2.1410 -2.1410 5.9749	1.4698 -1.3543 -1.3543 3.7650
2.8	7.0	1.9030 -1.7718 -1.7718 4.7529	1.1260 -1.0576 -1.0576 2.8347	0.6680 -0.5622 -0.5622 1.4774
	7.2	2.2964 -1.9554 -1.9554 5.3132	1.4099 -1.2953 -1.2953 3.5430	0.8311 -0.6688 -0.6688 1.7918
	7.4	3.6195 -3.8804 -3.8804 11.0877	2.0726 -2.1982 -2.1982 6.2552	1.1489 -0.9777 -0.9777 2.7145

Table 6: asymptotic variance and covariance of the estimates under Type-I censoring case.

τ	Stresses	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.0	S_1=0.1, S_2=0.7	2.0361 -2.0719	0.5392 0.8172	0.2459 -0.2600	6.2272 1.7964	1.0359 0.0060	0.4270 0.1554
	S_1=0.2, S_2=0.7	2.2951 -2.4250	0.3317 0.6202	0.1500 -0.1339	6.1108 1.8176	0.3126 0.0062	0.2837 0.1561
	S_1=0.2, S_2=0.9	2.1432 -1.7124	0.4343 1.4100	0.2062 -0.3884	6.2673 1.8285	0.2830 0.0203	0.2664 0.5434
2.2	S_1=0.1, S_2=0.7	2.0951 -2.1566	0.4843 0.7544	0.2241 -0.2298	6.5496 1.7958	0.9396 0.0062	0.3952 0.1581
	S_1=0.2, S_2=0.7	2.3705 -2.5560	0.2792 0.5352	0.1220 -0.0871	6.4192 1.7883	0.2785 0.0065	0.2647 0.1635
	S_1=0.2, S_2=0.9	2.2347 -1.8329	0.3616 1.1697	0.1724 -0.3454	6.4757 1.7950	0.2806 0.0189	0.2668 0.5170
2.4	S_1=0.1, S_2=0.7	2.1228 -2.2334	0.4564 0.6769	0.2138 -0.2023	6.6822 1.7496	0.9054 0.0076	0.3837 0.1772
	S_1=0.2, S_2=0.7	2.4385 -2.6658	0.2320 0.4718	0.0968 -0.0479	6.7219 1.7727	0.2526 0.0068	0.2574 0.1669
	S_1=0.2, S_2=0.9	2.2845 -1.9151	0.3094 1.0236	0.1539 -0.3160	6.6959 1.7523	0.2547 0.0167	0.2565 0.4831

Table 7: MLEs of (β_0, β_1), θ_1 and θ_2 with their MSEs and RABias for different stress changing times & different censoring times. Initial parameter values: $\beta_0 = 2.7$ & $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1,2$ with n=25 under Type-I censoring.

τ	Stresses	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.0	S_1=0.1, S_2=0.7	2.2785 -2.4018	0.3128 0.5093	0.1561 -0.1422	7.6781 1.8172	0.3449 0.0022	0.3326 0.1368
	S_1=0.2, S_2=0.7	2.6168 -2.8952	0.2145 0.5180	0.0308 -0.0340	7.6736 1.8043	0.1530 0.0023	0.2674 0.1400
	S_1=0.2, S_2=0.9	2.4316 -2.0368	0.2395 0.842	0.0994 -0.2726	7.5701 1.8193	0.3610 0.0087	0.3398 0.5250
2.2	S_1=0.1, S_2=0.7	2.2734 -2.4096	0.3109 0.4874	0.1580 -0.1394	7.6328 1.7980	0.3510 0.0024	0.3369 0.1441
	S_1=0.2, S_2=0.7	2.6285 -2.9194	0.2022 0.5137	0.0265 -0.0427	7.7265 1.7949	0.1540 0.0025	0.2633 0.1444
	S_1=0.2, S_2=0.9	2.4926 -2.1312	0.2013 0.6980	0.0768 -0.2389	7.8960 1.7763	0.1479 0.0078	0.2542 0.4904
2.4	S_1=0.1, S_2=0.7	2.3169 -2.5149	0.2727 0.4118	0.1419 -0.1018	7.8883 1.7444	0.3292 0.0030	0.3251 0.1641
	S_1=0.2, S_2=0.7	2.6559 -2.9785	0.1823 0.4899	0.0163 -0.0638	7.8474 1.7699	0.1533 0.0027	0.2478 0.1533
	S_1=0.2, S_2=0.9	2.4960 -2.1338	0.2040 0.6967	0.0755 -0.2379	7.9191 1.7783	0.1600 0.0078	0.2624 0.4915

Table 8: MLEs of (β_0, β_1), θ_1 and θ_2 with their MSEs and RABias for different stress changing times & different censoring times. Initial parameter values: $\beta_0 = 2.7$ & $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1, 2$ with n=50 under Type-I censoring.

τ	**Stresses**	$\hat{\beta}_0$ $\hat{\beta}_1$	$MSE(\hat{\beta}_0)$ $MSE(\hat{\beta}_1)$	$RABias(\hat{\beta}_0)$ $RABias(\hat{\beta}_1)$	$\hat{\theta}_1$ $\hat{\theta}_2$	$MSE(\hat{\theta}_0)$ $MSE(\hat{\theta}_1)$	$RABias(\hat{\theta}_0)$ $RABias(\hat{\theta}_1)$
2.0	S_1=0.1, S_2=0.7	2.3529 -2.5137	0.2262 0.3409	0.1286 -0.1022	8.1783 1.8098	0.1492 0.0010	0.3095 0.1329
	S_1=0.2, S_2=0.7	2.7310 -3.0589	0.1837 0.5075	0.0115 -0.0925	8.3244 1.8035	0.0924 0.0010	0.2501 0.1352
	S_1=0.2, S_2=0.9	2.5390 -2.1577	0.1602 0.6078	0.0596 -0.2294	8.2270 1.8167	0.0784 0.0041	0.2344 0.5183
2.2	S_1=0.1, S_2=0.7	2.3658 -2.5496	0.2111 0.3198	0.1238 -0.0894	8.2548 1.7879	0.1433 0.0011	0.3034 0.1429
	S_1=0.2, S_2=0.7	2.7156 -3.0471	0.1458 0.4226	0.0058 -0.0883	8.2169 1.7908	0.0753 0.0011	0.2199 0.1423
	S_1=0.2, S_2=0.9	2.5561 -2.1894	0.1415 0.5533	0.0533 -0.2181	8.3167 1.7963	0.0690 0.0038	0.2246 0.5016
2.4	S_1=0.1, S_2=0.7	2.3552 -2.5439	0.1982 0.2683	0.1277 -0.0915	8.1729 1.7762	0.1381 0.0011	0.2991 0.1483
	S_1=0.2, S_2=0.7	2.7036 -3.0393	0.1015 0.3221	0.0013 -0.0855	8.1316 1.7791	0.0438 0.0011	0.1823 0.1471
	S_1=0.2, S_2=0.9	2.5518 -2.2028	0.1231 0.5124	0.0549 -0.2133	8.2588 1.7671	0.0600 0.0035	0.1971 0.4773

Table 9: MLEs of (β_0, β_1), θ_1 and θ_2 with their MSEs and RABias for different stress changing times & different censoring times. Initial parameter values: $\beta_0 = 2.7$ & $\beta_1 = -2.8$, $\hat{\theta}_i = \exp(\hat{\beta}_0 + \hat{\beta}_1 S_i)$, $i = 1, 2$ with n=100 under Type-I censoring.

τ	Stresses	Approximate C.I.			Bootstrap C.I.		
		25	50	100	25	50	100
2.0	S_1=0.1,	-0.73, 4.81	0.45, 4.10	1.10, 3.65	1.25, 2.66	1.60, 3.04	1.77, 3.28
	S_2=0.7	-1.58, 5.65	-0.10, 4.66	0.78, 3.98	1.05, 2.85	1.39, 3.25	1.56, 3.51
	S_1=0.2,	-5.11, 9.69	0.49, 4.75	1.30, 4.15	1.49,3.05	1.88, 3.64	2.03, 4.00
	S_2=0.7	-7.37,11.96	-0.19, 5.38	0.87,4.58	1.25, 3.29	1.69, 3.95	1.79, 4.35
	S_1=0.2,	-2.03, 6.30	0.46, 4.47	1.18, 3.89	1.39, 2.79	1.79, 3.37	1.89, 3.64
	S_2=0.9	-3.28, 7.57	-0.17, 5.08	0.79, 4.39	1.18, 3.02	1.51, 3.55	1.63, 3.86
2.2	S_1=0.1,	-0.56, 4.75	0.59, 4.08	1.18, 3.59	1.36, 2.66	1.62, 3.05	1.83, 3.26
	S_2=0.7	-1.37, 5.57	-0.05, 4.55	0.82, 3.91	1.15, 2.85	1.43, 3.27	1.58, 3.49
	S_1=0.2,	-1.81, 6.55	0.56, 4.67	1.34, 4.09	1.59, 3.13	1.87, 3.68	2.01, 3.96
	S_2=0.7,	-3.09, 7.83	-0.04, 5.29	0.92, 4.51	1.35, 3.38	1.59, 3.96	1.72, 4.26
	S_1=0.2,	-0.91, 5.38	0.79, 4.26	1.27, 3.86	1.48, 2.89	1.79, 3.45	1.96, 3.58
	S_2=0.9	-1.87, 6.34	0.17, 4.80	0.88, 4.28	1.27, 3.14	1.54, 3.68	1.68, 3.87
2.4	S_1=0.1,	-0.24, 4.48	0.78, 3.85	1.24, 3.57	1.39, 2.69	1.68, 3.08	1.89, 3.11
	S_2=0.7	-0.96, 5.21	0.31, 4.32	0.85, 3.89	1.19, 2.89	1.47, 3.30	1.67, 3.33
	S_1=0.2,	-1.02, 5.89	0.75, 4.56	1.36, 4.04	1.68, 3.27	1.97, 3.73	2.08, 3.89
	S_2=0.7	-2.07, 6.95	0.16, 5.14	0.95, 4.45	1.44, 3.59	1.62, 4.07	1.81, 4.13
	S_1=0.2,	-0.41, 4.98	0.78, 4.26	1.35, 3.75	1.54, 2.98	1.84, 3.47	1.98, 3.54
	S_2=0.9	-1.24, 5.80	0.18, 4.85	0.98, 4.19	1.37, 3.19	1.59, 3.66	1.75, 3.78

Table 10: Confidence Intervals (C.I.) for the parameter β_0 under Type-II censoring. The 1st and 2nd lines of each rows indicates the confidence interval at 95% and 99% confidence levels, respectively.

τ	Stresse s	Approximate C.I.			Bootstrap C.I.		
		25	50	100	25	50	100
2.0	S_1=0.1,	-6.90, 2.79	-4.76,-0.04	-4.01,-0.98	-3.95,-1.17	-4.72, -1.35	-5.19, -1.45
	S_2=0.7	-8.38, 4.28	-5.48, 0.68	-4.58,-0.51	-4.33,-0.72	-5.25, -0.86	-5.77, -0.86
	S_1=0.2,	-19.6, 14.8	-6.15, 0.36	-5.07,-1.04	-4.69 -1.29	-5.68, -1.64	-6.28, -1.78
	S_2=0.7	-24.9, 20.1	-7.19, 1.38	-5.68,-0.43	-5.22,-0.79	-6.29, -1.05	-6.97, -1.05
	S_1=0.2,	-8.79, 5.38	-4.28, 0.23	-3.57,-0.78	-3.32,-0.89	-4.08, -1.16	-4.45, -1.24
	S_2=0.9	-10.96,7.5	-4.97, 0.92	-3.98,-0.33	-3.76 -0.53	-4.44, -0.79	-4.89, -0.75
2.2	S_1=0.1,	-6.54, 2.3	-4.76, 0.13	-3.98,-1.11	-4.05,-1.29	-4.85, -1.36	-5.29, -1.47
	S_2=0.7	-7.88, 3.57	-5.49, 0.50	-4.48,-0.67	-4.49,-0.85	-5.35, -0.87	-5.79, -0.91
	S_1=0.2,	-11.42, 6.3	-6.08, 0.29	-5.03,-1.06	-4.87, -1.42	-5.76, -1.65	-6.27, -1.78
	S_2=0.7	-14.13,9.07	-7.08, 1.29	-5.64,-0.45	-5.41, -0.89	-6.39, -1.02	-6.96, -1.04
	S_1=0.2,	-6.49, 2.84	-4.06,-0.23	-3.57,-0.87	-3.48, -1.04	-4.21, -1.26	-4.47, -1.28
	S_2=0.9	-7.96, 4.25	-4.65, 0.38	-3.91,-0.46	-3.89, -0.69	-4.69, -0.78	-4.99, -0.80
2.4	S_1=0.1,	-6.04, 1.57	-4.49,-0.56	-3.96,-1.12	-4.15, -1.28	-4.96, -1.68	-5.05, -1.59
	S_2=0.7	-7.27, 2.74	-5.10, 0.07	-4.42,-0.68	-4.59, -0.85	-5.45, -0.99	-5.59, -1.03
	S_1=0.2,	-9.56, 4.23	-5.91,-0.06	-5.05,-1.05	-5.23, -1.47	-5.92, -1.69	-6.17, -1.86
	S_2=0.7	-11.67,6.34	-6.82, 0.86	-5.63,-0.46	-5.85, -0.89	-6.57, -1.04	-6.84, -1.19
	S_1=0.2,	-5.68, 1.88	-4.07,-0.19	-3.46,-0.95	-3.68, -1.06	-4.23, -1.23	-4.49, -1.31
	S_2=0.9	-6.84, 3.03	-4.67, 0.44	-3.85,-0.56	-4.08, -0.66	-4.69, -0.74	-4.99, -0.84

Table 11: Confidence Intervals (C.I.) for the parameter β_1 under Type-II censoring. The 1st and 2nd lines of each rows indicates the confidence interval at 95% and 99% confidence levels, respectively.

τ	Stresses	n=25	n=50	n=100
2.0	S_1=0.1, S_2=0.7	1.9952 -2.1880 -2.1880 6.0745	0.8663 -0.5878 -0.5878 1.4521	0.4080 -0.2558 -0.2558 0.6071
	S_1=0.2, S_2=0.7	14.264 -31.634 -31.634 77.483	1.1676 -1.2606 -1.2606 2.7606	0.5261 -0.4977 -0.4977 1.0536
	S_1=0.2, S_2=0.9	4.4976 -6.3478 -6.3478 13.0508	1.0363 -0.7261 -0.7261 1.3176	0.4778 -0.2928 -0.2928 0.5097
2.2	S_1=0.1, S_2=0.7	1.8373 -1.7504 -1.7504 5.0081	0.7922 -0.5296 -0.5296 1.3727	0.3645 -0.2192 -0.2192 0.5386
	S_1=0.2, S_2=0.7	4.5538 -8.2896 -8.2896 20.4383	1.0864 -1.1532 -1.1532 2.6132	0.4911 -0.4662 -0.4662 1.0262
	S_1=0.2, S_2=0.9	2.5723 -2.7561 -2.7561 5.6463	0.8185 -0.5322 -0.5322 0.9683	0.4264 -0.2520 -0.2520 0.4518
2.4	S_1=0.1, S_2=0.7	1.4533 -1.3059 -1.3059 3.7749	0.6137 -0.3949 -0.3949 1.0195	0.3452 -0.2041 -0.2041 0.5271
	S_1=0.2, S_2=0.7	3.1116 -4.9616 -4.9616 12.3781	0.9441 -0.9752 -0.9752 2.2539	0.4668 -0.4491 -0.4491 1.0261
	S_1=0.2, S_2=0.9	1.8918 -1.8028 -1.8028 3.7050	0.8138 -0.5165 -0.5165 0.9826	0.3745 -0.2227 -0.2227 0.4117

Table 12: asymptotic variance and covariance matrix of the estimates under Type-II censoring.

5.1. Findings

We note the following points from Table 1-6:

1. In Table 1-3, for fixed value of τ and η, the MSE of β_0 and β_1 are decreases by increasing the sample size and the MSE of $\hat{\theta}_1$ and $\hat{\theta}_2$ approaches to zero. RABias of $\hat{\beta}_0$ and $\hat{\beta}_1$ are decreases except for some cases, this may be due to fluctuation in data.
2. The confidence interval for β_0 and β_1 obtained by the approximate method and bootstrap method for each values of τ at different value of censoring time(η) are given in Table 4-5, we observe that the bootstrap confidence intervals are narrower than the approximate confidence intervals for both the estimates of β_0 and β_1.
3. The asymptotic variance and covariance matrix for $\hat{\beta}_1$ and $\hat{\beta}_2$ are obtained in Table 6, here, we find that by increasing the sample size the asymptotic variances and covariance matrix is decreases.

From Tables 7-12, we observe the following:

4. In Table 7-9, for fixed value of τ at different stress levels, the MSE of $\hat{\beta}_0, \hat{\beta}_2, \hat{\theta}_1$ and $\hat{\theta}_2$ are decreasing as expected sample sizes are increasing. RABias of $\hat{\beta}_1$ and $\hat{\beta}_2$ are decreases for most of the stress combinations with stress changing time τ.
5. The confidence intervals for β_0 and β_1 obtained by the approximate method and bootstrap method for different values of τ at different stress levels are given in Table 10-11, we can see that the bootstrap confidence intervals are narrower than the approximate confidence intervals for both β_0 and β_1.

6. The asymptotic variance and covariance matrix for $\hat{\beta}_1$ and $\hat{\beta}_2$ are obtained given in Table 12, we can see that for fixed value of τ and for every combination of stress levels the asymptotic variance and covariance matrix is decreasing as sample sizes increases.

Finally, it is concluded that at censoring time (η) = 7.2 as compared to others for type-I and the stress combination $S_1 = 0.2, S_2 = 0.7$ in comparison to other sets under type-II censoring, the MLE's of both the parameters $\hat{\beta}_1$ and $\hat{\beta}_2$ performed well at each of the considered stress changing time (τ). Hence these particular values of censoring time and stress combination may work for getting better estimates in step stress ALT.

6. Conclusion

Accelerated life testing procedure is well accepted technique in reliability theory. It is useful in getting the required amount of failure time data in a shorter duration of time. Especially those units / items / products are operates from long time without fail i.e., such items are known as highly reliable.

In this paper, we attempted the problem on estimation in simple step-stress ALT for Rayleigh distribution under both the Type-I and Type-II censoring. The MLEs of the model parameters are obtained. The MSE and RABias of the estimators are obtained for three different values of censoring time and three different stress combinations with different sample sizes. It is also seen that for bootstrap confidence interval estimates are narrower in compare to approximate confidence interval estimates for both the parameters. As the sample size increases the asymptotic variance and covariance of the estimators decreases.

There are some open problems in this area need to be attempted for further research work, estimation problems for assumed strategy can be approached for progressive censoring, even the better estimates can be searched by using Bayesian approach also.

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