

A NOVEL THREE-STATE MODEL OF MARKOVIAN APPROACH TO RELIABILITY PREDICTION FOR PUBLIC TRANSIT TRAIN SYSTEMS

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Abstract

In this paper, a novel three-state reliability model of Markovian approach is introduced to calculate the MTBF (Mean Time Between Failure)/MTBSAF (Mean Time Between Service-Affecting Failures) for M-out-of-N redundant units consisting of three operational states: full operation state, functional operation state and failed state. In this three-state reliability model, a transition matrix is introduced to easily and correctly establish a set of state differentiating equations. Laplace transforms are also introduced to simplify the calculating process for the probability function. This model makes the Markovian analysis for three state operational systems become more simple, clear and robust. The concluded MTBF/MTBSAF formula is robust enough and can be used in all three-state models without performing another tedious calculation again. This model is illustrated by four different types of M-out-of-N Redundant units installed in the Public Transit Train: One of Two units, Two of Three voting units, Three of Four voting units and Two of Four voting units. The limitation and boundary of this method is also discussed and elaborated in this paper.

Key Words: Three-state Model, Transition Matrix, Laplace Transform, State Equation, Markovian Analysis, Availability, Reliability, Public Transit, M-out-of-N Redundancy.

Nomenclature

The following acronym, abbreviation and symbol are indispensable in this paper.

ATC	Automatic Train Control
OCC	Operation Control Center
MTBF	Mean Time Between Failure
MTBSAF	Mean Time Between Service Affecting Failure
MTTR	Mean Time To Repair
pphpd	Passengers per hour per direction
R(t)	Reliability Function
VATC	Vehicle Automatic Train Control
μ	Repair Rate (1/MTTR)
λ	Failure Rate

1. Introduction

This paper introduces a new three-state model of Markovian approach for reliability measure to M-out-of-N redundant system and applies this method in the reliability prediction (MTBSAF calculation) to a public transit train system.

2. Availability in Public Transit Train System

Availability is defined as the ability of a product to be in a state to perform a required function under given conditions at a given instant of time or over a given time interval assuming that the required external resources are provided (IEC 60050-191). In public transit system availability is a measure of the total quantity and quality of transportation service actually operated compared with that scheduled to be operated over a given time period.

Most public transit train systems face high availability requirements to meet pphpd. Availability is a combination of reliability and maintainability. For the consideration of robustness and easy maintenance, most on-board train equipment is designed to be capable of repair. Nevertheless, in the event that a failure occurs on the on-board train equipment during revenue service, the maintenance crew cannot immediately be accessible to take corrective action to diagnose and repair the failed unit(s). However, when a failure is detected on the operational train, the Operation Control Center (OCC) will assess the consequence of fault first and then decide to remove the train from revenue service at either of the following conditions:

- Next open station;
- After one round trip;
- At next fleet reduction;
- At the end of daily operation.

3. Three-State Model of Markovian Approach

For most units on board the train, the operational condition can be divided into three states as shown in figure 1: Full Operation, Functional Operation (service-affecting threshold by one or more failures) and Non-Functional Operation (train removal because of occurred failures interrupting service). Full Operation State indicates that all the equipment on the train operate in a healthy manner without any faults which is most desirable. Functional Operation State (Service-affecting threshold) indicates the system can still remain in operation without affecting the service despite one or more on board equipment failures, but any further fault will affect the service resulting in train removal from service. Non Functional Operation State (Train Removal) indicates that the fault (s) happened on board the train will affect the revenue service (i.e. causing delay) and need to move out of service.

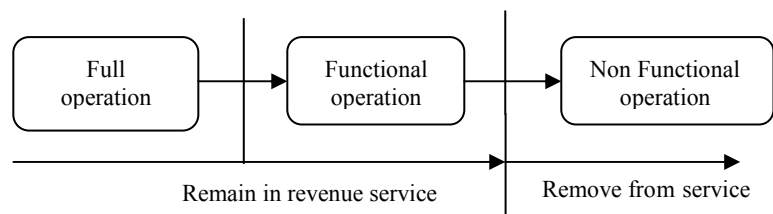


Figure 1 - Three-State Model of Train System Operation

Based on the above, the Three-State model in figure 2 shows Markovian state transitions:

- State 2 is where number of N identical and independent units are operational.
- State 1 is where N-M units are under failure with repair and number of M units operational.
- State 0 is the failed state where failed units are affecting the revenue service and need to move out of service.

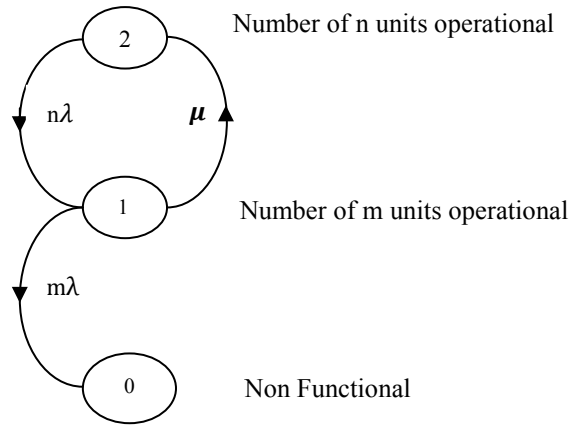


Figure 2 – Operational Three- State Diagram

From this state transition diagram we can write the transition matrix directly as follows:

	2'	1'	0'
2	1-nλΔt	nλΔt	0
1	μΔt	1-(mλ+μ) Δt	mλΔt
0	0	0	1

Note: ' indicates the instantaneous state after Δt.

From this transition matrix described above, we can write the following set of state equations:

$$P_2(t + \Delta t) = (1 - n\lambda\Delta t)P_2(t) + \mu\Delta tP_1(t) \tag{1}$$

$$P_1(t + \Delta t) = n\lambda\Delta tP_2(t) + (1 - (m\lambda + \mu)\Delta t)P_1(t) \tag{2}$$

$$P_0(t + \Delta t) = m\lambda\Delta tP_1(t) + P_0(t) \tag{3}$$

Where:

$$P_0(t) + P_1(t) + P_2(t) = 1 \tag{4}$$

Expanding and rearranging the state equations as follows:

$$\frac{P_2(t + \Delta t) - P_2(t)}{\Delta t} = \frac{dP_2(t)}{dt} = -n\lambda P_2(t) + \mu P_1(t) \quad (5)$$

$$\frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = \frac{dP_1(t)}{dt} = n\lambda P_2(t) - (m\lambda + \mu)P_1(t) \quad (6)$$

$$\frac{P_0(t + \Delta t) - P_0(t)}{\Delta t} = \frac{dP_0(t)}{dt} = m\lambda P_1(t) \quad (7)$$

Taking Laplace transforms:

$$sP_2(s) - P_2(0) = -n\lambda P_2(s) + \mu P_1(s) \quad (8)$$

$$sP_1(s) - P_1(0) = n\lambda P_2(s) - (m\lambda + \mu)P_1(s) \quad (9)$$

$$sP_0(s) - P_0(0) = m\lambda P_1(s) \quad (10)$$

Defining initial conditions:

$$P_0(0) = 0, P_1(0) = 0, P_2(0) = 1 \quad (11)$$

Substituting (11) into equations (8) - (10) and yields:

$$(s + n\lambda)P_2(s) - \mu P_1(s) = 1 \quad (12)$$

$$-n\lambda P_2(s) + (s + (m\lambda + \mu))P_1(s) = 0 \quad (13)$$

$$-m\lambda P_1(s) + sP_0(s) = 0 \quad (14)$$

The above simultaneous equations can be solved for $P_0(s)$:

$$P_0(s) = \frac{nm\lambda^2}{s(s^2 + ((n+m)\lambda + \mu)s + nm\lambda^2)} = \frac{k1}{s} + \frac{k2}{s - s_1} + \frac{k3}{s - s_2} \quad (15)$$

Where:

$$s_{1,2} = \frac{-((n+m)\lambda + \mu) \pm \sqrt{(n-m)^2\lambda^2 + 2(n+m)\lambda\mu + \mu^2}}{2} \quad (16)$$

$$s_1 s_2 = nm\lambda^2 \quad (17)$$

$$k1 = s \frac{nm\lambda^2}{s(s - s_1)(s - s_2)} \Big|_{s=0} = 1 \quad (18)$$

$$k2 = (s - s_1) \frac{nm\lambda^2}{s(s - s_1)(s - s_2)} \Big|_{s=s_1} = \frac{s_2}{s_1 - s_2} \quad (19)$$

$$k3 = (s - s_2) \frac{nm\lambda^2}{s(s - s_1)(s - s_2)} \Big|_{s=s_2} = -\frac{s_1}{s_1 - s_2} \quad (20)$$

$$P_0(s) = \frac{1}{s} + \frac{s_2}{s - s_1} - \frac{s_1}{s - s_2} \quad (21)$$

That implies

$$P_0(t) = 1 + \frac{s_2}{s_1 - s_2} e^{s_1 t} - \frac{s_1}{s_1 - s_2} e^{s_2 t} \quad (22)$$

Reliability function:

$$R(t) = P_1(t) + P_2(t) = 1 - P_0(t) = \frac{s_1 e^{s_2 t} - s_2 e^{s_1 t}}{s_1 - s_2} \quad (23)$$

Mean Time Between Failure:

$$\theta = \int_0^{\infty} R(t) dt = \frac{(n + m)\lambda + \mu}{nm\lambda^2} \quad (24)$$

3.1 Limitation of Three-State Model of Markovian Approach

The above described Three-State Model of Markovian approach is valid when $n - m = 1$; the validation of this approach when $n - m = 2, 3, \dots$ is questionable. We will further discuss and elaborate the solution when $n - m = 2$ or more later in this paper.

4. Application of Three-State Model of Markovian Approach in Public Transit Train System

In the following analysis we use a two-car train as an example to perform MTBSAF calculation for four types of M out of N redundant system.

4.1 One of Two Redundant Units

Train On Board ATC is designed as identical Master-Slave working units. In normal operational condition, the master unit will control the train to accelerate, decelerate, and stop the trains and open doors to transfer the passengers between stations. The master unit and slave unit continuously communicate with each other to monitor the healthy status of each other. In the event that the master unit has a fault and stops control operation, the slave unit will detect the fault of the master unit and take over to control the train.

Figure 3 shows the operation of two healthy VATC during the revenue service, and one of them has failed at point A of the operation. The slave unit will detect the fault through continuous heart beat monitoring and take over to control the train. Upon the detection of one VATC unit fault, OCC takes response to commence train removal from revenue service for maintenance.

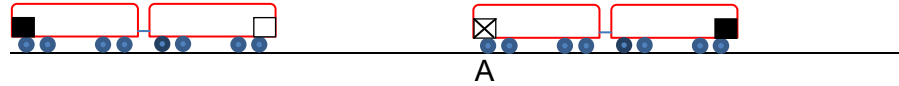


Figure 3 – One out of two VATC fails at point A

4.1.1 Three-State Model of Markovian Approach to One of Two Units

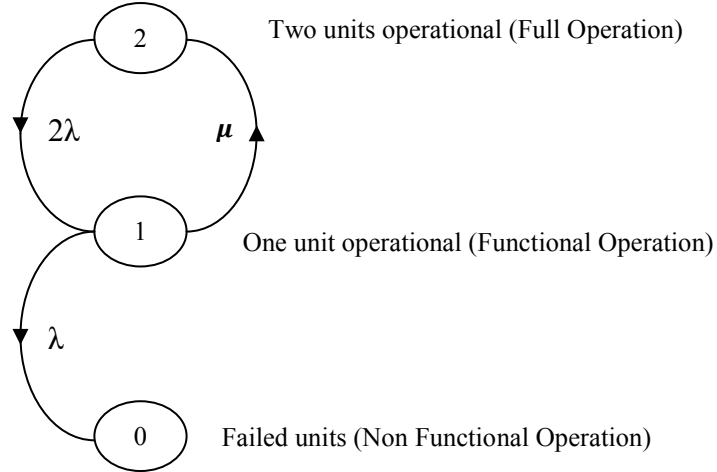


Figure 4 – Three-State Model for one out of two units

For one out of two units, n=2, m=1 and therefore, MTBSAF

$$\theta = \frac{3\lambda + \mu}{2\lambda^2} \tag{25}$$

4.2 Two of Three Majority Voting Units

Three doors are equipped per side of each car, total of 6 doors per car and 12 doors per train as shown in figure 5. If one set of doors fail and lose opening/closing function, an alarm will be transmitted to the OCC. Upon the detection of one door failure, the most popular solution is to isolate the failed door by maintenance crew, and still keep the train in service. The train will be moved out at the next fleet reduction or at the end of daily operation without interrupting service. When two out of three doors per side per car fail, the train shall be moved out of service for maintenance at the next open station because the failure consequences start to impact the revenue service.

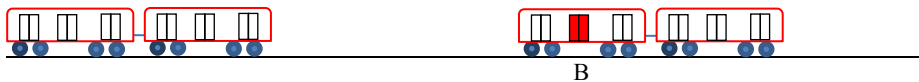


Figure 5 - One door fails at point of B

4.2.1 Three-State Model of Markovian Approach to Two of Three Units

The following Three-State Model is developed for the two out of three majority voting units in figure 6.

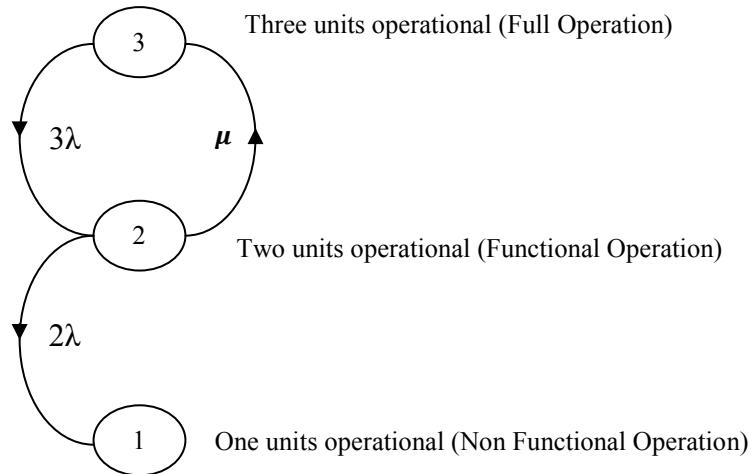


Figure 6 – Three-State Model for two out of three units

For two out of three majority voting units, $n=3$, $m=2$ and therefore, MTBSAF

$$\theta = \frac{5\lambda + \mu}{6\lambda^2} \tag{26}$$

4.3 Three of Four Majority Voting Units

Two propulsion drive units including traction motors and propulsion conversion unit are installed on each car, four of them per train. When one of four propulsion units fails (at point C in the figure 7), the train still remains operational in revenue. The train will be moved out of service after completing one round trip. In the event that two of four propulsion units fail, the fault will impact the revenue service and the OCC will disembark the passengers at the next open station and move the train out of service for maintenance.

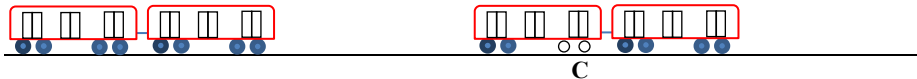


Figure 7 – One out of four propulsion units fails at point C

4.3.1 Three-State Model of Markovian Approach to Three of Four Units

A Three-State Model is developed for the three out of four majority voting units in the figure 8.

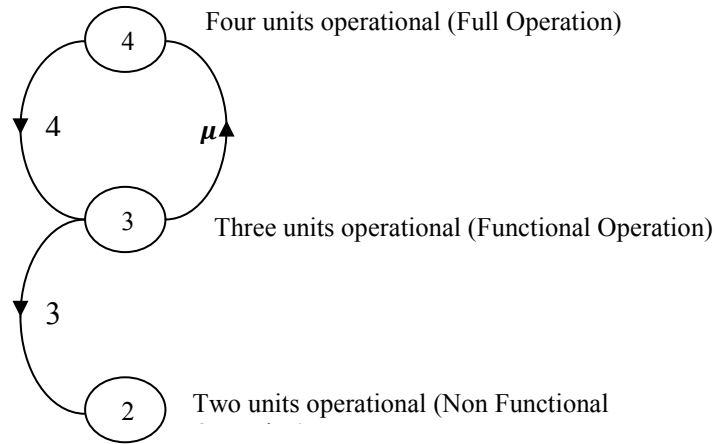


Figure 8 – Three-State Model for three out of four voting units

For three out of four voting majority units, n=4, m=3 and therefore, MTBSAF

$$\theta = \frac{7\lambda + \mu}{12\lambda^2} \tag{27}$$

4.4 Two of Four Majority Voting Units

It happens that when one propulsion unit fails and OCC decides to move the train out of service after completing one round trip another propulsion unit fails before the train removal. See figure 9.

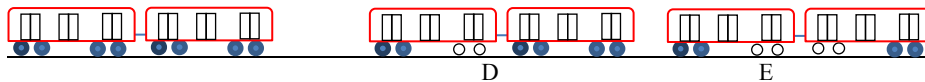


Figure 9 – Two out of four units fail at point E

4.4.1 Three-State Model of Markovian Approach to Two of Four Units

A Three-State Model is developed for the two out of four majority voting units in the figure 10.

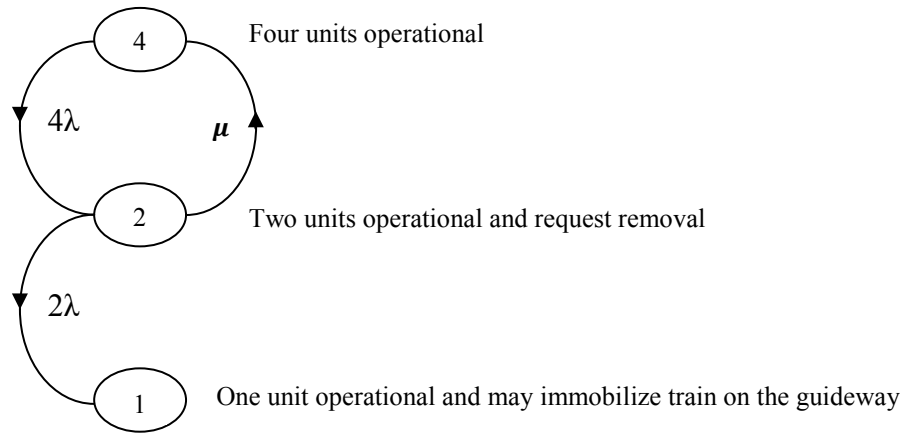


Figure 10 – Three-Sate Model for two out of four voting units

We have discussed in section 3.1, that Three-State model of Markovian Approach is not valid when $n - m = 2$ or more. A solution for this situation is to explode the Three-State transition $4 \rightarrow 2 \rightarrow 1$ to Four-State transition $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$ as shown in figure 11.

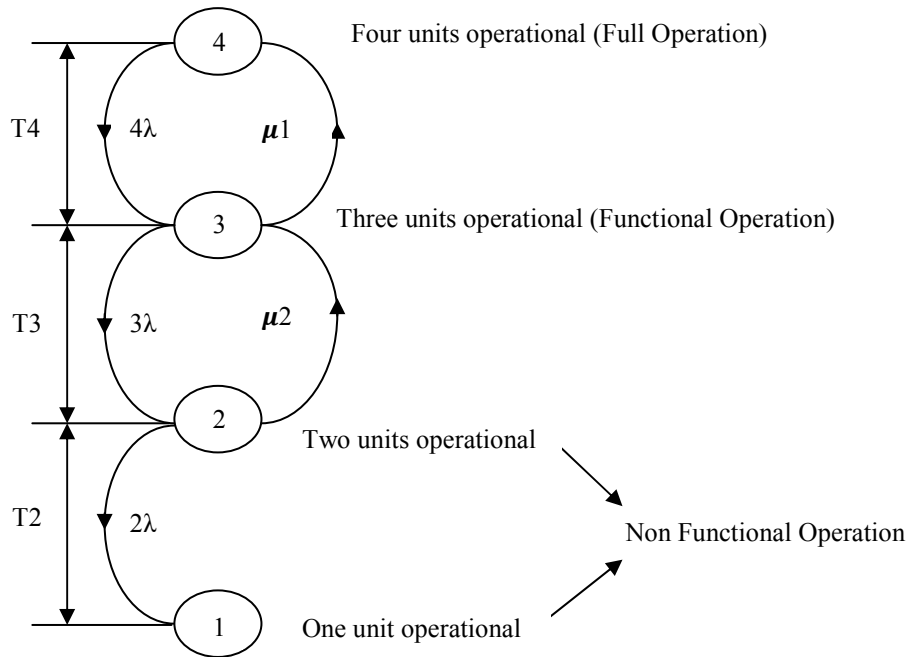


Figure 11 – Four-State Model for two out of four voting units

Perform the similar derivative process from the equation (1) to (24) to yield equation (28):

$$MTBSAF = T_2 + T_3 + T_4 = \frac{\mu_1 \times \mu_2}{4\lambda \times 3\lambda \times 2\lambda} = \frac{\mu_1 \times \mu_2}{24\lambda^3} \quad (28)$$

Hence, we can generalize the expression as equation 29 for M-out-of-N voting units:

$$MTBSAF = \frac{\mu^{(n-m)}}{n\lambda \times (n-1)\lambda \cdots \times m\lambda} \quad (29)$$

Where: n=number of active on-line units, m=minimum number of on-line active units which maintain functional operation, λ =failure rate of an individual on-line unit, μ =repair rate.

This result can be verified by the following equation 30 in the Book “Reliability Toolkit: Commercial Practices Edition”, page 161:

$$\lambda_{(n-q)/n} = \frac{n!(\lambda)^{q+1}}{(n-q-1)!\mu^q} \quad (30)$$

Where: n=number of active on-line units, q= number of on-line active units which are allowed to fail without system failure, λ =failure rate of an individual on-line unit, μ =repair rate.

5. Conclusion

Markovian Analysis is very powerful and effective reliability approximation considering the failure rate and repair rate in the calculating process. The weakness of Markovian analysis is it is difficult to establish precisely correct state equations, their parameters, and the validity of the approximation. In this new three-state model a transition matrix is introduced to establish a set of state equations. Laplace transforms simplify the calculating process of state probability functions. The MTBF/MTBSAF formula in this model can be used in similar systems consisting of three operational states: full operation state, functional operation state and failed state.

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