Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 7, Issue 2 (2014): 37-52

BAYESIAN ESTIMATION OF COMPONENT RELIABILITY USING PROGRESSIVELY CENSORED MASKED SYSTEM LIFETIME DATA FROM RAYLEIGH DISTRIBUTION

Jitendra Kumar, M.S. Panwar and Sanjeev K. Tomer[1]
Department of Statistics and DST-CIMS, Banaras Hindu University,
Varanasi-221005, INDIA
E Mail: [1]sktomer73@gmail.com

Received June 09, 2014
Modified December 10, 2014
Accepted December 19, 2014

Abstract

Progressive type-II censoring scheme is a very popular scheme adopted by contributors in the fields of reliability and life-testing. In this paper, we consider a problem when this scheme is applied to a life-testing experiment in which each unit under test is a series system and the investigator is interested in obtaining reliability estimates of individual components. Assuming the components lifetimes to be Rayleigh distribution, we present maximum likelihood and Bayesian approaches to estimate the reliability measures of individual components using masked system lifetime data. The Bayes estimates are evaluated using Lindley's approximation and Gibbs Sampler. The results are illustrated with the help of simulation study.

Key Words*:* Bayesian Estimation, Competing Risk, Gibbs Sampler, Masked Data, Maximum Likelihood Estimation, Rayleigh Distribution.

1. Introduction

The progressive type-II censoring scheme is a generalized censoring scheme which provides flexibility of withdrawal of units during the test and gives fixed number of failures after the termination of the test. The type-II censoring scheme is a special case of this scheme. This scheme has been considered by many authors for reliability estimation. Balakrishnan and Aggarwala (2000) provides a detailed literature and methodology for this scheme. See also Balakrishnan et. al. (2003), Balakrishnan (2007), Soliman (2005) and Ng et. al. (2005) for some citations.

The progressive type-II censoring scheme is described as follows. Let the random variable X denotes the lifetime of a unit. Suppose that n identical units are put to test and non-negative integers $R_1, R_2, ..., R_m$ are fixed in advance satisfying $R_1 + R_2 + ... + R_m = n - m$. At the time of first failure, R_1 of the remaining $n-1$ units are randomly removed. At the time of second failure, R_2 units out of the remaining $n-2-R_1$ units are randomly removed and so on. Finally, at the time of m^{th} failure the experiment is terminated by removing all remaining $R_m = n - m - R_1 + R_2 + ... + R_{m-1}$ units.

In case of life testing experiments designed for multi-component systems, investigators often face the problem of estimation of the reliability measures of individual components using system lifetime data. If the system is a series system, it fails as soon as any one of its components fail. Therefore, the observed data may consist of failure times of systems as well as an indicator denoting the component which causes the failure of the system. Such data can be analyzed using competing risk model in order to estimate the component reliabilities, mean life etc. Many authors have considered such problems in reliability and survival analysis [see Lawless (2003) and Sinha (1986) for some citations]. However, due to some unavoidable reasons such as lack of time, scarcity of funds etc., the cause of failure for some of the failed systems may not be observed. For example, suppose a system under life test caught fire and after its failure it is not possible to identify the exact cause of failure. The data, in such situation, remain incomplete since cause of failure of some of the systems is missing. Such data are also termed as 'masked data'.

The analysis of masked data under competing risk model is considered by many authors by assuming different lifetime distribution for component lifetimes. Miyakawa (1984) obtained the Maximum likelihood estimators (MLEs) for two-component series systems of exponential components when some of the sample observations are masked. Usher and Hodgson (1988) derived MLEs for three-component systems by taking into consideration the phenomena of exact and partial masking. Sarhan and El-Gohary (2003) derived the MLEs and Bayes estimates of reliability functions of components when system components have Pareto life data. In presence of masked data Reiser et. al. (1995) provided Bayesian solution for three component series systems having independent exponential components. Mukhopadhyay and Basu(1997) and Kuo and Yang (1999) presented the analysis of masked data when lifetimes of components follow Weibull distribution. They used Gibbs sampler and EM algorithm for computation. Tan (2007) studied the problem of estimating the reliability functions of the components of series and parallel systems. Xu and Tang (2009) performed Bayesian analysis of masked data assuming the Pareto reliability model as component lifetime. Singh and Tomer (2011) and Tomer et. al. (2013), respectively, derived the ML and Bayesian estimates of component reliabilities when component lifetimes follows a family of life time distributions.

In this paper, we discuss the analysis of progressively type-II censored masked system lifetime data under competing risk model. Assuming the lifetimes of the components to be Rayleigh distribution, we provide ML and Bayes estimates lifetime parameters, mean lives and reliability functions. Rest of the paper is organized as follows of components. In Section 2, we present the likelihood function and derive MLEs of parameters, mean life and reliability function. In Section 3, we give procedures to obtain Bayes estimates of these parametric functions using Lindley's approximation and Gibbs sampler. Finally, In Section 4, we carry out simulation study and conclude the findings.

2. Maximum Likelihood Estimation

Suppose that each system has J components in series and the lifetime of $j^{th}, (j = 1, 2, ..., J)$ component follows the Rayleigh distribution with parameter λ_j having probability density function (*pdf*) given by

$$f(x \mid \lambda_j) = \frac{x}{\lambda_j^{\,2}} \exp\left(-\frac{x^2}{2\lambda_j^{\,2}} \right); \quad x \ge 0, \lambda_j \ge 0. \tag{2.1}$$

The reliability function, hazard rate function and mean time to failure of j^{th} component for the model (2.1), at a specified mission time t (>0), are given, respectively, by

$$R(t \mid \lambda_j) = P(X > t)$$

$$= \exp\left(-\frac{t^2}{2\lambda_j^{\,2}} \right), \tag{2.2}$$

$$h(t \mid \lambda_j) = \frac{t}{\lambda_j^2},$$

and

$$\mu_j = \lambda_j \sqrt{\frac{\pi}{2}}\,. \tag{2.3}$$

Suppose that in an experiment n identical systems are put to test and the progressively type-II censoring scheme described in Section 1, is followed. After the termination of test, lifetimes of m failed systems $X_{1:m}, X_{2:m}, \ldots, X_{m:m}$, along with their cause of failures, are observed. That is, the data $(X_{i:m}, S_i), i = 1, 2, \ldots, m$ (denoted by $\underline{d}$ henceforth) are obtained, where S_i denotes the set of system's components which contain the corresponding cause of failure. Throughout the rest part of the paper, we use notation X_i instead of $X_{i:m}$. With these notations, we write the likelihood function of the data as follows.

$$L(\underline{\lambda} \mid \underline{d}) = \prod_{i=1}^{m} \sum_{j \in S_i} \left(f(x_i|\lambda_j) \prod_{\substack{k=1 \\ k \neq j}}^{J} R(x_i|\lambda_k) \right) \left(\prod_{\substack{k=1 \\ k \neq j}}^{J} R(x_i|\lambda_k) \right)^{R_i} \tag{2.4}$$

It may be noted that for i^{th} system if the set S_i is a singleton, we say that the cause of failure is exactly detected otherwise it is masked.

Since the analysis of the problem becomes very complicated for large number of components, we now assume that each system consists of two components. To proceed with this case, let out of m failures, m_1 and m_2, respectively denote the number of failures occur due to the failure of component 1 and component 2. Further, m_{12} are the number of failed systems for which the cause of failure is masked. It is evident that $m_1 + m_2 + m_{12} = m$. With these notations, the likelihood function (2.4) can be expressed as follows.

$$L(\lambda_1,\lambda_2\mid \underline{d})=\prod_{i=1}^{m_1}\{f(x_i|\lambda_1)R(x_i|\lambda_2)\}\prod_{i=1}^{m_2}\{f(x_i|\lambda_2)R(x_i|\lambda_1)\}\prod_{i=1}^{m_{12}}\{f(x_i|\lambda_1)R(x_i|\lambda_2)$$

$$+f(x_i|\lambda_2)R(x_i|\lambda_1)\}\prod_{i=1}^{m}\left\{\prod_{\substack{k=1\\k\neq j}}^{J}R(x_i|\lambda_k)\right\}^{R_i}. \tag{2.5}$$

Using (2.1) and (2.2), we obtain from (2.5), that

$$L(\lambda_1,\lambda_2\mid \underline{d})=\frac{1}{\lambda_1^{2m_1}\lambda_2^{2m_2}}\left(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}\right)^{m_{12}}\prod_{i=1}^{m}x_i\exp\left\{-\frac{1}{2}\left(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}\right)\sum_{i=1}^{m}x_i^2(R_i+1)\right\}. \tag{2.6}$$

Taking the logarithm of both sides of (2.6), differentiating it partially with respect to λ_1 and λ_2 and solving the likelihood equations, we obtain the expressions for MLEs of λ_1 and λ_2 given by

$$\hat{\lambda}_1=\left(\frac{(m_1+m_2)}{2mm_1}\sum_{i=1}^{m}x_i^2(R_i+1)\right)^{1/2}, \tag{2.7}$$

and

$$\hat{\lambda}_2=\left(\frac{(m_1+m_2)}{2mm_2}\sum_{i=1}^{m}x_i^2(R_i+1)\right)^{1/2}. \tag{2.8}$$

Remarks: Using the invariance property of MLE, the MLE of reliability function of j^{th} component, at time t, can be obtain as follows

$$\hat{R}(t\mid\lambda_j)=\exp\left(-\frac{t^2}{2\hat{\lambda}_j^{\,2}}\right),\ j=1,2.$$

3. Bayesian Estimation

In Bayesian paradigm, we consider $\lambda_j\ (j=1,2)$ to be a random variable. Let the prior density of λ_j with parameters (α_j,β_j), be given by

$$p(\lambda_j)\propto\left(\frac{1}{\lambda_j}\right)^{(2\alpha_j+1)}exp\left(-\frac{\beta_j}{2\lambda_j^2}\right);\quad \lambda_j,\alpha_j,\beta_j>0. \tag{3.1}$$

Now, assuming $\lambda_j's$ to be independent, the joint prior distribution of λ_1 and λ_2 can be written by $p(\lambda_1)p(\lambda_2)$. Combining this joint prior with likelihood function (2.6), via Bayes theorem, the joint posterior density of λ_1 and λ_2 comes out to be

$$\Pi(\lambda_1,\lambda_2\mid \underline{d})=\frac{1}{K\lambda_1^{2m_1+2\alpha_1+1}\lambda_2^{2m_2+2\alpha_2+1}}\left(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}\right)^{m_{12}}\exp\left\{-\sum_{j=1}^{2}\frac{1}{2\lambda_j^2}\left(T+\beta_j\right)\right\}, \tag{3.2}$$

where $T=\sum_{i=1}^{n}x_i^2(R_i+1)$ and K is the normalizing constant given by

$$K = \int_{\lambda_1}\int_{\lambda_2} \frac{1}{\lambda_1^{2m_1+2\alpha_1+1}\lambda_2^{2m_2+2\alpha_2+1}}\left(\frac{1}{\lambda_1^2}+\frac{1}{\lambda_2^2}\right)^{m_{12}} \exp\left\{-\sum_{j=1}^{2}\frac{1}{2\lambda_j^2}\left(T+\beta_j\right)\right\} d\lambda_2 d\lambda_1 .$$

From (3.2), we observe that the marginal distributions of λ_1 and λ_2 cannot be obtained in closed form, which is essential in order to obtain Bayes estimates of individual parameters or parametric functions. Therefore, for further analysis, we proceed with (i) Lindley approximation and (ii) Gibbs Sampler.

3.1 Bayesian Estimation Using Lindley's Approximation

According to Lindley's (1980) approximation, the posterior expectation of any parametric function $\omega(\lambda) = \omega(\lambda_1, \lambda_2)$ which is a ratio of two integrals given by

$$\tilde{\omega}(\lambda) = E(\omega(\lambda) \mid \underline{d}) = \frac{\int_{\lambda_1}\int_{\lambda_2} \omega(\lambda)\, L(\lambda_1,\lambda_2 \mid \underline{d}) p(\lambda_1,\lambda_2)\, d\lambda_1\, d\lambda_2}{\int_{\lambda_1}\int_{\lambda_2} L(\lambda_1,\lambda_2 \mid \underline{d}) p(\lambda_1,\lambda_2)\, d\lambda_1\, d\lambda_2}$$

can be obtained in the form of the following expression.

$$\tilde{\omega} = \hat{\omega}(\lambda) + \frac{1}{2}\left[A + l_{30}B_{12} + l_{03}B_{21} + l_{12}C_{12} + l_{21}C_{21}\right] + \rho_1 A_{12} + \rho_2 A_{21}, \quad (3.3)$$

where

$l = log(L)$, $A = \sum_{i=1}^{2}\sum_{j=1}^{2}\omega_{ij}\sigma_{ij}$, $l_{\eta\xi} = \dfrac{\partial^{\eta+\xi} l}{\partial\lambda_1^{\eta}\partial\lambda_2^{\xi}}$,

$\eta \,\&\, \xi = 0,1,2,3;\quad \eta + \xi = 3 \quad for\ i, j = 1,2.$

$\rho_i = \dfrac{\partial\rho}{\partial\lambda_i}$, $\omega_i = \dfrac{\partial\omega}{\partial\lambda_i}$, $\omega_{ij} = \dfrac{\partial^2\omega}{\partial\lambda_i\partial\lambda_j}$, where $\rho = log\,\pi(\lambda_1, \lambda_2)$ *and* *for* $i \neq j$

$A_{ij} = \omega_i\sigma_{ii} + \omega_j\sigma_{ji}$, $B_{ij} = (\omega_i\sigma_{ii} + \omega_j\sigma_{ij})\sigma_{ii}$, and

$C_{ij} = 3\omega_i\sigma_{ii}\sigma_{ij} + \omega_j(\sigma_{ii}\sigma_{jj} + 2\sigma_{ij}^2)$.

Here σ_{ij} is the $(i,j)^{th}$ element in the inverse of the matrix $\{-l_{ij}\}; i,j = 1,2,$ such that $l_{ij} = \dfrac{\partial^2 l}{\partial\lambda_i\partial\lambda_j}$. Let $\sigma_{11} = \dfrac{H}{N}$, $\sigma_{22} = \dfrac{G}{N}$ *and* $\sigma_{12} = \sigma_{21} = -\dfrac{I}{N}$, where $N = GH - I^2$.

With these notations, using the expression given by Nassar and Eissa (2004), (3.3) can be written as follows,

$$E(\omega(\lambda) \mid d) = \hat{\omega}(\lambda) + \omega_1\psi_1 + \omega_2\psi_2 + \phi, \quad (3.4)$$

where

$$\psi_1 = \frac{1}{N}(H\rho_1 - I\rho_2) + \frac{1}{2N^2}[H^2 l_{30} - IGl_{03} + (GH + 2I^2)l_{12} - 3IHl_{21}], \quad (3.5)$$

$$\psi_2 = \frac{1}{N}(G\rho_2 - I\rho_1) + \frac{1}{2N^2}[G^2 l_{03} - IHl_{30} + (GH + 2I^2)l_{21} - 3IGl_{12}] \quad (3.6)$$

and $$\phi = \frac{1}{2N}[H\omega_{11} - I(\omega_{12} + \omega_{21}) + G\omega_{22}].$$

Let $\delta = \left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \right)$ we drive following expressions for our estimation problem.

$$G = \frac{2m_{12}}{\lambda_1^6 \delta^2}\left[3\lambda_1^2\delta - 2\right] + \frac{2m_1}{\lambda_1^2} - \frac{3T}{\lambda_1^4},\ H = \frac{2m_{12}}{\lambda_2^6 \delta^2}\left[3\lambda_2^2\delta - 2\right] + \frac{2m_2}{\lambda_2^2} - \frac{3T}{\lambda_2^4} \text{ and } I = \frac{-4m_{12}}{\lambda_1^3\lambda_2^3\delta^2}.$$

$$l_{12} = \frac{4m_{12}}{\lambda_1^3\lambda_2^6\delta^3}[3\lambda_2^2\delta - 4], \qquad l_{21} = \frac{4m_{12}}{\lambda_1^6\lambda_2^3\delta^3}[3\lambda_1^2\delta - 4],$$

$$l_{30} = \frac{4m_{12}}{\lambda_1^9\delta^3}\left[-6\lambda_1^4\delta^2 + 9\lambda_1^2\delta - 4\right] - \frac{4m_1}{\lambda_1^3} + \frac{12T}{\lambda_1^5}, \text{ and}$$

$$l_{03} = \frac{4m_{12}}{\lambda_2^9\delta^3}\left[-6\lambda_2^4\delta^2 + 9\lambda_2^2\delta - 4\right] - \frac{4m_2}{\lambda_2^3} + \frac{12T}{\lambda_2^5}.$$

Using the fact that the Bayes estimator of any parameter, under squared error loss function, is its posterior mean, we obtain the Bayes estimates of λ_1, λ_2 and component reliabilities $R_1(t)$, and $R_2(t)$ by using (3.4) as follows.

(i) Bayes estimate of λ_1

When $\omega = \lambda_1$, we have $\omega_1 = 1,\ \omega_2 = 0$ and $\phi = 0$. Substituting these values in (3.4), the Bayes estimator of λ_1, is given by

$$\tilde{\lambda}_1 = \hat{\lambda}_1 + \psi_1.$$

(ii) Bayes estimate of λ_2

When $\omega = \lambda_2$, we have $\omega_1 = 0,\ \omega_2 = 1$ and $\phi = 0$. With these values we obtain the following Bayes estimator of λ_2, form (3.4)

$$\tilde{\lambda}_2 = \hat{\lambda}_2 + \psi_2.$$

(iii) Bayes estimate of $R_1(t)$

When $\omega = \exp\left(-\frac{t^2}{2\lambda_1^2}\right)$, the Bayes estimator of $R_1(t)$ form (3.4) comes out to be

$$\tilde{R}_1(t) = \hat{R}_1(t) + \psi_1\omega_1 + \phi_1,$$

where,

$$\phi_1 = \frac{1}{2N}H\omega_{11},\ \omega_1 = \frac{t^2}{\lambda_1^3} exp\left(-\frac{t^2}{2\lambda_1^2}\right) \text{ and } \omega_{11} = \frac{t^2}{\lambda_1^6}(t^2 - 3\lambda_1^2)\exp\left(-\frac{t^2}{2\lambda_1^2}\right).$$

(iv) Bayes estimate of $R_2(t)$,

If $\omega = \exp\left(-\frac{t^2}{2\lambda_2^2}\right)$, from (3.4) we get the following Bayes estimator of $R_2(t)$

$$\tilde{R}_2(t) = \hat{R}_2(t) + \psi_2\omega_2 + \phi_2,$$

where,

$\phi_2 = \frac{1}{2N} G\omega_{22}$, $\omega_2 = \frac{t^2}{\lambda_2^3} exp\left(-\frac{t^2}{2\lambda_2^2}\right)$ and

$\omega_{22} = \frac{t^2}{\lambda_2^6}(t^2 - 3\lambda_2^2)\exp\left(-\frac{t^2}{2\lambda_2^2}\right)$.

3.2 Bayesian Estimation using Gibbs sampler

Here, we use Gibbs sampling simulation procedure to get samples from the marginal posterior distributions. For implementing Gibbs sampling algorithm, the full conditionals of λ_1 and λ_2 obtained from (3.2) are given, respectively, by.

$$\pi(\lambda_1 \mid \lambda_2, \underline{d}) \propto \frac{1}{\lambda_1^{2m_1+2\alpha_1+1}}\left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}\right)^{m_{12}} \exp\left(-\frac{1}{2\lambda_1^2}(T + \beta_1)\right), \tag{3.7}$$

and

$$\pi(\lambda_1 \mid \lambda_2, \underline{d}) \propto \frac{1}{\lambda_2^{2m_2+2\alpha_2+1}}\left(\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}\right)^{m_{12}} \exp\left(-\frac{1}{2\lambda_2^2}(T + \beta_2)\right). \tag{3.8}$$

Using (3.7) and (3.8), we generate the Gibbs sequence $(\lambda_1^0, \lambda_2^0)$, $(\lambda_1^1, \lambda_2^1)$,…., $(\lambda_1^h, \lambda_2^h)$ as follows.

(1). Choose an initial value of λ_1, say λ_1^0,

(2). Generate λ_2^0 via $\pi(\lambda_2 \mid \lambda_1^0, \underline{d})$

(3). Generate λ_1^1 via $\pi(\lambda_1 \mid \lambda_2^0, \underline{d})$

(4). Generate λ_2^1 via $\pi(\lambda_2 \mid \lambda_1^0, \underline{d})$ and so on.

For a sufficiently large value of h, λ^h becomes a sample observation form marginal of λ. After the burn-in-process we obtain the samples from posterior distributions of λ_1 and λ_2. With these generated samples, we can evaluate the Bayes estimate of parameters or any parametric function.

5. Simulation study

In this section, we present a simulation study to observe the performance of estimators. With the parametric values $\lambda_1 = 2$ and $\lambda_2 = 2.2$, we generate n observations for each component from Rayleigh distribution through inverse transformation technique. As the system under consideration is a series system, the time to failure of the system x_i becomes the minimum of the simulated failure times of components. The component corresponding to minimum failure time is considered to be the cause of failure. From these observations, we draw a sample of m observations using progressive type-II censoring scheme. In this sample we mask the cause of failure of 30% observations and get the final form of the competing risk data with missing cause of failure. For such simulated data sets, we obtain ML estimates of parameters for different patterns of removals of observations in considered progressive type-II censoring scheme. For the computations purpose, the considered removal patterns and their notations which are used in tables, are as follows:

$S^{C}_{m:n}$:no unit is removed during the test; $S^{(1)}_{m:n}$:all (*n-m*) unites are removed at m^{th} failure; $S^{(2)}_{m:n}$:all (*n-m*) unites are removed at first failure; $S^{(3)}_{m:n}$, $S^{(3)}_{m:n}$ and $S^{(3)}_{m:n}$ indicates various possible patterns of removals that are scattered throughout the test.

For Bayesian study, we have chosen the values of prior parameters to be $\alpha_1 = 2$ $\beta_1 = 2,$ $\alpha_2 = 3$ and $\beta_2 = 3$. For these values, the Bayes estimates are evaluated using Lindley's approximation and Gibbs Sampler. We provide the average values of MLEs as well as Bayes estimates along with their mean square error mean (MSE's) based on 2000 repeated samples. The estimates of parameters are given in Tables 1 & 2, component reliabilities in Tables 3 & 4 and that of mean time to failure in Tables 5 & 6. From all the tables, we observe that the MSE of estimates increases as the number of failures *m* in the sample decreases. The MSE of Bayes estimates is less than the ML estimates. The Bayes estimates obtained through Lindleys approximation exhibit greater MSE than Bayes estimate obtained through Gibbs Sampler.

We also consider the analysis of a simulated data set to show how one can apply the results, obtained in the previous sections, to a real life problem. The data is simulated from the considered Rayleigh population by taking $\lambda_1 = 2$ and $\lambda_2 = 2.2$. We generated masked data under progressive type-II censoring schemes with five different patterns. This data is given in Table 7. For this data set the estimated values of parameters and mean time to failure are presented in Table 8. We also plotted reliability functions of individual components in Figures 1 and 2 for the considered two schemes.

				Bayes Estimate			
		MLE		Lindley		MCMC	
Schemes		$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
$S^{C}_{50:50}$	0*50	2.016 (0.048)	2.221 (0.076)	1.976 (0.043)	2.195 (0.068)	1.971 (0.016)	2.124 (0.025)
$S^{(1)}_{40:50}$	0*39, 10	2.020 (0.062)	2.233 (0.098)	1.969 (0.053)	2.199 (0.085)	1.961 (0.017)	2.114 (0.027)
$S^{(2)}_{40:50}$	10, 0*39	1.899 (0.068)	2.102 (0.099)	1.854 (0.071)	2.074 (0.094)	1.888 (0.030)	2.039 (0.047)
$S^{(3)}_{40:50}$	0*15, 1*10, 0*15	2.021 (0.064)	2.224 (0.099)	1.971 (0.055)	2.191 (0.086)	1.964 (0.018)	2.108 (0.029)
$S^{(4)}_{40:50}$	0*10, (0,1)*10, 0*10	1.912 (0.064)	2.102 (0.097)	1.866 (0.066)	2.074 (0.093)	1.896 (0.028)	2.040 (0.046)
$S^{(5)}_{40:50}$	(0,0,0,1)*10	1.946 (0.062)	2.147 (0.094)	1.898 (0.060)	2.117 (0.088)	1.917 (0.023)	2.065 (0.039)
$S^{(1)}_{30:50}$	0*29, 20	2.024 (0.082)	2.232 (0.135)	1.955 (0.067)	2.186 (0.111)	1.951 (0.019)	2.089 (0.033)
$S^{(2)}_{30:50}$	20, 0*29	1.807 (0.107)	2.007 (0.149)	1.751 (0.118)	1.973 (0.144)	1.829 (0.048)	1.972 (0.075)
$S^{(3)}_{30:50}$	0*5, 1*20, 0*5	2.034 (0.086)	2.244 (0.139)	1.963 (0.068)	2.197 (0.112)	1.952 (0.019)	2.096 (0.030)

$S^{(4)}_{30:50}$	0*5, (0,2)*10, 0*5	1.815 (0.105)	2.022 (0.147)	1.758 (0.115)	1.987 (0.141)	1.834 (0.047)	1.979 (0.073)
$S^{(5)}_{30:50}$	(0,1,1)*10	1.870 (0.092)	2.069 (0.137)	1.810 (0.096)	2.031 (0.128)	1.864 (0.037)	2.006 (0.060)
$S^{(1)}_{20:50}$	0*19, 30	2.042 (0.131)	2.270 (0.214)	1.931 (0.092)	2.191 (0.151)	1.933 (0.021)	2.067 (0.037)
$S^{(2)}_{20:50}$	30, 0*19	1.734 (0.180)	1.908 (0.249)	1.650 (0.196)	1.861 (0.235)	1.787 (0.067)	1.912 (0.110)
$S^{(3)}_{20:50}$	0*5, 3*10, 0*5	2.037 (0.132)	2.262 (0.218)	1.926 (0.093)	2.184 (0.155)	1.931 (0.021)	2.067 (0.038)
$S^{(4)}_{20:50}$	0*4, (0,5)*6, 0*4	1.759 (0.166)	1.972 (0.240)	1.674 (0.180)	1.917 (0.213)	1.801 (0.060)	1.937 (0.096)
$S^{(5)}_{20:50}$	(1,2)*10	1.817 (0.146)	2.023 (0.211)	1.726 (0.151)	1.964 (0.185)	1.829 (0.049)	1.962 (0.080)
$S^{(1)}_{10:50}$	0*9, 40	2.060 (0.267)	2.311 (0.431)	1.798 (0.142)	2.108 (0.188)	1.895 (0.025)	2.019 (0.050)
$S^{(2)}_{10:50}$	40, 0*9	1.780 (0.278)	1.993 (0.412)	1.568 (0.271)	1.847 (0.281)	1.806 (0.056)	1.925 (0.098)
$S^{(3)}_{10:50}$	0, 5*8, 0	2.053 (0.270)	2.304 (0.432)	1.791 (0.148)	2.100 (0.193)	1.894 (0.026)	2.018 (0.052)
$S^{(4)}_{10:50}$	0, (0, 10)*4, 0	1.850 (0.267)	2.056 (0.393)	1.622 (0.232)	1.901 (0.250)	1.828 (0.047)	1.945 (0.087)
$S^{(5)}_{10:50}$	4*10	1.827 (0.262)	2.045 (0.401)	1.607 (0.239)	1.890 (0.258)	1.824 (0.048)	1.939 (0.091)

Table 1: Average values of point estimates of λ_1 and λ_2 along their MSEs (in Brackets) for n=50.

				Bayes Estimate			
		MLE		Lindley		MCMC	
Schemes		$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
$S^{C}_{100:100}$	0*100	2.008 (0.024)	2.210 (0.036)	1.989 (0.022)	2.198 (0.034)	1.987 (0.012)	2.157 (0.018)
$S^{(1)}_{80:100}$	0*79,20	2.010 (0.028)	2.210 (0.044)	1.986 (0.026)	2.195 (0.042)	1.983 (0.013)	2.146 (0.020)
$S^{(2)}_{80:100}$	20,0*79	1.878 (0.041)	2.077 (0.054)	1.857 (0.045)	2.064 (0.055)	1.883 (0.028)	2.050 (0.040)
$S^{(3)}_{80:100}$	0*30, 1*20, 0*30	2.008 (0.029)	2.215 (0.045)	1.984 (0.027)	2.199 (0.042)	1.981 (0.014)	2.148 (0.019)
$S^{(4)}_{80:100}$	0*10, (0,0,1)*20, 0*10	1.894 (0.038)	2.093 (0.052)	1.872 (0.041)	2.080 (0.053)	1.895 (0.025)	2.062 (0.036)
$S^{(5)}_{80:100}$	(0, 0, 0, 1)*20	1.919 (0.034)	2.116 (0.049)	1.897 (0.036)	2.103 (0.050)	1.914 (0.021)	2.079 (0.033)
$S^{(1)}_{60:100}$	0*59,40	2.011 (0.039)	2.227 (0.063)	1.979 (0.035)	2.206 (0.058)	1.975 (0.015)	2.139 (0.022)
$S^{(2)}_{60:100}$	40,0*59	1.784 (0.080)	1.970 (0.104)	1.758 (0.089)	1.955 (0.107)	1.811 (0.052)	1.967 (0.075)
$S^{(3)}_{60:100}$	0*10, 1*40, 0*10	2.012 (0.038)	2.222 (0.063)	1.980 (0.035)	2.201 (0.057)	1.974 (0.015)	2.135 (0.023)

$S^{(4)}_{60:100}$	0*5, (0,0,4)*10, 0*5	1.800 (0.073)	1.991 (0.094)	1.773 (0.081)	1.976 (0.097)	1.823 (0.047)	1.982 (0.068)
$S^{(5)}_{60:100}$	(0,1,1)*20	1.831 (0.061)	2.022 (0.083)	1.804 (0.068)	2.006 (0.085)	1.847 (0.039)	2.003 (0.059)
$S^{(1)}_{40:100}$	0*39,60	2.011 (0.061)	2.240 (0.102)	1.961 (0.053)	2.206 (0.088)	1.958 (0.018)	2.115 (0.027)
$S^{(2)}_{40:100}$	60,0*39	1.730 (0.122)	1.915 (0.157)	1.692 (0.137)	1.895 (0.160)	1.777 (0.069)	1.925 (0.100)
$S^{(3)}_{40:100}$	0*5, 2*30, 0*5	2.019 (0.061)	2.232 (0.096)	1.969 (0.053)	2.199 (0.084)	1.960 (0.017)	2.111 (0.027)
$S^{(4)}_{40:100}$	0*5, (0,4)*15, 0*5	1.737 (0.119)	1.926 (0.150)	1.699 (0.133)	1.905 (0.153)	1.782 (0.067)	1.931 (0.096)
$S^{(5)}_{40:100}$	(1, 2)*20	1.779 (0.097)	1.965 (0.132)	1.739 (0.109)	1.943 (0.134)	1.811 (0.054)	1.956 (0.082)
$S^{(1)}_{20:100}$	0*19,80	2.037 (0.133)	2.263 (0.216)	1.926 (0.094)	2.185 (0.153)	1.932 (0.021)	2.066 (0.038)
$S^{(2)}_{20:100}$	80,0*19	1.649 (0.230)	1.845 (0.294)	1.574 (0.253)	1.803 (0.278)	1.744 (0.090)	1.879 (0.132)
$S^{(3)}_{20:100}$	0*5, 8*10, 0*5	2.047 (0.143)	2.261 (0.213)	1.934 (0.097)	2.183 (0.151)	1.931 (0.022)	2.064 (0.038)
$S^{(4)}_{20:100}$	0*2, 5*16, 0*2	1.719 (0.187)	1.907 (0.256)	1.637 (0.206)	1.859 (0.238)	1.779 (0.071)	1.909 (0.110)
$S^{(5)}_{20:100}$	4*20	1.759 (0.162)	1.941 (0.236)	1.673 (0.178)	1.890 (0.217)	1.799 (0.060)	1.926 (0.100)

Note: Here, *a***b* stand for *a, a, a…, b* times.

Table 2: Average values of Point estimates of λ_1 and λ_2 along with their MSEs (in Brackets) for *n*=100.

Schemes		MLE		Bayes Estimate Lindley		Bayes Estimate MCMC	
		$\hat{R}_1$	$\hat{R}_2$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_1$	$\tilde{R}_2$
$S^{C}_{50:50}$	0*50	0.544 (0.005)	0.603 (0.005)	0.526 (0.005)	0.589 (0.005)	0.530 (0.002)	0.578 (0.002)
$S^{(1)}_{40:50}$	0*39, 10	0.544 (0.006)	0.604 (0.006)	0.520 (0.006)	0.586 (0.006)	0.527 (0.002)	0.574 (0.003)
$S^{(2)}_{40:50}$	10, 0*39	0.502 (0.009)	0.567 (0.009)	0.479 (0.011)	0.549 (0.010)	0.500 (0.005)	0.551 (0.005)
$S^{(3)}_{40:50}$	0*15, 1*10, 0*15	0.544 (0.006)	0.602 (0.007)	0.521 (0.006)	0.584 (0.006)	0.527 (0.002)	0.572 (0.003)
$S^{(4)}_{40:50}$	0*10, (0,1)*10, 0*10	0.507 (0.008)	0.567 (0.009)	0.484 (0.010)	0.549 (0.010)	0.503 (0.004)	0.551 (0.005)
$S^{(5)}_{40:50}$	(0,0,0,1)*10	0.519 (0.007)	0.580 (0.008)	0.496 (0.008)	0.562 (0.008)	0.511 (0.003)	0.559 (0.004)
$S^{(1)}_{30:50}$	0*29, 20	0.546 (0.008)	0.604 (0.008)	0.514 (0.008)	0.579 (0.008)	0.523 (0.003)	0.568 (0.003)
$S^{(2)}_{30:50}$	20, 0*29	0.466 (0.016)	0.533 (0.016)	0.436 (0.020)	0.511 (0.018)	0.477 (0.008)	0.527 (0.009)
$S^{(3)}_{30:50}$	0*5, 1*20, 0*5	0.543 (0.008)	0.600 (0.009)	0.511 (0.008)	0.576 (0.008)	0.522 90.003)	0.566 (0.004)

$S^{(4)}_{30:50}$	0*5, (0,2)*10, 0*5	0.469 (0.016)	0.538 (0.015)	0.439 (0.019)	0.515 (0.017)	0.479 (0.007)	0.529 (0.009)
$S^{(5)}_{30:50}$	(0,1,1)*10	0.489 (0.013)	0.553 (0.013)	0.459 (0.015)	0.530 (0.015)	0.490 (0.006)	0.539 (0.007)
$S^{(1)}_{20:50}$	0*19, 30	0.543 (0.012)	0.603 (0.012)	0.495 (0.012)	0.566 (0.011)	0.515 (0.003)	0.558 (0.004)
$S^{(2)}_{20:50}$	30, 0*19	0.431 (0.029)	0.492 (0.030)	0.388 (0.036)	0.461 (0.034)	0.459 (0.011)	0.504 (0.014)
$S^{(3)}_{20:50}$	0*5, 3*10, 0*5	0.541 (0.012)	0.601 (0.013)	0.494 (0.012)	0.564 (0.012)	0.514 (0.003)	0.558 (0.004)
$S^{(4)}_{20:50}$	0*4, (0,5)*6, 0*4	0.442 (0.026)	0.512 (0.026)	0.398 (0.033)	0.479 (0.029)	0.465 (0.010)	0.513 (0.012)
$S^{(5)}_{20:50}$	(1,2)*10	0.464 (0.021)	0.531 (0.021)	0.419 (0.027)	0.496 (0.024)	0.476 (0.008)	0.522 (0.010)
$S^{(1)}_{10:50}$	0*9, 40	0.533 (0.022)	0.595 (0.023)	0.438 (0.026)	0.520 (0.022)	0.500 (0.004)	0.541 (0.006)
$S^{(2)}_{10:50}$	40, 0*9	0.435 (0.040)	0.500 (0.042)	0.346 (0.054)	0.434 (0.046)	0.466 (0.010)	0.507 (0.013)
$S^{(3)}_{10:50}$	0, 5*8, 0	0.530 (0.023)	0.592 (0.024)	0.435 (0.027)	0.517 (0.023)	0.500 (0.004)	0.541 (0.007)
$S^{(4)}_{10:50}$	0, (0, 10)*4, 0	0.461 (0.034)	0.521 (0.036)	0.369 (0.046)	0.453 (0.039)	0.474 (0.008)	0.515 (0.011)
$S^{(5)}_{10:50}$	4*10	0.453 (0.035)	0.517 (0.038)	0.362 0.048	0.449 (0.041)	0.473 (0.008)	0.513 (0.012)

Table 3: Average estimates of Reliabilities of component 1 and component 2 and their MSEs (in Brackets) for *n*=50.

				Bayes Estimate			
		MLEs		Lindley		MCMC	
	Schemes	$\hat{R}_1$	$\hat{R}_2$	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_1$	$\tilde{R}_2$
$S^{C}_{100:100}$	0*100	0.545 (0.002)	0.605 (0.003)	0.536 (0.002)	0.598 (0.003)	0.538 (0.001)	0.590 (0.002)
$S^{(1)}_{80:100}$	0*79,20	0.545 (0.003)	0.604 (0.003)	0.534 (0.003)	0.595 (0.003)	0.536 (0.002)	0.586 (0.002)
$S^{(2)}_{80:100}$	20,0*79	0.499 (0.006)	0.565 (0.005)	0.488 (0.007)	0.556 (0.006)	0.500 (0.004)	0.556 (0.004)
$S^{(3)}_{80:100}$	0*30, 1*20, 0*30	0.544 (0.003)	0.605 (0.003)	0.533 (0.003)	0.596 (0.003)	0.535 (0.002)	0.586 (0.002)
$S^{(4)}_{80:100}$	0*10, (0,0,1)*20, 0*10	0.505 (0.005)	0.570 (0.005)	0.494 (0.006)	0.561 (0.005)	0.505 (0.003)	0.560 (0.004)
$S^{(5)}_{80:100}$	(0, 0, 0, 1)*20	0.514 (0.004)	0.577 (0.004)	0.503 (0.005)	0.568 (0.005)	0.512 (0.003)	0.565 (0.003)
$S^{(1)}_{60:100}$	0*59,40	0.544 (0.004)	0.607 (0.004)	0.529 (0.004)	0.594 (0.004)	0.532 (0.002)	0.583 (0.002)
$S^{(2)}_{60:100}$	40,0*59	0.462 (0.012)	0.529 (0.011)	0.447 (0.014)	0.517 (0.013)	0.472 (0.008)	0.528 (0.009)
$S^{(3)}_{60:100}$	0*10, 1*40, 0*10	0.545 (0.004)	0.605 (0.004)	0.529 (0.004)	0.593 (0.004)	0.532 (0.002)	0.582 (0.002)

$S_{60:100}^{(4)}$	0*5, (0,0,4)*10, 0*5	0.468 (0.011)	0.536 (0.010)	0.454 (0.013)	0.525 (0.012)	0.477 (0.007)	0.533 (0.008)
$S_{60:100}^{(5)}$	(0,1,1)*20	0.481 (0.009)	0.546 (0.009)	0.466 (0.011)	0.535 (0.010)	0.486 (0.006)	0.540 (0.007)
$S_{40:100}^{(1)}$	0*39,60	0.544 (0.006)	0.604 (0.006)	0.520 (0.006)	0.586 (0.006)	0.526 (0.002)	0.573 (0.003)
$S_{40:100}^{(2)}$	60,0*39	0.438 (0.020)	0.506 (0.019)	0.416 (0.024)	0.490 (0.021)	0.457 (0.011)	0.511 (0.012)
$S_{40:100}^{(3)}$	0*5, 2*30, 0*5	0.541 (0.006)	0.606 (0.006)	0.518 (0.006)	0.587 (0.006)	0.525 (0.002)	0.575 (0.003)
$S_{40:100}^{(4)}$	0*5, (0,4)*15, 0*5	0.440 (0.019)	0.510 (0.018)	0.419 (0.023)	0.494 (0.020)	0.459 (0.011)	0.514 (0.012)
$S_{40:100}^{(5)}$	(1, 2)*20	0.458 (0.015)	0.523 (0.015)	0.436 (0.018)	0.507 (0.017)	0.471 (0.008)	0.523 (0.010)
$S_{20:100}^{(1)}$	0*19,80	0.541 (0.012)	0.601 (0.013)	0.494 (0.012)	0.564 (0.012)	0.515 (0.003)	0.558 (0.004)
$S_{20:100}^{(2)}$	80,0*19	0.396 (0.039)	0.468 (0.038)	0.356 (0.048)	0.439 (0.042)	0.441 (0.015)	0.491 (0.017)
$S_{20:100}^{(3)}$	0*5, 8*10, 0*5	0.543 (0.012)	0.601 (0.012)	0.496 (0.012)	0.564 (0.012)	0.514 (0.003)	0.557 (0.005)
$S_{20:100}^{(4)}$	0*2, 5*16, 0*2	0.425 (0.031)	0.491 (0.031)	0.383 (0.038)	0.460 (0.034)	0.456 (0.012)	0.503 (0.014)
$S_{20:100}^{(5)}$	4*20	0.442 (0.025)	0.504 (0.027)	0.398 (0.033)	0.472 (0.030)	0.464 (0.010)	0.509 (0.013)

Table 4: Average estimates of Reliabilities of component 1 and component 2 and their MSEs (in Brackets) for n=100.

Schemes		MLE		Bayes Estimate			
				Lindley		MCMC	
		$\hat{\mu}_1$	$\hat{\mu}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$
$S_{50:50}^{C}$	0*50	2.526 (0.076)	2.783 (0.119)	2.477 (0.068)	2.751 (0.107)	2.470 (0.026)	2.662 (0.039)
$S_{40:50}^{(1)}$	0*39, 10	2.532 (0.097)	2.798 (0.154)	2.468 (0.083)	2.756 (0.134)	2.458 (0.027)	2.650 (0.042)
$S_{40:50}^{(2)}$	10, 0*39	2.380 (0.107)	2.635 (0.155)	2.323 (0.111)	2.600 (0.148)	2.367 (0.047)	2.555 (0.074)
$S_{40:50}^{(3)}$	0*15, 1*10, 0*15	2.533 (0.100)	2.788 (0.155)	2.470 (0.086)	2.746 (0.136)	2.461 (0.028)	2.642 (0.045)
$S_{40:50}^{(4)}$	0*10, (0,1)*10, 0*10	2.396 (0.100)	2.634 (0.152)	2.339 (0.103)	2.599 (0.146)	2.377 (0.043)	2.557 (0.073)
$S_{40:50}^{(5)}$	(0,0,0,1)*10	2.439 (0.097)	2.690 (0.148)	2.379 (0.094)	2.653 (0.138)	2.403 (0.036)	2.588 (0.061)
$S_{30:50}^{(1)}$	0*29, 20	2.537 (0.129)	2.797 (0.211)	2.451 (0.106)	2.740 (0.175)	2.446 (0.029)	2.627 (0.048)
$S_{30:50}^{(2)}$	20, 0*29	2.264 (0.169)	2.213 (0.234)	2.194 (0.186)	2.473 (0.227)	2.292 (0.076)	2.472 (0.117)
$S_{30:50}^{(3)}$	0*5, 1*20, 0*5	2.549 (0.135)	2.813 (0.218)	2.461 (0.107)	2.753 (0.177)	2.445 (0.030)	2.618 (0.051)

$S_{30:50}^{(4)}$	0*5, (0,2)*10, 0*5	2.274 (0.165)	2.534 (0.230)	2.203 (.181)	2.490 (0.221)	2.298 (0.073)	2.480 (0.114)
$S_{30:50}^{(5)}$	(0,1,1)*10	2.344 (0.144)	2.593 (0.215)	2.268 (0.151)	2.546 (0.201)	2.336 (0.058)	2.514 (0.095)
$S_{20:50}^{(1)}$	0*19, 30	2.559 (0.205)	2.845 (0.336)	2.420 (0.145)	2.746 (0.237)	2.422 (0.032)	2.591 (0.058)
$S_{20:50}^{(2)}$	30, 0*19	2.174 (0.282)	2.392 (0.391)	2.068 (0.308)	2.333 (0.369)	2.240 (0.105)	2.396 (0.173)
$S_{20:50}^{(3)}$	0*5, 3*10, 0*5	2.553 (0.208)	2.835 (0.342)	2.414 (0.146)	2.737 (0.244)	2.420 (0.033)	2.590 (0.059)
$S_{20:50}^{(4)}$	0*4, (0,5)*6, 0*4	2.205 (0.260)	2.472 (0.376)	2.098 (0.282)	2.402 (0.335)	2.258 (0.095)	2.427 (0.150)
$S_{20:50}^{(5)}$	(1,2)*10	2.277 (0.229)	2.535 (0.332)	2.163 (0.237)	2.462 (0.291)	2.292 (0.077)	2.459 (0.126)
$S_{10:50}^{(1)}$	0*9, 40	2.582 (0.419)	2.897 (0.676)	2.253 (0.224)	2.642 (0.295)	2.376 (0.040)	2.531 (0.079)
$S_{10:50}^{(2)}$	40, 0*9	2.231 (0.436)	2.498 (0.647)	1.965 (0.426)	2.315 (0.442)	2.264 (0.088)	2.413 (0.091)
$S_{10:50}^{(3)}$	0, 5*8, 0	2.573 (0.425)	2.888 (0.679)	2.245 (0.233)	2.632 (0.303)	2.374 (0.041)	2.529 (0.082)
$S_{10:50}^{(4)}$	0, (0, 10)*4, 0	2.318 (0.420)	2.577 (0.618)	2.033 (0.364)	2.382 (0.392)	2.291 (0.074)	2.438 (0.137)
$S_{10:50}^{(5)}$	4*10	2.290 (0.411)	2.563 (0.630)	2.014 (0.376)	2.369 (0.405)	2.285 (0.075)	2.430 (0.143)

Table 5: Average estimates of Mean Time to Failures of component 1 and component 2 and their MSEs (in Brackets) for *n*=50.

Schemes		MLE		Bayes Estimate Lindley		Bayes Estimate MCMC	
		$\hat{\mu}_1$	$\hat{\mu}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$
$S_{100:100}^{C}$	0*100	2.516 (0.037)	2.770 (0.056)	2.492 (0.035)	2.754 (0.054)	2.490 (0.019)	2.703 (0.027)
$S_{80:100}^{(1)}$	0*79,20	2.519 (0.044)	2.770 (0.069)	2.489 (0.042)	2.751 (0.065)	2.485 (0.021)	2.690 (0.031)
$S_{80:100}^{(2)}$	20,0*79	2.354 (0.065)	2.603 (0.085)	2.327 (0.071)	2.587 (0.087)	2.360 (0.043)	2.570 (0.062)
$S_{80:100}^{(3)}$	0*30, 1*20, 0*30	2.516 (0.046)	2.776 (0.070)	2.486 (0.043)	2.756 (0.066)	2.482 (0.021)	2.692 (0.030)
$S_{80:100}^{(4)}$	0*10, (0,0,1)*20, 0*10	2.374 (0.059)	2.623 (0.081)	2.347 (0.064)	2.607 (0.083)	2.375 (0.039)	2.584 (0.057)
$S_{80:100}^{(5)}$	(0, 0, 0, 1)*20	2.405 (0.053)	2.652 (0.078)	2.378 (0.056)	2.635 (0.078)	2.399 (0.033)	2.605 (0.051)
$S_{60:100}^{(1)}$	0*59,40	2.521 (0.061)	2.792 (0.099)	2.480 (0.055)	2.765 (0.090)	2.475 (0.024)	2.681 (0.035)
$S_{60:100}^{(2)}$	40,0*59	2.235 (0.126)	2.468 (0.163)	2.203 (0.140)	2.450 (0.168)	2.270 (0.082)	2.465 (0.117)
$S_{60:100}^{(3)}$	0*10, 1*40, 0*10	2.522 (0.060)	2.785 (0.098)	2.481 (0.055)	2.758 (0.090)	2.474 (0.024)	2.676 (0.036)

$S^{(4)}_{60:100}$	0*5, (0,0,4)*10, 0*5	2.256 (0.115)	2.495 (0.148)	2.222 (0.128)	2.476 (0.152)	2.285 (0.074)	2.484 (0.106)
$S^{(5)}_{60:100}$	(0,1,1)*20	2.295 (0.096)	2.534 (0.131)	2.261 (0.107)	2.514 (0.134)	2.314 (0.061)	2.511 (0.092)
$S^{(1)}_{40:100}$	0*39,60	2.520 (0.096)	2.807 (0.160)	2.458 (0.084)	2.765 (0.138)	2.453 (0.028)	2.651 (0.043)
$S^{(2)}_{40:100}$	60,0*39	2.168 (0.192)	2.400 (0.246)	2.121 (0.215)	2.375 (0.252)	2.228 (0.108)	2.413 (0.157)
$S^{(3)}_{40:100}$	0*5, 2*30, 0*5	2.531 (0.096)	2.798 (0.151)	2.467 (0.083)	2.756 (0.131)	2.456 (0.027)	2.646 (0.043)
$S^{(4)}_{40:100}$	0*5, (0,4)*15, 0*5	2.177 (0.187)	2.414 (0.236)	2.129 (0.209)	2.388 (0.241)	2.233 (0.106)	2.421 (0.151)
$S^{(5)}_{40:100}$	(1, 2)*20	2.229 (0.152)	2.463 (0.207)	2.179 (0.172)	2.435 (0.210)	2.269 (0.084)	2.452 (0.128)
$S^{(1)}_{20:100}$	0*19,80	2.553 (0.209)	2.837 (0.339)	2.414 (0.148)	2.738 (0.241)	2.421 (0.033)	2.589 (0.059)
$S^{(2)}_{20:100}$	80,0*19	2.067 (0.361)	2.313 (0.462)	1.973 (0.397)	2.259 (0.437)	2.185 (0.141)	2.355 (0.207)
$S^{(3)}_{20:100}$	0*5, 8*10, 0*5	2.565 (0.224)	2.834 (0.334)	2.423 (0.153)	2.736 (0.237)	2.420 (0.034)	2.587 (0.060)
$S^{(4)}_{20:100}$	0*2, 5*16, 0*2	2.154 (0.294)	2.390 (0.402)	2.052 (0.323)	2.330 (0.373)	2.230 (0.111)	2.393 (0.173)
$S^{(5)}_{20:100}$	4*20	2.204 (0.255)	2.433 (0.371)	2.097 (0.280)	2.369 (0.341)	2.255 (0.095)	2.414 (0.157)

Table 6: Average estimates of Mean Time to Failures of component 1 and component 2 and their MSEs (in Brackets) for *n*=100.

Schemes			Components lifetimes
$S^{C}_{30:30}$	0*30	X_1	0.15, 0.50, 0.73, 0.75, 1.06, 1.46, 1.86, 1.86, 2.24, 2.42, 2.48, 2.71, 2.99.
		X_2	0.73, 1.18, 1.54, 1.58, 1.67, 1.78, 3.93, 4.49.
		X_{12}	3.52, 1.67, 1.93, 2.32, 4.21, 0.89, 1.02, 1.92, 2.31.
$S^{(1)}_{20:30}$	0*19, 10	X_1	0.15, 0.50, 1.06, 1.46, 1.67, 1.86, 1.93, 2.24.
		X_2	1.02, 1.18, 1.58, 1.67, 1.78, 1.92.
		X_{12}	0.73, 0.75, 1.54, 0.89, 1.86, 0.73.
$S^{(2)}_{20:30}$	10, 0*19	X_1	0.15, 1.06, 1.46, 2.31, 2.32, 2.42, 2.48, 2.71, 2.99.
		X_2	0.89, 1.67, 1.78, 3.93, 4.49.
		X_{12}	1.93, 1.54, 4.21, 0.73, 0.50, 1.92.
$S^{(3)}_{20:30}$	0*5, 1*10, 0*5	X_1	0.73, 0.75, 1.86, 1.86, 1.93, 2.32, 2.48, 2.99.
		X_2	0.73, 0.89, 1.02, 1.18, 1.78, 1.92.
		X_{12}	2.42, 1.06, 0.50, 2.24, 0.15, 1.67.
$S^{(4)}_{20:30}$	0*5, 0,2)*5, 0*5	X_1	0.15, 0.50, 0.73, 1.06, 1.86, 2.24, 2.48.
		X_2	0.73, 0.89, 1.02, 1.54, 1.58, 1.67, 1.78.
		X_{12}	1.92, 1.93, 1.18, 0.75, 2.42, 2.32.
$S^{(5)}_{20:30}$	(0, 1)*10	X_1	0.15, 0.50, 0.75, 1.86, 1.93, 2.31, 2.71
		X_2	0.89, 1.02, 1.18, 1.54, 1.58, 1.67, 1.92.
		X_{12}	0.73, 2.24, 1.78, 2.32, 1.06, 1.86.

Table 7: Generated data with 30% masking for different schemes when *n*=30 & *m*=20.

Schemes	MLE's				Bayes Estimate							
					Lindley				MCMC			
	$\hat{\lambda}_1$	$\hat{\lambda}_2$	$\hat{\mu}_1$	$\hat{\mu}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\mu}_1$	$\tilde{\mu}_2$
$S^{C}_{30:30}$	1.98	2.53	2.4	3.17	1.93	2.4	2.42	3.09	1.95	2.27	2.45	2.84
$S^{(1)}_{20:30}$	2.00	2.31	2.51	2.90	1.91	2.24	2.40	2.81	1.94	2.08	2.43	2.61
$S^{(2)}_{20:30}$	2.10	2.80	2.63	3.53	2.01	2.69	2.52	3.37	2.0	2.34	2.54	2.93
$S^{(3)}_{20:30}$	1.90	2.19	2.38	2.75	1.82	2.13	2.28	2.68	1.87	2.01	2.35	2.51
$S^{(4)}_{20:30}$	1.92	1.92	2.41	2.41	1.83	1.89	2.29	2.37	1.87	2.01	2.34	2.51
$S^{(5)}_{20:30}$	2.03	2.03	2.55	2.55	1.93	1.99	2.42	2.50	1.86	1.83	2.34	2.29

Table 8: Est. values of λ_1, λ_2 & Mean Time to failure of components (n=30 & m=20

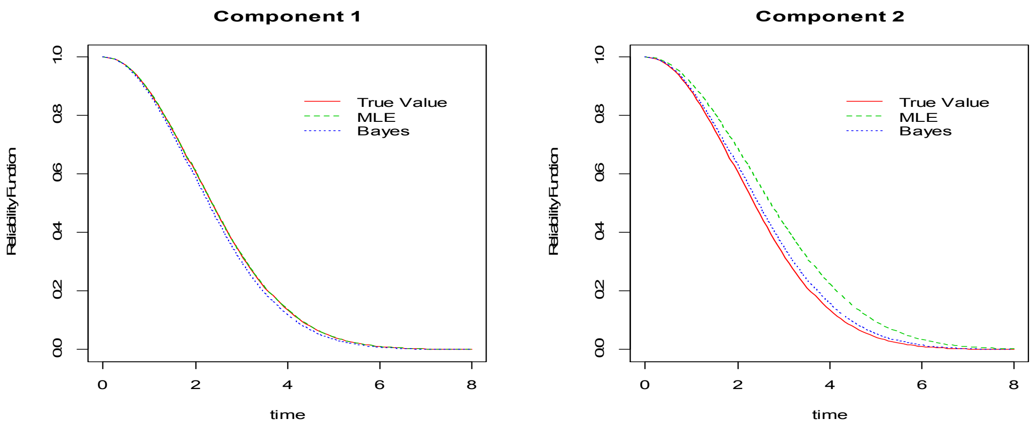

Fig. 1: Component Reliability curves under scheme $S^{(1)}_{20:30}$.

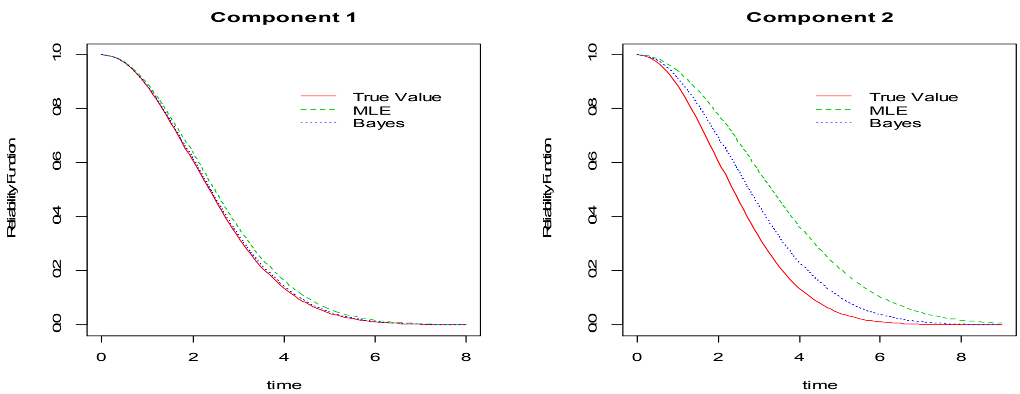

Fig. 2: Component Reliability curves under scheme $S^{(2)}_{20:30}$.

References

1. Balakrishana, N. (2007). Progressive censoring methodology: An appraisal (with discussions), Test, 16, p. 211-296.
2. Balakrishanan, N. and Aggarwala, R. (2000*).* Progressively censoring: Theory, Methods, and Application, Birkhauser, Boston, MA.
3. Balakrishnan, N., Kannan, N., Lin, C.T. and Ng, H.K.T. (2003). Point and Interval estimation for Gaussian distribution, based on progressively censored samples, IEEE Trans. Reliab., 52, p. 90–95.
4. L. Kuo and T. E. Yang. (2000). Bayesian reliability modeling for masked system lifetimedata, Statist. Probab. Lett. , 47, p.229–241.
5. Lawless, J. F. (2003). Statistical models and methods for lifetime data, Wiley, New York.
6. Lindley, D.V. (1980). Approximate Bayesian methods, Trabajos de Estadistica., 21, p. 223-237.
7. Mukhopadhyay, C. and Basu, A. P. (1997). Bayesian analysis of incomplete time and cause of failure data, Jour. Stat. Plan. Infer., 59, p. 79-100.
8. Miyakawa, M. (1984). Analysis of incomplete data in competing risks model, IEEE Trans. Reliability, 33, p. 293-296.
9. Nassar, M. M. and Eissa, F. H. (2005). Bayesian estimation for the exponentiated weibull model, Communications in Statistics - Theory and Methods, 33(10), p. 2343-2362,
10. Ng, H. K. T., Chan, P. S. and Balakrishnan, N. (2005). Optimal progressive censoring plans for the weibull distribution, Technometrics, 46, p. 470–481.
11. Reiser, B., Guttman, I., Lin, D.K.J., Guess, F.M. and Usher, J.S.(1995). Bayesian inference for masked system lifetime data, Appl. Stat. 44, p. 79-90.
12. Sarhan, A.M. and El-Gohary, A. (2003). Estimations of parameters in Pareto reliability model in the presence of masked data, Reliab. Eng. Syst. Safety, 82, p. 75-83.
13. Singh, A.K. and Tomer, S.K. (2011). Maximum Likelihood Estimation of component reliability using masked series system life time data, Cul. Stat. Assoc. Bull., 63, p. 249-252.
14. Sinha, S.K. (1986). Reliability and Life Testing. Wiley Eastern Ltd., New Delhi.
15. Soliman, A. A. (2005). Estimation of parameters of life from progressively censored data using Burr-XII model, IEEE Transactions on Reliability, 54(1), p. 34-42.
16. Tan, Z. (2007). Estimation of exponential component reliability from uncertain life data in series and parallel systems, Reliab. Eng. Syst. Safety, 92, p. 223-230.
17. Tomer, S. K., Singh, A. K. and Panwar, M. S. (2014). Bayesian analysis of masked series system lifetime data from a family of lifetime distributions, Inter. Jour. of System Assur. Engg. Mgmt., 5(4), p. 495-502.
18. Usher, J.S. and Hodgson, T.J. (1988). Maximum likelihood analysis of component reliability using masked system life-test data, IEEE Trans. Reliab., R-37, p. 550-555.
19. Xu, A. and Tang, Y. (2009). Bayesian analysis of Pareto reliability with dependent masked data, IEEE Trans. Reliab., 58(4), p. 583-588.