Journal of Reliability and Statistical Studies; ISSN (Print): 0974-8024, (Online):2229-5666
Vol. 7, Issue 1 (2014): 19-29

# AN ALTERNATIVE ESTIMATION OF THE SCALE PARAMETER FOR MORGENSTERN TYPE BIVARIATE LOG-LOGISTIC DISTRIBUTION USING RANKED SET SAMPLING

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Received October 26, 2013
Modified April 04, 2014
Accepted May 15, 2014

## Abstract

In this paper we have suggested some improved estimators of a scale parameter of Morgenstern type bivariate log-logistic distribution (MTBLLD) envisaged by Lesitha and Thomas (2012), based on the observations made on the units of ranked set sampling regarding the study variable $Y$ which is correlated with the auxiliary variable $X$, where $(X,Y)$ follows a MTBLLD. Numerical illustration is given in support of the present study.

***AMS Subject Classification***: 62G30; 62H12.

**Key Words**: Minimum Mean Squared Error Estimator, Shrinkage Estimator, Morgenstern Type Bivatiate Log-Logistic Distribution, Ranked Set Sample, Best Linear Unbiased Estimator, Extreme Ranked Set Sample.

## 1. Introduction

Ranked set sampling (RSS) is a method of sampling that can be advantageous when quantification of all sampling units is costly but a small set of units can be easily ranked, according to the character under investigation, without actual quantification. The technique was first introduced by McIntyre (1952) for estimating means pasture and forage yields. The theory and application of ranked set sampling is given by Chen *et al*. (2004). Suppose the variable of interest say $Y$, is difficult or much expensive to measure, but an auxiliary variable $X$ correlated with $Y$ is readily measureable and can be ordered exactly. In this case as an alternative to McIntyre (1952) method of ranked set sampling, Stokes (1977) used an auxiliary variable for the ranking of sampling units. Lesitha and Thomas (2012) used imperfect RSS to estimate the scale parameter $\alpha$ in MTBLLD. For a description of Morgenstern family of bivariate distributions, (see, Scaria and Nair (1999)).

A bivariate random vector $(X,Y)$ is said to have a Morgenstern type bivariate log-logistic distribution (MTBLLD)with parameters $\alpha,\beta,\omega,\eta$ if its cumulative distribution function is given by, see Lesitha and Thomas (2012)

$$F(x,y)=\frac{x^{\beta}}{\alpha^{\beta}+x^{\beta}}\frac{y^{\eta}}{\omega^{\eta}+y^{\eta}}\left[1+\lambda\left\{\left(1-\frac{x^{\beta}}{\alpha^{\beta}+x^{\beta}}\right)\left(1-\frac{y^{\eta}}{\omega^{\eta}+y^{\eta}}\right)\right\}\right], \tag{1}$$
$$-1\leq\lambda\leq 1,(x,y)>0,\alpha,\omega>0\ \beta,\eta>1.$$

Then the joint *pdf* of $(X,Y)$ is given by

$$f(x,y)=\frac{\beta\alpha^{\beta}x^{\beta-1}}{(\alpha^{\beta}+x^{\beta})^{2}}\frac{\eta\omega^{\eta}y^{\eta-1}}{(\omega^{\eta}+y^{\eta})^{2}}\left[1+\lambda\left\{\left(\frac{\alpha^{\beta}-x^{\beta}}{\alpha^{\beta}+x^{\beta}}\right)\left(\frac{\omega^{\eta}-y^{\eta}}{\omega^{\eta}+y^{\eta}}\right)\right\}\right], \tag{2}$$
$$-1\leq\lambda\leq 1,(x,y)>0,\alpha,\omega>0\ \beta,\eta>1.$$

Let $(X_i,Y_i), i=1,2,\ldots,n$ be a sequence of independent observations drawn from (1). Let $X_{[r:n]}$ be the concomitant of the *rth* order statistic $Y_{r:n}$ arising from (1). Then by using following representation described for the distribution of concomitants of order statistics by Scaria and Nair (1999), Lesitha and Thomas (2012) obtained the *pdf* $f_{[r:n]}(x)$ of $X_{[r:n]}, 1\leq r\leq n$ as:

$$f_{[r:n]}(x)=\frac{\beta\alpha^{\beta}x^{\beta-1}}{(\alpha^{\beta}+x^{\beta})^{2}}\left[1+\lambda\left(\frac{n-2r+1}{n+1}\right)\left(1-\frac{2x^{\beta}}{\alpha^{\beta}+x^{\beta}}\right)\right], x,\alpha>0\ \beta>1, 1\leq\lambda\leq 1\ . \tag{3}$$

Now the *kth* moment of the concomitant $X_{[r:n]}$ is given by

$$E\left[X_{[r:n]}^{k}\right]=\mu^{(k)}+\frac{\lambda(n-2r+1)}{n+1}\left(\mu^{(k)}-\mu_{2:2}^{(k)}\right), \tag{4}$$

where

$$\mu^{(k)}=E\left[X^{k}\right]=\alpha^{k}B\left(1-\frac{k}{\beta},1+\frac{k}{\beta}\right) \text{ and } \mu_{2:2}^{(k)}=E\left[X_{2:2}^{k}\right]=\alpha^{k}\Gamma\left(2+\frac{k}{\beta}\right)\Gamma\left(1-\frac{k}{\beta}\right).$$

So by (4) we get

$$E\left[X_{[r:n]}^{k}\right]=\alpha^{k}\Gamma\left(1-\frac{k}{\beta}\right)\Gamma\left(1+\frac{k}{\beta}\right)\left[1-\frac{\lambda(n-2r+1)}{n+1}\frac{k}{\beta}\right]. \tag{5}$$

Lesitha and Thomas (2012) used (5) to obtained mean and variance of $X_{[r:n]}$ respectively as:

$$E\left[X_{[r:n]}\right]=\alpha\xi_{r} \tag{6}$$

and

$$Var\left[X_{[r:n]}\right]=\alpha^{2}\delta_{rr}, \tag{7}$$

where

$$\xi_{r}=\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1+\frac{1}{\beta}\right)\left[1-\frac{\lambda(n-2r+1)}{n+1}\frac{1}{\beta}\right], \tag{8}$$

and

$$\begin{aligned}\delta_{rr} &= \Gamma\left(1-\frac{2}{\beta}\right)\Gamma\left(1+\frac{2}{\beta}\right)\left[1-\frac{\lambda(n-2r+1)}{n+1}\frac{2}{\beta}\right] \\ &-\left[\Gamma\left(1-\frac{1}{\beta}\right)\Gamma\left(1+\frac{1}{\beta}\right)\left[1-\frac{\lambda(n-2r+1)}{n+1}\frac{1}{\beta}\right]\right]^2.\end{aligned} \tag{9}$$

The reaming parts of the paper are organized as follows:

In sections 2 and 3, some improved estimators are described on the lines of Searls (1964), Singh *et al.* (1973) and Searls and Intarapanich (1990), the expressions of bias and mean squared error (MSE) are obtained and compared with usual unbiased estimators, given by Lesitha and Thomas (2012). In section 4, we have computed the relative efficiencies of different estimators numerically to evaluate their performance. Section 5, concludes the paper with final remarks.

## 2. Improved estimation of scale parameter $\alpha$ based on best linear unbiased estimator (BLUE) of RSS

Let $(X,Y)$ be a bivariate random variable having MTBLLD. Lesitha and Thomas (2012) used Stokes (1977) procedure for selection ranked set sample. At first select $n$ independent samples each with $n$ units from the population with distribution function given in (1). From *ith* sample the unit having the observation on $Y$ ranked as $i$ is chosen and measurement $X_{[r:n]}$ on $X$ is taken from that unit for $i=1,2,\ldots,n$. Then the RSS observations are $X_{[r:n]r}, r=1,2,\ldots,n$. Then from (6) Lesitha and Thomas (2012) observed that for each $r, \frac{X_{[r:n]r}}{\xi_r}$ is an unbiased estimator of $\alpha$. If the parameter $\beta$ is known, than Lesitha and Thomas (2012) defined BLUE of $\alpha$ as:

$$\hat{\alpha}_{rss} = \frac{\sum_{r=1}^{n}\left(\frac{\xi_r}{\delta_{rr}}\right)X_{[r:n]r}}{\sum_{r=1}^{n}\left(\frac{\xi_r^2}{\delta_{rr}}\right)} \tag{10}$$

and

$$Var(\hat{\alpha}_{rss}) = \frac{\alpha^2}{\sum_{r=1}^{n}\left(\frac{\xi_r^2}{\delta_{rr}}\right)} = \alpha^2 V_1, \tag{11}$$

where

$$V_1 = \frac{1}{\sum_{r=1}^{n}\left(\frac{\xi_r^2}{\delta_{rr}}\right)}.$$

We propose a class of estimators for the parameter $\alpha$ as

$$t_1 = A\hat{\alpha}_{rss}, \tag{12}$$

where $A$ is a suitably chosen constant, such that mean squared error of $t_1$ is minimum. The bias and mean squared error (MSE) of $t_1$ are respectively given by

$$B(t_1) = (A-1)\alpha \tag{13}$$

$$MSE(t_1) = \alpha^2\left[A^2(1+V_1) - 2A + 1\right]. \tag{14}$$

The $MSE(t_1)$ is minimum when

$$A = (1+V_1)^{-1}. \tag{15}$$

Now substitution of (15) in (12) yields the minimum MSE estimator of $\alpha$ as

$$t_{1m} = (1+V_1)^{-1}\hat{\alpha}_{rss}. \tag{16}$$

The bias and mean squared error (MSE) of $t_{1m}$ are respectively given by

$$B(t_{1m}) = -\alpha\frac{V_1}{1+V_1} \tag{17}$$

$$MSE(t_{1m}) = \alpha^2\frac{V_1}{1+V_1}. \tag{18}$$

From (11) and (18) we have

$$Var(\hat{\alpha}_{rss}) - MSE(t_{1m}) = \alpha^2\frac{V_1^2}{1+V_1} > 0.$$

Thus the proposed estimator $t_{1m}$ is more efficient than $\hat{\alpha}_{rss}$.

## 3. Improved estimation of scale parameter $\alpha$ based on best linear unbiased estimator (BLUE) of Extreme RSS (ERSS)

Lesitha and Chacko (2012) also described the BLUE of $\alpha$, using ERSS as:

$$\alpha^*_{erss} = \begin{cases} \dfrac{\sum_{r=1}^{n}\dfrac{\xi_1}{\delta_{11}}X_{[1:n]r} + \sum_{s=m+1}^{n}\dfrac{\xi_n}{\delta_{nn}}X_{[n:n]s}}{m\left[\dfrac{\xi_1^2}{\delta_{11}} + \dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ even\ ,i.e\ n = 2m \\[2ex] \dfrac{\sum_{r=1}^{m+1}\dfrac{\xi_1}{\delta_{11}}X_{[1:n]r} + \sum_{s=m+2}^{2m+1}\dfrac{\xi_n}{\delta_{nn}}X_{[n:n]s}}{\left[(m+1)\dfrac{\xi_1^2}{\delta_{11}} + m\dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ odd\ ,i.e\ n = 2m+1 \end{cases}. \tag{19}$$

and

$$Var\left(\alpha^{*}_{erss}\right)=\begin{cases}\dfrac{\alpha^{2}}{m\left[\dfrac{\xi_1^2}{\delta_{11}}+\dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ even\ ,i.e\ n=2m\\[2ex] \dfrac{\alpha^{2}}{\left[(m+1)\dfrac{\xi_1^2}{\delta_{11}}+m\dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ odd\ ,i.e\ n=2m+1\end{cases}=\alpha^{2}V_2, \tag{20}$$

where $m$ is any positive integer and

$$V_2=\begin{cases}\dfrac{1}{m\left[\dfrac{\xi_1^2}{\delta_{11}}+\dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ even\ ,i.e\ n=2m\\[2ex] \dfrac{1}{\left[(m+1)\dfrac{\xi_1^2}{\delta_{11}}+m\dfrac{\xi_n^2}{\delta_{nn}}\right]}, & when\ n\ is\ odd\ ,i.e\ n=2m+1\end{cases}.$$

We propose a class of estimators for the parameter $\alpha$ as

$$t_2=B\hat{\alpha}_{erss}, \tag{21}$$

where $B$ is a suitably chosen constant, such that mean squared error of $t_2$ is minimum. The bias and mean squared error (MSE) of $t_2$ are respectively given by

$$B(t_2)=(B-1)\alpha \tag{22}$$

$$MSE(t_2)=\alpha^{2}\left[B^{2}(1+V_2)-2B+1\right]. \tag{23}$$

The $MSE(t_2)$ is minimum when

$$B=(1+V_2)^{-1}. \tag{24}$$

Now substitution of (24) in (21) yields the minimum MSE estimator of $\alpha$ as

$$t_{2m}=(1+V_2)^{-1}\hat{\alpha}_{erss}. \tag{25}$$

The bias and mean squared error (MSE) of $t_{2m}$ are respectively given by

$$B(t_{2m})=-\alpha\frac{V_2}{1+V_2} \tag{26}$$

$$MSE(t_{2m})=\alpha^{2}\frac{V_2}{1+V_2}. \tag{27}$$

From (20) and (27) we have

$$Var(\hat{\alpha}_{erss}) - MSE(t_{2m}) = \alpha^2 \frac{V_2^2}{1+V_2} > 0 .$$

Thus the proposed estimator $t_{2m}$ is more efficient than $\hat{\alpha}_{erss}$ .

## 4. Relative efficiencies

To throw some light on the performances of various estimators $(\hat{\alpha}_{rss}, \hat{\alpha}_{erss}); (t_{1m}, t_{2m})$ of the scalar parameter $\alpha$ , we have computed the relative efficiencies by using the formulae as :

$$e_1 = RE(t_{1m}, \hat{\alpha}_{rss}) = 1 + V_1;$$

$$e_2 = RE(t_{1m}, \hat{\alpha}_{erss}) = \frac{V_2(1+V_1)}{V_1};$$

$$e_3 = RE(t_{2m}, \hat{\alpha}_{rss}) = \frac{V_1(1+V_2)}{V_2};$$

$$e_4 = RE(t_{1m}, \hat{\alpha}_{erss}) = 1 + V_2 \text{ and}$$

$$e_5 = RE(t_{2m}, t_{1m}) = \frac{V_1(1+V_2)}{V_2(1+V_1)} .$$

The values of $e_i, i = 1,2,\ldots,5$ for $n = 3(1)8$ , $\lambda = 0.25(0.25)1.00$ and $\beta = 2.5(0.5)5.0$ are shown in Tables 1 to 4.

| $n$ | $\beta$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|---|---|---|---|---|---|---|
| 3 | 2.5 | 1.4781 | 1.4758 | 1.4796 | 1.4773 | 1.0011 |
| | 3.0 | 1.2278 | 1.2259 | 1.2294 | 1.2275 | 1.0013 |
| | 3.5 | 1.1264 | 1.1245 | 1.1280 | 1.1262 | 1.0015 |
| | 4.0 | 1.0908 | 1.0891 | 1.0924 | 1.0907 | 1.0015 |
| | 4.5 | 1.0654 | 1.0637 | 1.0671 | 1.0653 | 1.0016 |
| | 5.0 | 1.0520 | 1.0503 | 1.0536 | 1.0519 | 1.0015 |
| 4 | 2.5 | 1.3583 | 1.3546 | 1.3611 | 1.3574 | 1.0020 |
| | 3.0 | 1.1708 | 1.1675 | 1.1735 | 1.1703 | 1.0024 |
| | 3.5 | 1.0947 | 1.0915 | 1.0977 | 1.0944 | 1.0027 |
| | 4.0 | 1.0681 | 1.0649 | 1.0710 | 1.0679 | 1.0027 |
| | 4.5 | 1.0490 | 1.0459 | 1.0521 | 1.0489 | 1.0029 |
| | 5.0 | 1.0389 | 1.0359 | 1.0419 | 1.0388 | 1.0029 |
| 5 | 2.5 | 1.2866 | 1.2814 | 1.2906 | 1.2854 | 1.0031 |
| | 3.0 | 1.1366 | 1.1320 | 1.1406 | 1.1360 | 1.0035 |
| | 3.5 | 1.0757 | 1.0711 | 1.0801 | 1.0754 | 1.0040 |
| | 4.0 | 1.0544 | 1.0500 | 1.0587 | 1.0542 | 1.0040 |
| | 4.5 | 1.0392 | 1.0347 | 1.0435 | 1.0390 | 1.0042 |
| | 5.0 | 1.0311 | 1.0268 | 1.0354 | 1.0310 | 1.0041 |
| 6 | 2.5 | 1.2387 | 1.2329 | 1.2435 | 1.2376 | 1.0038 |
| | 3.0 | 1.1138 | 1.1085 | 1.1185 | 1.1132 | 1.0043 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 3.5 | 1.0631 | 1.0577 | 1.0682 | 1.0628 | 1.0048 |
| | 4.0 | 1.0453 | 1.0401 | 1.0504 | 1.0451 | 1.0048 |
| | 4.5 | 1.0327 | 1.0274 | 1.0378 | 1.0325 | 1.0050 |
| | 5.0 | 1.0259 | 1.0208 | 1.0310 | 1.0258 | 1.0050 |
| 7 | 2.5 | 1.2046 | 1.1978 | 1.2102 | 1.2034 | 1.0047 |
| | 3.0 | 1.0975 | 1.0914 | 1.1031 | 1.0969 | 1.0051 |
| | 3.5 | 1.0541 | 1.0477 | 1.0601 | 1.0537 | 1.0057 |
| | 4.0 | 1.0389 | 1.0328 | 1.0448 | 1.0386 | 1.0057 |
| | 4.5 | 1.0280 | 1.0218 | 1.0341 | 1.0278 | 1.0059 |
| | 5.0 | 1.0222 | 1.0162 | 1.0282 | 1.0221 | 1.0058 |
| 8 | 2.5 | 1.1790 | 1.1719 | 1.1850 | 1.1779 | 1.0051 |
| | 3.0 | 1.0853 | 1.0788 | 1.0913 | 1.0848 | 1.0056 |
| | 3.5 | 1.0473 | 1.0405 | 1.0538 | 1.0470 | 1.0062 |
| | 4.0 | 1.0340 | 1.0274 | 1.0404 | 1.0338 | 1.0062 |
| | 4.5 | 1.0245 | 1.0178 | 1.0311 | 1.0243 | 1.0064 |
| | 5.0 | 1.0195 | 1.0129 | 1.0259 | 1.0193 | 1.0064 |

**Table 1: The values of $e_i, i = 1,2,\ldots,5$ for $\lambda = 0.25$**

| $n$ | $\beta$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|---|---|---|---|---|---|---|
| 3 | 2.5 | 1.4738 | 1.4635 | 1.4809 | 1.4705 | 1.0048 |
| | 3.0 | 1.2258 | 1.2175 | 1.2326 | 1.2243 | 1.0056 |
| | 3.5 | 1.1252 | 1.1171 | 1.1324 | 1.1243 | 1.0064 |
| | 4.0 | 1.0900 | 1.0824 | 1.0969 | 1.0893 | 1.0064 |
| | 4.5 | 1.0648 | 1.0573 | 1.0719 | 1.0643 | 1.0066 |
| | 5.0 | 1.0515 | 1.0443 | 1.0584 | 1.0511 | 1.0065 |
| 4 | 2.5 | 1.3545 | 1.3390 | 1.3660 | 1.3504 | 1.0085 |
| | 3.0 | 1.1689 | 1.1557 | 1.1804 | 1.1670 | 1.0098 |
| | 3.5 | 1.0936 | 1.0803 | 1.1060 | 1.0925 | 1.0113 |
| | 4.0 | 1.0673 | 1.0546 | 1.0794 | 1.0665 | 1.0113 |
| | 4.5 | 1.0485 | 1.0356 | 1.0608 | 1.0479 | 1.0118 |
| | 5.0 | 1.0385 | 1.0260 | 1.0507 | 1.0380 | 1.0117 |
| 5 | 2.5 | 1.2831 | 1.2602 | 1.3012 | 1.2780 | 1.0141 |
| | 3.0 | 1.1349 | 1.1152 | 1.1526 | 1.1326 | 1.0156 |
| | 3.5 | 1.0748 | 1.0550 | 1.0935 | 1.0734 | 1.0174 |
| | 4.0 | 1.0537 | 1.0350 | 1.0718 | 1.0528 | 1.0172 |
| | 4.5 | 1.0387 | 1.0199 | 1.0571 | 1.0380 | 1.0178 |
| | 5.0 | 1.0307 | 1.0125 | 1.0487 | 1.0302 | 1.0175 |
| 6 | 2.5 | 1.2356 | 1.2109 | 1.2560 | 1.2309 | 1.0165 |
| | 3.0 | 1.1123 | 1.0904 | 1.1324 | 1.1101 | 1.0181 |
| | 3.5 | 1.0622 | 1.0398 | 1.0838 | 1.0609 | 1.0203 |
| | 4.0 | 1.0447 | 1.0233 | 1.0657 | 1.0438 | 1.0201 |
| | 4.5 | 1.0322 | 1.0105 | 1.0537 | 1.0315 | 1.0208 |
| | 5.0 | 1.0256 | 1.0044 | 1.0467 | 1.0251 | 1.0205 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 7 | 2.5 | 1.2017 | 1.1718 | 1.2273 | 1.1967 | 1.0213 |
| | 3.0 | 1.0962 | 1.0696 | 1.1210 | 1.0938 | 1.0227 |
| | 3.5 | 1.0533 | 1.0262 | 1.0796 | 1.0519 | 1.0250 |
| | 4.0 | 1.0383 | 1.0125 | 1.0638 | 1.0374 | 1.0245 |
| | 4.5 | 1.0276 | 1.0016 | 1.0535 | 1.0269 | 1.0253 |
| | 5.0 | 1.0219 | 0.9967 | 1.0472 | 1.0214 | 1.0248 |
| 8 | 2.5 | 1.1764 | 1.1459 | 1.2030 | 1.1718 | 1.0226 |
| | 3.0 | 1.0841 | 1.0566 | 1.1101 | 1.0819 | 1.0240 |
| | 3.5 | 1.0466 | 1.0183 | 1.0744 | 1.0453 | 1.0266 |
| | 4.0 | 1.0335 | 1.0063 | 1.0605 | 1.0326 | 1.0261 |
| | 4.5 | 1.0241 | 0.9965 | 1.0518 | 1.0235 | 1.0270 |
| | 5.0 | 1.0192 | 0.9923 | 1.0463 | 1.0187 | 1.0266 |

**Table 2: The values of $e_i, i = 1,2,\ldots 5$ for $\lambda = 0.50$**

| $n$ | $\beta$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|---|---|---|---|---|---|---|
| 3 | 2.5 | 1.4662 | 1.4401 | 1.4842 | 1.4579 | 1.0123 |
| | 3.0 | 1.2222 | 1.2016 | 1.2394 | 1.2185 | 1.0140 |
| | 3.5 | 1.1231 | 1.1034 | 1.1409 | 1.1209 | 1.0159 |
| | 4.0 | 1.0885 | 1.0703 | 1.1055 | 1.0870 | 1.0156 |
| | 4.5 | 1.0637 | 1.0458 | 1.0808 | 1.0626 | 1.0161 |
| | 5.0 | 1.0506 | 1.0335 | 1.0672 | 1.0498 | 1.0157 |
| 4 | 2.5 | 1.3472 | 1.3100 | 1.3756 | 1.3376 | 1.0211 |
| | 3.0 | 1.1656 | 1.1343 | 1.1932 | 1.1612 | 1.0237 |
| | 3.5 | 1.0917 | 1.0605 | 1.1210 | 1.0891 | 1.0269 |
| | 4.0 | 1.0659 | 1.0365 | 1.0943 | 1.0641 | 1.0266 |
| | 4.5 | 1.0475 | 1.0179 | 1.0765 | 1.0461 | 1.0277 |
| | 5.0 | 1.0377 | 1.0091 | 1.0661 | 1.0367 | 1.0274 |
| 5 | 2.5 | 1.2764 | 1.2177 | 1.3246 | 1.2637 | 1.0378 |
| | 3.0 | 1.1319 | 1.0828 | 1.1772 | 1.1262 | 1.0400 |
| | 3.5 | 1.0730 | 1.0246 | 1.1202 | 1.0697 | 1.0440 |
| | 4.0 | 1.0525 | 1.0073 | 1.0974 | 1.0503 | 1.0427 |
| | 4.5 | 1.0378 | 0.9928 | 1.0831 | 1.0362 | 1.0437 |
| | 5.0 | 1.0300 | 0.9868 | 1.0739 | 1.0288 | 1.0426 |
| 6 | 2.5 | 1.2295 | 1.1675 | 1.2826 | 1.2179 | 1.0432 |
| | 3.0 | 1.1095 | 1.0562 | 1.1600 | 1.1043 | 1.0455 |
| | 3.5 | 1.0606 | 1.0070 | 1.1138 | 1.0575 | 1.0502 |
| | 4.0 | 1.0436 | 0.9929 | 1.0947 | 1.0415 | 1.0489 |
| | 4.5 | 1.0314 | 0.9805 | 1.0833 | 1.0298 | 1.0503 |
| | 5.0 | 1.0250 | 0.9757 | 1.0755 | 1.0238 | 1.0493 |
| 7 | 2.5 | 1.1962 | 1.1175 | 1.2666 | 1.1833 | 1.0589 |
| | 3.0 | 1.0936 | 1.0265 | 1.1591 | 1.0879 | 1.0598 |
| | 3.5 | 1.0518 | 0.9849 | 1.1197 | 1.0485 | 1.0646 |
| | 4.0 | 1.0373 | 0.9746 | 1.1016 | 1.0350 | 1.0620 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | 4.5 | 1.0268 | 0.9643 | 1.0917 | 1.0252 | 1.0631 |
| | 5.0 | 1.0213 | 0.9612 | 1.0839 | 1.0201 | 1.0613 |
| 8 | 2.5 | 1.1713 | 1.0926 | 1.2432 | 1.1598 | 1.0615 |
| | 3.0 | 1.0818 | 1.0134 | 1.1492 | 1.0766 | 1.0623 |
| | 3.5 | 1.0452 | 0.9763 | 1.1158 | 1.0423 | 1.0675 |
| | 4.0 | 1.0326 | 0.9675 | 1.0998 | 1.0305 | 1.0651 |
| | 4.5 | 1.0234 | 0.9581 | 1.0916 | 1.0219 | 1.0666 |
| | 5.0 | 1.0186 | 0.9554 | 1.0848 | 1.0175 | 1.0649 |

**Table 3: The values of $e_i, i = 1,2,\ldots,5$ for $\lambda = 0.75$**

| $n$ | $\beta$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|---|---|---|---|---|---|---|
| 3 | 2.5 | 1.4540 | 1.4016 | 1.4914 | 1.4377 | 1.0257 |
| | 3.0 | 1.2168 | 1.1762 | 1.2513 | 1.2095 | 1.0284 |
| | 3.5 | 1.1199 | 1.0817 | 1.1552 | 1.1158 | 1.0316 |
| | 4.0 | 1.0863 | 1.0514 | 1.1195 | 1.0835 | 1.0306 |
| | 4.5 | 1.0621 | 1.0281 | 1.0952 | 1.0601 | 1.0312 |
| | 5.0 | 1.0494 | 1.0171 | 1.0811 | 1.0479 | 1.0302 |
| 4 | 2.5 | 1.3350 | 1.2618 | 1.3930 | 1.3166 | 1.0434 |
| | 3.0 | 1.1602 | 1.1004 | 1.2146 | 1.1520 | 1.0469 |
| | 3.5 | 1.0886 | 1.0300 | 1.1454 | 1.0838 | 1.0522 |
| | 4.0 | 1.0639 | 1.0091 | 1.1181 | 1.0606 | 1.0510 |
| | 4.5 | 1.0459 | 0.9915 | 1.1009 | 1.0436 | 1.0526 |
| | 5.0 | 1.0366 | 0.9841 | 1.0899 | 1.0347 | 1.0515 |
| 5 | 2.5 | 1.2646 | 1.1402 | 1.3737 | 1.2386 | 1.0863 |
| | 3.0 | 1.1268 | 1.0269 | 1.2241 | 1.1156 | 1.0863 |
| | 3.5 | 1.0701 | 0.9740 | 1.1688 | 1.0638 | 1.0922 |
| | 4.0 | 1.0506 | 0.9624 | 1.1422 | 1.0463 | 1.0872 |
| | 4.5 | 1.0364 | 0.9498 | 1.1276 | 1.0334 | 1.0880 |
| | 5.0 | 1.0290 | 0.9466 | 1.1160 | 1.0267 | 1.0846 |
| 6 | 2.5 | 1.2182 | 1.0864 | 1.3396 | 1.1946 | 1.0996 |
| | 3.0 | 1.1048 | 0.9969 | 1.2130 | 1.0946 | 1.0979 |
| | 3.5 | 1.0579 | 0.9525 | 1.1686 | 1.0522 | 1.1046 |
| | 4.0 | 1.0418 | 0.9443 | 1.1451 | 1.0379 | 1.0992 |
| | 4.5 | 1.0301 | 0.9334 | 1.1337 | 1.0273 | 1.1006 |
| | 5.0 | 1.0240 | 0.9314 | 1.1234 | 1.0218 | 1.0971 |
| 7 | 2.5 | 1.1855 | 1.0095 | 1.3597 | 1.1579 | 1.1470 |
| | 3.0 | 1.0892 | 0.9476 | 1.2387 | 1.0776 | 1.1373 |
| | 3.5 | 1.0493 | 0.9127 | 1.1990 | 1.0429 | 1.1426 |
| | 4.0 | 1.0357 | 0.9110 | 1.1725 | 1.0314 | 1.1322 |
| | 4.5 | 1.0257 | 0.9033 | 1.1611 | 1.0226 | 1.1320 |
| | 5.0 | 1.0204 | 0.9043 | 1.1489 | 1.0181 | 1.1258 |
| 8 | 2.5 | 1.1611 | 0.9838 | 1.3414 | 1.1365 | 1.1553 |
| | 3.0 | 1.0776 | 0.9337 | 1.2318 | 1.0673 | 1.1430 |

| | | | | | |
|---|---|---|---|---|---|
| 3.5 | 1.0429 | 0.9029 | 1.1980 | 1.0372 | 1.1487 |
| 4.0 | 1.0310 | 0.9026 | 1.1734 | 1.0272 | 1.1380 |
| 4.5 | 1.0224 | 0.8956 | 1.1639 | 1.0196 | 1.1384 |
| 5.0 | 1.0178 | 0.8970 | 1.1526 | 1.0157 | 1.1324 |

**Table 4:** The values of $e_i, i = 1,2,\ldots 5$ for $\lambda = 1.00$:

It is observed from Tables 1 to 4 that

- For fixed $(n,\lambda)$ the values of $e_i, i = 1,2,\ldots 5$ decrease as $\beta$ increases.
- For fixed $(n,\beta)$ the values of $e_i, i = 1,2,\ldots 5$ decrease as $\lambda$ increases.
- For fixed $(\beta,\lambda)$ the values of $e_i, i = 1,2,\ldots 5$ decrease as $n$ increases.

It is further observed from Tables 1 to 4 that the higher gain in efficiencies are observed by using $t_{2m}$ over $\hat{\alpha}_{rss}$ for all values of $(n,\beta,\lambda)$, so we conclude that $t_{2m}$ are more efficient estimators as compared to Lesitha and Thomas (2012) estimators $(\hat{\alpha}_{rss}, \hat{\alpha}_{erss})$ and MMSE estimator $(t_{1m}, t_{2m})$ respectively.

## 5. Conclusion

In this paper motivated by Searls (1964), Singh *et al.* (1973) and Searls and Intarapanich (1990), we have suggested some improved estimators of the scale parameter $\alpha$ involved in (1) using ranked set sampling and obtained their biases and mean squared errors. It has been shown that the proposed estimators are better than the one recently suggested by Lesitha and Thomas (2012) estimators.

## Acknowledgement

The authors are highly grateful to the editor and referees for their constructive comments/ suggestions that helped in the improvement of the revised version of the paper.

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