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NESTED BALANCED N-ARY DESIGNS AND THEIR PB ARRAYS

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Abstract

This paper is concerned with the construction of nested balanced quaternary design (NBQD) through a set of BIBD & balanced ternary design (BTD) along with n-ary designs and their PB arrays considering a set of (n-1) balanced incomplete block designs. Two applications in relation to intercropping designs through PB arrays have also been added.

Key Words: Balanced Incomplete Block Designs, Balanced Ternary Design, Nested Balanced N-Ary Designs, PB Arrays, Strength.

1. Introduction

Initially, balanced n-ary designs were introduced by Tocher (1952) as a generalization of balanced incomplete block designs. A number of authors have given the methods for their construction (Agarwal & Das,1987; Agarwal & Kumar,1986; Billington,1984,1989; Dey,1970; Kageyama,1980; Khodkar,1992; Morgan,1977; Murthy & Das,1967; Saha & Dey,1973; Sarvate & Seberry,1993;Saha & Gujarathi,1989; Sharma & Agarwal,1976).

Nested balanced incomplete block (NBIB) designs were defined by Preece (1967) for statistical situations where there are two sources of variability and one source is nested within the other. That is, an NBIB design has two systems of blocks, the second nested within the first (each block from the first system, called super blocks, consisting of some blocks, called sub-blocks from the second) such that ignoring either system leaves a BIB design where blocks are those of the other system. Gupta and Kageyama (1994) showed an interesting application of NBIB designs in the form of universally optimal complete diallel crosses. Agrawal and Prasad (1983) have given the systematic methods for the construction of NBIB designs. Recently, Gupta ,Lee and Kageyama.(1995) have constructed nested balanced n –ary designs through a set of BIBDs. Sharma (2013) has considered the construction and applications of nested balanced ternary design through tactical configurations.

The purpose of this paper is to construct the nested balanced quaternary design (NBQD) through a set of BIBD and balanced ternary design(BTD) along with n-ary designs and their PB arrays considering a set of (n-1) balanced incomplete block designs giving an application in relation to intercropping designs.

2. Definitions and Notations

2.1 Balanced N-Ary Designs

A balanced n-ary block design has been defined by Tocher (1952) as one which has : (i) the incidence matrix $N=(n_{ij})$, where n_{ij} is the number of times the i^{th} treatment occurs in the j^{th} block, n_{ij} being one of the n identifiable numbers 0, 1,2,…, n-1; i=1,2,…, V; j= 1,2,…, B; and (ii) the property that the variances of comparisons between any two treatments are the same . He also gave some examples of BTD's and investigated their use as incomplete block designs. He did not, however, consider any method of construction of these designs.

The definition of n-ary designs given above with frequencies (0,1,2,…, n-1) replaced by a set of any n distinct positive integers $(f_0, f_1, \ldots, f_{v-1})$ is a more general idea of the definition of a balanced n-ary block design. It is obvious to take for an n-ary block design:

(i) $\sum^{v} n_{ij} = K$ a constant for all j

(ii) $\sum^{B} n_{ij} = R$ a constant for all i

(iii) $\sum_{j=1}^{B} n_{ij}\, n_{i'j} = \Lambda$ a constant for all i and i'; $i \neq i' = 1,2, \ldots, V$;

Each of the elements occurring i^{th} times in precisely Q_i blocks.

2.2 Nested Balanced N-Ary Designs

A nested balanced n-ary design with parameters V, B_{n-1}, B_{n-2} ,…B_1; Q_{n-1}, Q_{n-2},…Q_1; R_{n-1}, R_{n-2},…R_1;K_{n-1}, K_{n-2},…K_1; Λ_{n-1}, Λ_{n-2},…Λ_1; m) is an arrangement of V treatments each replicated R_n times with (n-1)th systems of blocking such that:

(i) the (n-1)th system is nested within the (n-2)th and subsequently with first and each block from the first system (subsequently referred to as whole block) contained exactly m blocks each from the first to (n-1)th system (sub-block);

(ii) ignoring the (n-1) th system leaves a balanced (n-1)-ary design with B_{n-1} blocks each of K_{n-1} unit and with Λ_{n-1} concurrences;

(iii) ignoring the first system leaves a nested balanced n-ary design with B_n blocks each of K_n units and with Λ_n concurrences, where $B_n=B_{n-1}\, B_{n-2} \ldots B_1$; $K_n= K_{n-1}+ K_{n-2}+ \ldots K_1$; and thus, VR =BK, $\Lambda_n(V-1)= R_n(K_n -1)-2Q_{n-1}$.

2.3 Partially Balanced (PB) Arrays

Let **A** be an m×N matrix, with elements 0, 1, 2,…, or s-1. Consider the s^t (1×t) vectors, $X' =(x_1,x_2,\ldots, x_t)$, which can be formed from t-rowed sub-matrix of **A** and associate with each (t×1) vector X a positive integer $\mu(x_1,x_2,\ldots, x_t)$, which is invariant under permutations of $(x_1,x_2,\ldots, x_t)$, where $x_i =0,1,2,\ldots,s-1$;

$i=1,2,\ldots, t$. If for every t-rowed sub-matrix of **A** the s^t distinct $(t\times 1)$ vectors X occur as columns $\mu(x_1,x_2,\ldots,x_t)$ times, then the matrix **A** is called a partially balanced (PB) array of strength t in N assemblies with m constraints, s symbols and the specified $\mu(x_1,x_2,\ldots,x_t)$, parameters.

In view of the fact that $\mu(x_1,x_2,\ldots,x_t)$ is invariant under permutation of $(x_1,x_2,\ldots,x_t)$ one can denote by

$$\mu\begin{matrix} i_1,i_2,\ldots,i_r \\ x_1,x_2,\ldots,x_r \end{matrix}\text{'s}$$

The number of repetitions of a fixed column of any $t\times N$ subarray of A, where the column contains i_1x_1's, i_2x_2's,…and

i_rx_r's, $(x_j=0,1,2,\ldots,s-1)$, $\sum_{j=1}^{r} i_j = t$, $r=\min(\{s,t\})$.

The set of all permutations

$$\mu\begin{matrix} i_1,i_2,\ldots,i_r \\ x_1,x_2,\ldots,x_r \end{matrix}$$

of an array of strength t in s symbols will be called the index set of the array and will be denoted by $\Lambda_{s,t}$. The array A will be represented as the PB arrays (m, N, s, t) with index set $\Lambda_{s,t}$.

The number of times a fixed column containing (i,j) , i,j=0,1,2 for every $t\times N$ sub matrix of A is defined as the frequency of the ordered pair (i,j) and is denoted by μ_{ij}.

3. Construction of Nested Balanced N-Ary Designs and PB Arrays

Gupta *et al.* (1995) have constructed NBTD using the BIB designs D_1 and D_2 with parameters $(v,b_1,r_1,k_1,\lambda_1)$ and $(k_1,b_2,r_2,k_2,\lambda_2)$ respectively. They obtained D as NBTD with parameters v, b_1b_2, $r_1(b_2+r_2)$, k_1+k_2, $\lambda=(2r_2+b_2+\lambda_2)\lambda_1$. It is to be noted that the number of treatments in D_2 equals k_1, the block size of D_1.

On the similar lines of Gupta *et al.* (1995), we have generalized the parameters of n-ary designs through a set of balanced incomplete block designs and thus we have the following theorems:

Theorem 3.1: The existence of a BIBD $(v,b_1,r_1,k_1,\lambda_1)$ and a BTD $(V, B, Q_1, Q_2, R, K, \Lambda)$ implies the existence of a balanced quaternary design provided the number of treatments in BTD must be equal to the block size of BIBD, with the following parameters. $V'=v$, $B'=b_1B$, $Q_1'=Q_1r_1$, $Q_2'=Q_2r_1$, $R'=(R+B)r_1$, $K'=K+k_1$, $\Lambda'=(2R+B+\Lambda)\lambda_1$, where Q_1 and Q_2 are the multiplicities of '1' and '2' in BTD respectively.

Proof: Let us construct a BTD using the treatment labels in the ith block of BIBD $(v,b_1,r_1,k_1,\lambda_1)$ and add the ith block to each of the blocks of this BIBD. Then the resulting design is a NBQD. Each parameter can be verified on the basis of similar lines of Gupta *et,el.*(1995).

Theorem 3.2: The existence of a set of three BIBD's with parameters ($v_1,b_1,r_1, k_1, \lambda_1$), ($k_1,b_2,r_2, k_2, \lambda_2$) and ($k_2,b_3,r_3, k_3, \lambda_3$) implies the existence of BQD (Balanced quaternary design) with the following parameters:

$V=v$, $B=b_1b_2b_3$, $Q_3= r_1r_2r_3$, $Q_2= r_1r_2(b_3-r_3)$, $Q_1= r_1(b_2-r_2) b_3$, $R = r_1r_2r_3 + r_1r_2b_3+ r_1b_2b_3$, $K=k_1+k_2+ k_3$,
$\Lambda= \lambda_1\lambda_2\lambda_3 +\lambda_1\lambda_2 (2r_3+ b_3)+\lambda_1(2r_2r_3 +2r_2b_3 +b_2b_3)$

Proof: Let us construct a BIBD ($k_2,b_3,r_3, k_3, \lambda_3$) using the treatment labels in the ith block of BIBD($k_1,b_2,r_2, k_2, \lambda_2$) and add the ith block to each of the blocks of this BIBD. Then the resulting design is a BTD. Using the Theorem 3.1, the resulting design is a NBQD. Each parameter can be verified on the basis of similar lines of Gupta *et,el.*(1995).

Theorem 3.3: The existence of a set of (n-1) BIBD's where $n\geq 5$, (i.e.) ($v,b_1,r_1, k_1, \lambda_1$), ($k_1,b_2,r_2, k_2, \lambda_2$), ($k_2,b_3,r_3, k_3, \lambda_3$)… ,($k_{n-2},b_{n-1},r_{n-1}, k_{n-1}, \lambda_{n-1}$) provide the existence of balanced n- ary design with the following parameters.

$V=v$, $B=b_1b_2b_3---b_{n-1}$, $Q_{n-1}= r_1r_2---r_{n-1}$, $Q_{n-2} = r_1r_2---r_{n-2}(b_{n-1}-r_{n-1})$,… $Q_1= r_1(b_2-r_2)b_3$,$K=k_1+k_2+---+ k_{n-1}$
$R = r_1r_2r_3---r_{n-1} + r_1r_2 ---r_{n-2}b_{n-1}+ r_1 r_2r_3 --- r_{n-3} b_{n-2}b_{n-1}+ r_1 r_2 --- r_{n-4} b_{n-3}b_{n-2}b_{n-1}+ r_1b_2b_3--- b_{n-2}b_{n-1}$,
$\Lambda= \lambda_1\lambda_2\lambda_3 --- \lambda_{n-1} +\lambda_1\lambda_2\lambda_3 --- \lambda_{n-1} (2r_{n-1}+b_{n-1})+\lambda_1\lambda_2 ---\lambda_{n-3} (2r_{n-2}r_{n-1} +2r_{n-2}b_{n-1} +b_{n-2} b_{n-1}) + \lambda_1\lambda_2 ---\lambda_{n-4} (2r_{n-3}r_{n-2}r_{n-1} + 2r_{n-3} r_{n-2} b_{n-1} +2r_{n-3} b_{n-2} b_{n-1}+--- +b_2 b_3 ---b_{n-2} b_{n-1})$

Proof: On the similar lines of Gupta *et al.* (1995) and Theorem 3.1 and Theorem 3.2.

Theorem 3.4: The column of A′ when treated as assemblies give rise to a partially balanced arrays with v treatments, 2b assemblies, five symbols and strength two, where A′ is given by $A' = [N' | M']$ where A′ denotes the transpose of A, N′ being the transpose of incidence matrix of BQD as per Theorem 3.2 and M′ is the matrix of images of N′.

Proof: Let us consider three BIBD's ($v_1,b_1,r_1, k_1, \lambda_1$), ($k_1,b_2,r_2, k_2, \lambda_2$) and ($k_2,b_3,r_3, k_3, \lambda_3$), then by Theorem 3.2, we have BQD.

Again let $N=n_{ij}$ be the incidence matrix of this BQD design where

n_{ij}	=	3	if the j^{th} treatment occurs thrice in the i^{th} block
	=	2	if the j^{th} treatment occurs twice in the i^{th} block
	=	1	if the j^{th} treatment occurs once in the i^{th} block
	=	0	otherwise

Evidently, N is a b×v array of symbols (0,1,2,3). Let any assembly of this array be designated by a row vector $z= (z_1, z_2,\ldots, z_v)$, $z_i = 0,1,2, 3$
Then we define the images of z as z* given by $z^*= (z_1^*, z_2^*, \ldots, z_v^*)$, $z_i+ z_i^*=4 \pmod 5$ for all i= 1,2, …v. Now let M be a b×v array of images of each of the assemblies of N. Then frequencies of all the treatment combinations in N′ are given below.

$\mu_{33} = \lambda_3\lambda_2\lambda_1$ $\mu_{32} = (r_3-\lambda_3)\,\lambda_2\lambda_1$ $\mu_{31} = r_3(r_2-\lambda_2)\lambda_1$
$\mu_{22} = (b_3 -2r_3+\lambda_3)\,\lambda_2\lambda_1$ $\mu_{21} = (b_3-r_3)\,(r_2-\lambda_2)\lambda_1$ $\mu_{11} = b_3(b_2-2r_2+\lambda_2)\lambda_1$
$\mu_{30} = b_3\lambda_2(r_1-\lambda_1)$ $\mu_{20} = b_3(r_2-\lambda_2)(r_1-\lambda_1)$ $\mu_{10} = b_3(b_2-r_2)(r_1-\lambda_1)$
$\mu_{00} = b_3\, b_2(b_1-2r_1+\lambda_1)$

In M′, we have
$\mu_{11} = \lambda_3\lambda_2\lambda_1$ $\mu_{12} = (r_3-\lambda_3)\,\lambda_2\lambda_1$ $\mu_{13} = r_3(r_2-\lambda_2)\lambda_1$
$\mu_{14} = b_3\lambda_2(r_1-\lambda_1)$ $\mu_{22} = (b_3 -2r_3+\lambda_3)\,\lambda_2\lambda_1$ $\mu_{23} = (b_3-r_3)\,(r_2-\lambda_2)\lambda_1$
$\mu_{24} = b_3(r_2-\lambda_2)(r_1-\lambda_1)$ $\mu_{33} = b_3(b_2-2r_2+\lambda_2)\lambda_1$ $\mu_{34} = b_3(b_2-r_2)(r_1-\lambda_1)$
$\mu_{44} = b_3\, b_2(b_1-2r_1+\lambda_1)$

In overall
$\mu_{00} =\mu_{44} = b_3\, b_2(b_1-2r_1+\lambda_1)$ $\mu_{33} =\mu_{11} = [\lambda_3\lambda_2+b_3\,(b_2-2r_2+\lambda_2)]\lambda_1$
$\mu_{22} =2(b_3 -2r_3+\lambda_3)\,\lambda_2\lambda_1$ $\mu_{21} =\mu_{12} = (b_3-r_3)\,(r_2-\lambda_2)\,\lambda_1= (r_3-\lambda_3)\,\lambda_2\lambda_1$
$\mu_{31} =\mu_{13} = r_3(r_2-\lambda_2)\lambda_1$ $\mu_{14} = b_3\lambda_2(r_1-\lambda_1)$
$\mu_{24} = b_3(r_2-\lambda_2)(r_1-\lambda_1)$ $\mu_{34} = b_3(b_2-r_2)(r_1-\lambda_1)$
$\mu_{32} = (r_3-\lambda_3)\,\lambda_2\lambda_1$ $\mu_{23} = (b_3-r_3)\,(r_2-\lambda_2)\lambda_1$
$\mu_{10} = \mu_{34}=b_3(b_2-r_2)(r_1-\lambda_1)$ $\mu_{20} =\mu_{24}= b_3(r_2-\lambda_2)(r_1-\lambda_1)$
$\mu_{30} =\mu_{14}= b_3\lambda_2(r_1-\lambda_1)$

4. Illustrative Examples

Example 4.1: Let us consider the incidence matrix of the Balanced Quaternary Design. For the parameters V=4, B= 24, Q_1=6, Q_2=6, Q_3=6, R=36, K=6, Λ=44 , using (4,4,3,3,2) (3,3,2,2,1) and (2,2,1,1,0), and applying the construction method given in Section 3 of this paper, we get X a PB array (m=4, N=48, s=5, t=2) with index set $\Lambda_{5,2.}$

$$A' = \left[\begin{array}{cccc|cccc} \multicolumn{4}{c|}{N'} & \multicolumn{4}{c}{M'} \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 0 & 2 & 1 & 3 & 4 \\ 3 & 1 & 2 & 0 & 1 & 3 & 2 & 4 \\ 2 & 1 & 3 & 0 & 2 & 3 & 1 & 4 \\ 1 & 3 & 2 & 0 & 3 & 1 & 2 & 4 \\ 1 & 2 & 3 & 0 & 3 & 2 & 1 & 4 \\ 0 & 3 & 2 & 1 & 4 & 1 & 2 & 3 \\ 0 & 2 & 3 & 1 & 4 & 2 & 1 & 3 \\ 0 & 3 & 1 & 2 & 4 & 1 & 3 & 2 \\ 0 & 2 & 1 & 3 & 4 & 2 & 3 & 1 \\ 0 & 1 & 3 & 2 & 4 & 3 & 1 & 2 \\ 0 & 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 0 & 3 & 2 & 3 & 4 & 1 & 2 \\ 1 & 0 & 2 & 3 & 3 & 4 & 2 & 1 \\ 2 & 0 & 3 & 1 & 2 & 4 & 1 & 3 \\ 3 & 0 & 2 & 1 & 1 & 4 & 2 & 3 \\ 2 & 0 & 1 & 3 & 2 & 4 & 3 & 1 \\ 3 & 0 & 1 & 2 & 1 & 4 & 3 & 2 \\ 2 & 1 & 0 & 3 & 2 & 3 & 4 & 1 \\ 3 & 1 & 0 & 2 & 1 & 3 & 4 & 2 \\ 1 & 2 & 0 & 3 & 3 & 2 & 4 & 1 \\ 1 & 3 & 0 & 2 & 3 & 1 & 4 & 2 \\ 3 & 2 & 0 & 1 & 1 & 2 & 4 & 3 \\ 2 & 3 & 0 & 1 & 2 & 1 & 4 & 3 \end{array}\right]$$

Example 4.2

Let us consider the incidence matrix of the nested balanced ternary designs as (V=3,B=6, Q_1=2, Q_2=2, R=6, K=3, □=4), so that N can be made.

N= {2,1,0}, { 1,2,0},{0,2,1},{0,1,2},{1,0,2},{2,0,1}

Let us consider an intercropping experiment using a main crop p and nine intercrops where the intercrops are partitioned into three groups T_1, T_2, and T_3 with 3 in each group viz., T_1= [1,2,3], T_2= [4,5,6], and T_3=[7,8,9]. Let us designate the symbols 0,1,2 of first row of PB array with inter crops 1,2,3 of T_1, second row with intercrops 4,5,6 of T_2, third row with intercrops 7,8,9 of T_3. Considering the column of the array as the plots of intercrop experiment in addition to the main crop p in each plot, the resulting intercropping experiment will consist of the following six plots :

(p,3,5,7) (p,2,6,7) (p,1,6,8) (p,1,5,9) (p,2,4,9) (p,3,4,8)

It is to be mentioned that this method provides intercropping design with one main crop and nine intercrops divided into three groups of three intercrops having fewer number of blocks and intercrops rather than other developed intercropping designs.

Example 4.3

Let us consider the incidence matrix of the nested balanced ternary designs as (V=4,B=12, Q_1=3, Q_2=6, R=15, K=5, □=16), so that N′ can be made. Taking the images of N′ as M′ using $z_i + z_i^* = 2 \pmod 3$ for all, i=1,2,…,v treatments. The blocks are given below:

$$A' = \left[\begin{array}{cccccccccccc|cccccccccccc} \multicolumn{12}{c|}{N'} & \multicolumn{12}{c}{M'} \\ 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 \\ 2 & 1 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 2 \\ 0 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Let us consider an intercropping experiment using a main crop p and 12 intercrops where the intercrops are partitioned into four groups T_1, T_2, T_3 and T_4 with 3 in each group viz., T_1= [1,2,3], T_2= [4,5,6], T_3=[7,8,9] and T_4= [10,11,12]. Let us designate the symbols 0,1,2 of first row of PB array with inter crops 1,2,3 of T_1, second row with intercrops 4,5,6 of T_2, third row with intercrops 7,8,9 of T_3, as fourth row with intercrops 10,11,12 of T_4. Considering the column of the array as the plots of intercrop experiment in addition to the main crop p in each plot, the resulting intercropping experiment will consist of the following 24 plots :

(p,3,6,8,10) (p,3,5,9,10) (p,2,6,9,10) (p,1,6,9,11) (p,1,6,8,12) (p,1,5,9,12)
(p,2,4,9,12)(p,3,4,9,11) (p,3,4,8,12) (p,3,5,7,12) (p,2,6,7,12) (p,3,6,7,11)
(p,1,4,8,12) (p,1,5,7,12) (p,2,4,7,12) (p,3,4,7,11) (p,3,4,8,10) (p,3,5,7,10)
(p,2,6,7,10) (p,1,6,7,11) (p,1,6,8,10) (p,1,5,9,10) (p,2,4,9,10) (p,1,4,9,11)

It is to be noted that this method provides intercropping design with one main crop and 12 intercrops divided into four groups of three intercrops each. This PB array can be used for conducting intercropping experiments when the intercrops are sub-divided into various groups based on agronomic practices including main crop assuming that some of the interactions among intercrops have less importance. It has been constructed following Rao and Rao (2001).

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